

# Monetary Policy and Inventory Investment

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Louis J. Maccini  
(Corresponding author)  
Department of Economics  
Johns Hopkins University  
3400 N. Charles Street  
Baltimore, MD 21218  
(410) 516-7607  
maccini@jhu.edu

Bartholomew Moore  
Department of Economics  
Fordham University  
441 East Fordham Road  
The Bronx, NY 10458  
(718) 817-4049  
bmoore@fordham.edu

Huntley Schaller  
Carleton University and Institute for Advanced  
Studies (Vienna)

Mailing Address:  
Department of Economics  
Carleton University  
1125 Colonel By Drive  
Ottawa, ON K1S 5B6  
(613) 520-3751  
schaller@ccs.carleton.ca

## Abstract

Declines in inventory investment account for a large fraction of the drop in output during a recession. But the relationship between monetary policy and inventories is unclear. Three main puzzles have been identified in the literature on monetary policy and inventory investment -- the mechanism puzzle, the sign puzzle, and the timing puzzle. First, the *mechanism* puzzle. Monetary policy changes the interest rate and should affect inventories, since the interest rate represents the opportunity cost of holding inventories. In fact, VAR studies find that monetary policy affects inventories. But 40 years of empirical literature on inventories has generally failed to find any significant effect of the interest rate on inventories. Second, the *sign* puzzle. Contractionary monetary policy raises the interest rate. An increase in the interest rate should **decrease** inventories through the increase in opportunity cost. VAR studies find that the short-term effect of contractionary monetary policy is to **increase** inventories. Third, the *timing* puzzle. Monetary policy induces transitory changes in the interest rate. The effect of monetary policy on the interest rate largely disappears within one year. But inventories begin to fall only after the transitory shock to the interest rate has largely dissipated. We use simulations of a theoretical model based on learning and regime shifts in the real interest rate to address all three puzzles.

## I. INTRODUCTION

Inventory investment tends to decline precipitously during recessions. Blinder and Maccini (1991) find that drops in inventory investment account for more than 80% of the fall in output during postwar recessions in the US. In their *Handbook of Macroeconomics* chapter, Ramey and West (1999) document the large declines in inventory investment during recessions across most of the G-7 countries.

This paper focuses on the role of inventories in the monetary policy transmission mechanism. Three main puzzles have been identified in the literature on monetary policy and inventory investment.

The first puzzle is the *mechanism* puzzle. Monetary policy changes the interest rate and should affect inventories, since the interest rate represents the opportunity cost of holding inventories. In fact, VAR studies find that monetary policy shocks affect inventories. But 40 years of empirical literature on inventories has generally failed to find any significant effect of the interest rate on inventories. So how does monetary policy affect inventories?

In our theoretical model, the real interest rate is subject to persistent and transitory shocks. Firms don't react much to transitory shocks, but they do react to persistent shocks (regime changes). The previous 40 years of empirical inventory research primarily used econometric techniques that emphasized high-frequency variation in the data, where there is much transitory variation in the interest rate without corresponding variation in inventories – and much transitory variation in inventories (due to their role in buffering sales shocks) without corresponding variation in the interest rate. Empirical tests based on cointegration techniques, which emphasize low-frequency (long-run)

movements in the variables, provide support for our model by showing a strong statistical relationship between the interest rate and inventories.

The second puzzle is the *sign* puzzle. Contractionary monetary policy raises the interest rate. An increase in the interest rate should **decrease** inventories through the increase in opportunity cost. VAR studies find that the short-term effect of contractionary monetary policy is to **increase** inventories.

Our solution to the puzzle is linked to the role of inventories in buffering demand (sales) shocks. Our empirical results show that demand shocks dominate the high-frequency movements in inventories. Sales drop rapidly in the first few months following a contractionary monetary policy shock. Inventories rise as they buffer negative sales shocks in the first few months following a contractionary monetary policy shock.

We test our solution to the sign puzzle by simulating the dynamic path of inventories in response to a monetary policy shock to assess whether the model produces the rise in inventories in the first few months after a contractionary shock that is observed in the actual data.

The third puzzle is the *timing* puzzle. Monetary policy induces transitory changes in the interest rate. The effect of monetary policy on the interest rate largely disappears within one year. But inventories begin to fall only after the transitory shock to the interest rate has largely dissipated.

Our solution to the timing puzzle works as follows. Because of learning, the Bayesian probabilities of being in a given interest rate régime respond slowly to a change in the interest rate (in simulations of our model). Although the effect of monetary policy

on the interest rate tends to be short-lived, the effect on the probabilities is persistent.

More than one third of the initial effect on the probabilities remains three years after the monetary policy shock.

The main elements of our theoretical model are learning and the behaviour of the real interest rate. The mean real interest rate tends to be highly persistent, with occasional large shifts. For example, the mean real interest rate was around 2% during the 1960s and early 1970s but negative from the mid-to late 1970s and much higher through much of the 1980s. As Garcia and Perron (1996) have shown, the behaviour of the real interest rate can be well captured by a Markov switching process with transitory fluctuations around persistent interest-rate regimes. Our model incorporates this stochastic process for the real interest rate into the optimization problem faced by the firm.

In the real world, no one posts a notice that the interest rate has shifted from a high-interest-rate regime to a low-interest-rate regime. Instead, firms must try to infer the expected path of interest rates from their best guess about the current interest rate regime. This best guess must be based on observable data, including current and past interest rates. Our theoretical model captures this by assuming that firms engage in a learning process.

The paper is organized as follows. Section II introduces the model. Section III describes how we identify monetary policy shocks and how we estimate the effect of monetary policy shocks on the Bayesian probabilities of being in a given interest rate régime. Section IV explains how we use the cointegrating regression for inventories to calibrate the model. Section V presents simulations of the effects of a monetary policy

shock. Section VI illustrates the pure interest rate effect and the broad interest rate effects (through sales and through costs) during a particularly interesting episode of recent U.S. macroeconomic history, the Volcker disinflation. Section VII provides a summary and conclusion.

## II. The Model

### The Firm's Optimization Problem

We begin by summarizing the basic model of the firm developed in Maccini, Moore and Schaller (2004). The representative firm is assumed to minimize the present value of its expected costs over an infinite horizon. Real costs per period are assumed to be quadratic and are defined as

$$C_t = \xi W_t Y_t + \frac{\theta}{2} Y_t^2 + \frac{\gamma}{2} (\Delta Y_t)^2 + \frac{\delta}{2} (N_{t-1} - \alpha X_t)^2 \quad (1)$$

where  $\theta, \gamma, \delta, \xi, \alpha > 0$ .  $C_t$  denotes real costs,  $Y_t$ , real output,  $N_t$ , end-of-period real finished goods inventories,  $X_t$ , real sales, and  $W_t$ , a real cost shock, which we will associate with real input prices. The level of real sales,  $X_t$ , and the real cost shock,  $W_t$ , are given exogenously. The first two terms capture production costs. The third term is adjustment costs on output. The last term is inventory holding costs, which balance storage costs and stockout costs, where  $\alpha X_t$  is the target stock of inventories.

Let  $\beta_t$  be a variable real discount factor, which is given by  $\beta_t = \frac{1}{1+r_t}$ , where  $r_t$

denotes the real rate of interest. The firm's optimization problem is to minimize the present discounted value of expected costs,

$$E_0 \sum_{t=0}^{\infty} \left[ \prod_{j=0}^{t-1} \beta_j \right] C_t, \quad (2)$$

subject to the inventory accumulation equation, which gives the change in inventories as the excess of production over sales,

$$N_t - N_{t-1} = Y_t - X_t. \quad (3)$$

The Euler equation that results from this optimization problem is

$$\begin{aligned} E_t \{ & \theta(Y_t - \beta_{t+1}Y_{t+1}) + \gamma(\Delta Y_t - 2\beta_{t+1}\Delta Y_{t+1} + \beta_{t+1}\beta_{t+2}\Delta Y_{t+2}) \\ & + \xi(W_t - \beta_{t+1}W_{t+1}) + \delta\beta_{t+1}(N_t - \alpha X_{t+1}) \} = 0 \end{aligned} \quad (4)$$

where from (3)  $Y_t = N_t - N_{t-1} + X_t$ . Observe that (4) involves products of the discount factor and the choice variables and products of the discount factor and the forcing variables. Linearizing these products around constant values, which may be interpreted as stationary state values or sample means, yields a linearized Euler equation:

$$\begin{aligned} E_t \{ & \theta(Y_t - \bar{\beta}Y_{t+1}) + \gamma(\Delta Y_t - 2\bar{\beta}\Delta Y_{t+1} + \bar{\beta}^2\Delta Y_{t+2}) + \xi(W_t - \bar{\beta}W_{t+1}) \\ & + \delta\bar{\beta}(N_t - \alpha X_{t+1}) + \eta r_{t+1} + c \} = 0 \end{aligned} \quad (5)$$

where  $\eta = \bar{\beta}(\theta\bar{Y} + \xi\bar{W}) > 0$ ,  $c = -\bar{r}\bar{\beta}(\theta\bar{Y} + \xi\bar{W}) < 0$ ,  $\bar{\beta} = \frac{1}{1+\bar{r}}$ , and a bar above a

variable denotes the stationary state value.

To interpret the Euler equation, ignore adjustment costs and the constant term for simplicity, define  $\eta = \bar{\beta}\hat{\eta}$ , and re-arrange (5) to get

$$E_t\{\theta Y_t + \xi W_t\} + E_t\bar{\beta}\{\delta(N_t - \alpha X_{t+1}) + \hat{\eta}r_{t+1}\} = E_t\bar{\beta}\{\theta Y_{t+1} + \xi W_{t+1}\}$$

Now,  $E_t\{\theta Y_t + \xi W_t\}$  is the marginal cost of producing a unit of output today,  $E_t\bar{\beta}\{\theta Y_{t+1} + \xi W_{t+1}\}$  is the discounted marginal cost of producing a unit of output tomorrow, and  $E_t\bar{\beta}\{\delta(N_t - \alpha X_{t+1}) + \hat{\eta}r_{t+1}\}$  are discounted marginal carrying costs of inventories, consisting of marginal holding costs plus the marginal interest charges. To have a unit of output available for sale tomorrow, the Euler equation thus states that the firm should equate the marginal cost of producing a unit of output today and carrying it in inventories to the discounted marginal cost of producing the unit of output tomorrow.

A key innovation in Maccini, Moore and Schaller (2004) is to assume that the real interest rate follows a three-state Markov switching process.<sup>1</sup> This is consistent with empirical patterns in real interest rates—See Garcia and Perron (1996) and the empirical work in Maccini, Moore and Schaller (2004). Specifically, we assume that the real interest rate follows

$$r_t = r_{S_t} + \sigma_{S_t} \cdot \varepsilon_t \tag{6}$$

where  $\varepsilon_t \sim \text{i.i.d. } N(0,1)$  and where  $S_t \in \{1, 2, 3\}$  follows a Markov switching process.

Let  $r_1 < r_2 < r_3$ , so that when  $S_t = 1$  the real interest rate is in the low-interest-rate regime, when  $S_t = 2$  the real interest rate is in a moderate interest rate regime, and when  $S_t = 3$  the real interest rate is in a high-interest-rate regime.  $S_t$  and  $\varepsilon_t$  are assumed to be

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<sup>1</sup> For a comprehensive discussion of Markov switching processes, see Hamilton (1994, Chapter 22).

independent. Denote the transition probabilities governing the evolution of  $S_t$  by  $p_{ij} = \text{Prob}(S_t = j | S_{t-1} = i)$ . Collecting these probabilities into a matrix we have

$$P = \begin{bmatrix} p_{11} & p_{21} & p_{31} \\ p_{12} & p_{22} & p_{32} \\ p_{13} & p_{23} & p_{33} \end{bmatrix}.$$

We assume that the firm knows the structure and parameters of the Markov switching process but does not know the true real interest rate regime. The firm must therefore infer  $S_t$  from observed interest rates. We denote the firm's current probability assessment of the true state by  $\pi_t$ . That is,

$$\pi_t = \begin{bmatrix} \pi_{1t} \\ \pi_{2t} \\ \pi_{3t} \end{bmatrix} = \begin{bmatrix} \text{Prob}(S_t = 1 | \Omega_t) \\ \text{Prob}(S_t = 2 | \Omega_t) \\ \text{Prob}(S_t = 3 | \Omega_t) \end{bmatrix},$$

where the firm's information set,  $\Omega_t$ , includes the current and past values of  $r_t$ . Here,  $\pi_{it}$  is the firm's estimate at date  $t$  of the probability that the real interest rate is in regime  $i$ .

To understand the learning process, consider how the firm uses its observation of the current real interest rate to develop its probability assessment,  $\pi_t$ . Beginning at the end of period  $t-1$  the firm uses  $\pi_{t-1}$  together with the transition probabilities in  $P$  to form beliefs about the period  $t$  interest rate state *prior to observing*  $r_t$ . That is the firm evaluates  $\pi_{it|t-1} \equiv \text{Prob}(S_t = i | \Omega_{t-1})$  for  $i = 1, 2, 3$  using

$$\begin{bmatrix} \pi_{1t|t-1} \\ \pi_{2t|t-1} \\ \pi_{3t|t-1} \end{bmatrix} = P\pi_{t-1} \quad (7)$$



Once the firm enters period  $t$  and observes  $r_t$ , it uses the prior probabilities from (7) together with the relevant conditional probability densities to update  $\pi_t$  according to Bayes' rule. Specifically,

$$\pi_{it} = \frac{\pi_{it|t-1} \cdot f(r_t | S_t = i)}{\sum_{j=1}^3 \pi_{jt|t-1} \cdot f(r_t | S_t = j)} \quad \text{for } i=1,2,3. \quad (8)$$

Thus, the firm uses Bayes' rule and its observations of the real interest rate to learn about the underlying interest rate regime.

