# Exploring the Role of Inventories in the Business Cycle*

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## Contents



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## 1. Introduction

It is not uncommon for commentators on the prospects for an economy to draw attention to recent inventory movements. Thus, if there has been a run down in stocks below what is perceived to be normal levels, this is taken as a sign of favourable output prospects in future periods; the reasoning behind this conclusion being that output not only needs to be produced to meet sales, but also to replenish stocks. Early in the history of business cycle research the question arose of whether the presence of inventory holdings by firms was a contributor to the "up and down" movements seen in economies. The classic analyses of this question were by Metzler (1941), (1947), who concluded that "An economy in which businessmen attempt to recoup inventory losses will always undergo cyclical fluctuations..". His model emphasised the fact that a business would attempt to keep inventories as a proportion of expected sales and so would re-build these if they declined below that target level. Given that sales had to be forecast from their past history, he showed that output would follow a second order difference equation which would have complex roots in many cases. Consequently his model produced a periodic cycle in output and this constituted the foundation of his conclusion. Of course the fact that a periodic cycle can be generated does not mean that it is an important one since the amplitude could be quite small.

After Metzler's work, interest in inventories shifted to deriving optimal rules for stock holdings that balanced the cost of being away from a target level against the cost of the sharp output changes that would be needed if any given level of sales was to be met automatically by rapid output adjustment. The classic work in this vein is by Holt et al. (1960) and a good summary of the type of model that results is Rowley and Trivedi(1975, ch 2). This strand of research produced optimal decision rules for inventory holdings that effectively rationalized the ad hoc rules that underlay Metzler's models. The hallmark of these models is that there are some exogenous driving forces such as sales and cost shocks and then optimal decisions are made in response to what is known about them. A large body of literature has used models of optimal inventory holdings in empirical work-See Blinder and Maccini (1991) and Ramey and West (1999) for surveys of the literature.

A fundamental problem with Metzler's analysis was that it concentrated upon the possibility of a periodic cycle in output. This view of the business cycle has long been rejected in economics since economic cycles are quite irregular and far from periodic. Instead the tradition in economics has been to describe cycles through their characteristics, such as the duration of time between successive peaks and troughs, in series taken to represent the level of aggregate economic activity. The latter is most commonly regarded as the level of GDP. Expansions and recessions are then defined by the fact that they are initiated by turning points (peaks and troughs) in the activity series. This is the way in which the NBER in the U.S., and the myriad of agencies around the world who follow their approach, measure the business cycle. When one looks at the cycle in this way there is no longer any need for output to follow a second order difference equation with complex roots in order to produce a business cycle. Indeed, it is a feature of GDP series around the world that such complex roots do not seem to be in the data, and yet there is an indisputable business cycle - see Pagan(1999).

Harding and Pagan (2002) set out a framework in which the dating of cycles through turning points can be formally analysed. Denoting the level of economic activity as $Q_t$, the turning points in $Q_t$ and $q_t = \ln Q_t$ are identical and Harding and Pagan showed that it was the DGP of either

$\Delta Q_t$ or $\Delta q_t$ that contains all the information needed to describe the cycle in economic activity. In particular, if one thinks of a linear model for $\Delta q_t$ then it is natural to summarize the DGP with the following parameters:

1. Long-run growth in output ($\mu$)

2. The volatility of output growth ( defined as proportional to the standard deviation of $\Delta q_t$, $\sigma_{\Delta q}$)

3. Parameters $\alpha_j$ describing any serial correlation in output growth

These parameters can be imbedded in a simple model which captures the AR(2) orientation of much of the early business cycle literature

$$(\Delta q_t - \mu) = \alpha_1(\Delta q_{t-1} - \mu) + \alpha_2(\Delta q_{t-2} - \mu) + \sigma_{\Delta q}\varepsilon_t, \tag{1.1}$$

where $\varepsilon_t$ is $i.i.d(0,1)$.

This model is quite a good description of GDP for many countries - see Pagan(1999). Consequently, it is not surprising that the data generating process (DGP) for $\Delta q_t$, when quantified using estimates of $\mu, \alpha_1, \alpha_2$, is capable of producing a good description of many of the features of the average business cycle for a number of countries. This does not mean that the match with business cycle characteristics is perfect, since the DGP in (1.1) would not generate a crucial feature of business cycles in some countries e.g. the US, which sees the average cycle exhibiting very fast recoveries from recessions, although this feature is not so noticeable in other countries- see Harding and Pagan (2001). Since the linear model above tends to produce a constant growth rate in activity during expansions and contractions, it would fail to produce such a feature. Hence some non-linearity in output growth (or an asymmetric density for $\varepsilon_t$) is needed to produce this characteristic, although it has not proven easy to find exactly what that non-linearity would be.

It is useful to think about questions regarding the business cycle in terms of the three sets of parameters given above. Such an analysis can be qualitative or quantitative. Thus on a qualitative level it might be expected that a rise in $\mu$, a fall in $\sigma$ and a reduction in positive serial correlation would lead to longer cycles. Quantitatively, once one has set out a DGP for $\Delta q_t$ it is possible to simulate data from the chosen model and to ask if the simulated characteristics are a good match with those seen in the data. To this end Harding and Pagan (2002) provided computer programs in the GAUSS language that could be used to perform such an analysis. In Harding and Pagan (2000) it was shown how to use this approach to examine whether some popular theories of the business cycle e.g. real business cycle and endogenous growth models in order to see if they are capable of producing realistic business cycles. In the Australian context the methods were used in Dungey and Pagan (2000) to explore what the influence of international factors was upon the Australian growth cycle.

It seems useful to re-examine the relation of inventories and the business cycle by utilizing the approach and techniques of the view of cycles described above i.e. as one reflecting turning points. That inventories may be important for understanding the US cycle can be seen from an analysis we have performed on U.S. data for sales and GDP using the BBQ algorithm in Harding and Pagan (2002). The data used is from 1947/1 until 2001/4. When we date turning points with BBQ we find that the average durations of GDP recessions and expansions are 3.2 and

20.1 quarters while those for sales are 3.4 and 33.5 i.e. expansions in sales are much longer than those in GDP. *Prima facie*, we would expect this since the mean growth rates are virtually the same (.008395 and .008371), but the standard deviations of the growth rates are quite different at .0101 and .0084 respectively. Thus the longer cycle in sales is expected since the probability of getting a negative growth rate in it is much lower than for GDP. One possibility to account for this difference in volatilities is that it is due to the way that sales movements get transformed into GDP changes through the presence of inventories in the system.

To understand these outcomes for GDP and inventories we need to build a model that is capable of being quantified and which can be used to investigate what type of cycles are generated when there are inventories in the system and when there are not. The model chosen is an extension of that in Humphreys et al (2001). It sees the objectives of firms as attempting to balance the costs of keeping raw material stocks in line with output, and finished goods stocks in line with sales, with the extra costs incurred by rapid adjustment in output and purchases of raw materials. Because of the presence of raw materials it has some additional driving forces such as the level of raw material prices as well as the traditional one of the sales of finished goods. The model also allows for a number of other shocks such as productivity and various cost shocks associated with inventories. Given that data on raw material usage is not readily available Humphreys et al collapsed the system down to a two equation one describing the optimal decisions relating to input and output inventories alone. By working with the original form of the model however we can solve for the optimal decision rules for material usage, raw material stocks and finished good stocks. These can then be used to determine value added, as this constitutes GDP. That introduces a complication as value added, output, raw materials and raw material prices enter into a non-linear relationship. Consequently we are forced to simulate data on these quantities and that points to indirect estimation methods as a way of quantifying the unknown parameters.

A number of questions will be addressed in this paper. First, on a general level, we want to examine the question of whether the presence of inventories is a major contributor to the business cycle. Second, there are some specific questions regarding the U.S. cycle that have arisen in recent literature which will be explored and analysed both generally and with the model above. One of these, which came out of the experience of the long expansion of the 1990's, is whether the business cycle has become longer i.e. whether the time between successive peaks (or troughs) has become longer. Qualitatively, if the GDP growth rate was described by (1.1) we would know that this would occur if the long run growth rate of GDP increases, the volatility of GDP growth decreases, or the degree of positive correlation in growth rates lessens. McConnell and Perez-Quiros (2000) found that the volatility in the growth rate in U.S. GDP seemed to shift after the mid 1980's and this observation also seems to be true for many other counties around the world, including Australia (although the date of this shift varies). Thus, such lower volatility should lead to a longer cycle. Pagan (2000) showed simulations of the business cycle under the assumption that $\sigma$ had effectively halved and these indicated that it would lengthen the business cycle quite substantially.

