Inventories, Fluctuations and Business Cycles

Louis J. Maccini
Johns Hopkins University

Adrian Pagan
Queensland University of Technology

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Abstract

The paper looks at the role of inventories in U.S. business cycles and fluctuations. It concentrates upon the goods producing sector and constructs a model that features both input and output inventories. A range of shocks are present in the model, including sales, technology and inventory cost shocks. It is found that the presence of inventories does not change the average business cycle characteristics in the U.S. very much. The model is also used to examine whether new techniques for inventory control might have been an important contributing factor to the decline in the volatility of US GDP growth. It is found that these would have had little impact upon the level of volatility.

1. Introduction

It is not uncommon for commentators on the prospects for an economy to draw attention to recent inventory movements. Thus, if there has been a run down in stocks below what is perceived to be normal levels, this is taken as a sign of favorable output prospects in future periods; the reasoning behind this conclusion being that output not only needs to be produced to meet sales, but also to replenish stocks. Early in the history of business cycle research the question arose of whether the presence of inventory holdings by firms was a contributor to the "up and down" movements seen in economies. The classic analyses of this question were by Metzler (1941), (1947), who concluded that "An economy in which businessmen attempt to recoup inventory losses will always undergo cyclical fluctuations..". His model emphasized the fact that a business would attempt to keep inventories as a proportion of expected sales and so would re-build these if they declined below that target level. Given that sales had to be forecast from their past history, he showed that output would follow a second order difference equation which would have complex roots in many cases. Consequently his model produced *oscillations* in output and this constituted the foundation of his conclusion.

Turning points in output can occur even if there are no oscillations. A peak in output will be attained if positive growth turns into negative for some period of time, and the chance of this depends on the magnitude of long-run growth, the extent to which current growth depends on past growth rates, and the size of the shocks that are impinging on the macro-economic system - see Harding and Pagan (2002). Because the extent of oscillations was determined by examining the roots of a difference equation for output (or output growth) their measured nature was independent of any shocks. Consequently, there can be quite major differences between the cycle characteristics established by studying turning points in a series that incorporates the shocks and those that come from analysing oscillations in the same series, which essentially ignore them.

Business cycles are distinct from fluctuations. The latter focusses not upon contractions and expansions but rather upon measures such as the standard deviations, covariances and serial correlation of series representing economic activity. There is a relationship between business cycles and fluctuations, since many business cycle characteristics such as the duration of expansions can be shown to depend upon the quantities studied in fluctuations work - see Harding and Pagan (2002). Viewed in this light business cycle analysis performed by examining turning points in a series on output simply combine together measures such as the means, variances and serial correlations of growth rates in order to yield information about the characteristics of expansions and contractions. Given that most knowledge of business cycles is expressed in terms of the latter e.g. NBER, IMF(2002) it seems useful to proceed in this way rather than just to use the moments up to second order of the series.

The question that then arises is what impact the presence of inventories has upon the nature of business cycles? An answer to this question is of interest for a number of reasons. First, such an analysis would generalize what Metzler did when he focussed upon whether oscillations emerged from simple models of aggregate output. Second, there have always been comments that a large fraction of the change in output was accounted for by inventory movements – see, for example, Blinder and Maccini (1991). This has often been interpreted in a causal way as suggesting that inventories are the cause of the cycle. So it is interesting to see if that is a correct interpretation. Since the evidence is generally expressed in terms of the extent to which the movement in inventory investment during recessions is close to the actual fall in output, a turning point perspective is needed, given that it is necessary to define periods of recession. Finally, in recent times, expansions seem to have lengthened around the world. One would expect that this would happen if the volatility of *output growth* declines, and so it has been suggested that advances in inventory technology have been a major contributor to the Great Moderation that has been seen in many countries after the 1980's. However, much of this work has not been with models that explain the *growth rate in output* and the mapping between model variables and data has often been less than transparent.

In this paper, we address the question of the impact of inventories on business cycles in two ways. One is to establish some facts about the nature of the business cycles and then ask how the cycles would change if inventories were not present in the system. A second involves undertaking an analysis of whether advances in inventory management techniques are responsible for the decline in the volatility of GDP growth since the early eighties in the US. Because both approaches involve questions that can only be answered with a model of output and inventory choices it is necessary to construct one that is a reasonable characterization of the data and which has enough structure to answer the questions posed above.

The model chosen is an extension of that in Humphreys et al (2001). It sees the objectives of firms as attempting to balance the costs of keeping raw material stocks in line with output, and finished goods stocks in line with sales, with the extra costs incurred by rapid adjustment in output and purchases of raw materials. Because of the presence of raw materials it has some additional driving forces, such as the level of raw material prices, as well as the traditional one of the sales of finished goods. The model also allows for a number of other shocks such as technology and various cost shocks associated with inventories. Optimal decision rules for value added (GDP), raw material stocks and finished good stocks are established, thereby distinguishing between input and output inventories, as the behavior of each has been quite different in recent times. We spend considerable time in ensuring that the model is compatible with the unit root and co-integration properties of the data, so as to avoid filtering operations such as Hodrick-Prescott (HP) that would be inconsistent with the nature of the data implied by the model. If one considers business cycles as defined through turning points it is not necessary to make any $I(1)$ series stationary, which seems to be a major motivation for the HP filtering of the data. If one does filter the data one can still study turning points in the resulting series, but now one is studying the growth cycle rather than the business cycle.

In order to utilize the model to investigate the questions mentioned above we need to assign parameter values. Often this is done via "calibration" e.g. as in Khan and Thomas (2007). But, as there is a scanty literature setting out values for the parameters of our model, it was decided to estimate them from data. A problem in doing this relates to the Great Moderation that has occurred in many economic variables in the US after 1983. Inevitably, this must mean that some of the values of the model parameters have changed. In order to handle such a difficulty one needs to either work with realizations from a time period in which the parameters were likely to have been constant or to make some assumptions about how the parameters changed. Since there is controversy about whether the Great Moderation occurred abruptly or slowly[1], it does not seem attractive to adopt the latter solution, as one is then working with not only a model of output and inventories but also a model of how the parameters changed. It seems much better to estimate from a sample period in which there is a reasonable chance that the parameters were constant. Given the features later seen in the data, this will involve using data prior to 1984.

The only argument against using a sample of data from 1959-1983 (the period we choose) is that the sample is relatively short. But, as will be seen, quite precise estimates of the parameters are obtained with it. Moreover, calibrating the model with information before the Great Moderation provides a good vehicle for analyzing the role of inventories in that phenomenon. The fact that we actually estimate parameters for inventory models is novel, since generally parameter values have just been assigned either by an appeal to previous literature or by utilizing some simple mappings to the data, e.g. as in Khan and Thomas. Finally, the fact that we utilize only pre-1984 data does not seem critical to answer the questions we want to address. After all, we can change these parameter values and observe what responses occur. If one wanted to use the model for forecasting then the situation might be very different.

At this point, it is useful to compare the approach that we take to explore whether the Great Moderation in the volatility of GDP growth is due to better inventory management techniques with that in the literature. A substantial body of the literature has used VAR models to analyze the broad question of whether the decline in volatility of GDP growth is due to better inventory management techniques, to better monetary policy, or to "good luck". Stock and Watson (2002, 2003) represent an example of this approach as well as a survey and critique of the literature.[2] In contrast, our approach develops a structural model to analyze whether changes in parameters that capture the effects on inventory management techniques can explain the decline in the volatility of GDP growth. An alternative

[1] See McConnell and Perez-Quiros (2000) for an example where the Great Moderation occurs as an abrupt shift and Blanchard and Simon (2001) and more recently Davis and Kahn (2007) for examples of where it occurs as a slow adjustment.

[2] Other studies that have used VAR approaches to look at the role of inventory management advances and the decline in the volatility of output growth include Ahmed, Levin and Wilson (2004), Blanchard and Simon (2001), Herrera and Pesavento (2005), Irvine and Schuh (2005), and McCarthy and Zakrajsek (2007).

and much more recent approach, illustrated, for example, by the recent paper of Khan and Thomas (2007), uses general equilibrium models that focus on the economy as a whole.[3] Our model is a partial equilibrium one that focuses on the goods sector of the economy.[4] Both the general equilibrium approach and our approach use structural models to analyze the same question. A disadvantage of general equilibrium approaches is that, precisely because they are models of the whole economy, assumptions must be made about the household sector, services,

[3] Our model and approach differs from Khan and Thomas (2007) in several respects, other than the fact that they use calibration nmethods and we use estimation methods to analyze the basic issue. The model of the firm that they develop holds only intermediate goods inventories motivated by fixed costs of acquisition and the production function for final goods depends on the stock of intermediate goods inventories. Our model emphasizes the distinction between materials inventories and finished goods inventories, which play different roles in the firm, and which behave very differently after 1984, and the production function for final goods depends on the usage of materials in production.

Iacoviello, Schiantarelli and Schuh (2007) also develop a general equilibrium model where both input and output inventories are held in the goods sector. Their model, however, differs from the one developed here in the motivation for holding input and output inventories, the use of different definitions of input and output inventories, and the lack of a distinction between gross output and value added.

Kahn, McConnell and Perez-Quiros (2002) develop a general equilibrium model that focuses on finished goods inventories and thus does not address differences in the role played by finished goods and materials inventories. Also, they undertake a calibration and simulation analysis, rather than an analysis based on estimation of the model.

Jung and Yun (2006) and Wen (2008) develop general equilibrium models with different types of inventories. Jung and Yun use the model to analyze the role that strategic complementarity plays in price-setting behavior. Wen introduces inventories into a standard RBC model via precautionary stockout avoidance motives. In early work, Fisher and Hornstein (2000) develop a general equilibrium model that focuses on (S,s) inventory policies in the reatil sector but does not consider materials inventories. However, the latter three papers do not address the question of the role that inventories play in the Great Moderation.

[4] In work with partial equilibrium models, Ramey and Vine (2006) develop a model with finished goods inventories alone and argue that a decline in the persistence of sales volatility contributed to a decline in GDP volatility in the automobile industry. However, their model ignores the role played by materials inventories, which we focus on.

Chang, Hornstein and Sarte (2006) develop a model of an industry where firms hold finished goods inventories to explore the response of employment to productivity shocks. However, the model ignores materials inventories.

In preliminary work, Kahn (2007) develops a model with unfilled orders and materials inventories and undertakes a calibration and simulation analysis of the durable goods sector of the economy. However, the model ignores finished goods inventories and makes assumptions about the representative firm, e.g., fixed coefficients in production, that are different from our model.

the conduct of monetary policy, etc. If these are incorrect the shocks entering into them may not be uncorrelated so that one cannot attribute variation in a variable to them in a unique way. A partial equilibrium model avoids these biases that stem from specification errors in the rest of the system at the cost of assuming some variables can be treated as exogenous to the sector being studied. Of course this may also be in error and can induce biases.[5] It seems sensible that one use both approaches. If the answers found are similar then that is reassuring. If not then one would want to look more closely at why this might be so. Our view is that partial and general equilibrium model outcomes are complementary in this context in that they take different routes to look at an important and subtle issue.