Given  $\pi_t$ , the expected real interest rate, which may be interpreted as an ex ante real interest rate, may then be computed as

$$E_t r_{t+1} = r'_v P \pi_t = \gamma_1 \pi_{1t} + \gamma_2 \pi_{2t} + \gamma_3 \pi_{3t} \quad (9)$$

where  $r'_v = [r_1, r_2, r_3]$ ,  $\gamma_1 \equiv p_{11}r_1 + p_{12}r_2 + p_{13}r_3$ ,  $\gamma_2 \equiv p_{21}r_1 + p_{22}r_2 + p_{23}r_3$ , and

$\gamma_3 \equiv p_{31}r_1 + p_{32}r_2 + p_{33}r_3$ . Since  $\pi_{1t} + \pi_{2t} + \pi_{3t} = 1$  by definition, we can eliminate  $\pi_{2t}$  from the right hand side of (9) to obtain

$$E_t r_{t+1} = (\gamma_1 - \gamma_2) \pi_{1t} + (\gamma_3 - \gamma_2) \pi_{3t} + \gamma_2 \quad (10)$$

Now, to isolate the expected real interest rate in the linearized Euler equation, partition (5) so that

$$\begin{aligned} E_t \left\{ \theta (Y_t - \bar{\beta} Y_{t+1}) + \gamma (\Delta Y_t - 2\bar{\beta} \Delta Y_{t+1} + \bar{\beta}^2 \Delta Y_{t+2}) + \xi (W_t - \bar{\beta} W_{t+1}) \right. \\ \left. + \delta \bar{\beta} (N_t - \alpha X_{t+1}) \right\} + \eta E_t r_{t+1} + c = 0. \end{aligned} \quad (11)$$

Then, substitute (10) into (11) to get

$$E_t \left\{ \theta (Y_t - \bar{\beta} Y_{t+1}) + \gamma (\Delta Y_t - 2\bar{\beta} \Delta Y_{t+1} + \bar{\beta}^2 \Delta Y_{t+2}) + \xi (W_t - \bar{\beta} W_{t+1}) \right. \\ \left. + \delta \bar{\beta} (N_t - \alpha X_{t+1}) \right\} + \eta (\gamma_1 - \gamma_2) \pi_{1t} + \eta (\gamma_3 - \gamma_2) \pi_{3t} + \eta \gamma_2 + c = 0. \quad (12)$$

We now derive the decision rule for optimal inventories that is implied by the firm's optimization problem. Assume now that sales and real input prices follow independent AR(1) processes and that the current information set of the firm includes lagged values of sales, and current and lagged values of input prices and the interest rate. Assume further that the firm carries out its production plans for time  $t$ , so that  $E_t Y_t = Y_t$ . Then the inventory accumulation equation, (3), implies that  $(N_t - E_t N_t) = -(X_t - E_t X_t)$ , which means in effect that inventories buffer sales shocks. Define  $u_t^x \equiv -(X_t - E_t X_t)$  as the sales forecast error. In the appendix, we show that the linearized Euler equation, (5), may be written as a fourth-order expectational difference equation. Denote  $\lambda_1$  and  $\lambda_2$  as the stable roots of the relevant characteristic equation. We then show in the appendix that the firm's actual inventory position is

$$N_t = \Gamma_0 + (\lambda_1 + \lambda_2) N_{t-1} - \lambda_1 \lambda_2 N_{t-2} + \Gamma_X X_{t-1} + \Gamma_W W_t + \Gamma_{\pi 1} \pi_{1t} + \Gamma_{\pi 3} \pi_{3t} + u_t^x. \quad (13)$$

where

$$\Gamma_X \begin{matrix} > \\ < \end{matrix} 0, \quad \Gamma_W < 0, \quad \Gamma_{\pi 1} > 0, \quad \Gamma_{\pi 3} < 0.$$

### III. MONETARY POLICY

## First-Stage VAR

We identify monetary policy shocks by using our data set to estimate the six-variable semi-structural VAR developed by Bernanke and Mihov (1998). The variables in the model are divided into two blocks of three variables each. The non-policy or “macroeconomic” block consists of real GDP, the GDP deflator, and a commodity price index, and is unrestricted. The policy block, which consists of total reserves, non-borrowed reserves, and the federal funds rate, is restricted using plausible assumptions about the market for bank reserves. These restrictions, together with the assumption that policy shocks only affect macroeconomic variables after a one-month lag, are sufficient to identify the unobserved structural monetary policy shocks and their dynamic effects on the macro economy.

The VAR model is

$$Y_t = \sum_{i=0}^k B_i Y_{t-i} + \sum_{i=0}^k C_i Z_{t-i} + A^Y v_t^Y \quad (14)$$

$$Z_t = \sum_{i=0}^k D_i Y_{t-i} + \sum_{i=0}^k G_i Z_{t-i} + A^Z v_t^Z \quad (15)$$

where  $Y$  denotes the vector of macroeconomic variables and  $Z$  denotes the vector of policy variables.  $B_i, C_i, A^Y, D_i, G_i$ , and  $A^Z$  are matrices,  $v_t^Y$  and  $v_t^Z$  are vectors of mutually uncorrelated structural shocks. The assumption that policy variables have no contemporaneous affect on macroeconomic variables<sup>2</sup> requires that  $C_0 = 0$ .

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<sup>2</sup> This assumption is plausible for the monthly data used in this paper, and for the monthly and bi-weekly data in Bernanke and Mihov (1998) but would be less plausible for lower frequency data.

Re-write equation (15) so that only lagged values of the policy variables appear on the right-hand-side. The result is

$$Z_t = (I - G_0)^{-1} \sum_{i=0}^k D_i Y_{t-i} + (I - G_0)^{-1} \sum_{i=1}^k G_i Z_{t-i} + u_t \quad (16)$$

where,

$$u_t = (I - G_0)^{-1} A^Z v_t. \quad (17)$$

Note that  $u_t$  is the portion of the residuals from the policy-block VAR that is orthogonal to the residuals from the non-policy block. If the elements of  $(I - G_0)^{-1} A^Z$  are known then we can use (17) to recover the unobservable structural shocks,  $v_t$ , from the observable VAR residuals.

To obtain the restrictions necessary to identify  $(I - G_0)^{-1} A^Z$  Bernanke and Mihov (1998) consider the market for federal funds. Omitting time subscripts let  $u_{\text{FFR}}$  denote innovations in the federal funds rate, and let  $v^d$  denote exogenous shocks to the demand for total reserves. Innovations in total reserve demand,  $u_{\text{TR}}$ , are then given by

$$u_{\text{TR}} = -\alpha u_{\text{FFR}} + v^d \quad (18)$$

where  $\alpha \geq 0$ . Also, if  $u_{\text{DISC}}$  denotes innovations in the discount rate, then  $u_{\text{BR}}$ , which denotes innovations in the demand for borrowed reserves, is given by

$$u_{\text{BR}} = -\beta(u_{\text{FFR}} - u_{\text{DISC}}) + v^b \quad (19)$$

where  $v^b$  denotes exogenous shocks to the demand for borrowed reserves and where  $\beta \geq 0$ . Innovations in the demand for non-borrowed reserves,  $u_{\text{NBR}}^D$ , are by definition

$$u_{\text{NBR}}^D = u_{\text{TR}} - u_{\text{BR}}. \quad (20)$$

Use (18) and (19) to substitute for the terms on the right-hand side of (20), assume that  $u_{\text{DISC}} = 0$ , and rearrange the result to obtain

$$u_{\text{FFR}} = \left( \frac{-1}{\alpha + \beta} \right) u_{\text{NBR}}^{\text{D}} + \left( \frac{v^d}{\alpha + \beta} \right) - \left( \frac{v^b}{\alpha + \beta} \right). \quad (21)$$

Innovations in the supply of non-borrowed reserves,  $u_{\text{NBR}}^{\text{S}}$ , are governed by Federal Reserve policy. Let

$$u_{\text{NBR}}^{\text{S}} = \phi^d v^d + \phi^b v^b + v^s. \quad (22)$$

Here  $v^s$  is an exogenous shock to the supply of non-borrowed reserves. The policy parameters,  $\phi^d$  and  $\phi^b$ , describe how the Fed will react to shocks to the demand for total reserves and borrowed reserves, respectively. Consider two examples. If the Fed is targeting non-borrowed reserves, it will set  $\phi^d = \phi^b = 0$  and, in so far as it is possible, hold the supply of non-borrowed reserves constant. If instead the Fed is targeting the federal funds rate, it will set  $\phi^d = 1$  and  $\phi^b = -1$ , and fully accommodate shocks to reserve demand. A positive shock to the demand for total reserves will be accommodated by an increase in the supply of non-borrowed reserves. A positive shock to the demand for borrowed reserves will be offset by a decline in the supply of non-borrowed reserves. Since  $u_{\text{NBR}}^{\text{D}} = u_{\text{NBR}}^{\text{S}}$  in equilibrium we can substitute the right-hand side of (22) for  $u_{\text{NBR}}^{\text{D}}$  in (21) to obtain

$$u_{\text{FFR}} = \left( \frac{1 - \phi^d}{\alpha + \beta} \right) v^d + \left( \frac{-1}{\alpha + \beta} \right) v^s - \left( \frac{1 + \phi^b}{\alpha + \beta} \right) v^b. \quad (23)$$

Combine equations (18), (22), and (23) to give equation (17) with

$$u'_t \equiv [u_{\text{TR}} \quad u_{\text{NBR}} \quad u_{\text{FFR}}], \quad v'_t \equiv [v^d \quad v^s \quad v^b], \text{ and}$$

$$(I - G_0)^{-1} A^Z = \begin{bmatrix} \left(\frac{-\alpha}{\alpha + \beta}\right)(1 - \phi^d) + 1 & \left(\frac{\alpha}{\alpha + \beta}\right) & \left(\frac{\alpha}{\alpha + \beta}\right)(1 + \phi^b) \\ \phi^d & 1 & \phi^b \\ \left(\frac{1 - \phi^d}{\alpha + \beta}\right) & \left(\frac{-1}{\alpha + \beta}\right) & -\left(\frac{1 + \phi^b}{\alpha + \beta}\right) \end{bmatrix}$$

Let  $\hat{\Omega}_T$  denote the estimated variance-covariance matrix of the policy-block residuals. That is,

$$\hat{\Omega}_T = \left(\frac{1}{T - k}\right) \sum_{t=1}^T \hat{u}_t \hat{u}_t'$$

where  $\hat{u}_t$  is the vector of (orthogonalized) policy-block residuals obtained by estimating the VAR in equations (14) and (15).<sup>3</sup> Since  $\hat{\Omega}_T$  is a (3x3) symmetric matrix it has six unique elements. Next, note from (17) that

$$E(uu') = \left[(I - G_0)^{-1} A^Z\right] E(vv') \left[(I - G_0)^{-1} A^Z\right]'$$

Since the elements of  $v_t$  are i.i.d. by assumption, we can also write

$$E(vv') = \begin{bmatrix} \sigma_d^2 & 0 & 0 \\ 0 & \sigma_s^2 & 0 \\ 0 & 0 & \sigma_b^2 \end{bmatrix}.$$

The matrix  $E(uu')$  is, of course, also (3x3) and symmetric. Equating  $E(uu')$  to  $\hat{\Omega}_T$  therefore places six restrictions on the seven unknown structural parameters:  $\alpha, \beta, \phi^d, \phi^b, \sigma_s^2, \sigma_d^2$ , and  $\sigma_b^2$ . At least one more restriction is needed to identify these parameters and, hence, the elements of  $(I - G_0)^{-1} A^Z$ .

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<sup>3</sup> Since we use a constant term plus 12 lags of the six variables,  $k = 73$ .

Bernanke and Mihov (1998) examine five alternative sets of identifying restrictions. Four of these sets impose two additional restrictions so that the model is over identified. For example, as explained above, the set of restrictions consistent with targeting the federal funds rate is  $\phi^d=1$  and  $\phi^b=-1$ . Bernanke and Mihov call their fifth set the “just identified” model as it imposes the single additional restriction that  $\alpha=0$ . This restriction is motivated by Strongin’s (1995) argument that the demand for total reserves is inelastic in the short run. Impulse-response functions show that a monetary policy shock has qualitatively similar effects under all five sets of restrictions. We therefore take the simplest approach to identification, the just identified model. We set  $\alpha=0$  and solve  $E(uu')=\hat{\Omega}_T$  for the remaining six structural parameters.

To estimate the VAR we use monthly data from December 1961 through February 1999. In the macroeconomic block we use the producer price index for crude materials as our commodity price index and obtain monthly observations of real GDP and the GDP deflator using the state-space interpolation procedure of Bernanke, Gertler, and Watson (1997).<sup>4</sup> In the policy block we render total reserves and non-borrowed reserves stationary by measuring each as a ratio to a 36-month moving average of total reserves.<sup>5</sup>

We report the results of our estimation in Table 1. Not surprisingly our estimates are similar to Bernanke and Mihov’s. The estimate of  $\beta$  is positive and significant indicating that an increase in the federal funds rate relative to the discount rate leads to an increase in the demand for borrowed reserves. The estimate of  $\phi^d$  is positive and the estimate of  $\phi^b$  is negative, though both are less than one in absolute value. This suggests

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<sup>4</sup> The interpolation procedure is described in the appendix (forthcoming).

<sup>5</sup> This follows Bernanke and Mihov (1998).

that the Fed dampens fluctuations in the federal funds rate by partially accommodating shocks to the demand for reserves.

Table 1: Parameter estimates for the just identified model (assumes  $\alpha = 0$ ).

$\beta$	$\phi^d$	$\phi^b$	$\sigma_d$	$\sigma_b$	$\sigma_s$
0.029 (0.009)	0.762 (0.063)	-0.350 (0.267)	0.009	0.014	0.011

Estimates for the sample 1961:12 through 1999:2. Standard errors in parentheses.

Having identified the structural parameters that characterize the money market it is then possible to identify the monetary policy shocks by inverting equation (17) to obtain

$$\begin{bmatrix} v^d \\ v^s \\ v^b \end{bmatrix} = \left[ (I - G_0)^{-1} A^z \right]^{-1} \begin{bmatrix} u_{\text{TR}} \\ u_{\text{NBR}} \\ u_{\text{FFR}} \end{bmatrix} \quad (24)$$

The middle row of this equation is

$$v^s = -(\phi^d + \phi^b)u^{\text{TR}} + (1 + \phi^b)u^{\text{NBR}} - (\alpha\phi^d - \beta\phi^b)u^{\text{FFR}} \quad (25)$$

Inserting the policy-block residuals for  $u^{\text{TR}}$ ,  $u^{\text{NBR}}$ , and  $u^{\text{FFR}}$  on the right-hand side of (25)

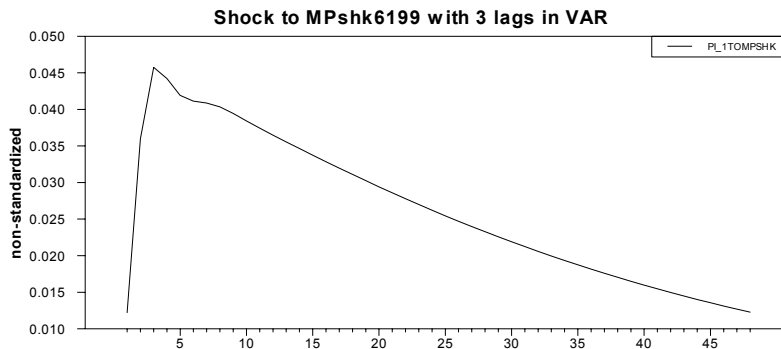
yields the time series of monetary policy shocks,  $\{v_t^s\}_{t=1}^T$ .

