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# Input and output inventories☆

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**Abstract**

This paper presents a new stage-of-fabrication inventory model with delivery, usage, and stocks of input materials that distinguishes between gross production and value added. It extends the linear-quadratic model of output inventories by adding the joint determination of input inventories. Empirically, input inventories are more important than output inventories. Maximum likelihood estimation of the decision rules yields correctly signed and significant parameter estimates using data for nondurable and durable goods industries, but the overidentifying restrictions of the model are rejected. The value added specification dominates because adjustment

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costs on materials usage are critical to fitting the data. 

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## 1. Introduction

Most firms produce goods in stages. A typical firm orders input materials from an upstream supplier, takes delivery, and combines them with other factor inputs to produce finished goods. Often during the production process the firm generates its own intermediate product as well. Many firms sell their finished goods to downstream firms, which view the goods as input materials. These stage-of-fabrication linkages—within and between firms—imply that rational, optimizing firms will be characterized by joint interaction among all aspects of production. Yet macroeconomic studies of firm behavior generally ignore such dynamic linkages, considering materials only to measure productivity. This paper begins to redress this oversight.

Nowhere is the neglect of stage-of-fabrication linkages more evident than in the inventory literature, where the vast bulk of work has focused almost exclusively on finished goods, or output, inventories. The literature, as summarized by Blinder and Maccini (1991), has been devoted primarily to understanding why the rational expectations version of the pure production smoothing model of output inventories seems to be inconsistent with the data in nondurable goods industries.[1] Such intense scrutiny of output inventory investment has "crowded out" consideration of stage-of-fabrication linkages, such as the ordering and usage of input materials. As a consequence, input inventories—defined here as raw materials and work-in-process—have been neglected almost entirely.

This neglect is problematic for two reasons. First, input inventories conceptually are the linchpin of the stage-of-fabrication production process. They arise whenever the delivery and usage of input materials differ, and firms generally do not synchronize deliveries and usage. Furthermore, since the usage of input

[1] Various authors working with aggregate data have modified the basic model. Blanchard (1983), West (1986), Kahn (1992) and Fuhrer et al. (1995) add a stockout avoidance motive; Maccini and Rossana (1984), Blinder (1986), Miron and Zeldes (1988), and Durlauf and Maccini (1995) add cost shocks in the form of real input prices; Eichenbaum (1989) and Kollintzas (1995) add unobservable technology shocks; Bils and Kahn (2000) allow for the effects of procyclical factor utilization on marginal cost; Ramey (1991) adds nonconvexities in production; and West (1988) adds backlogs of unfilled orders. A consensus explanation, however, is still lacking.

materials is a factor of production, decisions about smoothing production and output inventory investment inherently are related to decisions about input inventory investment. Second, input inventories are more important than output inventories empirically. Stylized facts indicate that input inventories are twice as large and three times more variable. Moreover, the dominance of input inventories occurs primarily in *durable goods* industries, which typically have been excluded from applied inventory research.[2]

Despite their conceptual importance and empirical dominance, the literature on input inventories is remarkably thin. Only Ramey (1989) has developed an optimizing model of inventories at different stages of fabrication. She, however, treats stocks of materials, work-in-process and finished goods inventories as factors of production and applies factor demand theory to derive demand functions for different types of inventories. This approach does not deal adequately with the stock-flow aspects of inventory holding behavior. It ignores the distinctions among the flow decision to order and take delivery of materials purchases, the flow usage of materials in the production process, and the benefits and costs to the holding of stocks of input inventories. Capturing the stock-flow aspects of input inventory decisions is integral to understanding the dynamics of movements in such inventories.[3]

This paper presents a new stage-of-fabrication inventory model with separate decisions to order, use, and stock input materials. As a first step, we assume that materials and intermediate goods are inputs purchased from outside the firm, and that there are no input delivery lags or output order backlogs.[4] The model then makes several advances. First, and most prominently, only the *flow usage* of input materials enters the production function, as in the productivity literature.[5]

[2] In addition, Durlauf and Maccini (1995) find that real materials prices influence finished goods inventory investment, which suggests possible interaction between materials inventories and finished goods inventories.

[3] Related literature includes Husted and Kollintzas (1987), who offer a rational expectations model of the purchase and holding of imported raw materials inventories but ignore interaction with work-in-process or finished goods inventories, and West (1988), who introduces order backlogs and work-in-process inventories into the standard output inventory model. See also the unpublished work of Mosser (1989) and Barth and Ramey (1997). Other work explaining interaction among inventory types includes Lovell (1961), Feldstein and Auerbach (1976), Maccini and Rossana (1984), Reagan and Sheehan (1985), Blinder (1986), Rossana (1990), and Bivin (1993), which rely on stock adjustment and reduced-form models.

[4] To some degree, of course, intermediate goods—and thus work-in-process inventories—are produced within the firm. Hence, an important extension of this paper is to model production of both intermediate and finished goods, which will require the firm to hold separate stocks of materials and work-in-process inventories. Extending the model to incorporate delivery lags and order backlogs may further improve the model's ability to fit the data.

[5] See Baily (1986) and Basu (1996) for discussions of the specification of materials in production functions and its role in explaining productivity movements.

Second, including the flow of materials admits alternative assumptions about the separability of materials in production: gross production (nonseparable) and value added (separable). Third, the firm simultaneously chooses output and input inventory investment, thus linking them with extensive cross-equation restrictions.

The model is fully structural with intertemporal cost minimization under rational expectations and is based on several quadratic approximations like those in conventional output inventory models. We estimate the model via maximum likelihood, conducting the first joint estimation of input and output inventory decision rules. Exploiting model identities, we overcome the lack of high-frequency data on deliveries and usage of materials and estimate gross production and value added versions of the model with data for nondurable and durable goods industries.

On balance, the data yield reasonable econometric support for the value added production model. All parameter estimates of the value added model are the correct sign and estimated very significantly—a degree of success quite uncommon for applied inventory models. In contrast, the model with gross production yields many insignificant and/or implausible parameter estimates. The relative success of the value added model appears to be attributable to the presence of adjustment costs on materials usage, which the standard gross production model omits. On the other hand, the data do reject the stage-of-fabrication model's overidentifying restrictions, like the vast majority of structural inventory models applied to aggregate data.

Several other important conclusions emerge. First, the results indicate that aggregate cost functions are convex so that marginal cost curves slope upward, even in durable goods industries. Second, the results are consistent with theoretical predictions regarding both real wages and real materials costs. Third, the model fits the data for the durable goods industry surprisingly well despite not including intermediate production. Differences between results for nondurable and durable goods industries seem sensible. Overall, the data reveal clear evidence of stage-of-fabrication interactions between inventory stocks, and among inventory stocks and other facets of production.

The paper proceeds as follows. Section 2 updates and expands the stylized facts about inventory movements at different stages of fabrication. Section 3 presents the new stage-of-fabrication inventory model. Section 4 describes the econometric specification and estimation, and Section 5 reports the econometric results. The paper concludes with a discussion of some implications for future research.

## 2. Motivation and stylized facts

This section presents key empirical facts about manufacturing production and inventory activity that motivate the stage-of-fabrication model developed in the next section.[6] To construct the facts, we mainly use monthly data for sales and inventories by stage of fabrication, which are in constant 1987 dollars, seasonally adjusted, and cover the period 1959:1 through 1994:5, except for deliveries and usage of materials for which there are no data at high frequencies. We also use annual data from the Bartelsman–Gray NBER Productivity Database, which includes data on usage and deliveries and covers the period 1959–1994. See the data appendix for details.