Section 2 first reports some basic facts about the nature of the business cycle and the behavior of inventories over it. Section 3 then describes the model of the representative firm used to address the issues of the paper. After quantifying the parameters in Section 4 we use the model in section 5 to simulate data and to study the questions raised earlier relating to the business cycle and how it has been changing. A final section concludes the paper.

The paper yields two main conclusions: First, the model suggests that there is little contribution of inventories to the length and amplitude of the business cycle. This is so despite the fact that the model produces predictions that are compatible with many of the features seen in the data relating to the the behavior of inventories over the business cycle. Second, advances in inventory management techniques seem to have contributed very little to the decline in the volatility of GDP growth since the early eighties and hence cannot be a source of the Great Moderation.

2. Some Cycle Characteristics

It seems useful to re-examine the relation of inventories and the business cycle by utilizing the approach and techniques of the view of cycles described above i.e.

[5] An incorrect assumption about the exogeneity of variables can have two effects. One is to render estimators of parameters inconsistent. However, we show that the MLE yields consistent estimates of the parameters of the model even if exogensity is not correct for our model. A second effect is that, even if the variables are exogenous, one would need a complete general equilibrium model if one wanted to isolate the contributions of a basic set of shocks. Thus we take sales and technology to be exogenous and independent of one another. It is likely however that sales will be influenced by technology and so part of the explanation of output attributed to sales may be in fact be due to technology. The two questions we address however do not require a precise decomposition so that this does not seem to be particularly important.

as one reflecting turning points. We begin by thinking of aggregate economic activity as being usefully summarized by GDP - see Burns and Mitchell (1946,p 72) for an early statement and NBER (2003) for a recent one. However, since inventories are principally used in the production of goods, determining their role in the business cycle would seem to begin by splitting GDP into its goods and non-goods (services and structures) components, and then looking at the cycle in the goods component. Designating Q^g and $Q^s = Q - Q^g$ as the goods and non-goods components of GDP respectively, a log linearization produces the approximation

$$\Delta q_t = \omega^g \Delta q_t^g + (1 - \omega^g)\Delta q_t^s$$

where ω^g is the average of the ratio $\frac{Q_t^g}{Q_t}$ and lower case letters indicate the log of a variable i.e. $q_t = \ln Q_t$. Over 1947/1-2005/4 this average was .31 with a standard deviation of .02. The first and last observations on it were .32 and .36 respectively. So it has been a fairly stable ratio. The approximate value of q_t follows the actual data quite closely so that one might build up the cycle in q_t by looking at those in the sub-aggregates q_t^g and q_t^s.

Table 1 sets out cycle characteristics using the modified BBQ algorithm set out in Harding and Pagan (2002) to find turning points in the series[6]. It is evident that the cycles in the sub-sectors are quite different, with the cycle in goods GDP being much shorter than that in the non-goods sector. Since most attention has been paid to cycles in the level of economic activity, as measured by variables such as GDP, it is interesting to examine the cycle in the sub-set of GDP that relates to goods. One of the striking features of the business cycle measured with GDP is that movements in this do not signal a recession in 2001, as there was a sequence of alternating positive and negative quarterly growth rates, with the positive ones always offsetting the negative ones, meaning that there was no decline in the level of GDP for two quarters. In contrast, there was a clear recession in the goods sector, starting in 2000/3 and finishing in 2001/3. Indeed it is always the case that recessions in the goods sector have been stronger and longer than those in aggregate GDP. It might be thought that this comes from a declining contribution to aggregate GDP of goods, but, as the ratios $\frac{Q_t^g}{Q_t}$ presented earlier show, the opposite has happened in the chain-weighted data. Of course in nominal terms the ratio may well have declined since the relative price of goods to

[6]The modification involves a more efficient algorithm developed by James Engel for locating turning points . GAUSS and MATLAB programs for it are available at http://www.ncer.edu.au/data/

non-goods has almost certainly declined. In any case because we will be modelling q_t^g the issue about how one assesses the importance of the goods sector does not arise.

Table 1
US Business Cycle Characteristics
Goods, Services and Aggregate GDP : 1947/1-2005/4

	Goods	Non-Goods[1]	Aggregate
Mean Duration (Quarters)			
Contractions	3.3	3.2	2.8
Expansions	12.3	36.5	20.7
Mean Amplitude (Percentages[2])			
Contractions	-4.2	-1.2	-2.0
Expansions	18.0	32.9	21.7

[1]The Non-Goods Sector includes Services and Structures.
[2]The amplitude in a contraction is the percentage decline in GDP from Peak to Trough and the amplitude in an expansion is the percentage increase in GDP from Trough to Peak.

Table 2 shows AR(2) processes of the form

$$\Delta q_t = \mu/(1-\rho_1-\rho_2) + \rho_1 \Delta q_{t-1} + \rho_2 \Delta q_{t-2} + \sigma \varepsilon_t$$

fitted to the growth rates in the three series over two periods - (I) 1947/1-1983/4 and (II) 1984/1-2005/4. It is clear that the long-run growth in all quantities (μ) is much the same over the first period but there is some difference in the second period, and it seems as if both the volatility of the growth rate in GDP and its pattern of serial correlation changed between these as well. What is also interesting is that the serial correlation seen in aggregate GDP comes from the "services" component - the t ratios for α_j for Δq_t^g are less than 1.61, so that there is virtually no serial correlation in it in either period [7].

[7]However there is quite a high standard deviation for $\hat{\rho}_j$ for the smaller sample, and so the point estimates are not significantly different from one another.

Table 2
AR(2) Processes Fitted to the Series over Two Sample Periods

	μ	ρ_1	ρ_2	σ
GDP-I[1]	.0088	.314	.050	.011
GDP-II[2]	.0079	.183	.358	.005
Goods-I[1]	.0086	.102	.074	.022
Goods-II[2]	.0096	.106	.260	.011
Services-I[1]	.0088	.40	.05	.009
Services-II[2]	.0071	.16	.34	.003

[1]Sample period I is 1947-1 to 1983-4
[2]Sample period II is 1984-1 to 2005-4

The different characteristics noted above in Table 2 work in different directions when it comes to determining the impact upon cycles. The much higher volatility in goods GDP growth will mean a shorter cycle in it than for services. Offsetting this however is the lower positive serial correlation in goods output, as simulations in Harding and Pagan (2002) point to this producing a longer cycle. Consequently, with the two factors operating in opposite directions, the relative length of cycles is indeterminate, although the very large differences in volatility and smaller differences in serial correlation suggest a much shorter cycle in goods GDP. Table 2 shows that this is indeed the case.

To explore the impact of inventories upon the cycle we focus upon Δq_t^g in this paper, since it is the behavior of this series which will be affected by the presence of inventories. On a broad level it is worth exploring the question of how inventories might affect the goods cycle by examining how the DGP of Δq_t^g is built up, i.e., what determines the parameters and the shocks in the AR(2) process. It would not be expected that the presence of inventories would affect long-run growth, μ, but we might expect that there could be some impact upon the dynamic response of q_t^g to shocks. Since ε_t will be built up from all the shocks of the macro-economy, and one of these could be inventory cost shocks, this is another way in which inventories could affect the cycle.

To begin the investigation we start with the identity

$$X_t = Y_t - \Delta N_t \tag{2.1}$$

where X_t is the level of gross sales in the goods sector, Y_t is the level of gross output in the goods sector, and N_t is the level of finished goods inventories. Then, if the holding of finished goods inventories is important to the cycle, we would expect that the cycle in $x_t = \ln X_t$ would be different to that in $y_t = \ln Y_t$. We constructed series on X_t and N_t and then found Y_t from the identity (2.1) -see Appendix A. It is worth mentioning that the X_t we construct here is not that referred to as " final sales" in the NIPA. The equivalent of the latter for the goods sector would be

$$FS_t = Q_t - \Delta N_t - \Delta M_t,$$

where M_t is the level of raw material inventories. Many investigations of inventories use FS_t to represent X_t, e.g. Wen (2005), but generally we cannot use the former as a proxy for the latter when attempting to quantify a model. It would seem that the series we use for X_t is only available for the goods sector of GDP, at least on a quarterly basis.

Value added Q_t^g is the goods contribution to GDP, and this relates to Y_t and M_t through the identities

$$\begin{aligned} Q_t^g &= Y_t - V_t U_t \\ \Delta M_t &= D_t - U_t, \end{aligned}$$

where U_t is usage of raw materials, D_t is deliveries of raw materials, V_t is the relative price of raw materials to the price of output and M_t is the stock of raw materials.[8] Thus inventories of raw materials may make the cycle in q_t^g different from that in y_t.[9]

From the discussion above it is clear that if inventories are to affect the cycle in Q_t^g (q_t^g) they must induce a change in the DGP of Δq_t^g from that of Δx_t.[10] Table 3 looks at the characteristics of the DGP of each of the series $\Delta x_t, \Delta y_t$ over the period 1959/3-2005/4, and it suggests that finished goods inventories have little role in cycles, because the DGP of Δx_t and Δy_t are little different. However, the move from output to value added does induce significant changes in the DGP of

[8] There are missing elements in this definition such as energy usage and imports. Also we have assumed that the price of materials used is the same as that of the stock of raw materials.

[9] We can measure the quantities in the identities above in the following way. First, V_t is taken to be the implicit price deflator for raw materials used in the private business sector divided by the implicit price deflator for goods GDP. Second, U_t may then be recovered from $\frac{Y_t - Q_t^g}{V_t}$.

[10] Cycles in Q_t^g and q_t^g are identical as the log transformation is monotonic and so turning points in each series will be at the same points in time.

Δq_t^g from that of Δy_t[11]. Whether this is because of the effect of raw material stocks or the fact that the nature of raw material prices has had an effect is what needs to be determined. To do so one needs to construct a model that can account for the separate effects.

Table 3
AR(2) Fitted to $\Delta \mathbf{x}_t, \Delta \mathbf{y}_t, \Delta \mathbf{q}_t^g$
1959/3-2005/4

	Δx_t		Δy_t		Δq_t^g
μ	.0082		.0081		.0093
ρ_1	.384		.379		.015
ρ_2	-.003		.002		.143
σ	.013		.013		.017

Turning to the decline in the volatility of Δq_t^g one factor proposed to account for this is the ability to economize on inventories with new technology. A quick look at the plausibility of this is available by looking at the ratios $\frac{N_t}{X_t}$ and $\frac{V_t M_t}{Y_t}$ over time.[12] Fig 1 gives a plot of these. It is clear that there has been little change in the first ratio, but the second has declined by about 50% after 1984, which is a substantial decline. These stylized facts again point to the potential importance of raw materials when looking at changes in cycles and, at least for the US, these seem to have been more significant than finished goods inventories. Whether changes in the levels of inventories that are held can in fact explain changes in the cycle is a different matter, and once again points to the need to develop a model that explains Δq_t^g and which formally incorporates raw materials.

[11] The data on x_t is not available before 1959/1.