### The Link between Monetary Policy and the Probabilities

We use a straightforward procedure to estimate the effect of monetary policy shocks on the probabilities: we estimate a three-variable vector autoregression, with monetary policy shocks,  $\pi_1$ , and  $\pi_3$ , with three lags of each variable. The graph below shows the impulse response function of  $\pi_1$  to a one-standard-deviation easing of



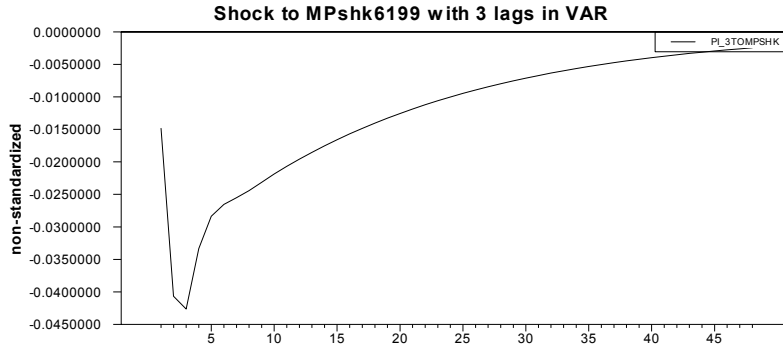
monetary policy. As the impulse response function shows, easing monetary policy increases the probability of the low interest rate state.



The effect of monetary policy on  $\pi_1$  is hump-shaped and peaks about two months after the shock. At the peak, a one-standard-deviation easing of monetary policy increases the probability of being in the low interest rate régime by about 0.045. To put this in perspective, the ergodic probability of being in the low interest rate régime is about 0.25. The effect of monetary policy on  $\pi_1$  is quite persistent, with half the peak effect still present two years after the shock.

Results are similar when we look at the effect of monetary policy easing on  $\pi_3$ . Loosening monetary policy reduces the probability of the high interest rate régime. The impulse response function is again hump-shaped, with the maximum response occurring two months after the shock. The maximum response is quantitatively similar to that of  $\pi_1$  (though, of course, in the opposite direction), as shown in the graph below. The ergodic probability of the high interest state is about 0.12. The 0.042 decrease in the probability of the high interest state caused by monetary policy easing therefore represents more than a 35% decrease relative to the ergotic probability. As in the case of the probability of low interest rate state, the effect of a monetary policy shock is quite

persistent. The results are qualitatively and quantitatively robust to variation in the number of lags in the vector autoregression.



#### IV. CALIBRATION

The coefficients in the decision rule ( $\lambda_1, \lambda_2, \Gamma_X, \Gamma_W, \Gamma_{\pi_1}$ , and  $\Gamma_{\pi_3}$ ) are known functions of the model's structural parameters. We therefore begin by using our previous empirical results to establish plausible values for those structural parameters.

We obtain the parameters of the stochastic process for the real interest rate, the elements of  $P$  and  $r_v$ , from our estimation of the three-state Markov-switching model.

Those estimates are

$$P = \begin{bmatrix} p_{11} = 0.98 & p_{21} = 0.01 & p_{31} = 0.00 \\ p_{12} = 0.02 & p_{22} = 0.98 & p_{32} = 0.04 \\ p_{13} = 0.00 & p_{23} = 0.01 & p_{33} = 0.96 \end{bmatrix}$$

and

$$r_v' \equiv \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} -1.71 \\ 1.61 \\ 5.15 \end{bmatrix}.$$

Together these estimates imply that the unconditional mean of the monthly real interest rate is  $\bar{r}=0.001$ , which gives  $\bar{\beta} = 0.999$ .

Next we set baseline values for  $\delta, \alpha, \gamma, \theta, \xi$  and  $\eta$ , the structural parameters from the cost function. We set  $\alpha = 1$  implying that the target level of inventories equals one month's sales. From the Euler equation, it follows that the parameters  $\delta, \alpha, \theta, \xi$  and  $\eta$  are uniquely identified only up to a multiplicative constant. We therefore adopt a widely used normalization and set  $\delta = 1$ . We then use our estimates of the cointegrating relationship among  $N_t$ ,  $X_t$ ,  $W_t$ ,  $\pi_{1t}$ , and  $\pi_{3t}$  to determine the values of  $\theta$ ,  $\xi$ , and  $\eta$ .

We show in the appendix that the linearized Euler equation can be re-written as

$$\begin{aligned}
 E_t \{ & \gamma (\Delta Y_t - 2\bar{\beta}\Delta Y_{t+1} + \bar{\beta}^2\Delta Y_{t+2}) - \bar{\beta}\theta(\Delta N_{t+1} + \Delta X_{t+1}) + \theta\Delta N_t - \bar{\beta}\delta\alpha\Delta X_{t+1} - \bar{\beta}\xi\Delta W_{t+1} \\
 & + \bar{\beta}\delta \left[ N_t - \left( \alpha - \frac{\theta(1-\bar{\beta})}{\bar{\beta}\delta} \right) X_t \right] + (1-\bar{\beta})\xi W_t \} \\
 & + \eta(\gamma_1 - \gamma_2)\pi_{1t} + \eta(\gamma_3 - \gamma_2)\pi_{3t} + \eta\gamma_2 + c = 0
 \end{aligned} \tag{26}$$

Suppose now that  $X_t$ ,  $W_t$ ,  $\pi_{1t}$  and  $\pi_{3t}$  are  $I(1)$ .<sup>6</sup> This implies that  $N_t$  will be  $I(1)$  and inventories, sales, the cost shock, and the probabilities will be cointegrated with cointegrating vector

$$\left[ 1, -\left( \alpha - \frac{\theta(1-\bar{\beta})}{\bar{\beta}\delta} \right), \frac{\xi(1-\bar{\beta})}{\bar{\beta}\delta}, \frac{\eta(\gamma_1 - \gamma_2)}{\bar{\beta}\delta}, \frac{\eta(\gamma_3 - \gamma_2)}{\bar{\beta}\delta} \right] \tag{27}$$

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<sup>6</sup> Since  $\pi_{1t}$  and  $\pi_{3t}$  have a restricted range, one might wonder whether it is better to model them as  $I(0)$  or  $I(1)$ . We note two points. First, in careful applied econometric research, variables with restricted ranges, such as the nominal interest rate, are modeled as  $I(1)$  variables when they are highly persistent. (See, e.g., Stock and Watson (1993) and Caballero (1994).) Second, unit root tests indicate that  $\pi_{1t}$  and  $\pi_{3t}$  are  $I(1)$ .

When the cointegrating vector is expressed in the form of a regression,  $X_t$ ,  $W_t$ ,  $\pi_{1t}$  and  $\pi_{3t}$  will be on the right hand side of the equation, so their coefficients will have signs opposite to those shown in the cointegrating vector above.

In Maccini, Moore, and Schaller (2004) we estimate this cointegrating regression. Equating the estimated coefficients on  $X_t$  and  $W_t$  from that regression to the corresponding composite parameters in the vector above we can, given the imputed values of  $\delta, \alpha$ , and  $\bar{\beta}$ , solve the resulting two equations for  $\theta$  and  $\xi$ . The parameters  $\bar{\beta}, \gamma_1, \gamma_2$ , and  $\gamma_3$  are determined from the estimation of the Markov-switching model of interest rates and  $\delta$  is set as a normalization. Equating the estimated coefficients on  $\pi_{1t}$  and  $\pi_{3t}$  from the cointegrating regression to the corresponding composite parameters in the vector above will therefore yield two separate restrictions on  $\eta$ . Since, in the cointegrating regression, the coefficient on  $\pi_{3t}$  is more precisely estimated<sup>7</sup> we equate that coefficient to  $\frac{\eta(\gamma_3 - \gamma_2)}{\bar{\beta}\delta}$  and solve the result for  $\eta$ . Since  $\gamma$  cannot be determined from the cointegrating vector, in the baseline simulations we assume that there are no adjustment costs and set  $\gamma = 0$ . The baseline parameter values that result from this exercise are reported in Table 2, below.

We then use the expressions, derived in the appendix, to determine the values of  $\lambda_1$ ,  $\lambda_2$ ,  $\Gamma_X$ ,  $\Gamma_W$ ,  $\Gamma_{\pi_1}$  and  $\Gamma_{\pi_3}$  from the structural parameters. To set  $\Gamma_0$ , note from the decision rule that the steady state level of inventories is given by

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<sup>7</sup> In our estimation of the cointegrating vector the coefficient on  $\pi_{3t}$  has a t statistic of  $-6.36$  and the coefficient on  $\pi_{1t}$  has a t statistic of  $1.73$ .

$$N_s = (\Gamma_0 + \Gamma_X X_s + \Gamma_W W_s + \Gamma_{\pi_1} \pi_{1s} + \Gamma_{\pi_3} \pi_{3s}) / [1 - (\lambda_1 + \lambda_2) + \lambda_1 \lambda_2] \quad (28)$$

where a subscript “s” denotes the steady-state value. Having determined  $\lambda_1$ ,  $\lambda_2$ ,  $\Gamma_X$ ,  $\Gamma_W$ ,  $\Gamma_{\pi_1}$  and  $\Gamma_{\pi_3}$ , we find  $\Gamma_0$  by substituting the sample mean values of  $N$ ,  $X$ , and  $W$  together with the ergodic probabilities into equation (28) and solve the result for  $\Gamma_0$ . The decision rule coefficients for the baseline parameter setting are reported in Table 4. The sample mean values of  $N$ ,  $X$ , and  $W$ , which are necessary to determine  $\Gamma_0$ , are reported in Table 3.

In section III above we identified the dynamic response of  $\pi_{1t}$  and  $\pi_{3t}$  to a monetary policy shock. We can now simulate how inventories respond to a monetary policy shock by using those values for  $\pi_{1t}$  and  $\pi_{3t}$  in the decision rule. Specifically, we initialize  $\pi_{1t}$  and  $\pi_{3t}$  at their ergodic values and then allow them to change as shown in the impulse response functions in section III. Since our purpose is to isolate the pure interest rate effect of monetary policy, we do not allow  $X_t$  and  $W_t$  to vary systematically in response to the policy shock. Instead, we model each of these variables as an exogenous random walk with zero drift and initialize each at their sample mean value. The standard deviations of the innovations in  $X_t$  and  $W_t$  are set equal to the standard deviations of  $\Delta X_t$  and  $\Delta W_t$ , respectively, as obtained from our sample. Given  $X_t$  and, from the decision rule,  $N_t$  we determine  $Y_t$  from the inventory accumulation identity,  $Y_t = X_t + N_t - N_{t-1}$ . In the simulations that follow we initialize the level of inventories at its steady state value, which is also its sample mean by construction.

Table 2: Parameters of the Cost Function, Baseline Values

$\delta$	$\alpha$	$\theta$	$\xi$	$\eta$	$\gamma$
1	1	765.09	302,001	$1.92 \times 10^6$	0

Table 3: Sample Mean Values

$\bar{N}$	$\bar{X}$	$\bar{W}$	$\bar{Y}$
53,312.21	107,276.34	110.24	107,393.97

Table 4: Coefficients in the Decision Rule

$\lambda_1$	$\lambda_2$	$\Gamma_X$	$\Gamma_W$	$\Gamma_{\pi 1}$	$\Gamma_{\pi 3}$	$\Gamma_0$
0.965	0	0.008	-10.55	7287.52	-5307.89	985.01

## V. SIMULATIONS OF THE EFFECTS OF A MONETARY POLICY SHOCK

Figure 1 shows the impulse response function of the stock of inventories to a one standard deviation stimulative monetary policy shock. It is important to understand that the impulse response function in Figure 1 shows the effect of monetary policy only through the narrow interest rate channel. In the simulations, we do not allow monetary policy to affect inventories through sales or costs, only directly through the interest rate. To the best of our knowledge, this is the first attempt to isolate the pure interest rate effect of monetary policy on inventories.

The response of the stock of inventories is hump-shaped, with the effect peaking about three years after the shock. Some intuition may be helpful. In a production

smoothing inventory model, the cost function is convex. This means that it is cheaper to produce at an intermediate level of output, rather than sometimes producing at low output and sometimes producing at high output. This makes the production level sticky: firms would prefer to produce at their usual (intermediate) output level, even when hit by transitory shocks. The more convex the cost function, the stickier output is.

In general, the previous literature has treated the interest rate as constant, so most of the standard intuition has been built up around sales and cost shocks. Interest rate shocks work a bit differently, but much of the standard intuition still applies. An interest rate shock changes the desired long-run inventory level. However, changing output (away from the usual level) is expensive because of the convexity of the cost function. If firms recognize that the interest rate shock is purely transitory, they will adjust output (and therefore the stock of inventories) little, if at all, since there are no sales or cost shocks. (We allow for sales and cost shocks in the simulations for Figure 1, but, over many iterations, positive and negative sales shocks cancel out, providing us with a picture of the effect of a pure monetary policy shock.) Because firms are reluctant to adjust output, the change in the stock of inventories is spread out over many months.

The shock to monetary policy illustrated in Figure 1 does not lead to a regime shift in the interest rate, but it does have some persistence. The result is that the stock of inventories gradually rises and then, as the effects of the monetary policy shock eventually die away, the stock of inventories gradually declines.

At the peak, inventories are higher by an amount that corresponds to a little more than 0.3% of output. A one standard deviation stimulative monetary policy shock reduces the interest rate by 0.37% (37 basis points).

Figure 2 shows the impulse response function of inventory investment to a monetary policy shock of the same magnitude. Inventory investment is of particular interest for monetary economics because it is fluctuations in inventory investment that contribute to fluctuations in aggregate demand. The response is again hump-shaped, with the effect peaking two months after the shock. The response of inventory investment is much quicker than the response of fixed investment. The stylized fact is that the peak response of fixed investment occurs more than a year after a shock.

The effect of monetary policy on inventory investment is persistent. About half the peak effect is still present nearly two years after the shock. It takes about three years for the effect to dissipate.

As noted above, it is the convexity of the cost function that spreads out the response of inventories to shocks. The convexity of the cost function is captured in the parameter  $\theta$ . In Figure 3, we illustrate the effect of changes in  $\theta$  on the impulse response function for the stock of inventories. If we set  $\theta$  equal to half the value implied by the cointegrating regression estimates, the peak effect on inventories is smaller – a little more than 0.2% of output, compared with somewhat more than 0.3% of output for the baseline value of  $\theta$ . The peak response occurs sooner with a lower value of  $\theta$  -- less than two years after the shock, compared with about three years after the shock for the baseline value of  $\theta$ .

We also illustrate the effect of a higher value of  $\theta$  in Figure 3. If we set  $\theta$  equal to twice the value implied by the cointegrating regression estimates, the peak response of inventories is nearly 0.6% of output. The reaction to the monetary policy shock is



considerably more sluggish with a higher value of  $\theta$ , with the peak response occurring more than four years after the shock.