### 2.1. Delivery and usage of input materials

One way to motivate the study of input inventories is to compare and contrast the usage and deliveries of input materials with production and sales of finished goods. Fig. 1 provides evidence from *annual* data—the only frequency available for deliveries and usage—on these variables in nondurable goods and durable goods industries. Note that the difference between production and sales equals output inventory investment, and the difference between deliveries and usage equals input inventory investment.

Virtually all prior inventory research focuses on the extent to which firms synchronize production and sales. Traditional output inventory models differ in their predictions about the variance of production versus sales, the central issue being whether firms should smooth production relative to sales. The second column of Fig. 1 shows that firms tend to synchronize production and sales quite closely (correlations of 0.96 in nondurables and 1.00 in durables; ratios of production variance to sales variance of 1.02 for nondurables and 0.99 for durables).

An analogous, but frequently overlooked, issue is the extent to which firms synchronize the deliveries and usage of materials. Usage is the upstream analogue of production because usage and production are very highly—though not perfectly—correlated (compare the solid lines in Fig. 1). Likewise, deliveries are the upstream analogue of sales. Conceptually, the difference is that production is supply in the downstream market and usage is demand in the upstream market. The first column of Fig. 1 shows that firms do not synchronize deliveries and usage nearly as closely as they do production and sales (correlations of 0.52 in nondurables and 0.79 in durables; ratios of usage variance to deliveries variance of 0.40 for nondurables and 0.53 for durables). The lack of synchronization of deliveries and usage is computed with annual data but it is unlikely to be reversed with higher frequency data.

The relatively weak synchronization of materials delivery and usage and the relatively strong synchronization of production and sales together imply that

[6] See Feldstein and Auerbach (1976), Ramey (1989), and Blinder and Maccini (1991) for prior studies that report basic facts. We extend these studies by comparing the facts for durable and nondurable industries, by reporting facts on deliveries as well as the usage of materials, and by updating the sample periods.

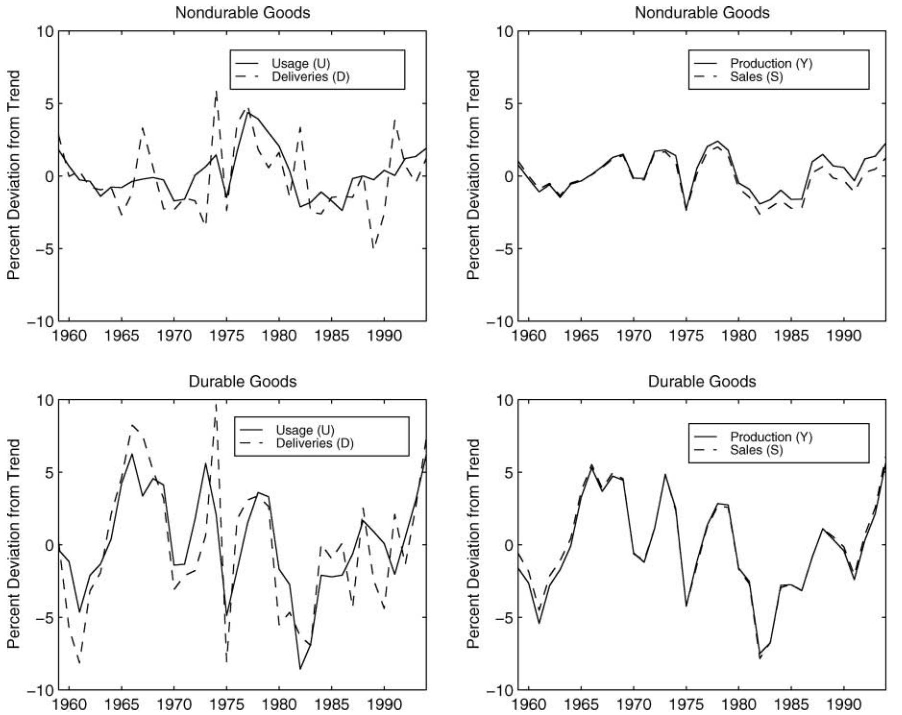


Fig. 1. Annual manufacturing production activity.

input inventory investment fluctuates considerable more over time than does output inventory investment. The next subsection confirms this implication directly from inventory data. However, without a structural model to interpret the data, such as the one advanced in this paper, it is impossible to infer anything about firms' cost functions or technologies that might explain the relative variability of input inventories.

### 2.2. Inventory investment

A second way to motivate the study of input inventories is to compare and contrast the behavior of various components of inventory investment. Table 1 reports the means and variances of inventory investment and inventory-to-sales ratios using *monthly* data on input and output inventories in nondurable goods and durable goods industries in U.S. manufacturing.

*Fact # 1*: *Input inventories are larger and more volatile than output inventories in manufacturing*.

As Table 1 indicates, input inventories are at least twice as large as output inventories in manufacturing, as measured by average inventory investment and

Table 1
Stylized facts about inventories[a]

| Mean inventory investment | Mfg. | (%) | Dur. | (%) | Non. | (%) |
|---|---|---|---|---|---|---|
| Total | 0.51 | 100.0 | 0.33 | 100.0 | 0.18 | 100.0 |
| Finished goods | 0.18 | 35.3 | 0.09 | 27.3 | 0.09 | 50.0 |
| Input | 0.33 | 64.7 | 0.24 | 72.7 | 0.09 | 50.0 |
| Work-in-process | 0.17 | 33.3 | 0.14 | 42.4 | 0.03 | 16.7 |
| Materials and supplies | 0.17 | 33.3 | 0.11 | 33.3 | 0.06 | 33.3 |
| **Variance decomposition of inventory investment** | **Mfg.** | **(%)** | **Dur.** | **(%)** | **Non.** | **(%)** |
| Total | 1.441 | 100.0 | 1.047 | 100.0 | 0.210 | 100.0 |
| Finished goods | 0.264 | 18.3 | 0.117 | 11.2 | 0.114 | 54.3 |
| Input | 0.958 | 66.5 | 0.793 | 75.7 | 0.079 | 37.6 |
| Covariance | 0.218 | 15.1 | 0.138 | 13.2 | 0.017 | 8.1 |
| Finished goods | 0.264 | 18.3 | 0.117 | 11.2 | 0.114 | 54.3 |
| Work-in-process | 0.417 | 28.9 | 0.384 | 36.7 | 0.018 | 8.6 |
| Materials and supplies | 0.387 | 26.9 | 0.276 | 26.4 | 0.058 | 27.6 |
| Covariance terms | 0.373 | 25.9 | 0.271 | 25.9 | 0.019 | 9.0 |
| **Mean inventory/sales ratio (Std. dev.)** | **Mfg.** | **(%)** | **Dur.** | **(%)** | **Non.** | **(%)** |
| Total | 1.66 | (0.11) | 1.98 | (0.20) | 1.29 | (0.05) |
| Finished goods | 0.54 | (0.03) | 0.52 | (0.04) | 0.57 | (0.03) |
| Input | 1.11 | (0.09) | 1.46 | (0.16) | 0.73 | (0.03) |
| Work-in-process | 0.53 | (0.05) | 0.83 | (0.10) | 0.19 | (0.01) |
| Materials and supplies | 0.58 | (0.04) | 0.63 | (0.07) | 0.53 | (0.03) |

[a]Statistics are calculated with monthly $1987 data over the period 1959:01 to 1994:05. In the lower panel, the denominator is sales for all inventory ratios.

inventory-to-sales ratios. Most importantly, the table shows that input inventory investment is more than three times more variable than output inventory investment in manufacturing. These facts suggest that analyses of manufacturing inventory investment should include, and focus on, input rather than output inventories.