The causes for this decline have been much debated and are surveyed and critiqued in Stock and Watson (2002)[1]. When this feature was observed it was natural that one look at

[1] Recent studies that have looked at the role of inventory management advances and the decline in the volatility of output growth include Ahmed, Levin and Wilson (2000), Blanchard and Simon (2001), Irvine and Schuh (2002), Kahn, McConnell and Perez-Quiros (2002), Kim and Nelson (1999), McCarthy and Zakrajsek (2002), and Ramey

what trends were in the economy which might lead to such an outcome. Since there had been great advances in inventory control methods, in particular the development of "just in time" philosophies relating to production, it seemed possible that this might be a source of the changes e.g. see Kahn et al (2002).

But the maintained stance in all this work has always been that the conditional volatility of GDP growth should be a constant, Accordingly in the following section we show that this is not so. Indeed it seems as if US GDP growth might be characterized as following a "square root " process in volatility, and this is a familiar one from the financial literature where the change in short term interest rates is known to exhibit "levels "volatility.

Section 3 of the paper sets out our extended version of the Humphreys et al model and the Euler equations. Section 4 estimates the parameters of this model. Section 5 conducts a number of experiments with it to gain some appreciation of what the role of inventories in the business cycle might be.

## 2. The DGP of GDP Growth

It is worth looking at the DGP of U.S. GDP growth in some detail. Let $Q_t$ be the level of real GDP and $q_t = \ln Q_t$. Then analysis has generally proceeded by characterizing the DGP of $\Delta q_t = \Delta \ln Q_t$. In contrast to this orientation many models, particularly in the inventory literature, are formulated in terms of "detrended" $Q(t)$ rather than of $q_t$. As we mentioned earlier the cycles in $Q_t$ and $q_t$ are identical. Since each is determined by the DGPs of either $\Delta Q_t$ or $\Delta q_t$, it is useful to think about relating the two processes.

Since the most common assumption has been that $q_t$ can be represented as an integrated process with constant volatility, we need to elicit the implications of this for the $\Delta Q_t$ process. Thus we begin by writing

$$\Delta q_t = \mu + \sigma \varepsilon_t \tag{2.1}$$

where $\varepsilon_t$ is taken to be $N(0,1)$.[2] Then, using the approximation $\Delta q_t = \frac{\Delta Q_t}{Q_{t-1}}$, which is quite a good one for GDP, we would expect that

$$\Delta Q_t = \mu Q_{t-1} + \sigma Q_{t-1} \varepsilon_t.$$

Thus if the DGP for $q_t$ was (2.1), and so has constant conditional volatility, then we would find that the DGP for $Q_t$ has a conditional volatility that rises with the level of $Q_t$. To see the significance of this for our work consider the series $\psi_t = \frac{\Delta Q_t}{\overline{Q}}$, where $\overline{Q}$ is the "mean" of $Q_t$. For our purpose the moments of $\Delta q_t$ might be well approximated by those of $\psi_t$. Of course, if $Q_t$ is a growing process, there is no such thing as the mean of $Q_t$ but it serves to provide some impression of the level of the $Q_t$ process.

Table 1 then presents some values for the standard deviations of $\Delta Q_t, \psi_t$ and $q_t$ over three sub-periods for the US- 1947/1-1958/4, 1959/1-1982/4 and 1983/1-2001/2. The latter two periods represent the data set we use later since we are unable to measure some of the quantities before 1959/1, while the first period is included for comparative analysis.

and Vine (2001).

[2] Of course we know that there is some weak serial correlation in $q_t$ but we want to make our points in a simple fashion. Later we will allow for the more general process.

Table 1: Alternative Measures of the Volatility of GDP Movements

| | 1947/1-1958/4 | 1959/1-1982/4 | 1983/1-2001/2 |
|---|---|---|---|
| $\sigma_{\Delta Q}$ | 6.45 | 10.48 | 9.94 |
| $\sigma_{\psi}$ | .0138 | .011 | .0056 |
| $\sigma_{\Delta q}$ | .0138 | .011 | .0058 |

A number of things stand out in this table. One is the decline in the volatility in $\Delta q_t$. A second is that our approximation reflects this outcome very accurately. The fact that this is so enables us to focus upon whether the decline in the volatility of $\Delta q_t$ comes from the numerator, $\Delta Q_t$, or the denominator. Comparing the second and third periods it is clearly the case that there has only been a very small reduction in the volatility of $\Delta Q_t$, and that almost all the decline in volatility that has been observed can be attributed simply to the fact that the average level of GDP in the second period is almost twice what it is in the first period. In contrast, between the first and second periods, there was an increase in the volatility of $\Delta Q_t$, so that the decline in the volatility of $\Delta q_t$ comes about despite this outcome.

Now this directs us to look at the question of the form of the conditional volatility in $\Delta Q_t$. We can start with the case when $q_t$ follows (2.1), implying that there is a "levels" effect in the conditional volatility in the DGP of $\Delta Q_t$ of the form

$$E_{t-1}((\Delta Q_t - \mu Q_{t-1})^2) = \sigma^2 Q_{t-1}^2.$$

Now in the finance literature such a levels effect is known to exist in short term interest rates but not with a square term as above. This suggests that we follow that literature and estimate a model of the form

$$\Delta Q_t = \mu Q_{t-1} + \sigma Q_{t-1}^{\delta} \varepsilon_t \tag{2.2}$$

to see if $\delta = 1$, as this value would correspond to the standard assumption that $q_t$ is a random walk. We will refer to (2.2) as an ARVL (*Autoregressive, Levels in Volatility*) model.

The parameters of the ARVL model can be estimated in two stages. First $\mu$ can be consistently estimated by regressing $\Delta Q_t$ on $Q_{t-1}$. This gives $\hat{\mu} = .008$ versus the sample mean of $q_t$ of .0083. In the second stage we note that, if $e_t = \sigma Q_{t-1}^{\delta} \varepsilon_t = \Delta Q_t - \mu Q_{t-1}$,

$$\begin{aligned} \ln(e_t^2) &= \ln \sigma^2 + \delta \ln(Q_{t-1}^2) + \ln \varepsilon_t^2 \\ &= (\ln \sigma^2 + E(\ln \varepsilon_t^2)) + \delta \ln Q_{t-1}^2 + \nu_t \end{aligned} \tag{2.3}$$

where $\nu_t = \ln \varepsilon_t^2 - E(\ln \varepsilon_t^2)$ has $E(\nu_t) = 0$.[3] Then, if the DGP for $\Delta Q_t$ is (2.2), we can consistently estimate $\delta$ by regressing $\ln \hat{e}_t^2$ against a constant and $\ln Q_{t-1}^2$,where $\hat{e}_t$ are the residuals from the first stage regression. This is not an efficient estimator since it ignores the fact that $v_t$ is not normally distributed. In fact the asymptotic efficiency of the estimator relative to the MLE can be computed as in Pagan and Ullah (1999,p22-23).

Using the two stage approach to estimate $\delta$ over the period 1959/1-2001/2 we find that $\hat{\delta} = .26$ with a standard error of .24.[4] Adopting HAC robust standard errors makes very little

[3] According to Maple $E(\ln \varepsilon_t^2) = -\ln(2) - \gamma$, where $\gamma$ is the Euler-Mascheroni constant which is approximately .57721566. See Heck (1993,p45) on the latter.