[12] We look at $\frac{V_t M_t}{Y_t}$ since a change in V_t would be expected to change $\frac{M_t}{Y_t}$ and so a change in the latter may simply reflect a response to relative price changes rather than a technological change. Of course even this ratio may not fully control for such an effect. Note that essentially the same pattern occurs for $\frac{M_t}{Y_t}$. It too declines after 1984 by about 45%, so it declines slightly less precipitously.

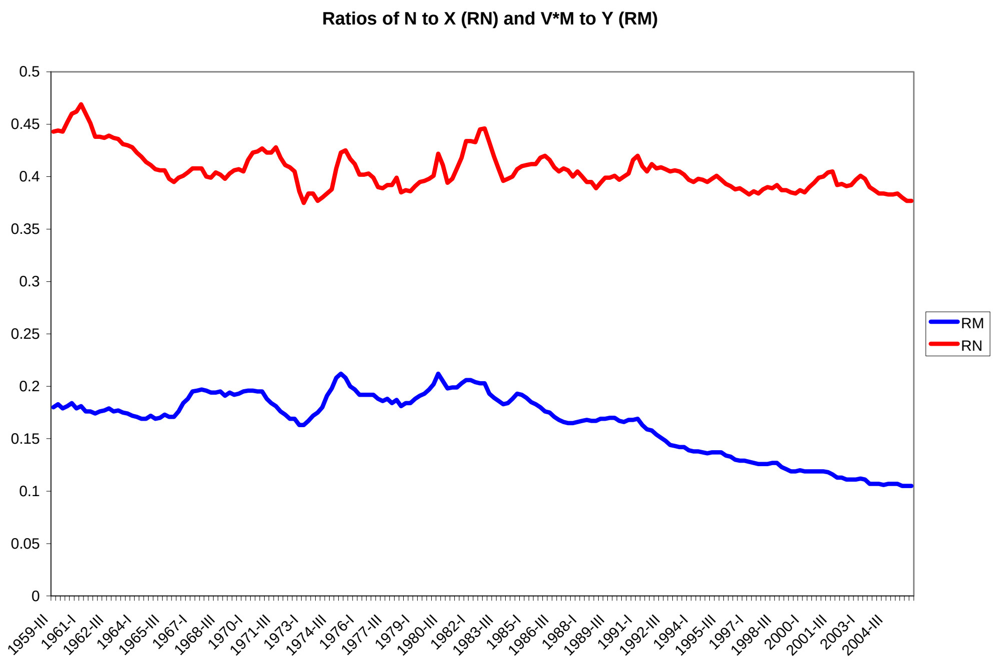

Figure 1

Finally it is worth dwelling upon inventory investment movements during business cycles. In particular, the view, expressed for example in Blinder and Maccini (1991), that a very large part of the reduction in GDP during recessions is associated with reductions in inventory investment. To measure this, it is useful to look at the ratio, $\frac{\Sigma_{t=1}^{T}(1-S_{gt})(\Delta^2 N_t+\Delta^2 M_t)}{\Sigma_{t=1}^{T}(1-S_{gt})\Delta Q_t^q}$, where $S_{gt}=1$ if q_t^g is in an expansion phase and $S_{gt}=0$ if it is in a contraction phase. Hence this records the average fraction of GDP decline across a recession that is associated with declines in inventory investment. It equals .77 for the 1959-2005 period (.78 over 1959-1983) and the equivalent ratio for just raw materials would be .27 (.31). Hence the effect is strongest for finished goods inventories. This quantity is often used to support the view that inventory investment movements seem to "account" for a large part of output decline. *Prima facie* this seems to conflict with our tentative conclusion expressed above that finished goods inventories play little role in business cycles but, in fact, there may be no inconsistency between the two perspectives. Because M, N are endogenous variables, it is possible that the absence of inventories might not change the cycle in Q_t^g much, but some common factor among the M, N series makes them cohere with Q_t^g. Again, a model is needed that enables one to see if it is possible to reconcile these differing "stylized facts".

3. The Model and its Euler Equations

The model of the representative firm that we use is an extension of the one developed by Humphreys et al.(2001). The model in Humphreys et al. has the advantage that it is a model of inventories broken down by stage of fabrication and thus distinguishes between finished goods or "output" inventories and materials and supplies or "input" inventories. The latter includes work in progress inventories as well; hereafter, we use the term materials inventories to refer to the sum of materials and supplies and work in progress inventories. The model thus permits an analysis of the role that each type of inventory stock plays in the production and sales process. This is an important advantage of the model as finished goods and materials inventories may have played very different roles in the reduction of the volatility of GDP growth. Figure 1 reported that the materials-output ratio had declined about 50% since the mid eighties, but the finished goods-sales ratio had remained about constant. This suggests that, to the extent that improved inventory management techniques have had a role to play in reducing the volatility of GDP growth, materials inventories may have been more important than finished goods inventories. Further, "just-in-time" techniques, which have become more widely used in recent years, are more applicable to materials inventory management than to that of finished goods.

3.1. The Production Function

We begin with a specification of the short-run production function, which is assumed to be Cobb-Douglas

$$\begin{aligned} Y_t &= F\left(L_t, U_t, \epsilon_{yt}\right) \\ &= L_t^{\gamma_1} U_t^{\gamma_2} e^{\epsilon_{yt}} \end{aligned} \tag{3.1}$$

where $\gamma_1 + \gamma_2 < 1$, which implies strict concavity of the production function in materials usage and labor. Here, Y_t is output, L_t is labor input, U_t is materials usage, and ϵ_{yt} is a technology shock. Note that U_t is the *flow* of materials used in the production process. When production and inventory decisions are made, the capital stock is assumed to be taken as given by the firm and to be growing at a constant rate, which will be captured by a deterministic trend in the empirical work. Finally, the firm is assumed to purchase intermediate goods (work-in-process) from outside suppliers rather than producing them internally.[13] Thus, intermediate goods are analogous to raw materials so work-in-process inventories can be lumped together with materials inventories. Because Y_t is gross output, we refer to equation (3.1) as the gross production function.

3.2. The Cost Structure

The firm's total cost structure consists of three major components: labor costs, inventory holding costs, and materials costs. This section describes each component.

3.2.1. Labor Costs

Labor costs are

$$\begin{aligned} LC_t &= W_t L_t + W_t \widetilde{A}\left(L_t, L_{t-1}\right) \\ &= W_t L_t + W_t A\left(\Delta l_t - \Delta \bar{l}\right) L_{t-1} \end{aligned} \tag{3.2}$$

where $\Delta l_t = \Delta \log L_t \approx \frac{\Delta L_t}{L_{t-1}}$ is the growth rate of labor and $\Delta \bar{l}$ is the steady state growth rate of labor. The first component, $W_t L_t$, is the standard wage bill where W_t is the real wage rate. The second component, $\widetilde{A}\left(L_t, L_{t-1}\right)$, is an adjustment cost function intended to capture the hiring and firing costs associated with changes in labor inputs. The adjustment cost function has the usual properties: Specifically,

$$A' \gtreqless 0 \qquad as \quad \Delta l_t \gtreqless \Delta \bar{l}$$

$$A(0) = A'(0) = 0 \qquad A'' > 0$$

Adjustment costs on labor accrue whenever the growth rate of the firm's labor force is different from the steady state growth rate. Further, adjustment costs exhibit rising marginal costs.

[13] To allow for production of intermediate goods within the firm requires extending the production function to incorporate joint production of final and intermediate goods. This extension is a substantial modification of the standard production process that we leave for future work.

3.2.2. Inventory Holding Costs

Inventory holding costs for finished goods inventories are:

$$HC_t^N = \Phi^N\left(N_{t-1}, X_t, \epsilon_{nt-1}\right) = \delta_1 \left(\frac{N_{t-1}}{X_t}\right)^{\delta_2} X_t + \delta_3 N_{t-1} + \epsilon_{nt-1} N_{t-1} \qquad (3.3)$$

$$\delta_1 > 0 \qquad \delta_2 < 0 \qquad \delta_3 > 0$$

where ϵ_{nt} is the shock to finished goods inventory holding costs. Inventory holding costs consist of two basic components. One, $\delta_1 \left(\frac{N_{t-1}}{X_t}\right)^{\delta_2} X_t$, captures the idea that, given sales, higher inventories reduce costs in the form of lost sales because they reduce stockouts. The other, $\delta_3 N_{t-1}$, captures the idea that higher inventories raise costs because they raise holding costs in the form of storage costs, insurance costs, etc.[14] The effects of technological advances that improve inventory management methods can be captured, for example, through a change in δ_2 and changes in the volatility of the shock to finished goods inventory holding costs.[15]

Inventory holding costs for materials and supplies inventories are:

$$\begin{aligned} HC_t^M &= \Phi^M\left(V_{t-1}M_{t-1}, Y_t, \epsilon_{mt-1}\right) \qquad (3.4) \\ &= \tau_1 \left(\frac{V_{t-1}M_{t-1}}{Y_t}\right)^{\tau_2} Y_t + \tau_3 V_{t-1}M_{t-1} + \epsilon_{mt-1} V_{t-1}M_{t-1} \end{aligned}$$

[14] These two components underlie the rationale for the quadratic inventory holding costs in the standard linear-quadratic model. The formulation above separates the components and assumes constant elasticity functional forms which facilitates log-linearization around constant steady states. We assume that the firm minimizes discounted expected costs and thereby abstract from market structure issues. See Bils and Kahn (2000) for a model that deals with market structure issues and also utilizes a constant elasticity specification of the benefits of holding finished goods inventories, though the benefits are embedded on the revenue side of the firm.

[15] Observe that (3.3) implies a "target stock" of finished goods inventories that minimizes finished goods inventory holding costs. Assuming $\epsilon_{nt-1} = 0$, the target stock, N_t^*, is

$$N_t^* = -\left(\frac{\delta_3}{\delta_1 \delta_2}\right)^{\frac{1}{\delta_2 - 1}} X_t$$

so that the implied target stock is proportional to sales. This is analogous to the target stock assumed in the standard linear-quadratic model. Note that the target stock is not the steady state stock of finished goods inventories. The steady state stock minimizes total costs in steady state whereas the target stock merely minimizes inventory holding costs. The steady state stock will be derived below.

$$\tau_1 > 0 \qquad \tau_2 < 0 \qquad \tau_3 > 0$$

where ϵ_{mt} is the shock to materials inventory holding costs. As above, there are two basic components: One, $\tau_1 \left(\frac{V_{t-1}M_{t-1}}{Y_t}\right)^{\tau_2} Y_t$, captures the idea that, given output, higher inventories reduce costs in the form of lost output because they reduce stockouts and disruptions to the production process. The other, $\tau_3 V_{t-1}M_{t-1}$, captures the idea that higher inventories raises costs because they raise holding costs in the form of storage costs, insurance costs, etc. Note that materials inventory holding costs depend on production, rather than sales. This is because stocking out of materials inventories entails costs associated with production disruptions – lost production, so to speak – that are distinct from the costs associated with lost sales. Lost production may be manifested by reduced productivity or a failure to realize production plans. Again, the effects of technological advances that improve inventory management methods can be captured, for example, through a change in τ_2 and changes in the volatility of the shock to materials inventory holding costs.[16]

The finished goods and material inventory holding costs differ because the firm holds the two inventory stocks for different reasons. The firm stocks finished goods inventories to guard against random demand fluctuations. Finished goods inventories thus facilitate sales. On the other hand, the firm stocks materials inventories to guard against random fluctuations in productivity, materials prices and deliveries, and other aspects of production. Materials stocks thus facilitate production. Although sales and production are highly positively correlated, they perform different roles in the firm and differ enough at high frequencies to justify different specifications.