*Fact #2: Durable goods inventories are larger and more volatile than nondurable goods inventories.*

Table 1 indicates that durable goods inventories are up to two times larger than nondurable goods inventories, as measured by average inventory investment and inventory-to-sales ratios. Moreover, the table shows that durable goods inventory investment is nearly five times more variable than nondurable goods inventory investment. These facts suggest that analyses of manufacturing inventory investment should include, and focus on, durable rather than nondurable goods inventories.

*Fact #3*: *Input inventories are much larger and more volatile than output inventories in durable goods industries, but input and output inventories are similar in size and volatility in nondurable goods industries.*

This fact is a byproduct of the first two. Table 1 indicates that input inventories are much larger than output inventories in durable goods industries, as measured by average inventory investment and inventory-to-sales ratios. Further, the table shows that input inventory investment is more than six times more variable than output inventory investment in durable goods industries. In nondurable goods industries, on the other hand, the magnitude and variability of input inventories are more even with those of output inventories. In nondurables, input inventory investment is a bit larger but a bit less variable than output inventory investment. The fact that output inventory investment is a bit more variable than input inventory investment provides some rationale for the literature's focus on output inventory investment in nondurable goods industries. Nevertheless, it is difficult to rationalize the nearly complete focus on output inventories in nondurables; instead, the focus should be on input inventories in durables.

*Fact #4*: *Interactions between input and output inventories are quantitatively significant, especially in durable goods industries.*

The middle panel of Table 1 quantifies the extent of inventory stock interaction. Fifteen percent of the variance in manufacturing inventory investment is accounted for by the covariance between input and output inventory investment. When the inventory stocks are disaggregated into the three stages of processing (materials, work-in-process, and finished goods), the covariance terms account for 26 percent of the variance. The table also shows that covariance among types of inventory investment is greater in durable goods industries than nondurable goods industries (26 percent versus 9 percent).

Together, these four stylized facts suggest the following main conclusions: (1) a complete analysis of total manufacturing inventory behavior requires the modeling of input inventories; (2) tests of inventory models should be conducted with durable goods, as well as nondurable goods, industries; and (3) interaction between input and output inventories is empirically evident and potentially a significant feature of firm behavior.

## 3. The stage-of-fabrication model

### 3.1. Overview

Fig. 2 provides a schematic illustration of the model, which focuses on flows through the stage-of-fabrication production process employed by a firm to

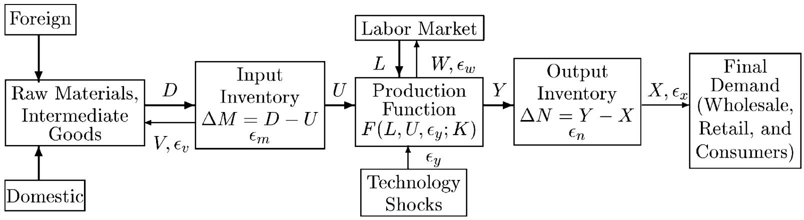


Fig. 2. Stage-of-fabrication model diagram.

transform input inventories (raw materials and work-in-process) into output inventories (finished goods). Each period, the firm combines labor ($L$), materials used in production ($U$), and capital ($K$) to produce finished goods. Materials used in production are obtained from the on-hand stock of input inventories ($M$), which is continually replenished by deliveries ($D$) of materials from foreign and domestic suppliers. Production ($Y$) of final goods is added to the stock of output inventories ($N$), which are used to meet final demand ($X$). The firm takes final demand, the price of labor ($W$), and the price of material deliveries ($V$) as exogenous (thin lines).

The firm optimizes in a dynamic stochastic environment. In the short run, with capital fixed, the firm chooses $U$, $M$, and $N$ to minimize the present value of total costs, given $V$, $W$, and $X$, for a total of six variables and equations in the model.[7] Six random shocks ($\varepsilon$)—one for each equation in the model—buffet the firm's production environment. One shock is a demand shock ($\varepsilon_x$). The other shocks comprise a disaggregation of the traditional "supply" shock: a technology shock ($\varepsilon_y$) affects the production function; inventory holding cost shocks ($\varepsilon_m$,$\varepsilon_n$) affect the costs of carrying inventory stocks; a real wage shock ($\varepsilon_w$) affects labor costs; and a real materials price shock ($\varepsilon_v$) affects material costs.

The model generalizes the traditional linear-quadratic model of output inventories. The central extension in the stage-of-fabrication model is the explicit introduction of input inventories, which must be chosen simultaneously with output inventories. Input inventory investment is controlled by varying the usage of materials in production and the deliveries of materials. Total costs—labor costs, inventory holding costs, and delivery costs—are approximated with a generalized quadratic form. Our stage-of-fabrication model differs from the few inventory papers that include input inventories by specifying the flow, rather than stock, of materials in the production function. For the purpose

[7] Actually, the firm chooses $L$, $U$, and $D$, but because there are no high-frequency data available on $U$ and $D$ we use the model flow identities and first-order conditions to recast the problem with inventory stocks as choice variables.

of testing the model with aggregate data, we adopt the convention of a representative firm, as is customary in the inventory literature.[8]

### 3.2. The production function

Following the literature on production functions and productivity—for examples, see Baily (1986) and Basu (1996)—we assume that the short-run production function contains as an input the flow of materials used in the production process. Specifically, the production function is

$$Y_t = F(L_t, U_t, \varepsilon_{yt}). \tag{1}$$

Note that $U_t$ is the *flow* of materials used in the production process, not the stock of materials inventories. Because $Y_t$ is gross output, we refer to Eq. (1) as the gross production function. Two assumptions are implicit in (1): First, the capital stock is a fixed factor of production with no short-run variation in utilization—an unrealistic assumption that should ultimately be relaxed in a more complete model of production. As a consequence, the remaining factors—materials usage and labor—possess positive and nonincreasing marginal products, and the short-run production function exhibits decreasing returns to scale. Second, the firm purchases intermediate goods (work-in-process) from outside suppliers rather than producing them internally.[9] Thus, intermediate goods are analogous to raw materials so work-in-process inventories can be lumped together with materials inventories.

An important specification issue for the production function is whether $U_t$ is additively separable from the other factors of production. For example, Basu and Fernald (1995) show that evidence of externalities caused by productive spillovers exists in value added data but not gross production data. If $U_t$ is separable, then the production function can be written as

$$Y_t - H(U_t) = G(L_t, \varepsilon_{yt}), \tag{2}$$

where $Y_t - H(U_t)$ is value added. For this paper, we make the strong simplifying assumption that $H(U_t) = U_t$. Consequently, Eq. (2) is a special case of Eq. (1) with the restrictions $F_U = 1$ and $F_{LU} = F_{\varepsilon_y U} = 0$. We refer to this form of the production function as the value added production function.

[8] In future work, we intend to estimate the stage-of-fabrication model with inventory data from the firm-level M3LRD data base originally developed by Schuh (1992). An advantage of working with individual firm data is that the accuracy of distinguishing inventories at different stages of fabrication is enhanced.

[9] To allow for production of intermediate goods within the firm requires extending the production function to incorporate joint production of final and intermediate goods. This extension is a substantial modification of the standard production process that we leave for future work.