[4] The fact that $\mu$ was estimated in the first stage does not affect the asymptotic distribution of $\hat{\delta}$ provided that the error term $\varepsilon_t$ is conditionally symmetric. The condition for asymptotic independence of $\hat{\delta}$ and $\hat{\mu}$ is that $E(\frac{\partial m_t}{\partial \mu}) = 0$ where $m_t = \ln Q_{t-1}^2(\ln e_t^2 - \alpha - \delta \ln Q_{t-1}^2)$ is the moment condition being used for estimation of $\delta$. This becomes $E(2\frac{e_t e_{t-1} \ln Q_{t-1}^2}{e_t^2}) = 0$.

change to the standard deviation. Hence we would reject the hypothesis that $\delta = 1$, suggesting that the model which sees $\ln Q_t$ as the basic random walk process is possibly incorrect. One needs to exercise some care here since, under the null hypothesis, $Q_{t-1}$ will be an $I(1)$ process and so the distribution of $\hat{\delta}$ may not be asymptotically normal. However if data is simulated from a process (2.1) that has $\mu = .0079$ and $\sigma = .008$, and $\delta$ is estimated using this data, the 5% significance level critical value seems to be around 2 i.e. the same as with stationary regressors. Probably this occurs since the time trend in $q_t$ dominates the stochastic trend. In any case the point estimate of $\delta$ is a long way from unity and much closer to that of zero. It implies that, because $\delta$ is well below unity, this would mean a decline in the volatility of $\Delta q_t$ as $Q_{t-1}$ rises. Hence this outcome is compatible with the long period of decline in volatility of $\Delta q_t$ observed by Blanchard and Simon(2001).

We need to recognize however that there is likely to be serial correlation in $\Delta Q_t$ since $q_t$ has often been modelled as an AR(2) process. Hence in all later results we fit the model

$$\Delta Q_t = \mu Q_{t-1} + a_1 \Delta Q_{t-1} + a_2 \Delta Q_{t-2} + \sigma Q_{t-1}^{\delta} \varepsilon_t.$$

We will refer to this as an ARVL(3) model in $Q_t$ in contrast to the ARVL(1) process that has a maximum lag of unity in $Q_t$. Now the estimate for $\delta$ over the period 1959/1-2001/2 is .41 with standard error of .22. Taking a longer perspective of 1947/1-2001/2 one would get $\hat{\delta} = .50$ with standard error of .14. It needs to be said however that this stability is not so evident in sub-periods e.g. if one estimates it using data from 1959/1-1982/4 it is 1.85 with standard error of .5 while 1983/1-2001/2 registers 2.58 and standard error of .84. It's clear that the estimate can be very sensitive to a few large observations and it needs a substantial sample size to overcome this lack of precision. A different way of seeing this is in Figure 2.1 which shows a recursive estimate of $\delta$ over the period 1947/1-2001/2. It is clear that there are some periods of time when the estimate of $\delta$ came close to being what was predicted if there had been a pure random walk in $q_t$ with constant volatility, but for most sample periods the estimate lies well away from unity and, in the past decade, it has been fairly stable around the value of .5.

A different way of seeing the same result is to think about starting with $q_t$ and looking at the volatility process in it. We therefore fit the following model

$$\Delta q_t = \mu + a_1 \Delta q_{t-1} + a_2 \Delta q_{t-2} + \sigma Q_{t-1}^{\gamma} \varepsilon_t \tag{2.4}$$

in the same two step approach as before, but now with residuals $\hat{e}_t = \Delta q_t - \hat{\mu} - \hat{a}_1 \Delta q_{t-1} - \hat{a}_2 \Delta q_{t-2}$. Now the lack of a levels effect in $\Delta q_t$ requires that $\gamma = 0$. The regression over 1959/1-2001/2 gives a value of $\hat{\gamma}$ of -.53 with a standard deviation of .2 so that once again one would reject the hypothesis that there is no levels effect in the volatility of the growth rates in $Q_t$. There is an obvious difficulty with the ARVL model that eventually shocks would eventually effectively die out if $\sigma$ remained constant. Of course that has not happened as seen for the 1947/1-1959/1 period. It's also the case that one has to think of any model as an approximation to the true DGP and so there may be some other feature that needs to be added to our specifications. Most notably if there is some non-linear structure to the conditional mean of $q_t$ then it would show up in the conditional variance. This is clearly a topic that needs to be examined in some greater depth elsewhere.

One might ask whether this is true of other variables. An important one in most inventory models is sales $X_t$. Table 2 is the analogue of Table 1 for sales with $x_t = \ln X_t$.

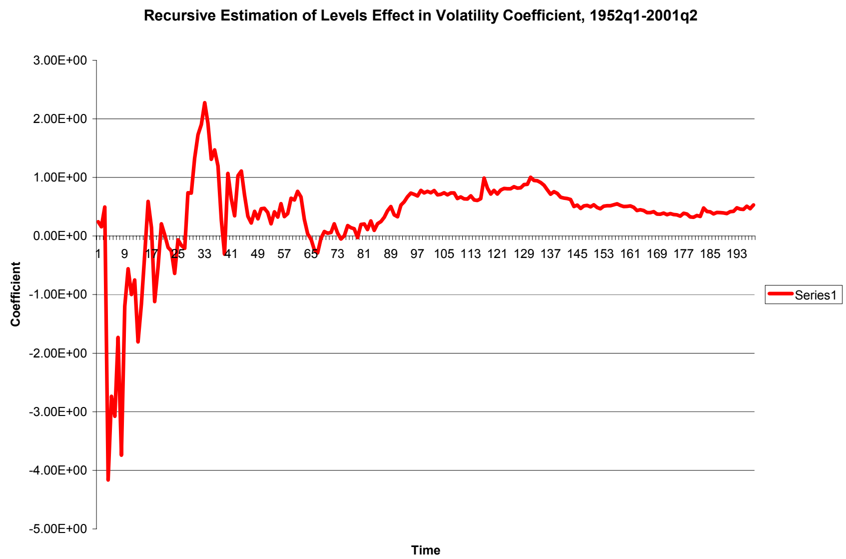


Figure 2.1:

Table 2: Alternative Measures of Sales Change Volatility

| | 1947/1-1958/4 | 1959/1-1982/4 | 1983/1-2001/2 |
|---|---|---|---|
| $\sigma_{\Delta X}$ | 5.62 | 8.38 | 8.39 |
| $\sigma_{\psi}$ | .0120 | .0091 | .0051 |
| $\sigma_{\Delta x}$ | .0119 | .0086 | .0051 |

The situation is clearly much the same for sales although the decline in the volatility in $\Delta x_t$ between the latter two periods is a little less marked then for GDP. This shows up in the value of $\hat{\gamma}$ in a regression like (2.4) being -.468 with a standard error of .2. One would reject a zero value for $\gamma$ but it is much closer than for the $\Delta q_t$ regression. The point estimates therefore suggest that one would see some reduction in the volatility in sales over time but not as much as for value added.

Once conditional volatilities are not taken to be constant we need to recognize that the probability of getting a fall in $Q_t$ or sales will depend not only upon the moments of $\Delta q_t$ but, as emphasized in Harding and Pagan (2002), the complete DGP of $q_t$ (or $x_t$) becomes important. To illustrate the importance of this we simulate data from two ARVL(1) processes for $Q_t$ whose parameters derive from data on GDP over the period 1947/1-2001/4. In the first $\sigma = .83077, \mu = .007708, \delta = .3343$ while, for the second, $\sigma = .01019, \mu = .00839, \delta = 1.$[5] The second ARVL model implies a random walk for $q_t$. The durations of recessions/expansions are 2.85/23.25 in the first case and 2.62/27.40 in the second. Therefore, it is clear that the levels

[5]In the case when $\delta$ is non-zero we also need to set a value for $Q_0$ and this is taken to be the value of GDP in 1947/1 i.e. 370 in our units.

effect in the ARVL model may be important in determining cyclical outcomes. It's hard to conduct an experiment that isolates this effect however since the models have such different volatility structures.

All of this suggests that we need to look at developing a reasonably general model of $\Delta Q_t$ (or $\Delta q_t$) to understand movements (or lack of them) in their volatilities. There may be some grounds for preferring the former since, if one computes the volatility of $\Delta Q_t$ over the period 1947/1 until 1959/1, it rose from 6.38 to the 10.48 in Table 1. Such a rise would interact with the levels effect - which is tending to reduce the volatility in $\Delta q_t -$ and so it may be worth splitting up the separate effects, and that requires a model of $\Delta Q_t$. Moreover, because the inventory literature has been so focussed upon the former, we do that in the remainder of the paper.