Figure 4 illustrates the effect of different values of  $\theta$  on inventory investment. Values of  $\theta$  that are twice as large – or half as large – as the value of  $\theta$  implied by the cointegrating regression estimates have almost no effect on the peak response of inventory investment. This variation in the convexity of the cost function also has no effect on the timing of the peak response of inventory investment, which still occurs two months after the shock. Variation in  $\theta$  does have an effect on the persistence of the effects of the shock. With  $\theta$  at half its baseline level, the effects of the monetary policy shock die out about a year and a half after the shock. With  $\theta$  at twice its baseline level, the effects of the monetary policy shock are still present four years after the shock.

Figure 5 provides a sensitivity analysis with respect to another parameter,  $\xi$ , the coefficient on input price cost shocks in the cost function. As Figure 5 illustrates, variations in  $\xi$  have little effect on either the magnitude or the timing of the peak response of the stock of inventories.

In the linearized Euler equation [equation (5)], the parameter on the interest rate is  $\eta$ . The decision rule parameters on  $\pi_1$  and  $\pi_3$  ( $\Gamma_{\pi_1}$  and  $\Gamma_{\pi_3}$ ) are functions of  $\eta$ . As noted above, we calibrate  $\eta$  using the coefficient on  $\pi_3$  in the cointegration regression, which is precisely estimated (with a t-statistic of 6.4). Nonetheless,  $\eta$  is an important parameter, and it may be of interest to explore the effect of variation in  $\eta$  on the impulse response function. Figure 6 illustrates the impulse response function with  $\eta$  set equal to

twice its baseline value and half its baseline value.<sup>8</sup> Variation in  $\eta$  makes a difference to the magnitude of the stock of inventories at the peak response, which is about 15% lower if we set  $\eta$  equal to half its baseline value and about 35% higher if we set  $\eta$  equal to twice its baseline value. Variation in  $\eta$  has no effect on the timing of the peak response of the stock of inventories, nor, more generally, on the shape of the impulse response function.

Figure 7 illustrates the sensitivity of inventory investment to variation in  $\eta$ . As for the stock of inventories, the only effect is on the magnitude of the peak response. The effects on the peak response are comparable to those for the stock of inventories.

In our baseline parameter settings, we include no adjustment costs on output. In other words, we set  $\gamma=0$ . The inventory literature has not reached a consensus on the existence of costs of adjusting output, let alone the precise value of  $\gamma$ . This is part of the reason we set  $\gamma=0$ . A second reason is that we cannot calibrate  $\gamma$  using the cointegrating regression. The intuition for this is simple. The cointegrating regression captures long-run relationships. Adjustment costs on output affect high-frequency dynamics but not the long-run relationship. Figure 8 explores the sensitivity of inventory investment to several possible values of  $\gamma$ . Unlike any of the other parameters we have considered so far, changes in  $\gamma$  affect the timing of the peak response of inventory investment. As Figure 8 shows, increasing  $\gamma$  from 0 to a value equal to twice the baseline value of  $\theta$  delays the peak response from two months after the shock to six

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<sup>8</sup> The impulse response function “confidence interval” (constructed by considering values of  $\eta$  calculated using the cointegrating regression coefficient on  $\pi_3$  plus two standard errors and minus two standard errors, respectively) lies well inside the impulse response functions based on setting  $\eta$  equal to twice its baseline value and half its baseline value.

months after the shock. Increasing  $\gamma$  also increases the peak response of inventory investment. Moving from  $\gamma=0$  to a  $\gamma$  of twice the baseline value of  $\theta$  increases the peak response by about 40%. An increase in  $\gamma$  also increases the persistence in the effect of a monetary policy shock on inventory investment, so that the effect no longer dies out after about three years, but instead lasts more than four years.

## VI. THE VOLCKER DISINFLATION

From January 1979 through the end of 1984, the U.S. economy experienced a recession, a brief recovery, the most serious recession since the Great Depression, and a period of rapid economic growth, so this is one of the more interesting episodes in post-war U.S. macroeconomic history. We can use our model of inventories with learning and regime switches, calibrated using the cointegrating regression for inventories, to show the pure interest rate effect and the broad interest rate effects through sales and through costs during this period.

Our methodology is straightforward. The model implies the decision rule for inventories, equation (13). We can use the Markov switching regression for the real interest rate to calculate  $\pi_1$  and  $\pi_3$  as they would have been calculated by agents in the model, who can observe interest rates but are not able to directly observe the interest rate regime. We can then substitute the actual path of  $\pi_1$  and  $\pi_3$ , as calculated using the Markov switching regression, into the decision rule for inventories. Holding sales and costs constant, this allows us to trace out the pure interest rate effect. We can then add the broad interest rate effect through sales by substituting the actual path of sales into the

decision rule. Finally, we can add the broad interest rate effect through costs by substituting the actual path of real input prices into the decision rule.<sup>9</sup>

Figure 9 shows the path of  $\pi_3$  over the Volcker disinflation. In January 1979 agents perceived a zero probability of currently being in the high interest rate regime. As monetary policy tightened, the probability briefly rose and fell and then rose again to a plateau (close to 1) that persisted for the remainder of the Volcker disinflation.

The line marked with Xs in Figure 10 shows the effect on the stock of inventories. The brief spike in  $\pi_3$  in 1979 leads to a small (but persistent) decline in inventories. When  $\pi_3$  rises to 1 in 1980, this initiates a long, gradual decline in inventories. This is the pure interest rate effect.

The line marked with circles adds the broad interest rate effect through sales. To better understand the effect through sales, look at Figure 11, which illustrates the path of detrended sales. As monetary policy tightens in 1979, sales fall. The effect of the drop in sales is initially buffered by inventories, which rise as sales fall. When sales again begin to rise following the first, brief recession, the effect on output is again buffered by inventories, which fall during the brief expansion between the 1980 and 1981-82 recessions. When the second phase of monetary tightening begins, sales again fall, this time remaining below trend for years during the unusually deep and lengthy 1981-82 recession. Because of their buffering role, inventories again rise in the initial stage of the recession, but to a lower level than in mid-1980 because the economy is now in the high interest rate regime. As the recession continues, the stock of inventories begins to

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<sup>9</sup> Of course, this is a simplification. The actual path of sales and costs was surely influenced by other shocks. But the Volcker disinflation is a period when many economists believe that monetary policy shocks played a relatively large role.

gradually fall. Although there are substantial fluctuations in sales, the level of sales is roughly flat over the period from mid-1981 to mid-1983. The stock of inventories is driven gradually downward over this period by the pure interest rate effect. The stock of inventories falls dramatically in the initial stages of the recovery as inventories again buffer shocks to sales. Because of the strong persistence in the inventory process – and because the interest rate is now in the high interest rate regime – inventories remain low during 1984.

The line marked with small rectangles in Figure 10 adds in the broad interest rate effect through costs. The most striking fact is how close this line is to the line that holds real input prices constant. Much of the inventory literature has failed to find a significant effect of observable cost shocks on inventories. When we estimate the cointegrating regression implied by our model, we obtain a t-statistic of 4.3 on  $W$  (real input prices). Even though we are able to find a highly significant long-run effect of costs on inventories, the estimated parameters imply that cost shocks have a small effect on the path of inventories, relative to the effect of sales.<sup>10</sup>

The magnitude of the pure interest rate effect is substantial during the Volcker period. About one-third of the deviation of inventories from their initial level can be attributed to the pure interest rate effect. This can be seen in Figure 10 by comparing the line (with Xs) showing the pure interest rate effect in early 1985 with the line (with small rectangles) showing the pure interest rate effect plus the broad interest rate effect through both sales and costs.

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<sup>10</sup> Interestingly, 1979 and 1980, years that saw large oil price shocks, are the period in which cost shocks have the largest effect on inventories, as shown by the distance between the path of inventories that includes the effect through costs and the path of inventories that includes the effect through sales but not through costs.

Figure 12 shows inventory investment during the Volcker period. The line marked with asterisks shows the pure interest rate effect. The line marked with Os includes the broad interest rate effect through both sales and costs. The pure interest rate effect on inventory investment is small. It is completely dominated by the effect of sales shocks, which clearly account for almost all of the high-frequency variation in inventory investment. Figure 12 is a dramatic illustration of one of the key points in Maccini, Moore, and Schaller (2004). It is virtually impossible to find the effect of the interest rate on inventories using techniques that emphasize high-frequency movements in the data. Figure 12 makes it clear that this is primarily because of sales shocks and the short-run buffering role of inventories.

## VII. CONCLUSION

Recent work, based on a theoretical model that incorporates learning and regime shifts in the real interest rate, has allowed us for the first time to identify and precisely estimate the relationship between inventories and the interest rate. The theoretical model implies a cointegrating relationship between inventories, sales, costs, and  $\pi_1$  and  $\pi_3$  (the probabilities of being in a given interest rate regime). In this paper, we simulate the pure interest rate effect of monetary policy on inventories using the theoretical model and parameter estimates from the cointegrating regression derived from the model.

The response of the stock of inventories to a monetary policy shock is hump-shaped, with the effect peaking about three years after the shock. At the peak, the stock of inventories is higher by an amount that corresponds to about 0.3% of output in

response to a one-standard-deviation stimulative shock to monetary policy (i.e., a 37 basis point decrease in the interest rate).

The response of inventory investment, a variable that is more directly relevant for business cycles, is also hump-shaped. The peak response of inventory investment to a one-standard-deviation monetary policy shock is about 0.025% of output. The response of inventory investment is quite persistent, with the effects still visible more than two years after the shock.

Variation in the convexity of the cost function (away from the parameter estimated from the cointegrating regression) has a substantial effect on both the magnitude and the timing of the peak response of the stock of inventories. Intuitively, this is because a more convex cost function makes output (and inventories) more sticky with respect to shocks, a point that is discussed in more detail in Section V. In contrast, plausible variation in the convexity of the cost function has little effect on either the magnitude or the timing of the peak response of inventory investment.

The sensitivity of the cost function to input prices is precisely measured in the cointegrating regression. Plausible variation in the relevant parameter has little effect on either the magnitude or timing of the peak response of the stock of inventories.

In the Euler equation for inventories [equation (5)], the parameter on the interest rate is  $\eta$ . The coefficient in the cointegrating regression that we use to calibrate  $\eta$  is precisely measured (t-statistic of 6.4), but, because  $\eta$  is an important parameter, we explore the effect of variation in  $\eta$ . Doubling  $\eta$  increases the peak response of the stock of inventories by about 35%; halving  $\eta$  decreases the peak response by about 15%. Variation in  $\eta$  has a comparable effect on the peak response of inventory investment but

no effect on the timing of the peak response nor on the shape of the impulse response function for either the stock of inventories or inventory investment.

The Volcker disinflation is one of the major -- and most interesting -- episodes in recent US economic history. We use our model to illustrate the pure interest rate effect and the broad interest rate effects through sales and through costs during the 1979-84 period. The pure interest rate effect accounts for about one-third of the low-frequency movement in the stock of inventories during the Volcker disinflation. The broad interest rate effect through costs plays a very modest role. The high-frequency movements in inventories are dominated by sales shocks, because inventories act as a buffer for sales.

The overall conclusion that emerges from the results is that monetary policy does, indeed, influence inventories through the pure interest rate effect, but the magnitude of the effect is fairly small in response to a typical monetary policy shock, and it takes a long time for the effect to be fully realized. The pure interest rate effect on inventory investment is very small and completely dominated by sales shocks.

The results have important implications for our understanding of the monetary policy transmission mechanism. Inventory investment accounts for a large proportion of the decline in output during a typical recession, but the results suggest this does not come about because tighter monetary policy raises the opportunity cost of holding inventories and therefore leads to a drop in inventory investment. Instead, the results suggest that the main impact of monetary policy on inventories occurs indirectly, through changes in final demand. These changes in final demand may come through some mix of conventional channels, such as the effect of an increase in the interest rate on business fixed investment



or residential investment, and the broad credit channel (e.g., through reduced cash flow for finance constrained firms).

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## APPENDIX A

### Derivation of Equation (13)

Using (3), the linearized Euler equation in (5) can be written as the following fourth- order difference equation in  $N_t$ :

$$\begin{aligned} E_t \left\{ \theta \left[ N_t - N_{t-1} + X_t - \bar{\beta} (N_{t+1} - N_t + X_{t+1}) \right] + \gamma \left[ N_t - N_{t-1} - (N_{t-1} - N_{t-2}) + X_t - X_{t-1} \right. \right. \\ \left. \left. - 2\bar{\beta} (N_{t+1} - N_t - (N_t - N_{t-1}) + X_{t+1} - X_t) + \bar{\beta}^2 (N_{t+2} - N_{t+1} - (N_{t+1} - N_t) + X_{t+2} - X_{t+1}) \right] \right. \\ \left. + \xi (W_t - \bar{\beta} W_{t+1}) + \delta \bar{\beta} (N_t - \alpha X_{t+1}) + \eta r_{t+1} + c \right\} = 0. \end{aligned}$$

Rearranging we have

$$E_t [f(L)N_{t+2}] = E_t \Psi_t \tag{A.1}$$

where

$$\begin{aligned} f(L) \equiv 1 - \frac{1}{\gamma \bar{\beta}} \left[ \theta + 2(1 + \bar{\beta})\gamma \right] L + \frac{1}{\gamma \bar{\beta}^2} \left[ \theta(1 + \bar{\beta}) + \gamma(1 + 4\bar{\beta} + \bar{\beta}^2) + \delta \bar{\beta} \right] L^2 \\ - \frac{1}{\gamma \bar{\beta}^2} \left[ \theta + 2\gamma(1 + \bar{\beta}) \right] L^3 + \frac{1}{\bar{\beta}^2} L^4 \end{aligned}$$

and

$$\begin{aligned} \Psi_t = -X_{t+2} + \frac{1}{\gamma \bar{\beta}} \left[ \theta + \gamma(2 + \bar{\beta}) \right] X_{t+1} - \frac{1}{\gamma \bar{\beta}^2} \left[ \theta + \gamma(1 + 2\bar{\beta}) - \alpha \delta \bar{\beta} \right] X_t \\ + \frac{1}{\bar{\beta}^2} X_{t-1} - \frac{\xi}{\gamma \bar{\beta}^2} (W_t - \bar{\beta} W_{t+1}) - \frac{\eta}{\gamma \bar{\beta}^2} r_{t+1} - \frac{c}{\gamma \bar{\beta}^2}. \end{aligned}$$

Let  $\lambda_i$ ,  $i = 1, 2, 3, 4$ , denote the roots of the fourth-order polynomial on the left-hand side

of (A.1). Order these roots as  $|\lambda_1| < |\lambda_2| < |\lambda_3| < |\lambda_4|$ . It follows that

$$\lambda_4 = \frac{1}{\beta \lambda_1} \text{ and } \lambda_3 = \frac{1}{\beta \lambda_2}, \text{ with } |\lambda_1|, |\lambda_2| < \frac{1}{\beta}. \text{ Suppose further that } |\lambda_1|, |\lambda_2| < 1.$$

Solve the unstable roots forward to obtain

$$E_t N_t = (\lambda_1 + \lambda_2) N_{t-1} - \lambda_1 \lambda_2 N_{t-2} + \frac{\bar{\beta} \lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)} \sum_{j=0}^{\infty} \left[ (\bar{\beta} \lambda_1)^{j+1} - (\bar{\beta} \lambda_2)^{j+1} \right] E_t \Psi_{t+j}. \quad ()$$

where

$$\begin{aligned} \Psi_{t+j} = & -X_{t+2+j} + \frac{1}{\gamma \bar{\beta}} \left[ \theta + \gamma (2 + \bar{\beta}) \right] X_{t+1+j} - \frac{1}{\gamma \bar{\beta}^2} \left[ \theta + \gamma (1 + 2\bar{\beta}) - \alpha \delta \bar{\beta} \right] X_{t+j} \\ & + \frac{1}{\bar{\beta}^2} X_{t-1} - \frac{\xi}{\gamma \bar{\beta}^2} (W_{t+j} - \bar{\beta} W_{t+1+j}) - \frac{\eta}{\gamma \bar{\beta}^2} r_{t+1+j} - \frac{c}{\gamma \bar{\beta}^2} \end{aligned} \quad ()$$

Note that  $\lambda_1$  and  $\lambda_2$  are either real or complex conjugates, so that  $\lambda_1 + \lambda_2$  and  $\lambda_1 \lambda_2$  are real.