By specifying the flow usage of input materials in the production function, we extend traditional inventory models to include an additional role for inventory dynamics (investment) in the stage-of-fabrication process. Previous inventory models focus solely on the role of inventory stocks as a convenience yield to the firm. Typically this convenience yield is interpreted as the savings of lost sales by the firm when it cannot satisfy customers, but it has also been interpreted as the savings in marketing costs (see Pindyck, 1994). The few inventory models that consider both input and output inventories, such as Ramey (1989) and Considine (1997), include inventory stocks in the production function. In this case, the benefits to holding inventory stocks are interpreted as benefits to the physical production process itself, for example the avoidance of production disruptions. None of these specifications, however, incorporates the flow dynamics implied by the delivery and usage of materials for input and output inventory investment.

### *3.3. The cost structure*

The firm's total cost structure consists of three major components: labor costs, inventory holding costs, and materials costs. This section describes each component.

#### *3.3.1. Labor costs*

Labor costs are

$$LC_t = W_t L_t + A(\Delta L_t) \tag{3}$$

with

$$A' \lesseqgtr 0 \text{ as } \Delta L_t \lesseqgtr 0$$
$$A'' > 0$$

where $\Delta L_t = L_t - L_{t-1}$. The first component, $W_t L_t$, is the standard wage bill. The second component, $A(\Delta L_t)$, is a standard adjustment cost function intended to capture the hiring and firing costs associated with changes in labor inputs. The adjustment cost function has the usual properties, including a rising marginal adjustment cost.

To focus on inventory decisions, we eliminate labor input. Inverting the production function, equation (1), yields the labor requirements function

$$L_t = L(Y_t, U_t, \varepsilon_{yt}). \tag{4}$$

Substituting (4) into (3) yields

$$LC_t = W_t L(Y_t, U_t, \varepsilon_{yt}) + A(L(Y_t, U_t, \varepsilon_{yt}) - L(Y_{t-1}, U_{t-1}, \varepsilon_{y,t-1})) \tag{5}$$

which is the central portion of the firm's cost function. Observe that, when materials usage is taken into account in the production function, the inverted production function implies that adjustment costs depend on the change in materials usage as well as the change in gross output, a feature which the standard model overlooks.

Following the inventory literature, we approximate Eq. (5) with a generalized quadratic function. Specifically, labor cost is

$$LC_t = \left(\frac{\gamma_1}{2}\right)Y_t^2 + \left(\frac{\gamma_2}{2}\right)U_t^2 + \gamma_3 Y_t U_t + W_t[\gamma_4 Y_t + \gamma_5 U_t] + \left(\frac{\varphi}{2}\right)[\gamma_6 \Delta Y_t + \gamma_7 \Delta U_t]^2 + \varepsilon_{yt}(\gamma_8 Y_t + \gamma_9 U_t) \quad (6)$$

with the parametric restrictions

$$\gamma_1,\gamma_2,\gamma_4 > 0 \quad \gamma_3 < 0 \quad \gamma_6,\gamma_9,(\gamma_1\gamma_2 - \gamma_3^2) \geqslant 0 \quad \gamma_5,\gamma_7,\gamma_8 \leqslant 0$$

implied by the production function.[10]

We focus on two production function specifications: gross production and value added. The cost function approximation is extensively parameterized, which makes it difficult to estimate all parameters precisely. Hence, some restrictions must be imposed. We choose restrictions that capture the essential features of our model but yield the standard output inventory model as a special case.

*Gross production*: To obtain the gross production specification, let

$$\gamma_5 = \gamma_7 = \gamma_9 = 0, \quad \gamma_6 = -\gamma_8 = 1.$$

Then the gross production ($g$) cost function is

$$LC_t^g = \left(\frac{\gamma_1}{2}\right)Y_t^2 + \left(\frac{\gamma_2}{2}\right)U_t^2 + \gamma_3 Y_t U_t + \gamma_4 W_t Y_t + \left(\frac{\varphi}{2}\right)(\Delta Y_t)^2 - \varepsilon_{yt} Y_t. \quad (7)$$

The standard output inventory model cost function is a special case of Eq. (7) and can be obtained by setting $\gamma_2 = \gamma_3 = 0$, an assumption implicit in the standard model. The restriction $\gamma_4 = 0$ usually is imposed as well, with Eichenbaum (1984) and Durlauf and Maccini (1995) being exceptions. This specification directly extends the standard model by allowing for materials usage in the production process, but not in adjustment costs.

[10] The restrictions are implied by the strict concavity of the short-run-production function and the strict concavity of the adjustment cost function.

*Value added*: To obtain the value added specification, let

$$\gamma_1 = \gamma_2 = -\gamma_3 = \gamma > 0, \quad \gamma_4 = -\gamma_5 > 0,$$
$$\gamma_6 = -\gamma_7 = -\gamma_8 = \gamma_9 = 1.$$

These restrictions make value added, $Y_t - U_t$, a factor in the inverted production function, rather than $Y_t$ and $U_t$ separately. Then the value added ($v$) cost function is

$$LC_t^v = \left(\frac{\gamma}{2}\right)(Y_t - U_t)^2 + \gamma_4 W_t(Y_t - U_t) + \left(\frac{\varphi}{2}\right)(\Delta Y_t - \Delta U_t)^2 - \varepsilon_{yt}(Y_t - U_t). \tag{8}$$

The standard output inventory model is also a special case of Eq. (8), and can be obtained by setting $U_t = 0$ for all $t$.

Comparing Eqs. (7) and (8) emphasizes that introducing materials usage makes the cost function critically dependent on the specification of the technology. The value added specification has several advantages. One is that it is consistent with the prevailing treatment of production technology. Another is that it is more parsimonious and thus potentially easier to estimate.

Finally, and most importantly, the value added specification highlights a very restrictive and previously unrecognized assumption implicit in standard output inventory models. The value added specification inherently imposes adjustment costs on the change in value added, which implies that adjustment costs depend on the change in materials usage ($\Delta U_t$) as well as on the change in gross output ($\Delta Y_t$). As we show later, the appearance of the change in materials usage in adjustment costs has important implications for the model's dynamic structure, especially for the persistence of input inventories.

### 3.3.2. Inventory holding costs

In line with much of the output inventory literature, holding costs for output inventories are a quadratic approximation to actual costs of the form

$$HC_t^N = (\delta_0 + \varepsilon_{nt})N_t + \left(\frac{\delta}{2}\right)(N_t - N_t^*)^2, \tag{9}$$

where $\varepsilon_{nt}$ is the white noise innovation to output inventory holding costs, $N_t^*$ is the target level of output inventories that minimizes output inventory holding costs, and $\delta > 0$. We adopt an analogous formulation for input inventories; holding costs for these stocks are a quadratic approximation of the form

$$HC_t^M = (\tau_0 + \varepsilon_{mt})M_t + \left(\frac{\tau}{2}\right)(M_t - M_t^*)^2, \tag{10}$$

where $\varepsilon_{mt}$ is the white noise innovation to input inventory holding costs, $M_t^*$ is the target level of input inventories that minimizes input inventory holding costs, and $\tau > 0$. The quadratic inventory holding cost structure balances two forces. Holding costs rise with the level of inventories, $M_t$ and $N_t$, due to increased storage costs, insurance costs, etc. But holding costs fall with $M_t$ and $N_t$ because—given expected $M_t^*$ and $N_t^*$—higher $M_t$ and $N_t$ reduce the likelihood that the firm will "stock out" of inventories.