## 3. The Model and its Euler Equations

The model we use is an extension of the one developed by Humphreys, et al.(2001). The model in Humphreys et al. has the advantage that it is a model of inventories broken down by stage of fabrication and thus distinguishes between finished goods or "output" inventories and materials and supplies or "input" inventories. The latter includes work in progress inventories as well; hereafter, we use the term materials inventories to refer to the sum of materials and supplies and work in progress inventories. The model thus permits an analysis of the role that each type of inventory stock plays in the production and sales process. This is an important advantage of the model as finished goods and materials inventories may have played very different roles in the reduction of the volatility of GDP growth. Figure 3.1 reports the ratio of finished goods inventories to sales and the ratio of materials inventories to output for the period 1959/1 through 2001/2. As the Figure indicates, the materials-output ratio has declined about 30% since the early eighties, but the finished goods-sales ratio has remained about constant. This suggests that, to the extent that improved inventory management techniques have had a role to play in reducing the volatility of GDP growth, materials inventories may have been more important than finished goods inventories. Further, "just-in-time" techniques which have become more widely used in recent years are more applicable to materials inventory management than to that of finished goods.

### 3.1. The Production Function

We begin with a specification of the short-run production function, which is

$$Y_t = F\left(L_t, U_t, \epsilon_{yt}\right) . \tag{3.1}$$

where $Y_t$ is output, $L_t$ is labor input, $U_t$ is materials usage, and $\epsilon_{yt}$ is a technology shock. Note that $U_t$ is the *flow* of materials used in the production process. The capital stock is assumed to be a completely fixed factor of production and is suppressed in the functional form. Further, materials usage and labor are assumed to possess positive and non-increasing marginal products, and the production function is assumed to be strictly concave in materials usage and labor. Finally, the firm is assumed to purchase intermediate goods (work-in-process) from outside suppliers rather than producing them internally.[6] Thus, intermediate goods are analogous to

[6] To allow for production of intermediate goods within the firm requires extending the production function to incorporate joint production of final and intermediate goods. This extension is a substantial modification of the

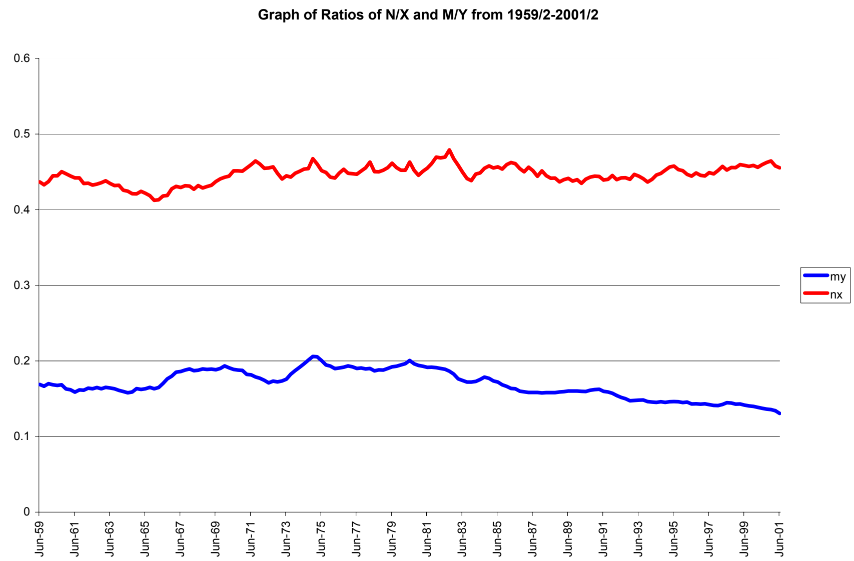


Figure 3.1:

raw materials so work-in-process inventories can be lumped together with materials inventories. Because $Y_t$ is gross output, we refer to equation (3.1) as the gross production function.

### 3.2. The Cost Structure

The firm's total cost structure consists of three major components: labor costs, inventory holding costs, and materials costs. This section describes each component.

#### 3.2.1. Labor Costs

Labor costs are

$$LC_t = W_t L_t + A(\Delta L_t) \tag{3.2}$$

with

$$\begin{array}{lll} A' & \lesseqgtr & 0 \quad \text{as} \quad \Delta L_t \quad \lesseqgtr \quad 0 \\ A'' & > & 0 \end{array}$$

where $\Delta L_t = L_t - L_{t-1}$. The first component, $W_t L_t$, is the standard wage bill where $W_t$ is the real wage rate. The second component, $A(\Delta L_t)$, is an adjustment cost function intended to capture the hiring and firing costs associated with changes in labor inputs. The adjustment cost function has the usual properties, including a rising marginal adjustment cost.

standard production process that we leave for future work.

To reduce the number of decision variables and focus on the inventory decisions, we eliminate labor input. Inverting the production function, equation (3.1), yields

$$L_t = L(Y_t, U_t, \epsilon_{yt}) \tag{3.3}$$

with

$$\begin{array}{rcrcl} L_Y & = & 1/F_L & > & 0 \\ L_U & = & -F_U/F_L & < & 0 \\ L_{\epsilon_y} & = & -(F_{\epsilon_y}/F_L) & < & 0 \end{array} \tag{3.4}$$

where $F_L$, $F_U$, and $F_{\epsilon_y}$ are the marginal products of production with respect to the factor inputs. Substituting (3.3) into (3.2) yields

$$LC_t = W_t L(Y_t, U_t, \epsilon_{yt}) + A(L(Y_t, U_t, \epsilon_{yt}) - L(Y_{t-1}, U_{t-1}, \epsilon_{y,t-1})) \tag{3.5}$$

which is the central portion of the firm's cost function.

To construct an econometric model, it is necessary to parameterize the labor cost function, equation (3.5). Here, we follow the tradition of the inventory literature, which exploits generalized quadratic approximations of the cost structure. Specifically, the quadratic approximation to labor costs is

$$\begin{aligned} LC_t = {} & \left(\tfrac{\gamma_1}{2}\right) Y_t^2 + \left(\tfrac{\gamma_2}{2}\right) U_t^2 + \gamma_3 Y_t U_t + W_t[\gamma_4 Y_t + \gamma_5 U_t] \\ & + \left(\tfrac{\varphi}{2}\right) [\gamma_6 \Delta Y_t + \gamma_7 \Delta U_t]^2 + \epsilon_{yt}(\gamma_8 Y_t + \gamma_9 U_t) \ . \end{aligned} \tag{3.6}$$

This equation omits products of inputs involving squared terms that would appear in a completely generalized quadratic approximation of equation (3.5). Nevertheless, the approximation captures the essential elements of the production and adjustment cost functions, and is comparable to the most general approximations found in previous inventory work. Note that the labor cost function is somewhat over-parameterized, but is left it in its most general form for now.

The signs of some, but not all, parameters in equation (3.6) are known without further assumptions. Parameters $\gamma_1$, $\gamma_6$, $\gamma_8$, and $\varphi$ are all positive from the assumed convexity of the production and adjustment cost functions. Abstracting from dynamics, $\partial LC/\partial Y = \gamma_1 Y + \gamma_4 W$ should be positive from (3.4); given $\gamma_1 > 0$, $\gamma_4 > 0$ is a sufficient, though not necessary, condition. But $\partial LC/\partial W = \gamma_4 Y$ should also be positive from the wage bill, which indicates that $\gamma_4$ must be positive. In contrast, the signs of $\gamma_2$, $\gamma_3$, $\gamma_5$, $\gamma_7$, and $\gamma_9$ are unknown *a priori* because they depend on the specification of the production function.