To resolve the forward sum on the right-hand side of () note that we assume that sales and input prices follow AR(1) processes:

$$X_t = \mu_x + \rho_x X_{t-1} + \varepsilon_{xt}, \text{ where } \varepsilon_{xt} \sim i.i.d.(0, \sigma_x^2)$$

$$W_t = \mu_w + \rho_w W_{t-1} + \varepsilon_{wt}, \text{ where } \varepsilon_{wt} \sim i.i.d.(0, \sigma_w^2)$$

where we allow for the special case of  $\rho_x = \rho_w = 1$ . Further, we assume that the real interest rate follows the Markov-switching process in (). Given these assumptions, we now proceed to derive the terms for each of the forcing variables:

1. Consider first the terms involving  $X$  on the right-hand side of (), which can be written as

$$\frac{\bar{\beta} \lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)} \left\{ \sum_{j=0}^{\infty} \left[ (\bar{\beta} \lambda_1)^{j+1} - (\bar{\beta} \lambda_2)^{j+1} \right] E_t \left( -X_{t+2+j} + a_1 X_{t+1+j} - a_0 X_{t+j} + \frac{1}{\bar{\beta}^2} X_{t-1+j} \right) \right\} \quad (\text{A.2})$$

where  $a_1 \equiv \frac{1}{\gamma\bar{\beta}} \left[ \theta + \gamma(2 + \bar{\beta}) \right]$  and  $a_0 \equiv \frac{1}{\gamma\bar{\beta}^2} \left[ \theta + \gamma(1 + 2\bar{\beta}) - \alpha\delta\bar{\beta} \right]$ .

Note that, for  $j = 0, 1, 2, \dots$ ,  $E_t X_{t+j} = \mu_x + \rho_x E_t X_{t-1+j}$

$E_t X_{t+j+1} = \mu_x(1 + \rho_x) + \rho_x^2 E_t X_{t-1+j}$ , and  $E_t X_{t+2+j} = \mu_x(1 + \rho_x + \rho_x^2) + \rho_x^3 E_t X_{t-1+j}$ .

It therefore follows that

$$E_t \left( -X_{t+2+j} + a_1 X_{t+1+j} - a_0 X_{t+j} + \frac{1}{\bar{\beta}^2} X_{t-1+j} \right) =$$

$$\left[ -(1 + \rho_x + \rho_x^2) + a_1(1 + \rho_x) - a_0 \right] \mu_x + \left( -\rho_x^3 + a_1 \rho_x^2 - a_0 \rho_x + \frac{1}{\bar{\beta}^2} \right) E_t X_{t-1+j}$$

and thus,

$$\sum_{j=0}^{\infty} (\bar{\beta}\lambda_i)^{j+1} E_t \left( -X_{t+2+j} + a_1 X_{t+1+j} - a_0 X_{t+j} + \frac{1}{\bar{\beta}^2} X_{t-1+j} \right) = \frac{\bar{\beta}\lambda_i \left[ -(1 + \rho_x + \rho_x^2) + a_1(1 + \rho_x) - a_0 \right]}{1 - \bar{\beta}\lambda_i} \mu_x$$

$$+ \bar{\beta}\lambda_i \left( -\rho_x^3 + a_1 \rho_x^2 - a_0 \rho_x + \frac{1}{\bar{\beta}^2} \right) \sum_{j=0}^{\infty} (\beta\lambda_i)^j E_t X_{t-1+j}. \quad (\text{A.3})$$

For the AR(1) process governing  $X_t$ , the forward sum in (A.?) is

$$\sum_{j=0}^{\infty} (\bar{\beta}\lambda_i)^j E_t X_{t-1+j} = \frac{\bar{\beta}\lambda_i \mu_x}{(1 - \bar{\beta}\lambda_i)(1 - \bar{\beta}\lambda_i \rho_x)} + \frac{1}{(1 - \bar{\beta}\lambda_i \rho_x)} X_{t-1}$$

This in (A.?) gives

$$\sum_{j=0}^{\infty} (\beta\lambda_i)^{j+1} E_t \left( -X_{t+2+j} + a_1 X_{t+1+j} - a_0 X_{t+j} + \frac{1}{\bar{\beta}^2} X_{t-1+j} \right) =$$

$$c(\rho_x, \lambda_i) \mu_x + \bar{\beta}\lambda_i \left( -\rho_x^3 + a_1 \rho_x^2 - a_0 \rho_x + \frac{1}{\bar{\beta}^2} \right) (1 - \bar{\beta}\lambda_i \rho_x)^{-1} X_{t-1} \quad (\text{A.4})$$

where

$$c(\rho_x, \lambda_i) \equiv \frac{\beta \lambda_i \left[ -(1 + \rho_x + \rho_x^2) + a_1(1 + \rho_x) - a_0 \right]}{1 - \beta \lambda_i} + \frac{(\bar{\beta} \lambda_i)^2 (-\rho_x^3 + a_1 \rho_x^2 - a_0 \rho_x + \bar{\beta}^{-2})}{(1 - \bar{\beta} \lambda_i)(1 - \bar{\beta} \lambda_i \rho_x)}.$$

Using (A.?) we can rewrite the term (A.?) as

$$\begin{aligned} \frac{\bar{\beta} \lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)} \left\{ \sum_{j=0}^{\infty} \left[ (\beta \lambda_1)^{j+1} - (\beta \lambda_2)^{j+1} \right] E_t \left( -X_{t+2+j} + a_1 X_{t+1+j} - a_0 X_{t+j} + \frac{1}{\bar{\beta}^2} X_{t-1+j} \right) \right\} \\ = c_X + \Gamma_X X_{t-1} \end{aligned} \quad (\text{A.5})$$

where  $c_X \equiv \frac{\bar{\beta} \lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)} [c(\rho_x, \lambda_1) - c(\rho_x, \lambda_2)] \mu_x$  and

$$\Gamma_X \equiv \bar{\beta}^2 \lambda_1 \lambda_2 \left( -\rho_x^3 + a_1 \rho_x^2 - a_0 \rho_x + \bar{\beta}^{-2} \right) \left[ \frac{1}{(1 - \bar{\beta} \lambda_1 \rho_x)(1 - \bar{\beta} \lambda_2 \rho_x)} \right].$$

2. Consider now the terms involving W on the right-hand side of (). Proceeding as with the terms in X, the terms involving W can be written as

$$\left( -\frac{\xi}{\gamma \bar{\beta}} \right) \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)} \left\{ \sum_{j=0}^{\infty} \left[ (\bar{\beta} \lambda_1)^{j+1} - (\bar{\beta} \lambda_2)^{j+1} \right] E_t (W_{t+j} - \bar{\beta} W_{t+1+j}) \right\} = c_W + \Gamma_W W_t \quad (\text{A.6})$$

where  $c_W \equiv \left( -\frac{\xi}{\gamma \bar{\beta}} \right) \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)} [c(\rho_w, \lambda_1) - c(\rho_w, \lambda_2)] \mu_w$ ,

$$c(\rho_w, \lambda_i) \equiv \left[ \frac{-\bar{\beta}^2 \lambda_i}{1 - \bar{\beta} \lambda_i} + \frac{(\bar{\beta} \lambda_i)^2 (1 - \bar{\beta} \rho_w)}{(1 - \bar{\beta} \lambda_i)(1 - \bar{\beta} \lambda_i \rho_w)} \right] \mu_w, \text{ and}$$

$$\Gamma_W \equiv \left( -\frac{\xi}{\gamma} \right) \lambda_1 \lambda_2 \left[ \frac{(1 - \bar{\beta} \rho_w)}{(1 - \bar{\beta} \lambda_1 \rho_w)(1 - \bar{\beta} \lambda_2 \rho_w)} \right]. \text{ Note that if } \lambda_1 \lambda_2 > 0 \text{ then } \Gamma_W < 0.$$

3. Consider next the terms involving expected future interest rates. Here we assume that the real interest rate follows the Markov-switching process in (). Consider the expectation of a firm that knows the structure and parameters of () but does not know the true state. Recall that we denote the firm's current probability assessment of the true state by  $\pi_t$ , where

$$\pi_t = \begin{bmatrix} \pi_{1t} \\ \pi_{2t} \\ \pi_{3t} \end{bmatrix} = \begin{bmatrix} \text{Prob}(S_t = 1 | \Omega_t) \\ \text{Prob}(S_t = 2 | \Omega_t) \\ \text{Prob}(S_t = 3 | \Omega_t) \end{bmatrix}$$

and let  $\pi_{jt+m|t} \equiv \text{Prob}(S_{t+m} = j | \Omega_t)$ . Note that, in general,  $\pi_{jt+m|t} = \sum_{i=1}^3 p_{ij} \pi_{it+m-1|t}$ . Thus,

$\pi_{t+m|t} = P^m \cdot \pi_t$ . It follows that  $E(r_{t+m} | \Omega_t) = r_v' [P^m \pi_t]$ , where  $r_v' \equiv [r_1 \ r_2 \ r_3]$ . We can

therefore write that, for  $i = 1, 2$ ,

$$\sum_{j=0}^{\infty} (\bar{\beta} \lambda_i)^{j+1} E_t r_{t+1+j} = \bar{\beta} \lambda_i \sum_{j=0}^{\infty} (\bar{\beta} \lambda_i)^{j+1} r_v' P^{j+1} \pi_t = \bar{\beta} \lambda_i r_v' \sum_{j=0}^{\infty} (\bar{\beta} \lambda_i P)^j P \pi_t = \bar{\beta} \lambda_i r_v' [I - \bar{\beta} \lambda_i P]^{-1} P \pi_t.$$

Using this result we have

$$\left( -\frac{\eta}{\gamma \bar{\beta}} \right) \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)} \left\{ \sum_{j=0}^{\infty} \left[ (\bar{\beta} \lambda_1)^{j+1} - (\bar{\beta} \lambda_2)^{j+1} \right] E_t r_{t+1+j} \right\} = \Gamma \pi_t \quad (\text{A.})$$

where

$$\Gamma_{(1 \times 3)} \equiv [\Gamma_1 \ \Gamma_2 \ \Gamma_3] = \left( \frac{-\eta}{\gamma \bar{\beta}} \right) \left( \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} \right) \bar{\beta} r_v' \left\{ \lambda_1 [I - \bar{\beta} \lambda_1 P]^{-1} - \lambda_2 [I - \bar{\beta} \lambda_2 P]^{-1} \right\} P.$$

Since  $\pi_{1t} + \pi_{2t} + \pi_{3t} = 1$  by definition, we can use  $\pi_{2t} = 1 - (\pi_{1t} + \pi_{3t})$  to eliminate  $\pi_{2t}$  from the right hand side of (A.). The resulting expression is,

$$\left(-\frac{\eta}{\gamma\bar{\beta}}\right)\frac{\lambda_1\lambda_2}{(\lambda_1-\lambda_2)}\left\{\sum_{j=0}^{\infty}\left[(\bar{\beta}\lambda_1)^{j+1}-(\bar{\beta}\lambda_2)^{j+1}\right]E_t r_{t+1+j}\right\}=\Gamma_2+\Gamma_{\pi 1}\pi_{1t}+\Gamma_{\pi 3}\pi_{3t} \quad (\text{A.7})$$

where we define  $\Gamma_{\pi 1}\equiv(\Gamma_1-\Gamma_2)$  and  $\Gamma_{\pi 3}\equiv(\Gamma_3-\Gamma_2)$ .

4. Finally, the constant term is

$$\left(\frac{-1}{\gamma\bar{\beta}^2}\right)\frac{\bar{\beta}\lambda_1\lambda_2}{(\lambda_1-\lambda_2)}\left\{\sum_{j=0}^{\infty}\left[(\bar{\beta}\lambda_1)^{j+1}-(\bar{\beta}\lambda_2)^{j+1}\right]c\right\}=\frac{-\lambda_1\lambda_2}{\gamma(1-\bar{\beta}\lambda_1)(1-\bar{\beta}\lambda_2)}c. \quad (\text{A.8})$$

5. Now, assume that the firm carries out its production plans for time  $t$ , so that  $E_t Y_t = Y_t$ .

Then the inventory accumulation equation, (3), implies that  $(N_t - E_t N_t) = -(X_t - E_t X_t)$ .