Finally, it remains to specify the inventory target stocks. Again following the literature, the output inventory target stock is

$$N_t^* = \alpha X_t, \tag{11}$$

where $\alpha > 0$. The output inventory target depends on sales because the firm incurs costs due to lost sales when it stocks out of output inventories. For the input inventory target stock, we assume that the target stock depends on production, rather than sales. In particular,

$$M_t^* = \theta Y_t, \tag{12}$$

where $\theta > 0$. The input inventory target depends on production ($Y_t = X_t + \Delta N_t$) because stocking out of input inventories also entails costs associated with production disruptions—lost production, so to speak—that are distinct from the cost of lost sales. Lost production may be manifested by reduced productivity or failure to realize production plans.

To summarize, the input and output inventory targets differ because the firm holds the two inventory stocks for different reasons. The firm stocks output inventories to guard against random demand fluctuations, but it stocks input inventories to guard against random fluctuations in productivity, materials prices and deliveries, and other aspects of production. Although sales and production are highly positively correlated, they differ enough at high frequencies to justify different target stock specifications.

### *3.3.3. Input materials costs*

Input materials costs consist of purchase and adjustment costs. Specifically, input materials costs are

$$MC_t = V_t D_t + \left(\frac{\phi}{2}\right) D_t^2. \tag{13}$$

The first term on the right side of Eq. (13) is the cost of ordering and purchasing input materials at the "base" price each period. This term is the only one in the model without a parameter, and it permits identification of all remaining parameters (except target stock parameters, which are identified separately) relative to the units in which $V_t$ is measured.[11] The second term is a quadratic approximation for adjustment costs on purchases of materials and supplies.

On adjustment costs, two cases may be distinguished:

1. *Increasing marginal cost*: $\phi > 0$. In this case, the firm faces a rising supply price for materials purchases. The firm thus experiences increasing marginal costs to purchasing materials due to higher premia that must be paid to acquire materials more quickly. A rationale for such a rising supply price is that the firm is a monopsonist in the market for materials. This is most likely to occur when materials are highly firm or industry specific and the firm or industry is a relatively large fraction of market demand.[12] The rising marginal cost of course gives rise to the "smoothing" of purchases.
2. *Constant marginal cost*: $\phi = 0$. In this case, firms are price takers in competitive input markets and purchase all the raw materials needed at the prevailing market price.

### 3.4. Cost minimization

To focus on the cost minimization problem, we assume that inventories do not enter the firm's revenue function, and that the materials price and wage are both exogenous to the firm. Thus, the firm chooses $\{U_t, M_t, N_t\}_{t=0}^{\infty}$ to minimize the discounted present value of total costs (TC),

$$E_0 \sum_{t=0}^{\infty} \beta^t (LC_t + HC_t^N + HC_t^M + MC_t), \tag{14}$$

where $\beta = (1+r)^{-1}$ is the discount factor implied by the constant real rate of interest $r$. The two laws of motion governing inventory stocks,

$$\Delta N_t = Y_t - X_t, \tag{15}$$

$$\Delta M_t = D_t - U_t, \tag{16}$$

can be used to substitute for production ($Y_t$) and deliveries ($D_t$).

### 3.5. Euler equations

The model yields Euler equations for $U_t$, $M_t$, and $N_t$. However, because there are no high-frequency data on usage, $U_t$ must be eliminated from the Euler

[11] See West (1993) for a discussion of identification in inventory models.

[12] This is analogous to the literature on adjustment cost models for investment in plant and equipment where external adjustment costs are imposed in the form of a rising supply price for capital goods.

equations for empirical work to proceed. We thus use the Euler equation for materials usage to eliminate $U_t$ from the system. After some straightforward but tedious algebra, collecting terms around common parameters, and imposing the relevant restrictions, the Euler equations for each production specification can be derived.[13]

To present the Euler equations in a concise fashion, define the lag operator as $L$, which works as a lead operator when inverted (e.g., $L^{-1}Y_t = Y_{t+1}$), and a variable $Z_{ij}$ that denotes quasi-differences $(1-\beta L^{-1})$ of model variables. Subscript $i$ indicates the variable being quasi-differenced, subscript $j$ indicates the number of quasi-differences, $(1-\beta L^{-1})^j$, and $\Delta = (1-L)$ is the standard first-difference operator. Three examples clarify the notation: $Z_{Y1} = (1-\beta L^{-1})Y_t$ is the quasi-first difference of $Y_t$; $Z_{Y2} = (1-\beta L^{-1})^2 Y_t$ is the quasi-second difference of $Y_t$, and $\Delta Z_{Y2} = (1-L)(1-\beta L^{-1})^2 Y_t = (1-\beta L^{-1})^2 \Delta Y_t$ is the change in the quasi-second difference. Similar notation applies for all other variables except the inventory target terms.

Using this notational convention, the Euler equations for the two production specifications can be presented as follows:

*Gross production model*: The Euler equation for input inventories is

$$E_t\{\gamma_2\phi Z_{\Delta M1} + \gamma_2 Z_{V1} - \gamma_3\phi Z_{Y1} + \tau(\gamma_2+\phi)[M_t - \theta Y_t] + (\gamma_2+\phi)Z_{\varepsilon_m 1}\} = 0 \tag{17}$$

and the Euler equation for output inventories is

$$E_t\{(\gamma_1 - \zeta\gamma_3)Z_{Y1} + \varphi Z_{Y2} + \delta[N_t - \alpha X_t] + \gamma_4 Z_{W1} - \zeta Z_{V1} - \tau\theta[M_t - \theta Y_t - \beta(M_{t+1} - \theta Y_{t+1})] - \phi\zeta Z_{\Delta M1} + Z_{\varepsilon_y 1} + Z_{\varepsilon_n 1}\} = 0 \tag{18}$$

where $\zeta = \gamma_3/(\gamma_2+\phi)$.

*Value added model*: The Euler equation for input inventories is

$$E_t\{\gamma\phi Z_{\Delta M1} + \varphi\phi Z_{\Delta M2} + \gamma Z_{V1} + \varphi Z_{V2} + \phi\gamma_4 Z_{W1} + \phi\gamma Z_{Y1} + \phi\varphi\Delta Z_{Y2} + \tau(\gamma+\phi)(M_t - \theta Y_t) + \varphi\tau[\Delta M_t - \theta\Delta Y_t - \beta(\Delta M_{t+1} - \theta\Delta Y_{t+1})] - \phi Z_{\varepsilon_y 1} + (\gamma_2+\phi)Z_{\varepsilon_m 1} + \varphi\Delta Z_{\varepsilon_m 2} + \tau_0\} = 0 \tag{19}$$

and the Euler equation for output inventories is

$$E_t\{\delta(\gamma+\phi)(N_t - \alpha X_t) + \delta\varphi(\Delta N_t - \alpha\Delta X_t) - \tau(\gamma+\phi)[(1+\theta)(M_t - \theta Y_t) - \theta\beta(M_{t+1} - \theta Y_{t+1})]$$

[13] See the unpublished appendix for the details of the derivation.

$$- \tau\varphi[(1+\theta)(\Delta M_t - \theta\Delta Y_t) - \theta\beta(\Delta M_{t+1} - \theta\Delta Y_{t+1})]$$
$$+ (\gamma+\phi)Z_{\varepsilon_n 1} + \varphi Z_{\varepsilon_n 2} - (\gamma+\phi)Z_{\varepsilon_m 1} - \varphi Z_{\varepsilon_m 2} + \delta_0\} = 0, \qquad (20)$$

where $\gamma > 0$ is the parameter attached to the cost of producing value added.