### 3.2.2. Inventory Holding Costs

In line with much of the output inventory literature, holding costs for output inventories are a quadratic approximation to actual costs of the form

$$HC_t^N = (\delta_0 + \epsilon_{nt}) N_t + \left(\frac{\delta}{2}\right) (N_t - N_t^*)^2 \tag{3.7}$$

where $\epsilon_{nt}$ is the white noise innovation to output inventory holding costs, $N_t^*$ is the target level of output inventories that minimizes output inventory holding costs, and $\delta > 0$. We adopt an analogous formulation for input inventories; holding costs for these stocks are a quadratic approximation of the form

$$HC_t^M = (\tau_0 + \epsilon_{mt})M_t + \left(\frac{\tau}{2}\right)(M_t - M_t^*)^2 \tag{3.8}$$

where $\epsilon_{mt}$ is the white noise innovation to input inventory holding costs, $M_t^*$ is the target level of input inventories that minimizes input inventory holding costs, and $\tau > 0$. The quadratic inventory holding cost structure balances two forces. Holding costs rise with the level of inventories, $M_t$ and $N_t$, due to increased storage costs, insurance costs, etc. But holding costs fall with $M_t$ and $N_t$ because – given expected $M_t^*$ and $N_t^*$ – higher $M_t$ and $N_t$ reduce the likelihood that the firm will "stock out" of inventories.

Finally, it remains to specify the inventory target stocks. Again following the literature, the output inventory target stock is

$$N_t^* = \alpha X_t \tag{3.9}$$

where $\alpha > 0$. The output inventory target depends on sales because the firm incurs costs due to lost sales when it stocks out of output inventories. For the input inventory target stock, we assume that the target stock depends on production, rather than sales. In particular,

$$M_t^* = \theta Y_t \tag{3.10}$$

where $\theta > 0$. The input inventory target depends on production ($Y_t = X_t + \Delta N_t$) because stocking out of input inventories also entails costs associated with production disruptions – lost production, so to speak – that are distinct from the cost of lost sales. Lost production may be manifested by reduced productivity or failure to realize production plans.

To summarize, the input and output inventory targets differ because the firm holds the two inventory stocks for different reasons. The firm stocks output inventories to guard against random demand fluctuations, but it stocks input inventories to guard against random fluctuations in productivity, materials prices and deliveries, and other aspects of production. Although sales and production are highly positively correlated, they differ enough at high frequencies to justify different target stock specifications.

### 3.2.3. Materials Costs

Finally, we turn to the cost of purchasing materials and supplies. We assume that the real cost of purchasing materials and supplies is given by

$$MC_t = P_t^M(V_t, D_t)D_t = \phi_1 V_t D_t + (\phi_2/2)D_t^2 \tag{3.11}$$

where $P_t^M(V_t, D_t) = \phi_1 V_t + (\phi_2/2)D_t$ is a "price schedule" which states that the real price the firm pays per unit of the quantity of materials purchased and delivered depends on a real "base price", $V_t$, and a premium or discount, $(\phi_2/2)D_t$, that depends on the amount purchased and delivered. A natural normalization here is of course to set $\phi_1 = 1$, which is done later in empirical work. Three cases may be distinguished:

1. Increasing Marginal Cost: $\phi_2 > 0$. In this case, the firm faces a rising supply price for materials purchases. The firm thus experiences increasing marginal costs to purchasing

materials due to higher premia that must be paid to acquire materials more quickly. A rationale for such a rising supply price is that the firm is a monopsonist in the market for materials. This is most likely to occur when materials are highly firm or industry specific and the firm or industry is a relatively large fraction of market demand.[7] The rising marginal cost of course gives rise to the "smoothing" of purchases.

2. Decreasing Marginal Cost: $\phi_2 < 0$. In this case, the firm faces a falling supply price for materials purchases. This captures the idea that the firm can obtain quantity discounts in purchasing materials and supplies, so that the marginal cost falls with larger purchases. To rationalize a falling supply price, presumably the firm again possesses some monopsony power. The falling marginal cost gives rise to the "bunching" of purchases. Here, conditions are needed to ensure that over-all costs are minimized.

3. Constant Marginal Cost: $\phi_2 = 0$. In this case, firms are price takers in competitive input markets and purchase all the raw materials needed at the prevailing market price.

### 3.3. Cost Minimization

Assume that sales ($X_t$) and factor prices ($V_t$ and $W_t$) are exogenous and that the firm's optimization problem is to minimize the discounted present value of total costs ($TC$),

$$E_0 \sum_{t=0}^{\infty} \beta^t TC_t = E_0 \sum_{t=0}^{\infty} \beta^t (LC_t + HC_t + MC_t) \ , \tag{3.12}$$

where $\beta = (1+r)^{-1}$ is the discount factor implied by the constant real rate of interest $r$. Two identities determine the evolution of stocks. That for finished goods inventories is

$$\Delta N_t = Y_t - X_t \tag{3.13}$$

while that for materials and supplies inventories is

$$\Delta M_t = D_t - U_t. \tag{3.14}$$

(3.13) and (3.14) can be used to substitute out production ($Y_t$) and deliveries ($D_t$) in $TC_t$, leaving the firm's problem as choosing $\{U_t, M_t, N_t\}_{t=0}^{\infty}$ to minimize equation (3.12). Taking the derivatives of equation (3.12) with respect to $U_t$, $M_t$, and $N_t$ produces the Euler equations which we will solve.

Designating $\Delta^2$ as the second-difference operator, i.e., $\Delta^2 X_t = \Delta X_t - \Delta X_{t-1}$, the Euler equation for materials usage, $U_t$, is

$$\begin{gathered} E_t\{-\beta\varphi\gamma_7[\gamma_6(\Delta X_{t+1} + \Delta^2 N_{t+1}) + \gamma_7 \Delta U_{t+1}] \\ +\gamma_2 U_t + \gamma_3 (X_t + \Delta N_t) + \gamma_5 W_t + \varphi\gamma_7[\gamma_6(\Delta X_t + \Delta^2 N_t) + \gamma_7 \Delta U_t] \\ +\phi_1 V_t + \phi_2(\Delta M_t + U_t) + \gamma_9 \epsilon_{yt} + c_1\} = 0. \end{gathered} \tag{3.15}$$

[7] This is analogous to the literature on adjustment cost models for investment in plant and equipment where external adjustment costs are imposed in the form of a rising supply price for capital goods. See, e.g. Gould (1968) or Abel( ).

This optimality condition shows that the firm balances the marginal cost of ordering and using materials this period (second line) against the marginal cost of using materials next period (first line). Note that in period $t+1$ the only cost is the marginal cost of usage in production. All inter-temporal costs associated with materials occur through the input inventory optimality condition.

The Euler equation for materials inventories, $M_t$, is

$$\begin{gathered} E_t \{ -\beta[\phi_1 V_{t+1} + \phi_2(\Delta M_{t+1} + U_{t+1})] \\ +\phi_1 V_t + \phi_2(\Delta M_t + U_t) + \tau(M_t - \theta(\Delta N_t + X_t)) + \epsilon_{mt} + c_2 \} = 0 \; . \end{gathered} \tag{3.16}$$

This optimality condition shows that the firm balances the marginal cost of ordering and holding input inventories this period (second line) against the cost of ordering input inventories next period (first line).

Finally, the Euler equation for finished goods inventories, $N_t$, is

$$\begin{gathered} E_t \left\{ \beta^2 \varphi \gamma_6 [\gamma_6(\Delta X_{t+2} + \Delta^2 N_{t+2}) + \gamma_7 \Delta U_{t+2}] \right. \\ -\beta[\gamma_1(X_{t+1} + \Delta N_{t+1}) + \gamma_3 U_{t+1} + \gamma_4 W_{t+1} \\ +2\varphi\gamma_6(\gamma_6(\Delta X_{t+1} + \Delta^2 N_{t+1}) + \gamma_7 \Delta U_{t+1}) \\ +\gamma_8 \epsilon_{y,t+1} - \theta\tau(M_{t+1} - \theta(\Delta N_{t+1} + X_{t+1}))] \\ +\gamma_1(X_t + \Delta N_t) + \gamma_3 U_t + \gamma_4 W_t + \varphi\gamma_6(\gamma_6(\Delta X_t + \Delta^2 N_t) + \gamma_7 \Delta U_t) \\ \left. -\theta\tau(M_t - \theta(\Delta N_t + X_t)) + \delta(N_t - \alpha X_t) + \gamma_8 \epsilon_{yt} + \epsilon_{nt} + c_3 \right\} = 0 \end{gathered} \tag{3.17}$$

This optimality condition shows that the firm balances the marginal cost of producing a good and storing it as output inventory this period (last two lines) against the cost of producing the good in the future (first three lines). The presence of adjustment costs on labor introduces an additional period over which costs are balanced.