[16] Similarly, observe that (3.4) implies a "target stock" of materials and supplies inventories that minimizes materials and supplies inventory holding costs. Assuming $\epsilon_{mt-1} = 0$, the target stock, $V_tM_t^*$, is

$$V_tM_t^* = -\left(\frac{\tau_3}{\tau_1\tau_2}\right)^{\frac{1}{\tau_2-1}} Y_t,$$

so that the implied target stock is proportional to output. Note here as well that the target stock of materials inventories is not the steady state stock of materials inventories. The latter will be derived below.

3.2.3. Materials Costs

Finally, we turn to the cost of purchasing materials and supplies. We assume that the real cost of purchasing materials and supplies is given by

$$\begin{aligned} MC_t &= V_t D_t + V_t \widetilde{\Gamma}\left(V_t D_t, Y_t\right) = V_t D_t + V_t \Gamma\left(\frac{V_t D_t}{Y_t}\right) D_t \qquad (3.5) \\ &= V_t D_t \left[1 + \Gamma\left(\frac{V_t D_t}{Y_t}\right)\right] \end{aligned}$$

$$\Gamma' \gtreqqless 0 \qquad \Gamma'' > 0$$

$$\Gamma(0) = 0 \qquad \Gamma(\overline{\tfrac{VD}{Y}}) = 0 \qquad \Gamma'(\overline{\tfrac{VD}{Y}}) = 0$$

where V_t is a real "base price" for raw materials. The term, $V_t D_t$, is the value of purchases and deliveries valued at the base price. The term, $V_t \widetilde{\Gamma}\left(V_t D_t, Y_t\right)$, represents a premium that may need to be paid over and above the base price to undertake the level of purchases and deliveries, D_t. It is assumed to rise at an increasing rate with the amount purchased and delivered.

Two cases may be distinguished:

1. Increasing Marginal Cost: $\Gamma' > 0$. In this case, the firm faces a rising supply price for materials purchases. When purchases are high relative to current stocks, the firm thus experiences increasing marginal costs due to higher premia that must be paid to acquire materials more quickly. A rationale for such a rising supply price is that the firm is a monopsonist in the market for materials. This is most likely to occur when materials are highly firm or industry specific and the firm or industry is a relatively large fraction of market demand.[17] The rising marginal cost of course gives rise to the "smoothing" of purchases.

2. Constant Marginal Cost: $\Gamma' = 0$. In this case, the firm is in effect a price taker in competitive input markets and is able to purchase all the raw materials it needs at the prevailing market price.

[17] This is analogous to the literature on adjustment cost models for investment in plant and equipment where external adjustment costs are imposed in the form of a rising supply price for capital goods. See, e.g., Abel(1979).

3.3. Cost Minimization

Assume that the representative firm takes sales (X_t) and factor prices (V_t and W_t) as exogenous. The firm's optimization problem is to minimize the discounted present value of total costs (TC),

$$E_0 \sum_{t=0}^{\infty} \beta^t TC_t = E_0 \sum_{t=0}^{\infty} \beta^t (LC_t + HC_t + MC_t) \ , \tag{3.6}$$

where $\beta = (1+r)^{-1}$ is the discount factor. The constraints are the production function, (3.1), and the two laws of motion governing inventory stocks. The identity for finished goods inventories is

$$\Delta N_t = Y_t - X_t \tag{3.7}$$

and the identity for materials and supplies inventories is

$$\Delta M_t = D_t - U_t. \tag{3.8}$$

The firm chooses $\{L_t, U_t, Y_t, M_t, N_t, D_t\}_{t=0}^{\infty}$ to minimize equation (3.6) subject to the constraints (3.1), (3.7), and (3.8).

3.4. Optimality Conditions

Assume that, when the representative firm makes decisions, current values of exogenous variables are in its information set. Then, the optimality conditions are:

$$W_t + \Lambda_{1t} F_L \left(L_t, U_t, \epsilon_{yt}\right) + W_t \widetilde{A}_1(L_t, L_{t-1}) + E_t \beta W_{t+1} \widetilde{A}_2(L_{t+1}, L_t) = 0 \tag{3.9}$$

$$\Lambda_{1t} F_U \left(L_t, U_t, \epsilon_{yt}\right) + \Lambda_{3t} = 0 \tag{3.10}$$

$$V_t + V_t \widetilde{\Gamma}_D (V_t D_t, Y_t) - \Lambda_{3t} = 0 \tag{3.11}$$

$$\Phi_Y^M (V_{t-1} M_{t-1}, Y_t, \epsilon_{mt-1}) + \widetilde{\Gamma}_Y (V_t D_t, Y_t) - \Lambda_{1t} - \Lambda_{2t} = 0 \tag{3.12}$$

$$\Lambda_{2t} - \beta E_t \Lambda_{2t+1} + \beta E_t \Phi_N^N(N_t, X_{t+1}, \epsilon_{nt}) = 0 \quad (3.13)$$

$$\Lambda_{3t} - \beta E_t \Lambda_{3t+1} + \beta E_t \Phi_M^M(V_t M_t, Y_{t+1}, \epsilon_{mt}) = 0 \quad (3.14)$$

together with the production function, (3.1), and the accumulation equations, (3.7) and (3.8). Note that $\Lambda_{1t}, \Lambda_{2t}$, and Λ_{3t} are the Lagrangian multipliers associated with (3.1), (3.7) and (3.8) respectively.

To interpret the optimality conditions, observe that (3.9)-(3.11) imply that

$$\frac{F_L(L_t, U_t, \epsilon_{yt})}{F_U(L_t, U_t, \epsilon_{yt})} = \frac{W_t\left[1 + \widetilde{A}_1(L_t, L_{t-1}) + \beta E_t \frac{W_{t+1}}{W_t} \widetilde{A}_2(L_{t+1}, L_t)\right]}{V_t\left[1 + \widetilde{\Gamma}_D(V_t D_t, Y_t)\right]},$$

which states that the relative marginal products of labor and materials utilization must equal their relative marginal costs. In the case of labor, the latter is the current wage rate together with the marginal adjustment costs of hiring and firing labor. In the case of materials utilization, the marginal cost is the purchase price of an additional unit of materials together with any marginal premia due to rising supply prices that must be paid to acquire materials.

The optimality condition, (3.13) can be summed forward to get

$$\Lambda_{2t} = -E_t \sum_{i=0}^{\infty} \beta^{i+1} \Phi_N^N(N_{t+i}, X_{t+1+i}, \epsilon_{nt+i})$$

which states that the imputed value to the firm of holding an additional unit of finished goods inventories must equal (minus) the expected discounted value of current and future marginal finished goods inventory holding costs.[18] The minus sign indicates that at the optimum the firm must operate on the downward sloping component of the inventory holding cost function for finished goods inventories. The latter is the segment where the marginal value of holding finished goods inventories to avoid lost sales exceeds the marginal storage costs. Similarly, the optimality condition, (3.14) can be summed forward to get

[18] This assumes that the transversality condition, $\lim_{T\to\infty} \beta^T E_t \Lambda_{2t+T} = 0$, is satisfied.

$$\Lambda_{3t} = -E_t \sum_{i=0}^{\infty} \beta^{i+1} \Phi_M^M (V_{t+i} M_{t+i}, Y_{t+1+i}, \epsilon_{mt+i})$$

which states that the imputed value to the firm of holding an additional unit of materials inventories must equal (minus) the expected discounted value of current and future marginal material inventory holding costs.[19] Again, the minus sign indicates that at the optimum the firm must operate on the downward sloping component of the inventory holding cost function for materials inventories. The latter is the segment where the marginal value of holding materials inventories to avoid lost output due to stockouts and disruptions of the production process exceeds the marginal storage costs.

Now, define the following shares and ratios

$$\begin{array}{llll} S_{L,t} = \frac{W_t L_t}{Y_t} & S_{U,t} = \frac{V_t U_t}{Y_t} & R_{N,t} = \frac{N_t}{X_t} & R_{M,t} = \frac{V_t M_t}{Y_t} \\ R_{D,t} = \frac{V_t D_t}{Y_t} & R_{Y,t} = \frac{Y_t}{X_t} & R_{\Lambda_3,t} = \frac{\Lambda_{3t}}{V_t} & \end{array} \tag{3.15}$$

It is assumed that the variables in (3.15) are stationary with finite expected values even if the variables they are constructed from have unit roots. It will also be assumed that V_t is a stationary random variable and this will make $\Lambda_{3t}, \Lambda_{1t}$ and Λ_{2t} stationary. Then, applying the functional form assumptions in (3.1), (3.3), (3.4), and (3.5), the resulting optimality conditions can be log-linearized around constant expected steady state values. Appendix B provides the details. On notation, lower case letters are logarithms of upper case letters, so, for example, $l_t = \log L_t$,and thus the growth rate of a variable is $\Delta l_t = \Delta \log L_t \approx \frac{\Delta L_t}{L_{t-1}}$. A "bar" above a variable denotes a constant expected steady state value. A "^" above an upper case letter denotes a log-deviation, while above a lower case letter denotes a simple (i.e., non-logarithmic) deviation. So, for example, the log-deviation of $R_{D,t}$ from its expected value is $\hat{R}_{D,t} = \log R_{D,t} - \log \overline{R}_D$, where $\overline{R}_D = E(R_{D,t})$, while the simple deviation of the growth rate of sales is $\Delta\widehat{x}_t = \Delta x_t - \Delta\overline{x}$. Similar notation applies to other variables. The log-linearized optimality conditions are then

$$\widehat{S}_{Lt} - \widehat{\Lambda}_{1t} + \varphi\Delta\widehat{l}_t - \beta\varphi E_t \Delta\widehat{l}_{t+1} = 0 \tag{3.16}$$

$$\widehat{S}_{Ut} - \widehat{\Lambda}_{1t} + \widehat{R}_{\Lambda_3,t} = 0 \tag{3.17}$$

[19] This assumes that the transversality condition, $\lim_{T\to\infty} \beta^T E_t \Lambda_{3t+T} = 0$, is satisfied.