An important issue on which the inventory literature has focused is the slope of the marginal cost of production. The traditional linear-quadratic output inventory model assumes rising marginal cost due to diminishing returns to the variable factors in the short-run cost function, which induces a production smoothing motive. The slope of marginal cost in the stage-of-fabrication model, obtained from the second derivative of the dynamic total cost function, is

$$\partial^2 TC/\partial Y^2 = \gamma_1 + (1+\beta)\varphi \qquad (21)$$

for the gross production models; the formula for value added is the same except that $\gamma$ replaces $\gamma_1$. For both models, the slope of marginal cost should be positive from the concavity of the production function.

### 3.6. Interpretation of Euler equations[14]

The input inventory Euler equations, (17) and (19), which represent new contributions to the literature, embody standard economic behavior for a generic quadratic regulator problem. Consider first the gross production case. The firm attempts to set the input inventory stock equal to its target subject to several dynamic frictions. First, the adjustment costs associated with purchases and deliveries of materials, quantified by $\phi$, prevent the firm from instantaneously eliminating input inventory gaps, $M_t - \theta Y_t$. Second, time variation in expected materials prices gives the firm an incentive to intertemporally substitute deliveries of input materials. Bargains on input materials must be large enough to more than offset adjustment and stockout costs. Finally, higher output inventory stocks induce the firm to raise gross production, which in turn requires higher materials usage and thus tends to draw down materials inventory stocks.

Consider next the input inventory Euler equation in the value added case. It contains the same forces at work as in the gross production case, albeit with the value added restrictions $\gamma_1 = \gamma_2 = -\gamma_3 = \gamma$ imposed. The key extension of the value added case is that, because adjustment costs depend on the change in value added, they depend on changes in materials usage as well as changes in gross output. Hence, changes in the second differences of the relevant variables

[14] In principle, it would be informative to solve analytically for the decision rules of the complete system. These rules would show analytically the impact of the dynamic linkages imposed by the model on production and inventory investment. Unfortunately, however, the models are a sixth-order difference equation system in $M$ and $N$, and such systems are very difficult, perhaps impossible, to solve analytically.

that appeared in the gross production equation enter the value added equation as well. These are captured by the terms involving the slope of the marginal adjustment cost of labor, $\varphi$. These forces tend to impart additional persistence on materials stocks.

The output inventory Euler equations, (18) and (20), extend the standard output inventory model in the literature by explicitly introducing input materials. The first line of Eq. (18), which assumes gross production, represents the most general standard model. Thus, introducing input materials expands the standard model in two ways. First, it adds a quasi-difference of the input inventory gap to the equation. Second, the adjustment costs associated with materials deliveries affect the extent to which materials can be used in producing output and thus the accumulation of output inventory stocks. By omitting these variables, the standard output inventory model implicitly imposes theoretical restrictions that may contribute to its poor econometric performance.

In contrast, Eq. (20), which assumes value added, differs markedly from the standard output inventory model. In particular, only input and output inventory gap terms appear in the Euler equation because the value added restrictions eliminate the nongap terms. As a result, the value added output inventory model reduces to a relatively simple case of balancing input and output inventory gaps. Interestingly, the cost of frictions in the value added model are manifest through changes in the inventory gaps rather than through changes in production, as in the standard model.

Input and output inventory stocks interact directly and indirectly in the stage-of-fabrication model, and the modes of interaction are essentially the same in the gross production and value added versions of the model. Input inventories *directly* affect output inventories through the input inventory gap in the output inventory Eqs. (17) and (19). All else equal, an increase in the input inventory gap raises current and, due to adjustment costs, future output inventories.

The intuition behind this result is simple. Suppose the firm starts with zero inventory gaps. Then an increase in the input inventory gap involves a stockout cost. Because the stockout costs for both inventory types are quadratic, it is cost-minimizing to spread the stockout costs between inventory stocks rather than have one zero and one nonzero gap. The firm spreads excess input stocks between input and output inventories by drawing down input inventories through increased usage (and, hence, production). Given sales, this action necessarily raises the output inventory gap. Obviously, the extent of stock spreading that occurs depends on the actual magnitudes of production, adjustment, and output inventory stockout costs relative to input inventory stockout costs.

On the other hand, output inventories *indirectly* affect input inventories through the input inventory target stock, $M_t^*$, in Eqs. (18) and (20). All else equal, an increase in output inventories raises production, $Y_t$, and thus $M_t^*$ by a factor of $\theta$, thereby reducing the input inventory gap. The firm's optimal response to this change is to increase input inventories, albeit less than completely due to adjustment cost frictions. This indirect interaction is the main source of cross-equation restrictions in the model. In addition to this force, higher output inventory stocks generate higher production of finished goods which raises materials usage and thus draws down materials inventory stocks.

## 4. Econometric specification and estimation

Following Blanchard (1983), Eichenbaum (1984), and Fuhrer et al. (1995) (FMS), we estimate the stage-of-fabrication model by applying maximum likelihood to the decision rules rather than GMM to the Euler equations. Three factors argue for the maximum likelihood approach. First, instrumental variables estimators such as GMM tend to exhibit substantial biases and imprecision in small samples.[15] Second, FMS demonstrates that maximum likelihood estimates of a benchmark linear-quadratic output inventory model are less biased and more significant than GMM estimates in small samples. Third, our (unreported) attempts to estimate the model with GMM produced typical difficulties.[16]

Our estimation of the new stage-of-fabrication model is the most comprehensive to date. We estimate structural parameters from decision rules for output and input inventories jointly, imposing all cross-equation restrictions and transversality conditions. Furthermore, the maximum likelihood estimation permits examination of the dynamic properties of the inventory system.

The stage-of-fabrication model is a system of five equations: two Euler equations for the endogenous inventory stocks, $M_t$ and $N_t$, and three autoregressive auxiliary models for the variables $V_t$, $W_t$, and $X_t$. Following the bulk of the output inventory literature, we treat sales as exogenous in the estimation. In future work it will be important to relax this assumption. The system can be solved using the procedure developed by Anderson and Moore (1985), which generalizes Blanchard and Kahn (1980).[17] We use a two-step approximation to full-information maximum likelihood, in which parameters of the auxiliary models are estimated with OLS in the first step. This estimator is less efficient but asymptotically equivalent to full-information estimation and considerably faster—a major consideration given the complexity of the joint model.

[15] See West and Wilcox (1994) and FMS, and references therein.

[16] The GMM parameter estimates are mostly insignificant and highly sensitive to variations in normalization, instrument set, and other asymptotically irrelevant specifications. See also Humphreys (1995) for a discussion of problems with GMM estimation of a similar inventory model.

[17] For details on solution and estimation, see FMS and Humphreys et al. (1997).

We estimate the gross production and value added versions of the model with aggregate data for nondurable goods and durable goods industries.[18] Following the bulk of the applied inventory literature, we use data log detrended with linear and quadratic trends; results are qualitatively similar for data detrended with an HP filter. All regressions cover the period 1959:1 to 1994:5, less appropriate lags. The discount factor, $\beta$, is preset at 0.995, a common practice for structural estimation of this sort.[19] Standard errors are calculated using the method of Berndt et al. (1974).

## 5. Econometric results

This section reports econometric results for the stage-of-fabrication inventory models. Table 2 contains the joint maximum likelihood estimates for the gross production (GP) and value added (VA) models, plus a generalized gross production (GGP) model that is explained later in this section. $\partial^2 TC/\partial Y^2$ is the estimate of the slope of marginal cost. $2(\mathscr{L} - \mathscr{L}^{\mathscr{R}})$ is the $\chi^2$ statistic from the likelihood ratio test of the model's overidentifying restrictions, where $\mathscr{L}^{\mathscr{R}}$ denotes the likelihood of the restricted stage-of-fabrication model and $\mathscr{L}$ is the likelihood of the unrestricted reduced form of the stage-of-fabrication model. The *p*-values are in parentheses.