Humphreys et al (2001) discuss a variety of restrictions that might be applied to the model. Here we use a variant of what they refer to as the parsimonious gross production model, option II. This employs the following parametric restrictions.

$$\begin{aligned} \gamma_4 &= -\gamma_5 \\ \gamma_9 &= -\gamma_8 = 1 \\ \phi_1 &= 1 \\ \gamma_6 &= 1 \end{aligned}$$

The restriction that $\phi_1 = 1$ is a normalization. As is well known, linear-quadratic models of the type used here require some normalization to identify parameters. This normalization implies that the parameters of the Euler equations are measured in terms of units of materials prices. The other restrictions are made in the interest of specifying a labor cost function that is parsimonious in the parameters to simplify the estimation process.

## 4. Solution and Estimation of the Model

### 4.1. Solution

The Euler equations can be written in the form

$$y_t = Ay_{t-1} + E_t\{B_1 y_{t+1} + B_2 y_{t+2} + T_1\xi_{t+1} + T_2\xi_{t+2}\} + T_0\xi_t + T_{-1}\xi_{t-1}$$

where $y_t = \begin{bmatrix} N_t \\ U_t \\ M_t \end{bmatrix}, \xi_t = \begin{bmatrix} X_t \\ W_t \\ V_t \\ \varepsilon_{yt} \\ \varepsilon_{nt} \\ \varepsilon_{mt} \end{bmatrix}$. The solution for $y_t$ is then found using the approach in Binder and Pesaran (1995). In the first stage this method determines a $P$ such that $y_{t-1}$ is eliminated from the system, leaving a system in variables $Z_t = y_t - Py_{t-1}$ and $\xi_t$. Then, after reduction to first order, one gets

$$\zeta_t = SE_t(\zeta_{t+1}) + Q_0\xi_t + Q_1E_t(\xi_{t+1}) + Q_2E_t(\xi_{t+2}) + Q_1\xi_{t-1}, \tag{4.1}$$

where $\zeta_t$ is constructed in a known way from $Z_t$ and its lags. Thereafter a specification is made of the nature of the forcing processes in $\xi_t$ and it is possible to solve the forward equation for $\zeta_t$. When $\xi_t$ is a VAR the final solution will involve a VAR in $y_t$ and $\xi_t$ but with contemporaneous values of $\xi_t$ appearing in it as well. A nice feature of the algorithm is that one can re-specify the process generating $\xi_t$ and this only affects the second stage i.e. the solution of (4.1), and not the derivation of $P$.

In what follows the observable processes are assumed to have the form

$$\begin{aligned} \Delta X_t &= a\varepsilon_{yt} + \varepsilon_{xt} \\ \Delta W_t &= e_{wt} \\ \Delta V_t &= e_{vt}. \end{aligned}$$

Thus the "sales" series is taken to be driven by a "productivity shock" as well as a "demand shock". Treating sales in this way is always going to be an issue once one is working with large aggregates rather than a single firm so, although we will refer to $\varepsilon_{xt}$ as a demand shock, it should be borne in mind that our model is not sufficiently detailed for one to have much confidence in such a description. The unobservable shocks are made white noise.[8]

As outlined above the solution method enables one to determine $N_t, M_t$ and $U_t$. But we are interested in value added $Q_t$ (GDP) and that is fundamentally connected to the variables solved for earlier through the relation

$$Q_t = Y_t - V_tU_t = X_t + \Delta N_t - V_tU_t. \tag{4.2}$$

Because this involves a non-linearity one might find an equation for $Q_t$ by linearizing it but, as we expect that $U_t$ is $I(1)$, it is unclear how effective that would be. Instead we resort to

[8] We did try making productivity an $I(1)$ process but the results were the same as we report here since the estimated standard deviation of these shocks under either of the specifications is effectively zero.

simulation methods: for a given set of values for the parameters of the model it is possible to simulate data on $U_t, N_t, M_t$ etc. from the solved Euler equations after which we obtain $Y_t$ from $X_t = Y_t - \Delta N_t$ and then $Q_t$ from (4.2).

### 4.2. Aggregation and Data

In the model above it has been assumed that the representative firm behaves as if it is vertically integrated, so that it is representative of the whole economy. The representative firm holds materials inventory stocks which it uses in conjunction with labor (and capital) to produce output of finished goods, which it adds to finished goods inventories. The finished goods inventories may be held by manufacturers, wholesalers or retailers. In effect, we treat the representative firm as managing the inventory stocks of finished goods whether they are held on the shelves of the manufacturer, the wholesaler, or the retailer.

Accordingly, we construct an aggregate stock of finished goods inventories by summing the real value of finished goods inventories in manufacturing, wholesale trade and retail trade. The aggregate stock of materials inventories is constructed by adding up the materials and supplies and work in progress inventories held by manufacturers.[9] Value added or GDP is the real value of aggregate GDP. The data are quarterly, seasonally-adjusted, (1996) chain-weighted series in billions of dollars, and cover the period 1959/1 through 2001/4. GDP is of course the flow of value added over the quarter, and inventories are measured as end-of-quarter stocks.

The sales data are the (1996) chain-weighted final sales of domestic product. Essentially this equals $Q_t - \Delta M_t - \Delta N_t$. Note that this will not be the same as $X_t = Y_t - \Delta N_t$. There is yearly data on $Y_t$ but no quarterly data is available. We will refer to this sales variable as $X_t^m = Q_t - \Delta M_t - \Delta N_t$. i.e. "sales as measured" as distinct from *latent sales* $X_t$. The real wage rate is real compensation per hour for the business sector. Nominal materials prices is an implicit price deflator, which was constructed by dividing the nominal value of materials inventories by their real value. Real material prices ($V_t$) were obtained by dividing the nominal value by the PPI for the business sector

### 4.3. Estimation

Because we are forced to simulate from our theoretical model it is logical that we estimate the parameters utilizing the ideas of minimum distance - Chamberlain (1982), Kodde et al (1990)- and indirect estimation - Gourieroux and Monfort (1993) and Smith (1993). In these approaches an auxiliary model is selected and its parameters $\eta$ are fitted to the data to give $\hat{\eta}$. The model parameters $\theta$ will imply some value for $\eta, \eta_M(\theta)$, and one chooses $\theta$ to minimize $(\hat{\eta} - \eta_M(\theta))'(\hat{\eta} - \eta_M(\theta))$. We only report point estimates here as our objective was to get some feel for what parameter values would be appropriate for the model rather than to conduct tests on it. Because we have some doubts about the quality of the wage data and the raw material prices we decided that the auxiliary model would only be based on the observables $Q_t, N_t, M_t$ and $X_t^m$. Hence we used a VAR(1) in these variables as the auxiliary model, assuming that $X_t$ and $X_t^m$ were strongly exogenous. To perform the estimation we need to produce the analogue of

[9] Note that there are no materials and supplies and work in progress inventories in wholesale trade or retail trade.

$X_t^m$ from the model, and this is simply done by computing $Q_t - \Delta M_t - \Delta N_t$ from the simulated data.

Since there are sixteen unknown parameters in the model, and there are only twelve parameters in the VAR(1), once the strong exogeneity of $X_t^m$ is imposed, we need to specify some other features that are to be encompassed. We choose the standard deviations of the changes in all four variables of the VAR along with the estimated coefficients from the regressions of $N_t$ on $X_t^m$ and $M_t$ on $Q_t$. If we had not used the last two parameters then our estimation strategy would be exactly indirect estimation, where the auxiliary model is being estimated by MLE under the assumption that the shocks are normally distributed and uncorrelated with one another. It deviates from standard minimum distance estimation in that the model being used as the auxiliary model is not thought of as being the DGP.