$$\eta\widehat{R}_{D,t} - \widehat{R}_{\Lambda_3,t} = 0 \tag{3.18}$$

$$\mu_2\overline{R}_M\left(1-\Delta\overline{y}\right)\left[\widehat{R}_{M,t-1} - \Delta\widehat{y}_t\right] + \eta\overline{R}_D\widehat{R}_{D,t} + \overline{\Lambda}_1\widehat{\Lambda}_{1t} + \overline{\Lambda}_2\widehat{\Lambda}_{2t} = 0 \tag{3.19}$$

$$\overline{\Lambda}_2\widehat{\Lambda}_{2t} - \beta\overline{\Lambda}_2E_t\widehat{\Lambda}_{2t+1} + \beta\mu_1E_t\left(\widehat{R}_{N,t} - \Delta\widehat{x}_{t+1}\right) + \beta\epsilon_{nt} = 0 \tag{3.20}$$

$$\widehat{R}_{\Lambda_3,t} - \beta E_t\left[\Delta\widehat{v}_{t+1} + \widehat{R}_{\Lambda_3,t+1}\right] + \beta\mu_2E_t\left[\widehat{R}_{M,t} - \Delta\widehat{y}_{t+1}\right] + \beta\epsilon_{mt} = 0 \tag{3.21}$$

$$\Delta\widehat{y}_t = \gamma_1\Delta\widehat{l}_t + \gamma_2\Delta\widehat{u}_t + \epsilon_{yt} - \epsilon_{yt-1} \tag{3.22}$$

$$\widehat{R}_{N,t} - \left(1-\Delta\overline{x}\right)\widehat{R}_{N,t-1} + \left(1-\Delta\overline{x}\right)\Delta\widehat{x}_t - \frac{\overline{R}_Y}{\overline{R}_N}\widehat{R}_{Y,t} = 0 \tag{3.23}$$

$$\widehat{R}_{M,t} - \left(1+\Delta\overline{v}-\Delta\overline{y}\right)\left[\widehat{R}_{M,t-1} + \Delta\widehat{v}_t - \Delta\widehat{y}_t\right] - \frac{\overline{R}_D}{\overline{R}_M}\widehat{R}_{D,t} + \frac{\overline{S}_U}{\overline{R}_M}\widehat{S}_{U,t} = 0 \tag{3.24}$$

where $\varphi = A''(0)$, $\eta = \Gamma''(\overline{R}_D)\overline{R}_D^2$, $\mu_1 = \left(\delta_2-1\right)\delta_2\delta_1\left[\overline{R}_N\left(1-\Delta\overline{x}\right)\right]^{\delta_2-1}$, and $\mu_2 = \left(\tau_2-1\right)\tau_2\tau_1\left[\overline{R}_M\left(1-\Delta\overline{y}\right)\right]^{\tau_2-1}$.

Finally, other quantities can be derived in a similar way. Since we will ultimately want to re-construct Q_t^g it is useful to find an expression for the log-deviation of the ratio of GDP to gross output for the goods sector. Specifically, define $R_{Q^g,t} = \frac{Q_t^g}{Y_t}$. Then, to obtain $\widehat{R}_{Q^g,t}$, recall that the definition of GDP is

$$Q_t^g = Y_t - V_tU_t,$$

so that

$$\frac{Q_t^g}{Y_t} = 1 - \frac{V_tU_t}{Y_t}.$$

Then, using the definition of $R_{Q^g,t}$ and the utilization share, $S_{U,t} = \frac{V_tU_t}{Y_t}$, we have that

$$R_{Q^g,t} = 1 - S_{U,t}.$$

A log-linear approximation of this then yields

$$\overline{R}_{Q^g}\widehat{R}_{Q^g,t} = -\overline{S}_U\widehat{S}_{U,t}. \tag{3.25}$$

4. Quantifying Model Parameters

4.1. Matching Data and Model Variables

The model above is that of a representative firm. To apply the model to the goods sector as a whole, we assume that the representative firm behaves as if it is vertically integrated, so that it is representative of the whole goods sector of the economy. The representative firm holds materials inventory stocks which it uses in conjunction with labor (and capital) to produce output of finished goods, which it adds to finished goods inventories. The finished goods inventories may be held by manufacturers, wholesalers or retailers. In effect, we treat the representative firm as managing the inventory stocks of finished goods whether they are held on the shelves of the manufacturer, the wholesaler, or the retailer. Appendix A deals with the exact measurement of the variables in the data.

4.2. Estimation Strategy

As pointed out in the introduction we seek to quantify the model parameters using data over the period 1959/1-1983/4. Ideally one would like to have begun with 1947/1 but quarterly data were not available on M_t and N_t over that earlier period. The arguments for using a longer sample normally relate to increased precision of estimators, but, as will become apparent, the parameters are quite precisely estimated. Once estimated the model can then be used to explore some of the questions raised in the introduction.

In (3.16)-(3.24) we will treat the variables $v_t, w_t, x_t,\ \varepsilon_{yt}, \varepsilon_{nt}, \varepsilon_{mt}$ as exogenous processes. v_t is assumed stationary but, based on unit root tests, w_t and x_t are $I(1)$ (ADF tests of -2.2 and -.72)). Some simplification of (3.16)-(3.24) is possible. In particular we can eliminate $\hat{R}_{\Lambda_{3,t}}$ and $\widehat{\Lambda}_{1t}$ by using (3.17) and (3.18). This leaves us with seven equations to determine the ten endogenous variables

$$z_t^{*\prime} = \left[\begin{array}{cccccccccc} \widehat{S}_{L,t} & \widehat{S}_{U,t} & \Delta\hat{y}_t & \widehat{R}_{D,t} & \widehat{\Lambda}_{2t} & \widehat{R}_{N,t} & \widehat{R}_{M,t} & \Delta\hat{u}_t & \Delta\hat{l}_t & \hat{R}_{Y,t} \end{array} \right].$$

Three extra equations connecting endogenous and exogenous variables are available from identities describing $\Delta S_{L,t}, \Delta S_{U,t}$ and $\Delta R_{Y,t}$; for example, $\Delta\hat{S}_{L,t} = \Delta\hat{w}_t + \Delta\hat{l}_t - \Delta\hat{y}_t$. This system of ten equations therefore has the form

$$z_t^* = Az_{t-1}^* + BE_t(z_{t+1}^*) + G\eta_t, \tag{4.1}$$

where $\eta_t' = \begin{bmatrix} \Delta\hat{x}_t & \Delta\hat{w}_t & v_t & \Delta\varepsilon_{yt} & \varepsilon_{nt} & \varepsilon_{nt} \end{bmatrix}$. Hence it can be solved by standard methods to give the solution

$$z_t^* = Pz_{t-1}^* + E_t(\sum_{j=0}^{\infty} F^j G\eta_{t+j}) \tag{4.2}$$

where F is a function of A, B - see for example Binder and Pesaran (1995). When η_t follows a VAR(1) this becomes

$$z_t^* = Pz_{t-1}^* + K\eta_t. \tag{4.3}$$

Further, given the solution for $\widehat{S}_{U,t}$ that is an element of z_t^* in (4.3), a solution for $\widehat{R}_{Q^g,t}$ may be obtained from (3.25).

In our model derivations we have implicitly assumed that there are two co-integrating relations among any $I(1)$ variables. These are given by the finished goods inventory-sales ratio, $R_{N,t} = \frac{N_t}{X_t}$, and the raw material ratio, $R_{M,t} = \frac{V_t M_t}{Y_t}$. ADF tests on $lnR_{M,t}$ and $lnR_{N,t}$ were -3.09 and -3.17, easily rejecting the null hypothesis of a unit root at the 5% level of significance.[20]. Labour usage was not in our data set but it was assumed that the labour share $S_{L,t}$ is $I(0)$. Taking these three ratios to be $I(0)$, we show in Appendix B that $S_{U,t}$, $R_{D,t}$, $R_{Y,t}$, $R_{\Lambda_{3t}}$, Λ_{1t}, and Λ_{2t} will then be $I(0)$ as well. Not unexpectedly unit root tests applied to n_t, m_t, y_t, x_t and w_t reveal these series to have unit roots. Two of these variables, x_t and w_t, are exogenous, and so can be taken as the permanent components underlying the $I(1)$ series. But, since there are two cointegrating vectors among the five $I(1)$ variables there must be another common permanent component, and we identify this as the technology shock i.e. ε_{yt} has a unit root.

It is necessary to describe the evolution of η_t. Since Δx_t and Δw_t are observable we can fit processes to these series. An AR(1) was fitted to each producing AR coefficients of .33 and .02 respectively.[21] $\Delta\varepsilon_{yt}, \varepsilon_{nt}$ and ε_{mt} were all assumed to be

[20]These tests are based upon using the SBC criterion to choose the best lag length for the ADF test. The result is lags of 2 and 3 respectively.

[21]Ramey and Vine (2005) argue that there has been a change in the persistence of sales in the automobile industry and that this led to a decline in output volatility. They find that there was a shift in persistence in 1984/1. Since we impose a unit root on the x_t process we do not allow for any such decline. However, fitting the same regression as they used (an AR(1) with shifts in the intercepts and deterministic time trend) to x_t, which is (the logarithm of) sales in the goods sector as a whole, over their sample period of 1967/1-2004/4, we find that the t ratio for a shift in the degree of persistence is just .6. Moreover, the point estimate for the lag coefficient parameter actually *rose* by .03 in the post 1984 period.

AR(1) processes with coefficients ρ_y, ρ_n and ρ_m respectively and these are to be estimated by MLE. v_t seemed closer to stationarity, having an AR(2) of the form

$$v_t = 1.21v_{t-1} - .29v_{t-2} + .0053\varepsilon_t^v.$$

The v_t process is a very persistent one but the sharp rise in oil prices (and associated raw material prices) in 1974 had a major effect upon the unit root tests. Because of this we decided to treat it as $I(0)$.

The parameters in the Euler equations can be divided into three groups:

$$(I)\ \theta_1 = [\beta, \gamma_1, \gamma_2, \overline{R}_Y, \overline{R}_D, \overline{R}_M, \overline{R}_N] \quad (4.4)$$

$$(II)\ \theta_2 = [\tau_3, \delta_3, \overline{\Lambda}_1, \overline{\Lambda}_2], \quad (4.5)$$

$$(III)\ \theta_3 = [\tau_1, \tau_2, \delta_1, \delta_2, \varphi, \eta, \sigma_y, \sigma_n, \sigma_m, \rho_y, \rho_n, \rho_m], \quad (4.6)$$

where σ_y, σ_n and σ_m are the standard deviations of the unobervable shocks $\varepsilon_y, \varepsilon_n$ and ε_m respectively. θ_1 is either pre-set in the case of $\beta = .99$, $\gamma_1 = .22, \gamma_2 = .66$ or estimated using sample means for the ratios $\overline{R}_Y$ etc. The γ_j parameters were set because the absence of capital in the model makes it difficult to estimate the parameters of a production process.[22]

Estimates of the four parameters in θ_2 are found from appropriate steady state conditions - see (8.36)-(8.44) in Appendix B. To obtain values for θ_2 from those steady state equations requires values for θ_3. Once a set of values for θ_3 is established, θ_1 and θ_2 are then fixed, so that it is only necessary to estimate θ_3 from the data. Finally, from the optimality conditions in (3.16) and (3.17) it is evident that τ_1, τ_2, δ_1 and δ_2 enter only through μ_1 and μ_2, so only two of these parameters can be identified. Consequently we set $\delta_1 = \tau_1 = 1$, which are common normalizations in the empirical literature on inventories. Doing so leaves the final set of parameters to be estimated as $\xi = [\tau_2, \delta_2, \varphi, \eta, \sigma_y, \sigma_n, \sigma_m, \rho_y, \rho_n, \rho_m]$.