### 5.1. General results for GP and VA

The overall impression conveyed by Table 2 is that the parameter estimates for the VA model are consistent with the predictions of the model in both industry groups but those from the GP model are not. Every VA parameter is estimated significantly at the 5 percent level or better, and all estimates are the correct sign predicted by the model. In contrast, half or fewer of the GP parameters are estimated significantly, and some are the incorrect sign ($\gamma_3$ should be negative and $\gamma_4$ should be positive). Moreover, the magnitudes of the VA parameters are much more plausible and quite different than those of the GP parameters.

Quantitatively, the main differences between the GP and VA models arise in the target stock ($\alpha$,$\theta$), adjustment cost ($\varphi$), and delivery cost ($\phi$) parameter

[18] A worthwhile extension of this paper would be to estimate the models using more disaggregated data, such as the twenty nondurable and durable goods 2-digit SIC industries. However, the estimation process is extremely difficult and time consuming because of the extensive cross-equation restrictions, so we leave this substantial task for future work.

[19] Also, time variation in the discount rate makes the model nonlinear in variables, which our solution and estimation methodology does not allow. Ultimately, however, it would be preferable to incorporate a time-varying discount rate, as in Bils and Kahn (2000).

Table 2
Stage-of-fabrication model estimates[a]

| | Nondurables | | | Durables | | |
|---|---|---|---|---|---|---|
| Parameter | GP | VA | GGP | GP | VA | GGP |
| $\alpha$ | 7.85[b] | 0.76[b] | 0.99[b] | 4.84 | 0.77[a] | 0.22[b] |
| | (1.08) | (0.09) | (0.21) | (7.33) | (0.07) | (0.06) |
| $\theta$ | 2.42[b] | 1.09[b] | 1.37[b] | 1.48 | 1.52[a] | 0.84[b] |
| | (0.35) | (0.08) | (0.20) | (2.08) | (0.09) | (0.06) |
| $\gamma$ | | 2.15[b] | | | 1.09[b] | |
| | | (0.42) | | | (0.16) | |
| $\gamma_1$ | 0.23 | | 6.44 | 1.01[b] | | 2.67[b] |
| | (0.51) | | (6.92) | (0.40) | | (1.57) |
| $\gamma_2$ | 0.15 | | 3.29 | 0.47[b] | | 1.16[b] |
| | (0.88) | | (3.26) | (0.17) | | (0.55) |
| $\gamma_3$ | 0.001 | | 5.84[b] | 0.02[b] | | 1.88[b] |
| | (0.07) | | (2.10) | (0.01) | | (0.85) |
| $\gamma_4$ | − 0.03 | 9.21[b] | 4.04 | 0.02 | 34.2[b] | − 9.02[b] |
| | (0.05) | (4.41) | (3.92) | (0.05) | (2.90) | (3.64) |
| $\delta$ | 0.002 | 0.79[b] | 5.08[b] | 0.010[b] | 0.65[b] | 1.70[b] |
| | (0.20) | (0.13) | (1.78) | (0.004) | (0.07) | (0.40) |
| $\tau$ | 0.00009 | 0.89[b] | 0.98[b] | 0.0006[b] | 0.32[b] | 2.94[b] |
| | (0.001) | (0.10) | (0.41) | (0.0001) | (0.03) | (0.42) |
| $\varphi$ | 0.12 | 2.02[b] | 8.16[b] | 0.28 | 6.21[b] | 2.64[b] |
| | (2.40) | (0.17) | (2.21) | (0.19) | (0.34) | (0.58) |
| $\phi$ | 134.2[b] | 1.20[b] | 7.39[b] | 86.9 | 2.08[b] | 3.30[b] |
| | (1.35) | (0.03) | (1.78) | (197.4) | (0.11) | (0.83) |
| $\partial^2 TC/\partial Y^2$ | 0.47 | 6.19[b] | 22.7[b] | 1.57[b] | 13.5[b] | 7.95[b] |
| | (0.51) | (0.26) | (4.54) | (0.59) | (0.73) | (0.95) |
| $2(\mathscr{L} - \mathscr{L}^{\mathscr{R}})$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

[a]The models are estimated with maximum likelihood over the period 1959:1 through 1994:5, less appropriate lags. GP denotes gross production, VA denotes value added, and GGP denotes generalized gross production. Asymptotic standard errors are in parentheses except for $2(\mathscr{L} - \mathscr{L}^{\mathscr{R}})$, which is the $p$-value.

[b]Significance at the 5 percent level. See the text for details.

estimates. The VA target stock estimates are highly significant and close to the average inventory-sales ratios reported in Table 1. Also, the estimate of $\alpha$ is quite consistent with estimates from standard output inventory models reported in the literature. In contrast, GP estimates of $\alpha$ imply that firms aim to hold inventory stock five to eight times larger than monthly sales, which clearly is implausible. GP estimates of $\theta$ are more reasonable, but still two and one-half times larger in nondurables; both GP target stock parameters in durables are insignificant. GP estimates of $\phi$ are 40 to 100 times larger than the VA estimates, while the GP estimates of $\varphi$ are about an order of magnitude smaller and insignificant.

These substantial differences in parameter estimates between the two models illustrate the econometric consequences of failing to include materials usage in the labor adjustment cost specification of the GP model. To understand this, it is important to note that the inventory gaps, $M_t - \theta Y_t$ and $N_t - \alpha X_t$, are extraordinarily persistent, so inventory stocks deviate from their targets for very long periods—often many years.

It is well recognized that this persistence requires a cost of changing gross production in the standard output inventory model to justify sluggish inventory adjustment and fit the data. Because they include another (persistent) inventory stock, the stage-of-fabrication models also require an additional source of adjustment costs to fit the data. Implicitly, the GP model contains a cost of adjusting input inventory stocks via delivery costs, $(\phi/2)D_t^2 = (\phi/2)(U_t + \Delta M_t)^2$. But absent a cost of *changing materials usage*, the firm can vary usage and value added quickly and costlessly.

These characteristics help explain the econometric estimates. In the GP model, absence of an extra smoothing motive for usage and value added leads to very high estimates of delivery costs to justify persistent input inventory behavior. These costs are estimated to be so large that the firm maintains enormous output inventory stocks to guard against demand shocks. Such shocks would require substantial changes in production, which can be accommodated easily through changes in usage even though changes in labor are implicitly costly. The only way to prevent this output adjustment is to keep deliveries from changing much, and this explains the very high estimates of slope of marginal delivery costs ($\phi$). Without adjustment costs on changing materials usage, the cost of adjusting labor is improperly specified and thus its slope ($\varphi$) is estimated to be small and insignificant.

In contrast, the VA model includes a cost of changing materials usage. Note that, using the identity $U_t = D_t - \Delta M_t$, placing an adjustment cost on changing materials usage ($\Delta U_t$) implicitly places a cost on adjusting the change in input inventory *investment* ($\Delta^2 M_t$). This imparts additional persistence on input inventory stocks and alleviates the need to get persistence through higher estimates of the slope of marginal delivery costs. Instead, adjustment costs are spread evenly through the production process, and estimates of the slopes of marginal delivery costs and marginal labor adjustment costs are more reasonable. As a practical matter, the VA model includes more lags and more variables, both of which provide supplementary channels by which to capture persistence.