The variables were all detrended by regressing each one against a constant and $t$. We use a linear trend rather than the quadratic one of most of the inventory literature since it means that the volatility of $\Delta Q_t^*$ should then be close to that of $\Delta Q_t$, where $Q_t^*$ is the detrended quantity.

The model was estimated with quarterly data over the second of the periods analysed in section 2, namely, 1959/1-1982/4. We use only the second period to ensure that we are estimating the model over a period where the parameters of the model can be taken to be stable. This would not be so if, for example, advances in inventory management techniques brought about important changes in paramters after 1982. Table 3 gives the point estimates.

Table 3 Estimated Model Parameters

| | | | |
|---|---|---|---|
| $\alpha$ | .24 | $\gamma_4$ | 18.7 |
| $\delta$ | .26 | $\gamma_7$ | -.054 |
| $\theta$ | .31 | $a$ | -1.94 |
| $\phi_2$ | .32 | $\tau$ | .79 |
| $\varphi$ | .32 | $\sigma_y$ | .0002 |
| $\gamma_1$ | 1.32 | $\sigma_n$ | .52 |
| $\gamma_2$ | -.55 | $\sigma_m$ | .05 |
| $\gamma_3$ | .18 | $\sigma_x$ | .99 |

The parameters are of correct sign and of reasonable magnitude. The large value of $\gamma_4$ seems to be due to the units of measurement of the data. It is apparent that productivity and material cost shocks have little role in helping the model interpret the data.

Below we compare the estimates of the VAR coefficients, the standard deviations of the variables $\Delta Q_t, \Delta X_t^m, \Delta N_t, \Delta M_t$, and the regression coefficients from the model and the data. The VAR has variables $N, Q, M$ and $X_t^m$ in that order and the $P$ matrix below is the 4x4 matrix connected to the lag values. Thus the element in the second row and first column of $P$ is that of $N_{t-1}$ in the $Q_t$ equation. We have assumed that $X_t^m$ is strongly exogenous and so have dropped the row corresponding to it. While it is not necessary that this assumption be correct, as the auxiliary model need not be the DGP, it is actually hard to find much evidence against it.[10] The subscript $M$ means that these are the values predicted by the model. The unsubscripted values are those for the data.[11]

[10] It is interesting that regressing $X_t^m$ against $Q_{t-1}, M_{t-1}, N_{t-1}$ and $X_{t-1}$ (all being detrended) the F test statistic that $M_{t-1}, N_{t-1}$ and $Q_{t-1}$ can be deleted is 2.14 with a p value of .1. The same is true when adding in $W_{t-1}$ and $V_{t-1}$, so it does seem as if the strong exogeneity assumption for $X_t$ is reasonable.

[11] The data was scaled during estimation so that all parameters of the auxiliary model lay between -1 and 1.1.

$$
\begin{aligned}
\hat{P} &= \begin{bmatrix} .851 & .170 & -.033 & -.106 \\ -.389 & .478 & -.389 & .706 \\ -.091 & .088 & .934 & -.000 \end{bmatrix}, \hat{P}_M = \begin{bmatrix} .838 & .161 & -.064 & -.111 \\ -.391 & .465 & -.373 & .706 \\ -.082 & .059 & .891 & -.015 \end{bmatrix} \\
V &= \begin{bmatrix} \sigma_{\Delta N} & 3.58 \\ \sigma_{\Delta Q} & 10.45 \\ \sigma_{\Delta M} & 2.42 \\ \sigma_{\Delta X^m} & 8.38 \end{bmatrix}, V_M = \begin{bmatrix} \sigma_{\Delta N} & 3.37 \\ \sigma_{\Delta Q} & 10.48 \\ \sigma_{\Delta M} & 2.56 \\ \sigma_{\Delta X^m} & 8.38 \end{bmatrix} \\
R &= \begin{bmatrix} \text{Reg}, N \text{ on } X^m & .242 \\ \text{Reg } M \text{ on } Q & .190 \end{bmatrix}, R_M = \begin{bmatrix} \text{Reg}, N \text{ on } X^m & .242 \\ \text{Reg } M \text{ on } Q & .195 \end{bmatrix}
\end{aligned}
$$

The model clearly provides an excellent prediction of the standard deviation of $\Delta Q_t$ during the 1959/1-1982/4 period. This is used as the benchmark in the static experiments of the next section.

# 5. Looking at Some Cycle Issues

## 5.1. Some Static Experiments

The model gives us a way of analysing some cyclical issues. As in all cycle analyses it is useful to start with examining what has happened to the mean and variance of $\Delta Q_t$ (or $\Delta q_t$) and to try to explain these movements with the model. Given that the mean of $\Delta Q_t$ has risen from the 1959/1-1982/4 period, while the standard deviation has remained constant, an amelioration in the business cycle might have been expected. Now the model is rather silent on why the mean of $\Delta Q_t$ has changed, due to its use of detrended data, and mostly it focuses upon explaining changes in the standard deviation of $\Delta Q_t$ But if the model was linear, then we would expect that $\Delta Q_t = k\Delta X_t^m$, where the multiplier $k$ comes from the model. Consequently, if we pass a trend in $X_t$ through the model we can gain some appreciation for how the mean in $\Delta Q_t$ would change as that for $\Delta X_t^m$ does. Doing so we find that the rise in the mean of $\Delta X_t^m$ from the second to the third sub-period should have increased the mean of $\Delta Q_t$ from 7.2 to 14.82. This compares with the observed movement of 7.03 to 14.96. Hence this determinant of business cycle outcomes owes more to the unexplained trend changes in an exogenous variable, sales, than to the model.

Turning to the volatility we might ask what factors impact upon the volatility of $\Delta Q_t$ and whether the changes in these factors have been important historically. Some of these factors are changes in the technologies relating to inventories. Such technical change might be regarded as enabling a lower ratio of stocks to sales or perhaps an ability to forecast current sales more accurately. The latter may come from electronic ticketing, which enables firms to match orders and current sales more closely on a day by day basis, effectively implying that, on a quarterly basis, one might assume that current sales are known. The model can be run in different modes that emulate these trends either by changing the target inventory/sales ratios or by changing

In this table we have multiplied the standard deviations by 10 to make them consistent with statistics given earlier.

the information available to firms when making their decisions. The effects on the volatility of $\Delta Q_t$ can then be assessed.

A number of experiments were therefore run with the model, consisting of 10% and 50% reductions in $\alpha, \theta, \delta, \tau, \sigma_{\Delta X}, \sigma_{\Delta W}, \sigma_{\Delta V}, \sigma_n$ and 10% and 50% increases in $\sigma_m$ and $\sigma_y$, ( increases being selected given the small magnitudes of the estimates of these latter parameters). The resulting predictions of $\sigma_{\Delta Q}$ are given in Table 4. From this table it is clear that changes in inventory technology, as summarized by changes in $\alpha$ and $\theta$, would be unlikely to have a major impact upon $\sigma_{\Delta Q}$, as would changes in the volatility of real wages and materials prices. Essentially the only factor that matters for the volatility in $\Delta Q_t$ is that in $\Delta X_t^m$. Consequently, since the volatility in $\Delta X_t^m$ remained constant between the two sub-periods one would expect that the volatility of $\Delta Q_t$ would remain constant, and that was a feature of the data. Improved inventory management techniques may very well have brought about a substantial decline in the materials inventory-output ratio, as the data shows, but this seems to have had little effect on the volatility of $\Delta Q_t$.