Define  $u_t^x \equiv -(X_t - E_t X_t)$  as the sales forecast error, then the relationship that defines the firm's actual inventory position is

$$N_t = (\lambda_1 + \lambda_2)N_{t-1} - \lambda_1\lambda_2 N_{t-2} + \frac{\bar{\beta}\lambda_1\lambda_2}{(\lambda_1 - \lambda_2)}\sum_{j=0}^{\infty}\left[(\bar{\beta}\lambda_1)^{j+1} - (\bar{\beta}\lambda_2)^{j+1}\right]E_t \Psi_{t+j} + u_t^x \quad ()$$

where again  $\Psi_{t+j}$  is defined by (). Substituting (A.5), (A.6), (A.) and (A.) into (), we

have that

$$N_t = \Gamma_0 + (\lambda_1 + \lambda_2)N_{t-1} - \lambda_1\lambda_2 N_{t-2} + \Gamma_X X_{t-1} + \Gamma_W W_t + \Gamma_{\pi 1}\pi_{1t} + \Gamma_{\pi 3}\pi_{3t} + u_t^x \quad ()$$

where

$$\Gamma_0 = c_W + c_X + \Gamma_2 + \frac{-\lambda_1\lambda_2}{\gamma(1-\bar{\beta}\lambda_1)(1-\bar{\beta}\lambda_2)}c$$



$$\Gamma_X \begin{matrix} > \\ < \end{matrix} 0, \quad \Gamma_W < 0, \quad \Gamma_{\pi 1} > 0, \quad \Gamma_{\pi 3} < 0.$$

### Model without Adjustment Costs

In the model with  $\gamma = 0$  equation (3) in (5) yields a second-order difference equation in  $N_t$  that has one stable and one unstable root. Denoting the stable root by  $\lambda_1$ , equation () becomes

$$N_t = \Gamma_0 + \lambda_1 N_{t-1} + \Gamma_X X_{t-1} + \Gamma_W W_t + \Gamma_{\pi 1} \pi_{1t} + \Gamma_{\pi 3} \pi_{3t} + u_t^x \quad ()$$

where  $\Gamma_X$  and  $\Gamma_W$  are defined as in (19') and where  $\Gamma_{\pi 1} \equiv \Gamma_1 - \Gamma_2$ ,  $\Gamma_{\pi 3} \equiv \Gamma_3 - \Gamma_2$ , and

$$[\Gamma_1 \quad \Gamma_2 \quad \Gamma_3] \equiv \left( \frac{-\eta}{\theta} \right) \lambda_1 \Gamma_v' [I - \bar{\beta} \lambda_1 P]^{-1} P.$$

Figure 1: Impulse response function of the stock of inventories

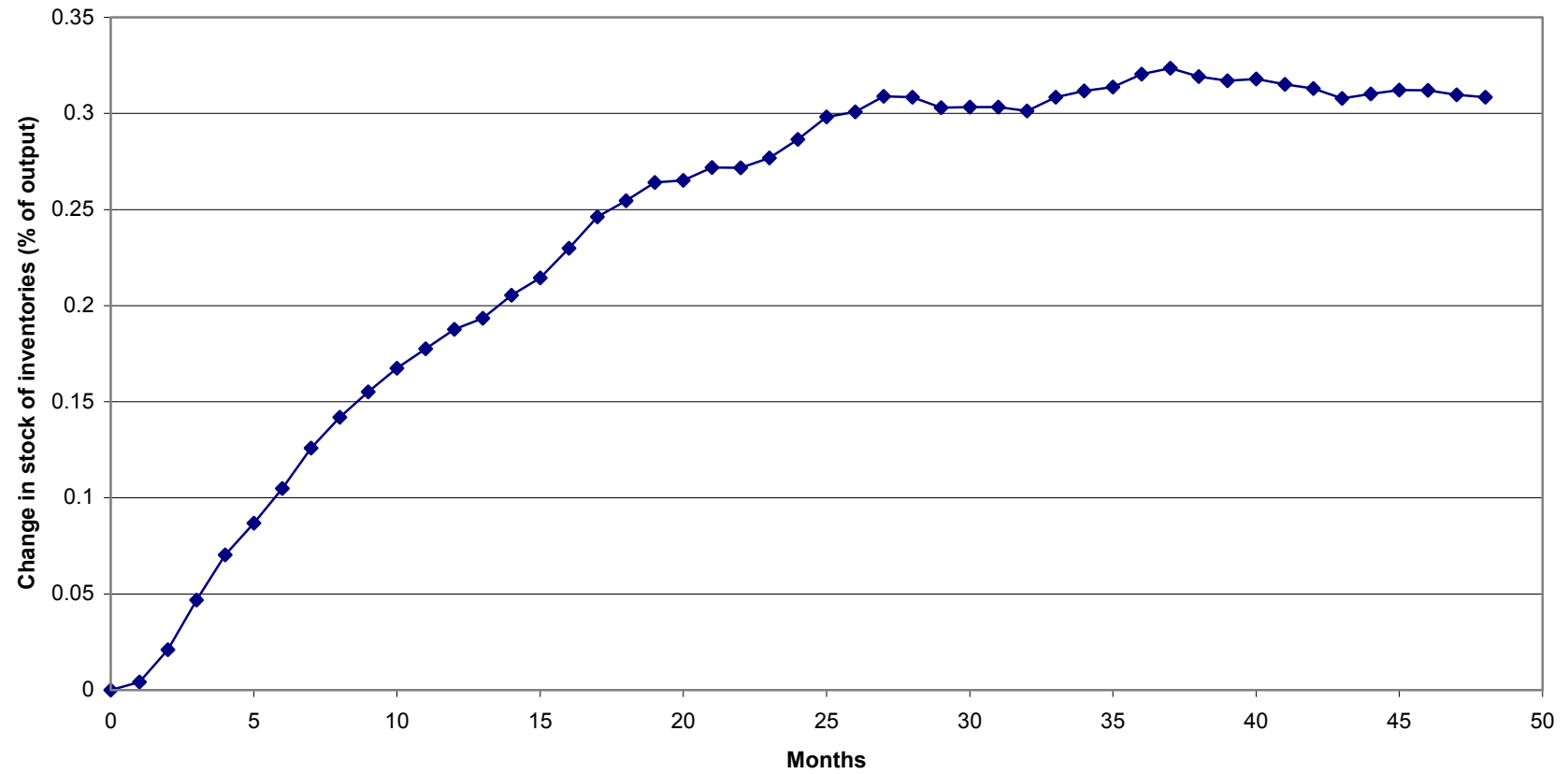
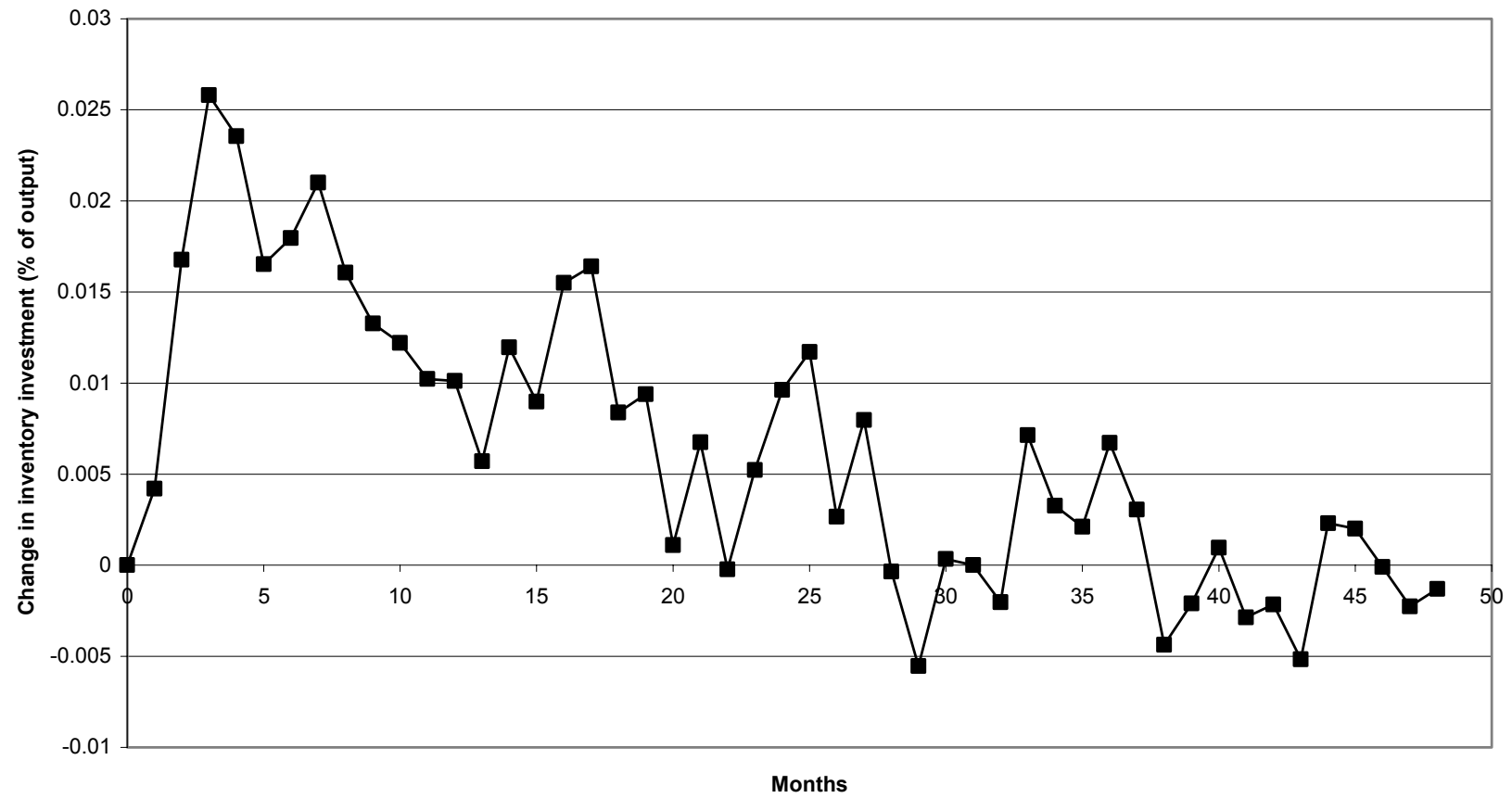