Maximum likelihood estimation is then done to get estimates of ξ. A complication arises from the fact that we only have data on six variables - $z_t = [S_{U,t}, R_{N,t}, R_{M,t}, \Delta x_t, v_t, \Delta w_t]$. If z_t^* had been observed then the likelihood would be just that of the VAR in z_t^*. When not all variables in z_t^* are observed it is necessary to add an observation equation connecting the observed and unobserved variables

[22] If we had set $\gamma_2 = .71$ to reflect the share of raw materials as measured in our data then it seems virtually impossible to allow for a role for capital, as a realistic share for labour would mean that the sum of the raw material and labour shares would exceed unity. In addition, it was found that setting γ_j below the pre-set values above resulted in a substantial decline in the likelihood, so these values seemed a reasonable compromise.

i.e. $z_t = Hz_t^*$, to the state dynamics equations, as summarized by the VAR in z_t^*. The likelihood of the resulting state space form formed from this equation and (4.3) is then that for the observed z_t, and it is found recursively by an application of the Kalman predictor. This is a standard way of performing MLE on DSGE models e.g. see Ireland (2004), and is adopted in the DYNARE program that we utilize for estimation.[23]

Maximum Likelihood Estimates of the parameters are available in Table 4. The implied estimates of τ_3 and δ_3 from the steady state equations are .07 and .04 respectively.

Table 4
Model Parameter Estimates

		τ_2	δ_2	φ	η	σ_y
Estimate		-.015	-.026	1.16	5.7	.008
t-Statistic		2.8	2.9	4.3	4.9	10.1
		σ_n	σ_m	ρ_n	ρ_m	ρ_y
Estimate		.002	.005	.96	.88	.11
t-Statistic		3.5	3.6	26.5	10.6	5.0

It is worth commenting on the exogeneity assumptions in force. In a general equilibrium (GE) model, where consumer and policy choices are modelled, there would be an extra set of equations describing the expanded choices, and these would augment our equations for the goods sector. In addition there may be some extra shocks that are either common or specific to the agents being considered. Unless strategic behavior was involved the equations for the goods sector would not change. Thus the solution for the goods sector still has the structure in (4.2). The impact of a GE model would be that agents in the goods sector would now form expectations of η_{t+j} using an expanded information set. Thus the GE version of (4.2) would be

$$z_t^* = Pz_{t-1}^* + E_t^+(\sum_{j=0}^{\infty} F^j G\eta_{t+j}), \tag{4.7}$$

[23] We use version 3.065 of DYNARE written by S. Adjemian, M. Juillard and O. Kamenik.

where "+" indicates the expanded information set. Hence to reconcile (4.7) with (4.2) it is necessary that an error ζ_t be added on to (4.2) so that the equation becomes

$$z_t^* = z_t^* = Pz_{t-1}^* + E_t(\sum_{j=0}^{\infty} F^j G\eta_{t+j}) + \zeta_t$$

where $\zeta_t = E_t^+(\sum_{j=0}^{\infty} F^j G\eta_{t+j}) - E_t(\sum_{j=0}^{\infty} F^j G\eta_{t+j})$. This changes the VAR solution to

$$z_t^* = Pz_{t-1}^* + K\eta_t + \zeta_t$$

Now $E_t(\zeta_t) = 0$ since the information used in the goods model would be a sub-set of the expanded information set. This also implies that $E(z_{t-1}^*\zeta_t) = 0$ and so we can legitimately estimate the parameters in the partial equilibrium (PE) model using the VAR that ignores ζ_t.[24] Hence it is not consistent estimation of the Euler equation parameters that is affected by the use of a PE model but rather issues of efficiency and interpretation.

What is lost in using the PE approach is the potential for a gain in efficiency owing to the fact that ζ_t can be eliminated (which must of course be balanced against the biases that can arise if the rest of the model is poorly specified) and a more precise interpretation of η_t owing to the fact that it might be a function of variables that do not appear in the goods (partial equilibrium (PE)) model. Efficiency does not seem to be of great concern here given the precision of the estimators in Table 4. Therefore, it is the interpretation which we focus upon. Put simply, in a broader model Δx_t (say) would be determined not only by z_{t-1}^* and the shocks identified in the PE model, but also lagged values of other variables e.g. interest rates and other shocks. A complete model is needed to define such a relationship. If only the PE model is used then any decomposition attributing output variation to sales will be unable to precisely identify what shock it was that was driving Δx_t. Because the questions we seek to resolve do not depend critically on any such decomposition the partial equilibrium approach provides valuable information that does not require the complexity (and possible errors)

[24] It is necessary to ensure that we find $E_t\eta_{t+j}$ correctly however i.e. all information in the partial equilibrium model needs to be used to forecast items like Δx_t ,Δw_t and v_t. We did run regressions of Δx_t against lags of variables such as n_t, m_t, y_t etc but these provided little explanatory power over the own lagged values, which is why we describe Δx_t etc as exogenous. Our methodology does not however require that such series be strongly exogenous. It was the nature of the data that lead to this assumption.

induced by a complete model. Of course if one was seeking to provide a model for q_t rather than q_t^g it would be necessary to have a GE model.

At this point, it is useful to look at the adequacy of the model in re-producing some of the more interesting characteristics of the data. First the model implies that the standard deviation for Δq_t^g, $\sigma_{\Delta q^g}$, would be .0243, versus the value of .021 in the data.[25] The test statistic that these are different is 2.4 so that the model seems to produce a reasonable fit to the variance of Δq_t^g, although a little too high. The parameter estimates imply a first-order serial correlation coefficient in Δq_t^g of .15, whereas the data says there is virtually no serial correlation. The standard error on this estimate is however .10 and so there is no significant difference between them. In terms of cycle outcomes the durations of expansions and contractions for Q^g are 9.45/3.43 (data) and 9.28/4.23 (model).[26]

Other features of interest relate to the first order serial correlation coefficients in Δm_t and Δn_t. There is quite high serial correlation in these, in contrast to the situation for Δq_t^g. Specifically they are .625 and .318 respectively. The model predicts them to be .610 and .344 respectively, so it captures the fact that the serial correlation patterns in inventories and GDP are quite different.

Finally, as we observed earlier, one of the reasonably constant features of the data is the association between the decline in inventory investment and the fraction of the decline in GDP over recessions. For 1959-1983 this was .78 for total inventories and .31 for raw materials. The model predicts these to be .88 and .39 respectively (found by averaging over 500 sets of simulated data with 98 observations in each set). The range of variation that comes out of the simulations is quite large, meaning that the predictions are consistent with the data at an extremely low level of significance. Consequently the model seems to capture many of the features of inventory movements and their relations to the goods cycle quite well.

[25] The characteristics of Δq_t^g reported in Table 2 for the sample period 1947-1983 are virtually the same as those from 1959-1983.

[26] Note this is for the sub-period 1959/3-1983/4 and it is quite a short period to measure cycle characteristics.

5. Experiments with the Model

5.1. Analysis of Fluctuations and the Great Moderation

The causes for the great moderation have been much debated in the literature. When this feature was observed it was natural that one look at what changes were taking place in the economy which might lead to such an outcome. Since there had been great advances in inventory control methods, in particular the development of “just in time” philosophies relating to production, it seemed possible that this might be a source of the changes e.g. see Kahn et al (2002). To assess this possibility we first adopt the above estimated model as the "base model" and then ask how $\sigma_{\Delta q^g}$ varies with changes in selected parameters of the model. In particular, we are interested in what happens as δ_2 and τ_2 change so as to mean that less finished goods inventories and raw materials are held as a ratio of sales and output respectively. Consider, for example, a decline in the absolute value of τ_2, which shifts the materials inventory holding cost function. Intuitively, such a decline captures the idea that computerization, just-in time procedures, or other technological advances in inventory management techniques imply that the firm can experience the same level of lost production with a smaller level of materials inventories, given the level of output. Or, alternatively, a given materials inventory/output ratio will generate a smaller amount of lost production. A similar interpretation applies to a decline in the absolute value of δ_2. To compute values of Δq_t^g, when a parameter changes, we reverse the estimation strategy, and now solve for ratios such as R_N and R_M as functions of the estimated model parameters. Also of interest is the magnitude of the impact of changes in the volatility of the observed and unobserved shocks.

$|\delta_2|$ was arbitrarily reduced by 10% while $|\tau_2|$ was reduced by 20%. The latter produced a decline in the $\frac{VM}{Y}$ ratio that roughly matches what was seen over the period 1984/1-2005/4. Similarly the reductions in standard deviations of all shocks was set to 50%, as that was roughly what happened to the observable shocks over that period. As noted above, $\frac{N_t}{X_t}$ changed only minimally after 1984, and so we look at only a small change in $|\delta_2|$ to see whether this could have had any impact on volatility. So these experiments are about how we might have expected fluctuations to have changed over the second period given that the volatility reduction in observed shocks was matched by that in the unobserved ones. The experiments involving reductions in the standard deviations of shocks also give some insight into what the main sources of fluctuations would be. Finally, we present an experiment in which δ_2 and τ_2 are just one-hundreth of the values in

Table 3. Such a reduction means that inventories have very low value to the firm in the sense of reducing the costs of lost sales or lost output and thus emulates a situation where inventories are not present in the system. Table 5 gives the results of these experiments.[27]

Table 5
Effects on $\sigma_{\Delta q^g}$ of Parameter Perturbations

		$\sigma^*_{\Delta q^g}$	$\rho^*_{\Delta q^g}$
Base		.0243	.158
$.9\delta_2$		.0242	.156
$.8\tau_2$		.0241	.156
$.5\sigma_x$		.0204	.014
$.5\sigma_w$		.0243	.158
$.5\sigma_v$		.0233	.16
$.5\sigma_n$		.0242	.157
$.5\sigma_m$		.0241	.16
$.5\sigma_y$		.0194	.315
$.01(\delta_2, \tau_2)$		.0208	.156

*Effect of cuts in parameters

It is clear that, if the change in inventory technology can be thought of as involving a change in the magnitude of τ_2, so that smaller materials stocks relative to output are an optimal choice, then this produces only slightly smaller fluctuations in goods GDP. As Table 5 indicates, a 20% decline in τ_2 produces only a 1% reduction in $\sigma_{\Delta q^g}$. Hence this cannot be the source of the reduced volatility in Δq^g_t. Further, decomposing the variance of Δq^g_t into the various shocks: 48% is due to the technology shock, 39% is due to the sales shock, 10% is due to the raw

[27] If one wishes to find the impact of combinations of parameter changes rather than a single one, the approach used in assessing the sensitivity of computable general equilibrium model solutions to parameters can be followed. Given the model solution as a function of the parameters, $y_t = g(\theta)$, where θ are the parameters, linearization around some base set of parameters produces $y_t - y^* = \frac{\partial g}{\partial \theta}(\theta^*)(\theta - \theta^*)$. Of course this is only a local approximation but experiments show it is quite accurate for the likely range of variation in the coefficients. $\frac{\partial g}{\partial \theta}(\theta^*)$ can be measured from Table 4 e.g. a 50% reduction in σ_x leads to be 16% reduction in $\sigma_{\Delta q^g}$ so that a 1% change σ_x would lead to a .32% change in $\sigma_{\Delta q^g}$.

material price shock, and 1% is due to the raw materials inventory shock. Neither wages nor the finished goods inventory shocks are important.