Table 4 Some Static Experiments

Effects on the Volatility of $\Delta Q$

| | 10% Change in Parameter | | 50% Change in Parameter | |
|---|---|---|---|---|
| Parameter | $\sigma_{\Delta Q}$ | % Change | $\sigma_{\Delta Q}$ | % Change |
| $\alpha$ | 10.48 | - .6% | 10.81 | - 2.9% |
| $\theta$ | 10.50 | .25% | 10.60 | 1.1% |
| $\delta$ | 10.45 | -.29% | 10.29 | -1.8% |
| $\tau$ | 10.49 | .1% | 10.56 | .8% |
| $\sigma_{\Delta X}$ | 9.621 | - 8.2% | 6.493 | -38.0% |
| $\sigma_{\Delta W}$ | 9.951 | 0% | 9.94 | - .1% |
| $\sigma_{\Delta V}$ | 10.476 | 0% | 10.476 | 0% |
| $\sigma_n$ | 10.424 | -.5% | 10.278 | - 1.9% |
| $\sigma_m$ | 10.476 | 0% | 10.476 | 0% |
| $\sigma_y$ | 10.485 | .1% | 10.547 | .7%- |
| Baseline $\sigma_{\Delta Q}$ | 10.476 | | 10.476 | |

### 5.2. Some Dynamic Experiments

A limitation of the analysis in the previous sub-section is that the nature of the business cycle depends not only upon the first two moments of $\Delta Q_t$ but also on its dynamics. Hence, it is possible that the dynamics can be modified by the presence or absence of inventories, Consequently, we now turn to look at some "dynamic" experiments in this sub-section. We might ask what would have happened if various shocks had been missing. Similar parametric variations to those in Table 4 can also be made. Table 5 contains these experiments.

Elimination of the shocks is seen to have very little impact upon the nature of the cycle. Changes in inventory technology have some effect. Thus, with a halving of $\alpha$, expansions became one quarter longer on average, and the same is true for $\delta$. Perhaps somewhat surprisingly, changes in $\theta$ and $\tau$ have essentially no effect. These seem to be small effects for such a large change in the parameters. As mentioned earlier, an alternative approach might be that this technology makes possible a more accurate prediction of current sales. In the model above these have been assumed to be known. But one might want to compare the cycle based on two scenarios: in one only past shocks are known while in the other current shocks are known. We will perform such a comparison at a later time.

We can take the process of reducing model parameters to an extreme by considering what would happen if no inventories were held. To achieve this one can set the parameters $\alpha = 0, \theta = 0$ and the parameters $\delta, \tau$ to very large numbers. Then $Q_t = X_t^m$ and the GDP cycle and the measured sales cycle would be identical. So we can actually measure the effect of inventories in the system by measuring the characteristics of the measured sales cycle and comparing that to the GDP cycle. To determine the nature of the cycle in measured sales we simply pass the data on $X_t^m$ generated in the basic experiment in Table 5 through BBQ. One then finds that contractions in sales ($X_t^m$) are of much the same length as for GDP, at 2.7 quarters, but expansions are longer at 25.0 quarters. Hence the presence of inventories acts to modify the business cycle, making the complete cycle around a year shorter than if they were absent. In this sense changes in inventory technology can be thought of as influencing the business cycle

What is the source of this difference in cycles?. One contributor is the attentuation in volatility that is caused by the model i.e. the predicted volatility of GDP change is amplified from the volatility of sales change and this would lead to a shorter cycle given that the mean of $\Delta Q_t$ and $\Delta X_t^m$ are virtually the same. Another potential explanation is that a white noise process for $\Delta X_t$ might become an autocorrelated one in $\Delta Q_t$ and, as Harding and Pagan (2002) observe, such dynamics can modify the cycle quite dramatically. In this instance however there is no evidence of any serial correlation in the model-generated $\Delta Q_t$ when there is none in the $\Delta X_t$ that is passed into it. Thus it is simply the attenuation in volatility that causes the difference in the cycle.

Table 5: Some Dynamic Experiments

Effects on Business Cycle Durations for GDP

| | Effect on Recessions (in Quarters) | Effect on Expansions (in Quarters) |
|---|---|---|
| Set $\epsilon_n = 0$ | 2.7 | 21.8 |
| Set $\epsilon_{V_t} = 0$ | 2.7 | 21.4 |
| Set $\epsilon_{W_t} = 0$ | 2.7 | 22.3 |
| 50% Reduction in $\alpha$ | 2.7 | 22.5 |
| 50% Reduction in $\theta$ | 2.7 | 21.3 |
| 50% Reduction in $\delta$ | 2.7 | 22.4 |
| 50% Reduction in $\tau$ | 2.7 | 21.4 |
| Baseline | 2.7 | 21.4 |

One issue that is worth looking at is the effects of having different processes for the exogenous variables. Above they have just been treated as random walks with constant volatility. But the evidence tendered earlier was that this was not an adequate characterization. As an alternative then ARVL(3) models were fitted to the series on *detrended* values of Sales ($X_t^m$), real wages ($W$) and real raw material prices ($V$) over 1959/1-2001/2, . although the volatility structure depended upon the *levels* of the series and not the detrended value.[12] These produce parameter values:

$$\begin{aligned} Sales &: \mu = -.0035, a_1 = .213, a_2 = .317, \sigma = .079, \delta = .65 \\ Real\ Wages &: \mu = -.0284, a_1 = .12, a2 = .18, \sigma = .079, \delta = .26 \\ Raw\ Mats &: \mu = 0, a_1 = .026, a_2 = .146, \sigma = .094, \delta = -.36 \end{aligned}$$

Data was simulated from these processes and then passed through the model to produce "detrended" values of $Q(t)$. The observed trend in $Q(t)$ was then added back to this series and the business cycle analysis was conducted on the resulting $Q(t)$. A similar process was followed to generate a comparable set of data on the level of sales. Using such output the average duration of recessions/expansions were 3.1/24.9 for GDP and 3.7/28.3 for sales. These are to be compared to the estimated durations of 3.7/23.8 for GDP and 4/28.5 for sales (using BBQ for dating over the period 1958/1-2001/4). Hence the model produces a good representation of the duration of the business cycle. In particular it is interesting that it correctly predicts a longer cycle in sales than in GDP.

It was mentioned earlier that a characteristic of business cycles in the U.S. and Australia is the fast growth in the early stages of an expansion. One can measure this by looking at the extent to which the growth rate in these phases remains constant with the period of time spent in the phase. As explained in Harding and Pagan (2002) the latter is summarized by an "excess" index that measures the extent to which the actual path which output follows in expansions or contractions differs from a triangle. The value of this excess statistic for GDP is 1.1. It should be said here that a number like 1.1 is a very significant departure from the benchmark of a constant growth rate at every point in the expansion. Linear models of GDP cannot produce such a feature. Since the inventory models that we construct above also make GDP growth a linear stochastic process, it is clear that they are not capable of reproducing this outcome. For the U.S. Sichel (1994) has claimed that, once the inventory change is removed from GDP, then there is no evidence that growth in the early stages of an expansion is different to the later stages. Now removal of inventory changes produces measured sales so that one way to assess this claim is to compute the "excess" statistics for the measured sales and GDP cycles. For sales the statistic has the value of 2.34 so that the effect is even stronger once inventories are removed. Thus our conclusion would be that any non-linearity in the GDP process arises from the nature of the sales process and is not obviously due to inventories (although inventories might modify it a little). How then does one reconcile this outcome with Sichel's contention?. The answer seems to be that Sichel compares the average growth in sales over the early and later parts of an expansion using the GDP expansion dates and not the sales expansion dates. Given that these cycles are very different in the U.S., it is clear that doing the comparison in this way is rather misleading.

[12] Of course since $X_t^m$ is not the same as $X_t$ we cannot really assume that the ARVL process for the former is the same as for the latter.

## 6. Conclusions

This paper seeks to shed light on the nature of the volatility of GDP growth and on the role that inventories play in the business cycle. We begin with an analysis of the form of the data generating process for GDP growth. We argue that the data generating process for the change in the level of GDP is well described by an ARVL–an Autoregressive, Levels in Volatility–Model whereby the conditional volatility of the change in the level of GDP rises with the level of GDP. This in turn implies that the DGP for the growth rate of GDP is governed by a "square root" process where the conditional volatility of GDP growth declines with the level of GDP.

We then used a model of inventory holding behavior to investigate the degree to which advances in inventory holding techniques can be responsible for the decline in volatility of GDP growth. The model is well-suited to the task in that it distinguishes between finished goods inventories and materials inventories. We estimate the model and expose it to a number of experiments. We find that even substantial changes in important parameters governing inventory holding behavior have rather small effects on the volatility of changes in the level of GDP and on the length of the business cycle. This suggests that, while advances in inventory management techniques may have reduced materials/output ratios, they have had little effect on the reduction of the volatility of output growth.

## 7. References


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