Figure 2: Impulse response function of inventory investment



**Figure 3: Impulse response function of the stock of inventories**  
**Variation in convexity of cost function**

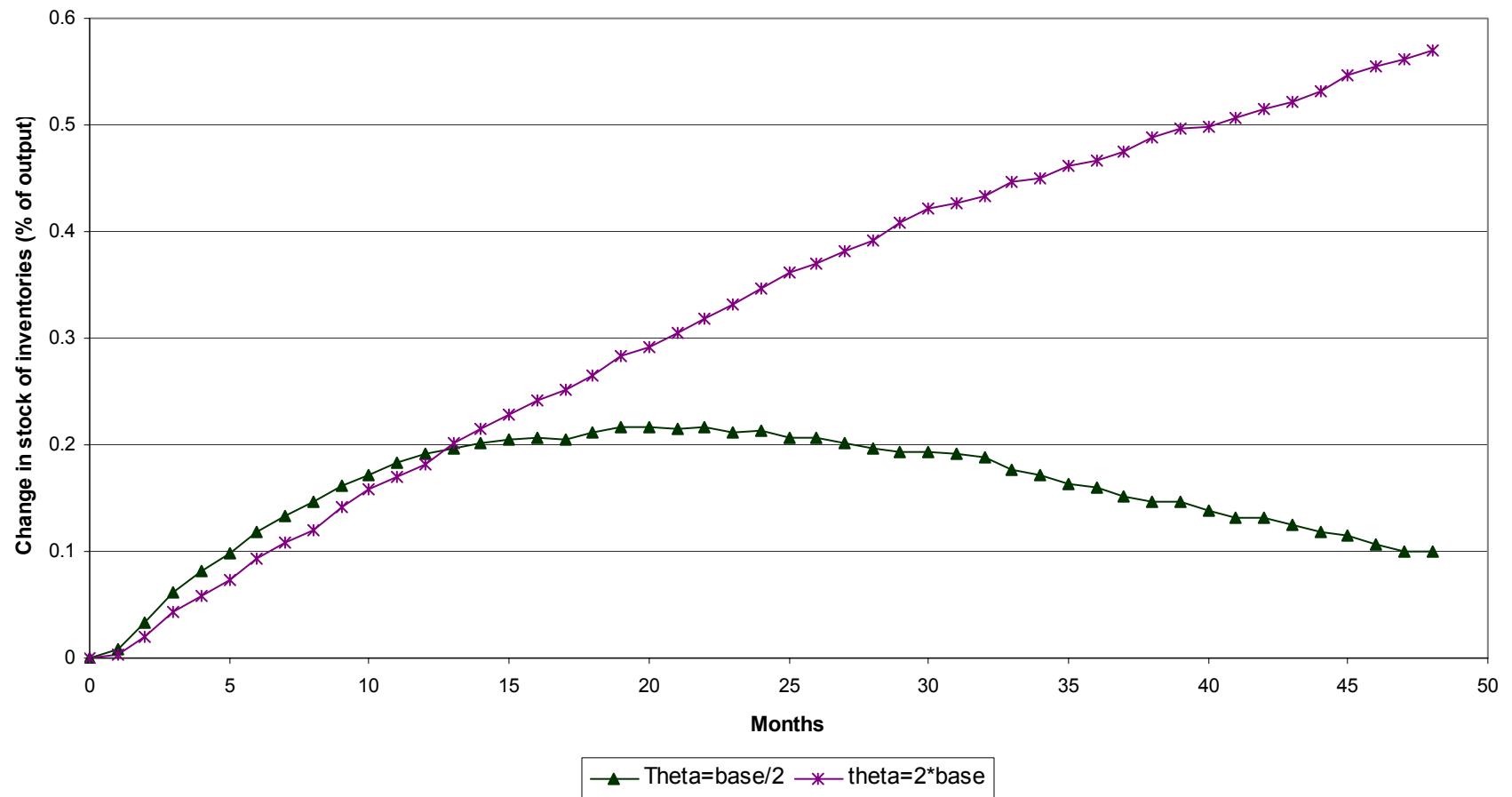
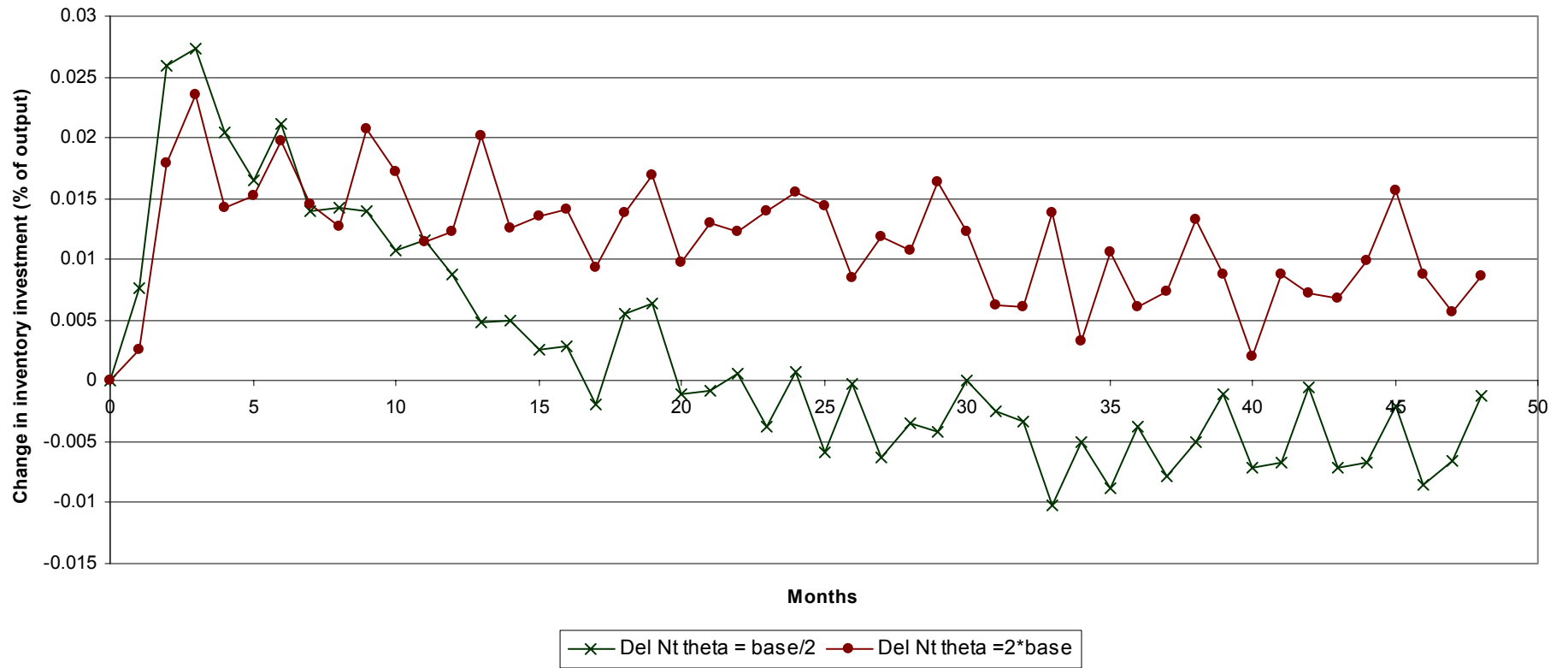
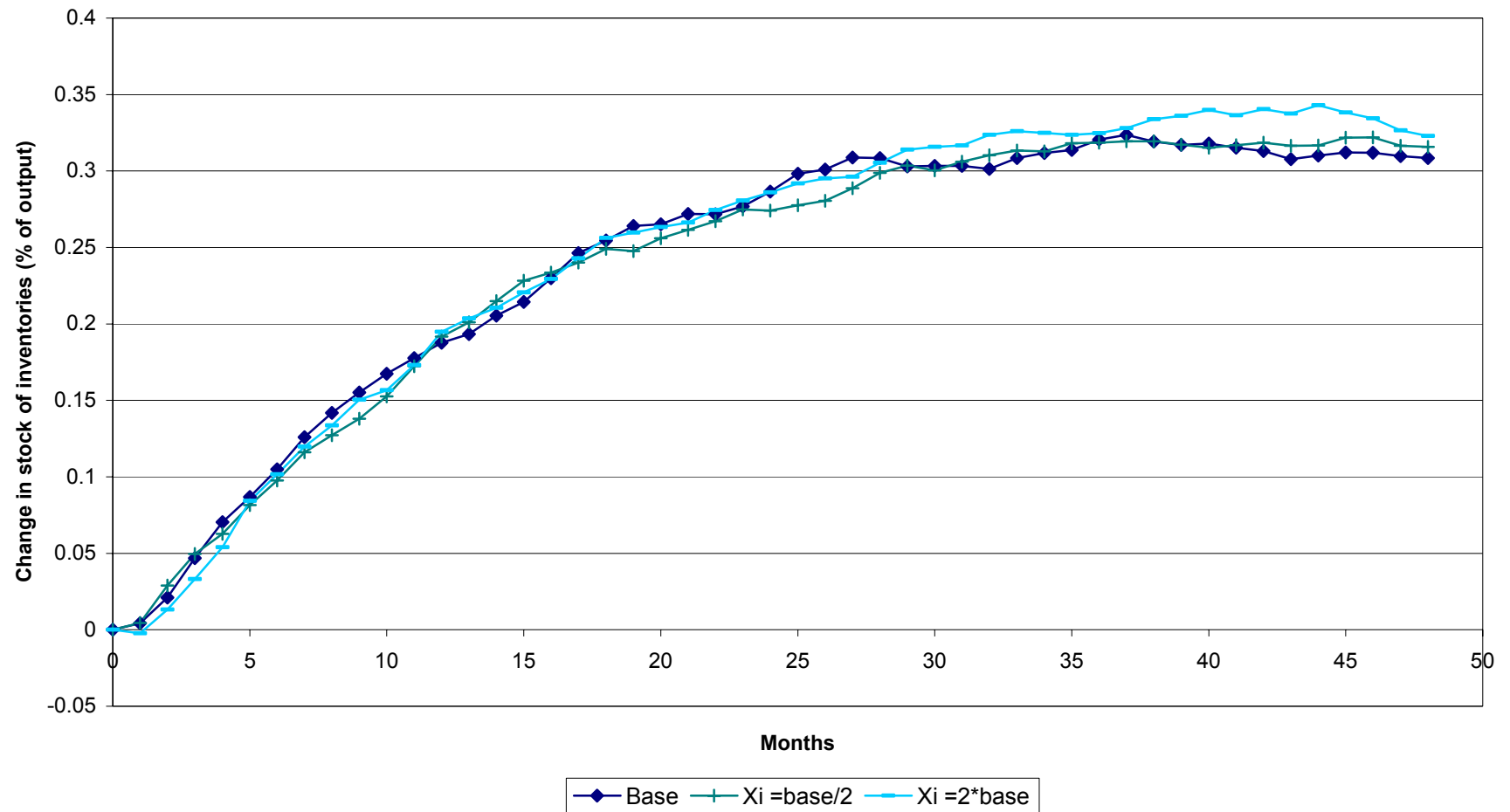


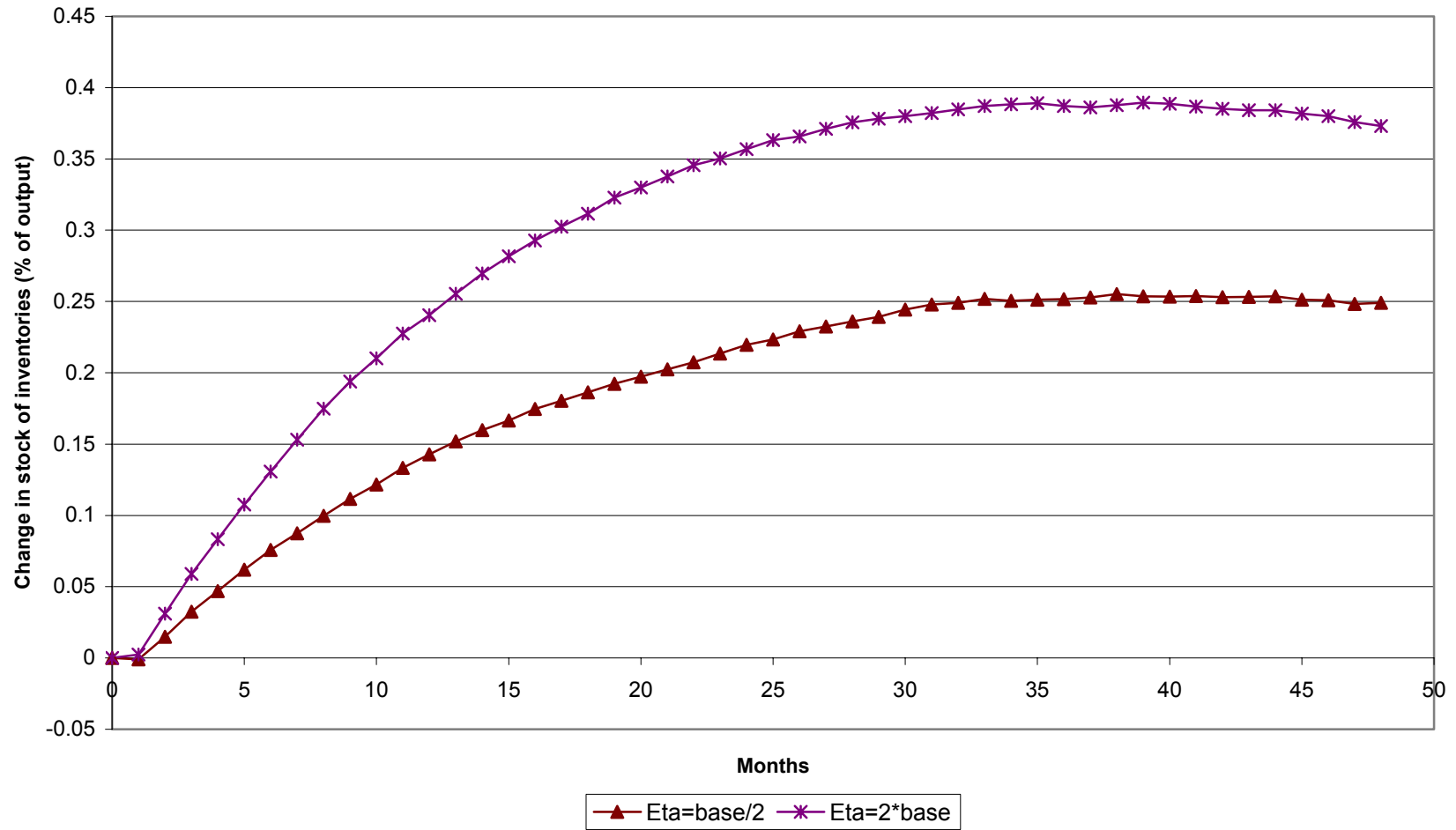
Figure 4: Impulse response function of inventory investment  
Variation in convexity of cost function



**Figure 5: Impulse response function of the stock of inventories**  
**Variation in sensitivity of costs to input prices**



**Figure 6: Impulse response function of the stock of inventories**  
**Variation in sensitivity of inventories to the interest rate**



**Figure 7: Impulse response function of inventory investment**  
**Variation in sensitivity of inventories to the interest rate**

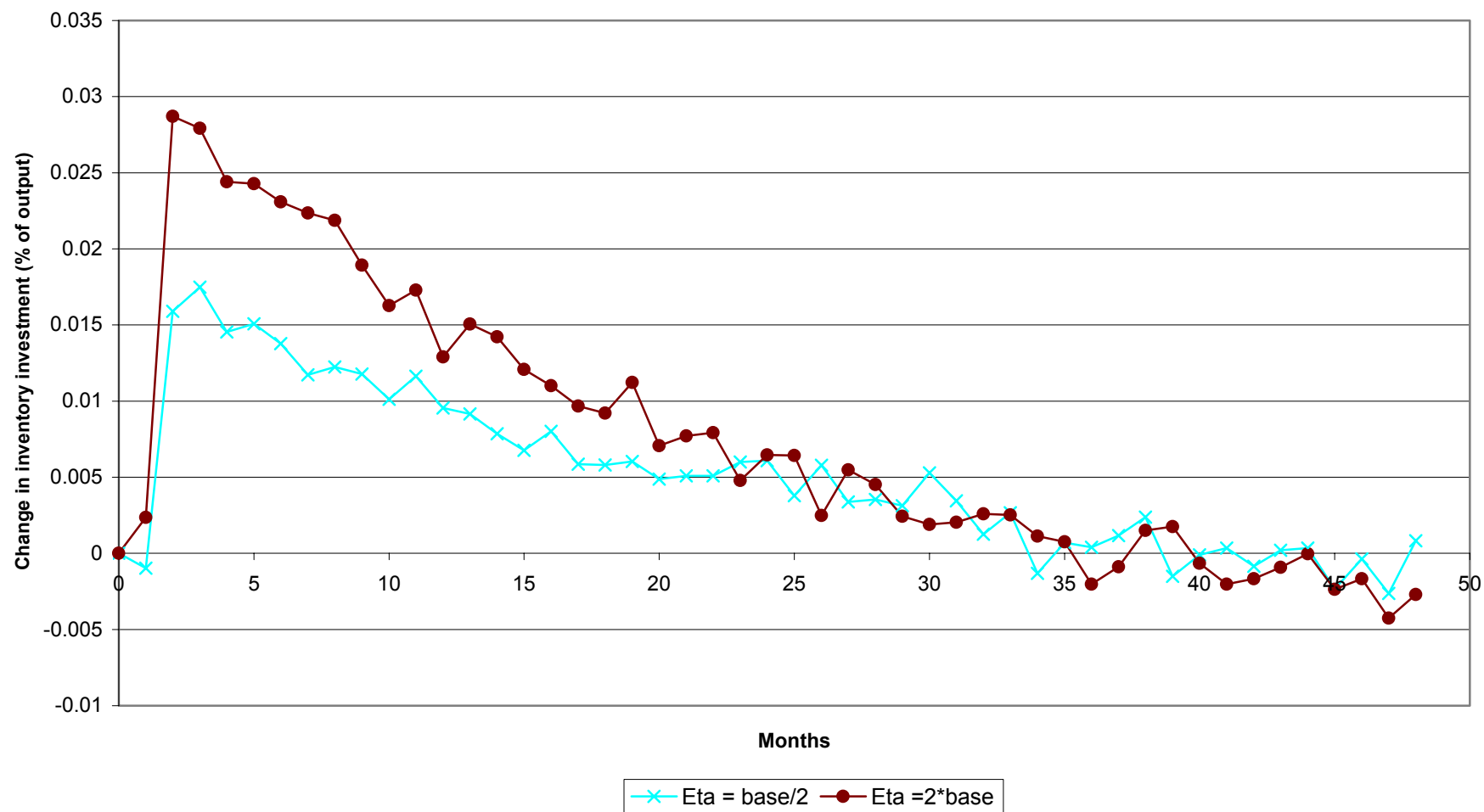




Figure 8: Impulse response function of inventory investment  
Variation in adjustment costs on output