If the volatility of all shocks was reduced by 50%, $\sigma_{\Delta q^g}$ would become .012 which is very close to the actual reduction over the 1984/1-2005/4 period - see Table 5. If only the observed ones were reduced, $\sigma_{\Delta q^g}$ would only have dropped to .019 so that the unobservable shocks are critical to explain this phenomenon. Specifically, a large decline in the volatility of technology shocks will be needed.

An alternative viewpoint, expressed in Kahn et al (2002), is that the information set of firms may have changed as a consequence of computerization. In particular it may be that sales are now known with greater accuracy. To assess this we considered an experiment in which it was assumed that only $\{x_{t-j-1}\}_{j=0}^{\infty}$ rather than $\{x_t\}_{j=0}^{\infty}$ was known in the period 1959/1-1983/4.[28] In the post-1983/4 period however x_t was taken to be part of the information set. To conduct this experiment the model was re-estimated using the new information set and the implied $\sigma_{\Delta q^g}$ was still found to be .0243. Hence the reduction in the volatility of Δq_t^g as a result of improved information about sales is negligible, since the base case in Table 4 represents what it would be with the expanded information.

In light of these results it is useful to consider the debate over whether monetary policy had an impact on $\sigma_{\Delta q^g}$. One might expect that this effect would work through sales and, although the decline in the volatility of the latter has made a contribution, it would not have led to the observed decline in volatility if technology shocks had not changed as well. Thus it is hard to see monetary policy as being the major driving force in the reduction in the volatility in the goods sector.

5.2. Analysis of Cycles

Whilst the nature of the business cycle depends upon the volatility of Δq_t^g it also depends upon the mean of this process and the nature of any serial correlation in it. Consequently, the experiments above were repeated to determine their effects upon the cycle in q_t^g. Table 6 shows how the durations and amplitudes of expansions and contractions in q_t^g would change. It should be noted that over the period of estimation, 1959/1-1983/4, the duration of contractions and expansions were 3.43 and 9.45 quarters, and so the length of expansions using the estimated model parameters (the "base" simulation) is quite close to that actually observed.

[28] This means that $E_t(\Delta\hat{x}_{t+1})$ in the Euler equations is replaced by $\rho^2\Delta\hat{x}_{t-1}$ rather than $\rho\Delta\hat{x}_t$.

Table 6
Effects on Cycles of Parameter Perturbations

		Durations (Quarters)*		Amplitudes (Percentages)*	
		Contractions	**Expansions**	**Contractions**	**Expansions**
Base		4.23	9.28	-5.84	16.02
$.9\delta_2$		4.24	9.32	-5.81	16.03
$.8\tau_2$		4.23	9.34	-5.77	16.02
$.5\sigma_x$		3.80	11.19	-4.14	15.6
$.5\sigma_w$		4.23	9.34	-5.84	16.09
$.5\sigma_v$		4.22	9.57	-5.58	16.02
$.5\sigma_n$		4.24	9.27	-5.81	15.99
$.5\sigma_m$		4.23	9.31	-5.79	16.02
$.5\sigma_y$		4.11	9.86	-4.73	15.29
$.01(\delta_2, \tau_2)$		4.31	10.57	-4.89	15.60

*Effect of cuts in parameters

Given that we know cycle length depends upon the volatility and serial correlation properties of Δq_t - Harding and Pagan (2002)- the results in Table 6 are largely predictable by the outcomes in Table 4. An exception occurs for the relative effects of the experiments involving a reduction in sales and technology shock volatilities. Here the cycle becomes longer with the first experiment, even though the volatility decrease was slightly less than in the second experiment. This shows that the degree of serial correlation in Δq_t^g is also important for cycle outcomes. The final experiment shows that the presence of inventories in the system does create less cycles although on average they are only one quarter longer. Overall, the importance of inventories to the average cycle is limited, even though it may be that for particular cycles their presence has a greater effect. It should be noted that in no case are there complex roots in the Δq_t^g process and so no oscillations.

6. Conclusion

In this paper, we have developed a model of the optimal holding of finished goods and raw material input inventories by a goods producing firm and have used it to analyze a number of questions that have come up about the role of inventories.

The paper yields two main conclusions. First, we showed that inventories have had only a small effect upon the average duration of expansions and contractions of the business cycle in the U.S. economy. To show the latter, we looked at business cycles in terms of the turning points in the level of goods GDP, which is a very different perspective from the traditional work on inventory cycles that has looked at oscillations in activity. This conclusion emerges despite the fact that the model produces predictions that are consistent with many of the features observed in the data regarding the behavior of inventories over the business cycle. Second, we showed that changes in inventory management technology have had little effect upon the volatility of GDP growth in the goods sector of the US economy. The model we developed allows for a role for raw material prices in producing cycles and we found that the latter did have some impact, although the main drivers of changes in the business cycle were technology and sales variations.

The approach we take in this paper is to develop a partial equilibrium model of the goods sector of the economy. This enables us to focus on the basic question of whether advances in inventory technology were responsible for the decline in the volatility of GDP growth in the goods sector with a minimal number of assumptions about the rest of the economy. Our approach is quite different from the VAR approaches and the more recent general equilibrium models of the economy as a whole. Nonetheless, the different approaches that have been used to address the basic question appear to be coming to similar conclusions, namely, that advances in inventory management techniques have played a relatively small role in the Great Moderation. This is reassuring in that knowledge is advanced when very different approaches arrive at essentially similar conclusions regarding important questions.

7. References

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8. Appendix A: Data Description

In the model above we assumed that the representative firm behaves as if it is vertically integrated, so that it is representative of the goods sector of the whole economy. We assumed that the representative firm holds materials inventory stocks which it uses in conjunction with labor (and capital) to produce output of finished goods, which it adds to finished goods inventories. The finished goods inventories may then be held by manufacturers, wholesalers or retailers. In effect, we treat the representative firm as managing the inventory stocks of finished goods whether they are held on the shelves of the manufacturer, the wholesaler, or the retailer.

The data on inventories, sales, and GDP are quarterly, seasonally-adjusted, (2000) chain-weighted series in billions of dollars, and cover the period 1959/1 through 2005/4. To construct aggregates of chain-weighted data, we used the appropriate Tornqvist approximation for chain-weighted data as suggested by Whelan (2002). Accordingly, we constructed an aggregate stock of finished goods inventories by appropriately aggregating the real value of finished goods inventories in manufacturing, wholesale trade and retail trade.[29] The aggregate stock of materials inventories is constructed by appropriately aggregating the materials and supplies and work in progress inventories held by manufacturers.[30] Value added or GDP is the real value of GDP for the goods sector of the economy. A data series for gross output of the goods sector is not available from government statistical agencies. We therefore constructed an approximate measure of gross output for the goods sector by appropriately aggregating gross sales or shipments for the manufacturing and trade sectors of the economy and the changes in finished goods inventory investment for those sectors. GDP is of course the flow of value added over the quarter, and inventories are measured as end-of-quarter stocks.

The series on W is the ratio of the average hourly earnings for goods producing industries divided by the implicit price deflator for sales of the business sector. V is found by dividing the implicit price deflator for input inventories by the implicit price deflator for the sales of the business sector.

[29] The Bureau of Economic Analysis series on manufacturing and trade inventories experienced discontinuities in the late nineties due to the conversion from SIC to NAICS classification systems. We dealt with this problem by constructing inventory stock series by extending backwards each series from current stocks using growth rates.

[30] Note that there are no materials and supplies and work in progress inventories in wholesale trade or retail trade.

8.1. Appendix B: Derivation of Optimality Conditions

Recall again that lower case letters are the logarithms of an upper case letter, so, for example, $l_t = \log L_t$,and thus the growth rate of a variable is $\Delta l_t = \Delta \log L_t \approx \frac{\Delta L_t}{L_{t-1}}$. Then, using the functional form assumptions, the optimality conditions, (3.9)-(3.14) together with the production function, (3.1), and the accumulation equations, (3.7) and (3.8) become:

$$W_t + \Lambda_{1t}\gamma_1 \frac{Y_t}{L_t} + W_t A' \left(\Delta l_t - \Delta \bar{l}\right) + \beta E_t W_{t+1} A \left(\Delta l_{t+1} - \Delta \bar{l}\right) \\ -\beta E_t W_{t+1} \frac{L_{t+1}}{L_t} A' \left(\Delta l_{t+1} - \Delta \bar{l}\right) = 0 \tag{8.1}$$

$$\Lambda_{1t}\gamma_2 \frac{Y_t}{U_t} + \Lambda_{3t} = 0 \tag{8.2}$$

$$V_t \left[1 + \Gamma(\frac{V_t D_t}{Y_t}) + \frac{V_t D_t}{Y_t}\Gamma'(\frac{V_t D_t}{Y_t})\right] - \Lambda_{3t} = 0 \tag{8.3}$$

$$(1-\tau_2)\,\tau_1 \left(\frac{V_{t-1}M_{t-1}}{Y_t}\right)^{\tau_2} - \left(\frac{V_t D_t}{Y_t}\right)^2 \Gamma'(\frac{V_t D_t}{Y_t}) - \Lambda_{1t} - \Lambda_{2t} = 0 \tag{8.4}$$

$$\Lambda_{2t} - \beta E_t \Lambda_{2t+1} + \beta E_t \delta_2 \delta_1 \left(\frac{N_t}{X_{t+1}}\right)^{\delta_2 - 1} + \beta\delta_3 + \beta\epsilon_{nt} = 0 \tag{8.5}$$

$$\Lambda_{3t} - \beta E_t \Lambda_{3t+1} + \beta E_t \tau_2 \tau_1 V_t \left(\frac{V_t M_t}{Y_{t+1}}\right)^{\tau_2 - 1} + \beta V_t \tau_3 + \beta V_t \epsilon_{mt} = 0$$

$$Y_t = (L_t)^{\gamma_1}(U_t)^{\gamma_2} e^{\epsilon_{yt}} \tag{8.6}$$

$$N_t - N_{t-1} - Y_t + X_t = 0 \tag{8.7}$$

$$M_t - M_{t-1} - D_t + U_t = 0 \tag{8.8}$$

Re-arranging terms, we have that

$$1+\Lambda_{1t}\gamma_1\frac{Y_t}{W_tL_t}+A'\left(\Delta l_t-\Delta\bar{l}\right)+\beta E_t\frac{W_{t+1}}{W_t}\left[A\left(\Delta l_{t+1}-\Delta\bar{l}\right)\right] \qquad (8.9)$$
$$-\beta E_t\frac{W_{t+1}}{W_t}\frac{L_{t+1}}{L_t}A'\left(\Delta l_{t+1}-\Delta\bar{l}\right)=0$$

$$\Lambda_{1t}\gamma_2\frac{Y_t}{V_tU_t}+\frac{\Lambda_{3t}}{V_t}=0 \qquad (8.10)$$