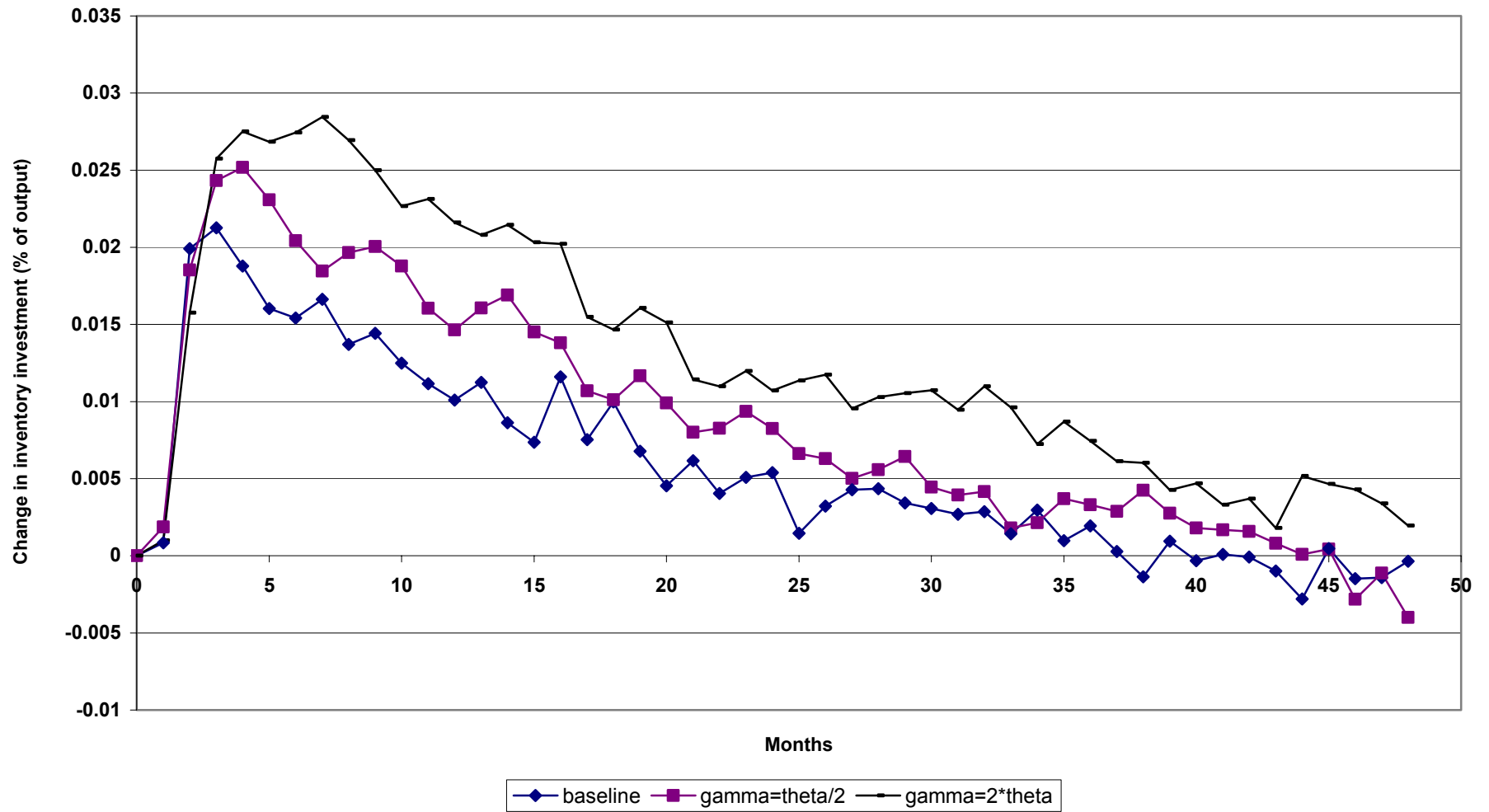


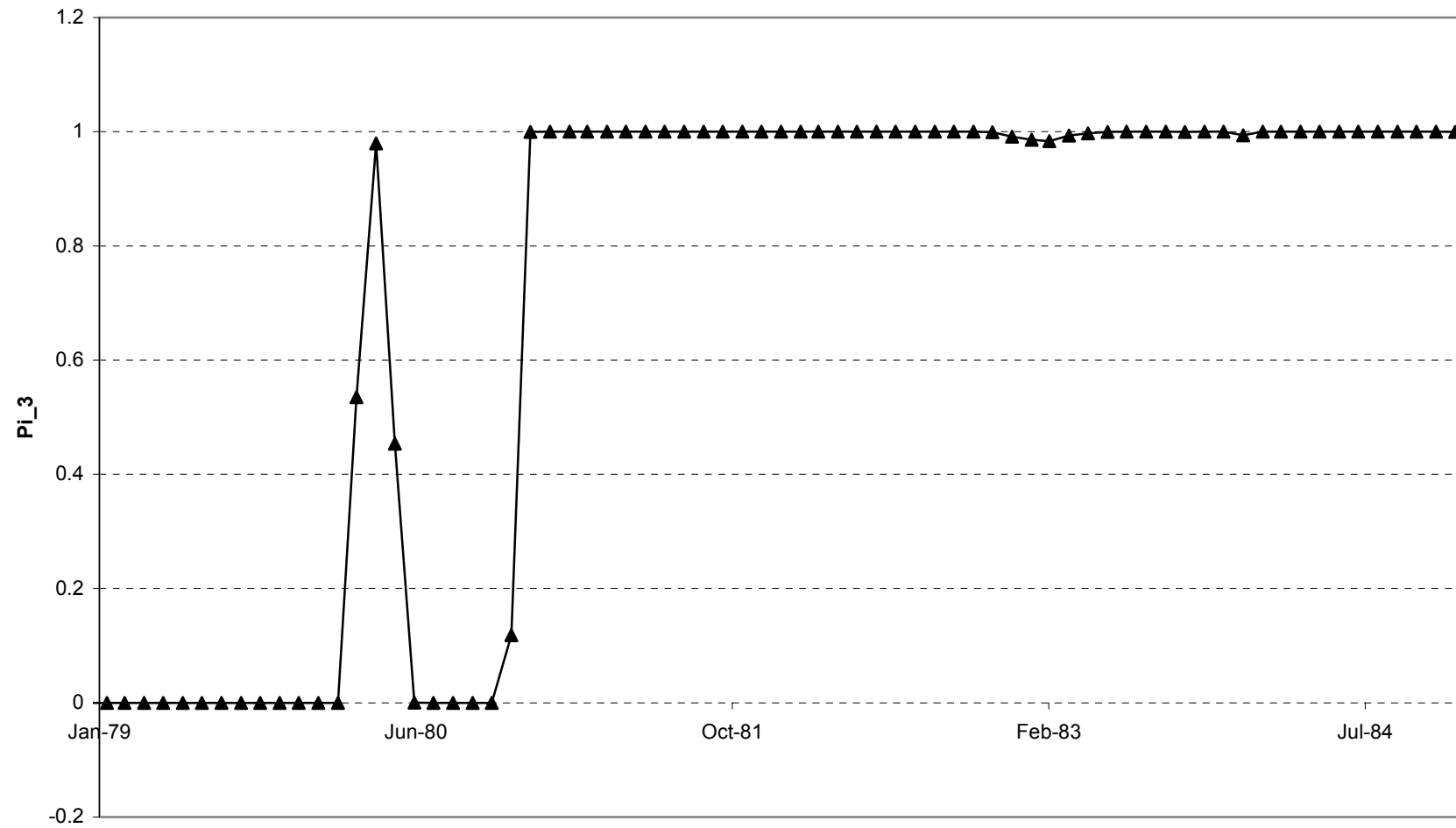
Figure 9:  $\Pi_3$  during the Volcker Disinflation

Figure 10: The stock of inventories during the Volcker Disinflation

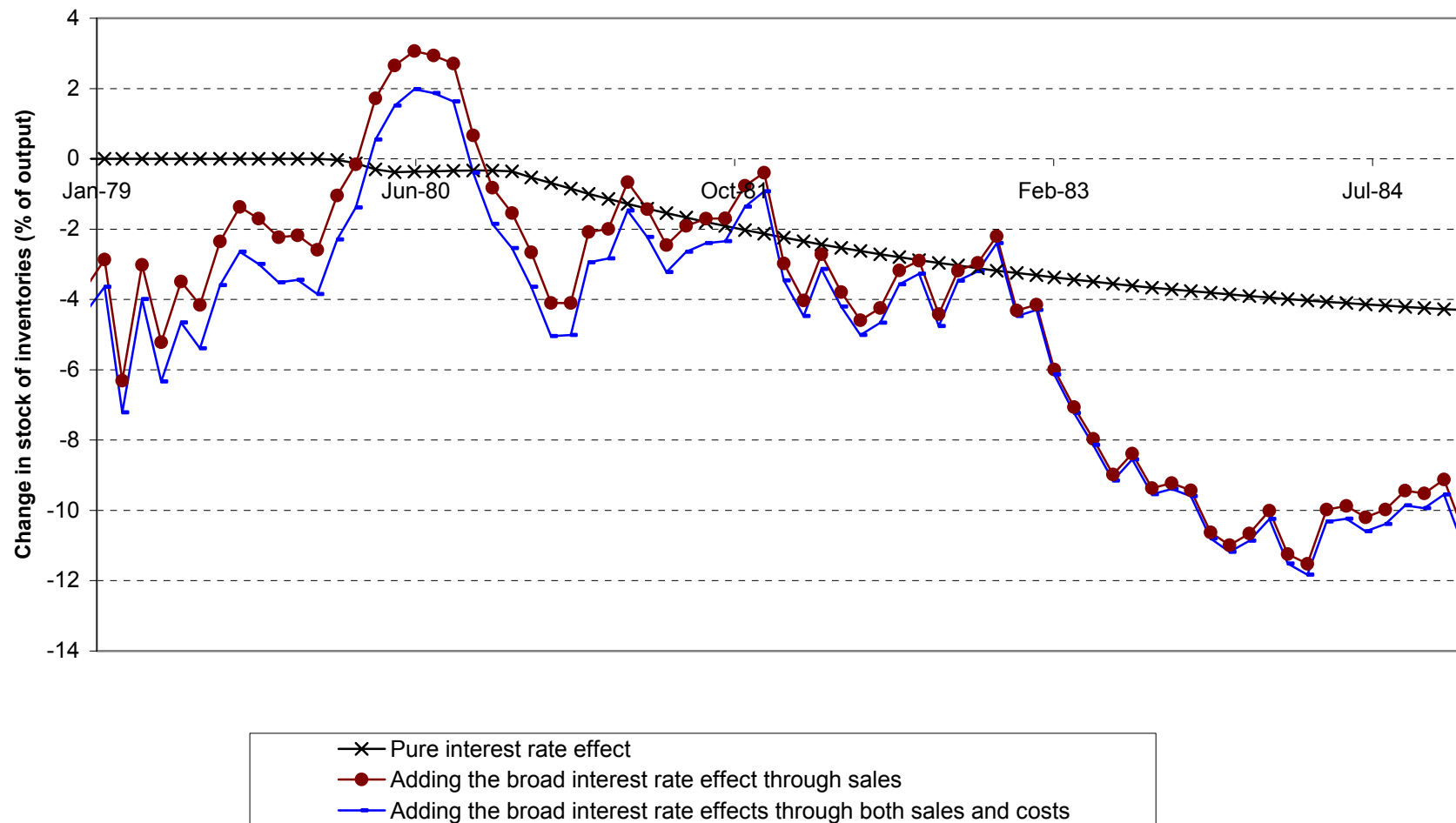


Figure 11: Actual and trend sales during the Volcker Disinflation

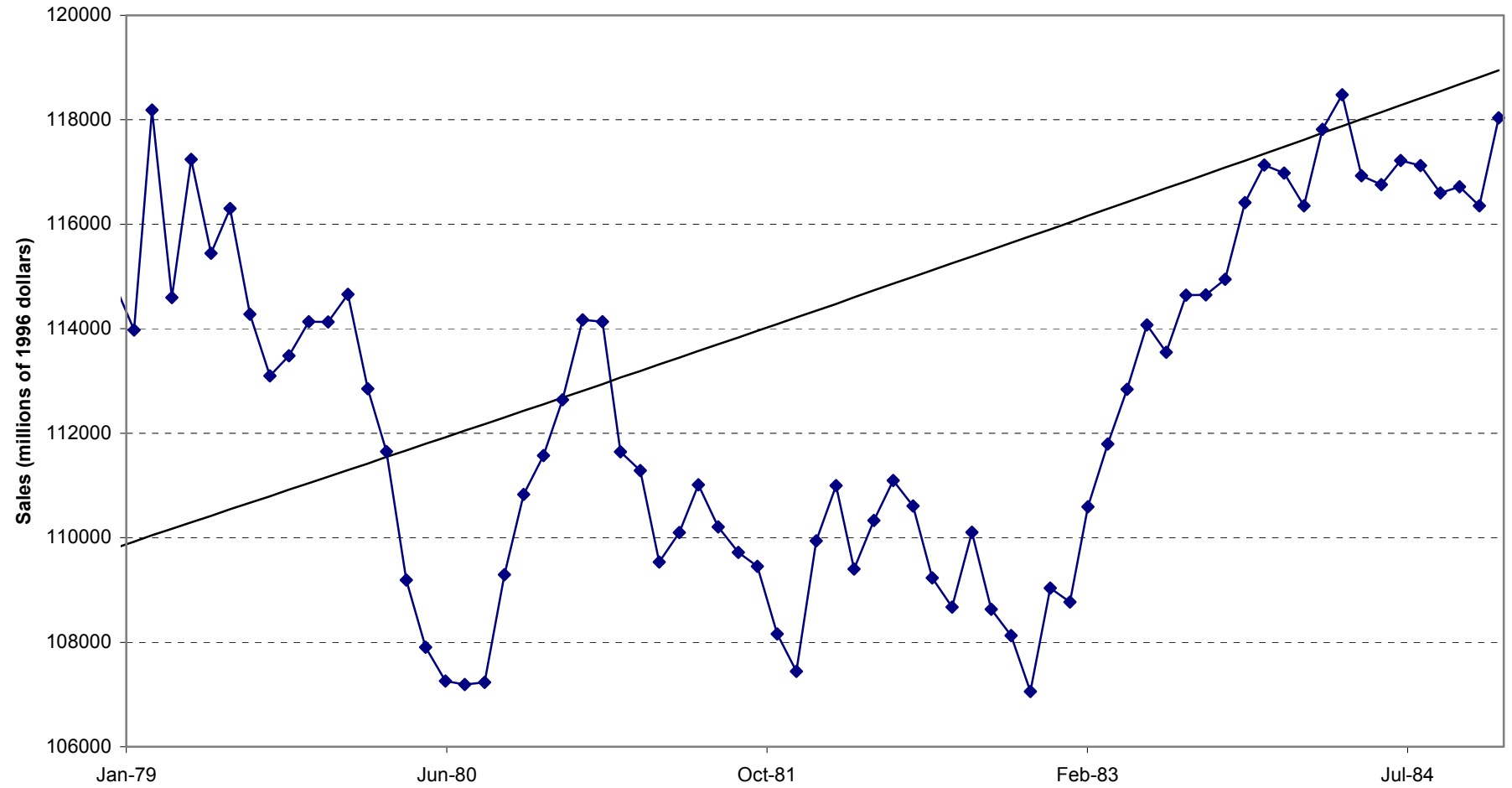


Figure 12: Inventory investment during the Volcker Disinflation