$$1+\Gamma(\frac{V_tD_t}{Y_t})+\frac{V_tD_t}{Y_t}\Gamma'(\frac{V_tD_t}{Y_t})-\frac{\Lambda_{3t}}{V_t}=0 \qquad (8.11)$$

$$(1-\tau_2)\,\tau_1\left(\frac{V_{t-1}M_{t-1}}{Y_{t-1}}\frac{Y_{t-1}}{Y_t}\right)^{\tau_2}-\left(\frac{V_tD_t}{Y_t}\right)^2\Gamma'(\frac{V_tD_t}{Y_t})-\Lambda_{1t}-\Lambda_{2t}=0 \qquad (8.12)$$

$$\Lambda_{2t}-\beta E_t\Lambda_{2t+1}+\beta E_t\delta_2\delta_1\left(\frac{N_t}{X_t}\frac{X_t}{X_{t+1}}\right)^{\delta_2-1}+\beta\delta_3+\beta\epsilon_{nt}=0 \qquad (8.13)$$

$$\frac{\Lambda_{3t}}{V_t}-\beta E_t\frac{V_{t+1}}{V_t}\frac{\Lambda_{3t+1}}{V_{t+1}}+\beta E_t\tau_2\tau_1\left(\frac{V_tM_t}{Y_t}\frac{Y_t}{Y_{t+1}}\right)^{\tau_2-1}+\beta\tau_3+\beta\epsilon_{mt}=0 \qquad (8.14)$$

$$\left(\frac{Y_t}{Y_{t-1}}\right)=(\frac{L_t}{L_{t-1}})^{\gamma_1}(\frac{U_t}{U_{t-1}})^{\gamma_2}e^{\epsilon_{yt}-\epsilon_{yt-1}} \qquad (8.15)$$

$$\frac{N_t}{X_t}-\frac{N_{t-1}}{X_{t-1}}\frac{X_{t-1}}{X_t}-\frac{Y_t}{X_t}+1=0 \qquad (8.16)$$

$$\frac{V_tM_t}{Y_t}-\frac{V_t}{V_{t-1}}\frac{V_{t-1}M_{t-1}}{Y_{t-1}}\frac{Y_{t-1}}{Y_t}-\frac{V_tD_t}{Y_t}+\frac{V_tU_t}{Y_t}=0 \qquad (8.17)$$

Now, define the growth rate of, for example, X_t , by $X_t=(1+\Delta x_t)\,X_{t-1}$, and similarly for other variables. Further, use the approximation, $\frac{1}{1+\Delta x_t}\approx 1-\Delta x_t$ to re-write the optimality conditions as

$$1+\Lambda_{1t}\gamma_1\frac{Y_t}{W_tL_t}+A'\left(\Delta l_t-\Delta\bar{l}\right)+\beta E_t\left(1+w_{t+1}\right)A\left(\Delta l_{t+1}-\Delta\bar{l}\right) \qquad (8.18)$$
$$-\beta E_t\left(1+w_{t+1}\right)\left(1+l_{t+1}\right)A'\left(\Delta l_{t+1}-\Delta\bar{l}\right)=0$$

$$\Lambda_{1t}\gamma_2\frac{Y_t}{V_tU_t}+\frac{\Lambda_{3t}}{V_t}=0 \qquad (8.19)$$

$$1+\Gamma(\frac{V_tD_t}{Y_t})+\frac{V_tD_t}{Y_t}\Gamma'(\frac{V_tD_t}{Y_t})-\frac{\Lambda_{3t}}{V_t}=0 \qquad (8.20)$$

$$\left(1-\tau_2\right)\tau_1\left(\frac{V_{t-1}M_{t-1}}{Y_{t-1}}\left(1-\Delta y_t\right)\right)^{\tau_2}-\left(\frac{V_tD_t}{Y_t}\right)^2\Gamma'(\frac{V_tD_t}{Y_t})-\Lambda_{1t}-\Lambda_{2t}=0 \qquad (8.21)$$

$$\Lambda_{2t}-\beta E_t\Lambda_{2t+1}+\beta E_t\delta_2\delta_1\left(\frac{N_t}{X_t}\left(1-\Delta x_{t+1}\right)\right)^{\delta_2-1}+\beta\delta_3+\beta\epsilon_{nt}=0 \qquad (8.22)$$

$$\frac{\Lambda_{3t}}{V_t}-\beta E_t\left(1+\Delta v_{t+1}\right)\frac{\Lambda_{3t+1}}{V_{t+1}}+\beta E_t\tau_2\tau_1\left(\frac{V_tM_t}{Y_t}\left(1-\Delta y_{t+1}\right)\right)^{\tau_2-1}+\beta\tau_3 \qquad (8.23)$$
$$+\beta\epsilon_{mt}=0$$

$$\Delta y_t=\gamma_1\Delta l_t+\gamma_2\Delta u_t+\epsilon_{yt}-\epsilon_{yt-1} \qquad (8.24)$$

$$\frac{N_t}{X_t}-\frac{N_{t-1}}{X_{t-1}}\left(1-\Delta x_t\right)-\frac{Y_t}{X_t}+1=0 \qquad (8.25)$$

$$\frac{V_tM_t}{Y_t}-\left(1+\Delta v_t\right)\left(1-\Delta y_t\right)\frac{V_{t-1}M_{t-1}}{Y_{t-1}}-\frac{V_tD_t}{Y_t}+\frac{V_tU_t}{Y_t}=0 \qquad (8.26)$$

Then, using the definitions of the shares and ratios stated in (3.15), the optimality conditions may be written as

$$1+\Lambda_{1t}\gamma_1 S_{L,t}^{-1}+A'\left(\Delta l_t-\Delta\bar{l}\right)+\beta E_t\left(1+\Delta w_{t+1}\right)A\left(\Delta l_{t+1}-\Delta\bar{l}\right) \\ -\beta E_t\left(1+\Delta w_{t+1}\right)\left(1+\Delta l_{t+1}\right)A'\left(\Delta l_{t+1}-\Delta\bar{l}\right)=0 \tag{8.27}$$

$$\Lambda_{1t}\gamma_2 S_{U,t}^{-1}+R_{\Lambda_3,t}=0 \tag{8.28}$$

$$1+\Gamma\left(R_{D,t}\right)+R_{D,t}\Gamma'\left(R_{D,t}\right)-R_{\Lambda_3,t}=0 \tag{8.29}$$

$$\left(1-\tau_2\right)\tau_1\left(R_{M,t-1}\left(1-\Delta y_t\right)\right)^{\tau_2}-R_{D,t}^2\Gamma'\left(R_{D,t}\right)-\Lambda_{1t}-\Lambda_{2t}=0 \tag{8.30}$$

$$\Lambda_{2t}-\beta E_t\Lambda_{2t+1}+\beta E_t\delta_2\delta_1\left(R_{N,t}\left(1-\Delta x_{t+1}\right)\right)^{\delta_2-1}+\beta\delta_3+\beta\epsilon_{nt}=0 \tag{8.31}$$

$$R_{\Lambda_3,t}-\beta E_t\left(1+\Delta v_{t+1}\right)R_{\Lambda_3,t+1}+\beta E_t\tau_2\tau_1\left(R_{M,t}\left(1-\Delta y_{t+1}\right)\right)^{\tau_2-1}+\beta\tau_3 \\ +\beta\epsilon_{mt}=0 \tag{8.32}$$

$$\Delta y_t=\gamma_1\Delta l_t+\gamma_2\Delta u_t+\epsilon_{yt}-\epsilon_{yt-1} \tag{8.33}$$

$$R_{N,t}-R_{N,t-1}\left(1-\Delta x_t\right)-R_{Y,t}+1=0 \tag{8.34}$$

$$R_{M,t}-\left(1+\Delta v_t-\Delta y_t\right)R_{M,t-1}-R_{D,t}+S_{U,t}=0 \tag{8.35}$$

where again $\Lambda_{1t}, \Lambda_{2t}$, and Λ_{3t} are Lagrangian multipliers associated with (3.1), (3.7) and (3.8) respectively.

In Section 4.2 it was assumed that $R_{N,t}$, $R_{M,t}$, and $S_{L,t}$ are all $I(0)$. If these three ratios are $I(0)$, S_{Ut}, $R_{D,t}$, $R_{Y,t}$, $R_{\Lambda_{3t}}$, Λ_{1t}, and Λ_{2t} are $I(0)$ as well. To see this, observe that if $R_{N,t}$, $R_{M,t}$,and $S_{L,t}$ are $I(0)$, then Λ_{1t} is $I(0)$, (8.32) makes Λ_{2t} $I(0)$, (8.29) has $R_{D,t}$ as $I(0)$, (8.30) makes $R_{\Lambda_3,t}$ $I(0)$, (8.28) makes $S_{U,t}$ $I(0)$, (8.34) makes $R_{Y,t}$ an $I(0)$ variable.

We assume that in steady state the expected values of the factor input shares, $S_{U,t}$ and $S_{L,t}$, the ratios $R_{N,t}$, $R_{M,t}$, $R_{D,t}$, $R_{Y,t}$, and $R_{\Lambda_3,t}$, and the growth rates of variables are constants. The non-stochastic steady state conditions are then:

$$
\begin{aligned}
1+\overline{\Lambda}_1\gamma_1\overline{S}_L^{-1} &= 0 && (8.36)\\
1+\overline{\Lambda}_1\gamma_2\overline{S}_U^{-1} &= 0 && (8.37)\\
\overline{R}_{\Lambda_3} &= 1 && (8.38)\\
(1-\tau_2)\,\tau_1\left[\overline{R}_M\,(1-\Delta\overline{y})\right]^{\tau_2}-\overline{\Lambda}_1-\overline{\Lambda}_2 &= 0 && (8.39)\\
(1-\beta)\,\overline{\Lambda}_2+\beta\delta_2\delta_1\left[\overline{R}_N\,(1-\Delta\overline{x})\right]^{\delta_2-1}+\beta\delta_3 &= 0 && (8.40)\\
\left[1-\beta\,(1+\Delta\overline{v})\right]+\beta\tau_2\tau_1\left[\overline{R}_M\,(1-\Delta\overline{y})\right]^{\tau_2-1}+\beta\tau_3 &= 0 && (8.41)\\
\gamma_1\Delta\overline{l}+\gamma_2\Delta\overline{u} &= \Delta\overline{y} && (8.42)\\
1+\Delta\overline{x}\overline{R}_N &= \overline{R}_Y && (8.43)\\
\overline{R}_D+(\Delta\overline{v}-\Delta\overline{y})\,\overline{R}_M &= \overline{S}_U && (8.44)
\end{aligned}
$$

Again, in Section 4.2 above on estimation strategy, we stated that estimates of the four parameters in θ_2 are found from appropriate steady state conditions. Specifically, they are found from the four equations corresponding to the steady state conditions (8.37) and (8.38)-(8.41).

Now, log-linearizing the optimality conditions, (8.27)-(8.35), around the constant steady state values, (8.36)-(8.44) yields the log-linearized optimality conditions stated above in (3.16)-(3.24).