### 5.2. Exploring a hybrid model

Table 2 also includes results for a generalized gross production (GGP) model, a hybrid of the GP and VA models designed to determine why the

VA model produces better estimates. Like the VA model, the GGP model incorporates the change in materials usage in the adjustment cost component of the labor cost function, but it does not impose the VA restriction on production. Thus the GGP results help determine whether adjustment costs on the change in materials usage are responsible for the improved estimates.[20]

Estimation of the GGP model provides a test of this hypothesis for the difference between estimates of the GP and VA model. If the hypothesis is correct, then the value added production specification ($\gamma_1 = \gamma_2 = -\gamma_3 = \gamma$) should *not* be the reason the model has trouble fitting the data. Instead, the assumption that adjustment costs do not depend on the change in materials usage ($\gamma_7 = 0$) should be the reason.

As the table shows, the GGP model estimates are much closer to the VA model estimates. In particular, the GGP target stock estimates are considerably smaller and much more plausible. Also, the estimates of $\phi$ are much smaller and the estimates of $\varphi$ much larger, though they are still larger and smaller, respectively, than in the VA model. GGP estimates of both of these parameters are significant as well. Despite these improvements, however, the GGP estimates are not quite as supportive of the model as the VA estimates. Several GGP estimates are still insignificant, plus the $\gamma_3$ and $\gamma_4$ signs are still mostly incorrect. Thus, there appears to be additional benefit to imposing the VA restriction on the production process, which leads to notably different structure of the Euler equations.

[20] To obtain the generalized gross production specification, let

$$\gamma_4 = -\gamma_5, \quad \gamma_6 = -\gamma_7 = -\gamma_8 = \gamma_9 = 1.$$

Then the generalized gross production ($g^*$) cost function is

$$LC_t^{g^*} = (\tfrac{\gamma_1}{2})Y_t^2 + (\tfrac{\gamma_2}{2})U_t^2 + \gamma_3 Y_t U_t + \gamma_4 W_t(Y_t - U_t) + (\tfrac{\varphi}{2})(\Delta Y_t - \Delta U_t)^2 - \varepsilon_{yt} Y_t. \tag{22}$$

A more general specification would leave $\gamma_5$ and $\gamma_7$ unrestricted, but we adopt the VA model restrictions on these parameters to isolate the effect of the value added restriction on the production function only. The Euler equation for input and output inventories, respectively, are

$$\begin{aligned} E_t\{ &-\phi\gamma_3 Z_{y_1} + \phi\varphi Z_{y_2} + \gamma_2\phi Z_{\Delta M_1} + \phi\varphi Z_{\Delta M_2} + \gamma_2 Z_{v_1} + \varphi Z_{v_2} + \phi\gamma_4 Z_{w_1} \\ &+ (\gamma_2 + \phi)\tau[M_t - \theta Y_t] + \varphi\tau[\Delta M_t - \theta\Delta Y_t] - \phi Z_{\varepsilon_y 1} + (\gamma_2 + \phi)Z_{\varepsilon_m 1} + \varphi Z_{\varepsilon_m 2}\} = 0 \end{aligned} \tag{23}$$

$$\begin{aligned} E_t\{&[\gamma_1(\gamma_2 + \phi) - \gamma_3^2]Z_{y_1} + \varphi(\gamma_2 + \phi + 2\gamma_3 + \gamma_1)Z_{y_2} - \gamma_3\phi Z_{\Delta M_1} + \varphi\phi Z_{\Delta M_2} \\ &- \gamma_3 Z_{v_1} + \varphi Z_{v_2} + \gamma_4(\gamma_2 + \gamma_3 + \phi)Z_{w_1} - \theta\tau(\gamma_2 + \phi)[M_t - \theta Y_t - \beta(M_{t+1} - \theta Y_{t+1})] \\ &- \theta\tau\varphi[\Delta M_t - \theta\Delta Y_t - \beta(\Delta M_{t+1} - \theta\Delta Y_{t+1})] + \delta(\gamma_2 + \phi)[N_t + \alpha X_t] \\ &+ \delta\varphi[\Delta N_t - \alpha\Delta X_t] - [\gamma_2 + \gamma_3 + \phi]Z_{\varepsilon_y 1} + \varphi Z_{\varepsilon_n 2} + (\gamma_2 + \phi)Z_{\varepsilon_n 1}\} = 0 \end{aligned} \tag{24}$$

Derivation of this model is available in the unpublished appendix.

### 5.3. Specific results

Beyond the general conclusion that the VA model fits the data better, several more specific conclusions emerge from the VA results:

*Convexity*—The slope of marginal cost is positive and significant, indicating that aggregate cost functions are convex and providing additional evidence against Ramey's (1991) claim to the contrary. Our results extend the evidence against nonconvex aggregate costs in two ways. First, aggregate costs are even more convex in durable goods industries, where nonconvexities are most often surmised to arise, at least at the micro level.[21] Second, the results point to convex costs even in the presence of input inventories. If material costs are linear, or there are fixed ordering costs, input inventories would follow nonconvex $(S, s)$ rules that presumably could spill over into production behavior through stage-of-fabrication linkages.

*Wages and prices*—The results are generally consistent with theoretical predictions regarding real wages and materials prices. Marginal labor costs, $\gamma_4$, are positive and significant in the VA model. Although there are no specific parameters associated with real materials costs, the forward-looking speculative behavior implied by the model receives support from the overall success of the VA model in particular. Thus, the stage-of-fabrication model differs from many previous attempts to include real wages and materials prices in inventory models.

*Industrial heterogeneity*—Several differences arise between the results for nondurable and durable goods industries. Marginal adjustment costs are larger for durable goods industries. Labor adjustment costs ($\varphi$) are about three times larger and delivery adjustment costs ($\phi$) are about twice as large. In contrast, the marginal cost of inventory stockout costs ($\delta$,$\tau$) are smaller in durable goods industries, and the marginal cost of input inventory stockouts ($\tau$) is smaller relative to the marginal cost of output inventory stockouts in durables. Marginal wage costs ($\gamma_4$) are about three times larger in the durable goods industry. Finally, the slope of marginal cost appears to be higher in the durable goods industry, at least for the VA model.

*Overidentifying restrictions*—The only substantive shortcoming of the model is that the overidentifying restrictions are overwhelmingly rejected. This rejection is a well-known problem that plagues not only inventory models but most structural macroeconomic models applied to aggregate data. The reason for the rejection is that the model residuals are extremely persistent, as they are in standard output inventory models. Schuh (1996) and Krane and Braun (1991)

[21] Bresnahan and Ramey (1994) report evidence of nonconvexities in auto production plants resulting from the fixed costs.

find that firm-level and detailed industry-level data, respectively, do *not* reject the overidentifying restrictions of a conventional output inventory model. This suggests the problem may be attributable to aggregation, and motivates future testing of this model with disaggregated data.

### 5.4. Dynamic properties

This section explores some of the dynamic properties of the stage-of-fabrication model applied to nondurable and durable goods industries. We only use the value added model because it yielded the best econometric estimates. Fig. 3 plots the impulse responses of inventories to sales and materials price shocks, $\varepsilon_x$ and $\varepsilon_v$, which are the most relevant and interpretable.

Input and output inventories rise, ultimately, in response to a positive sales shock. Initially, output inventories decline slightly, especially in durables, because the firm does not increase production in the short run as much as the sales shock due to adjustment costs. Not bound by the same constraint, input inventories rise immediately. The response of input inventories is bigger and

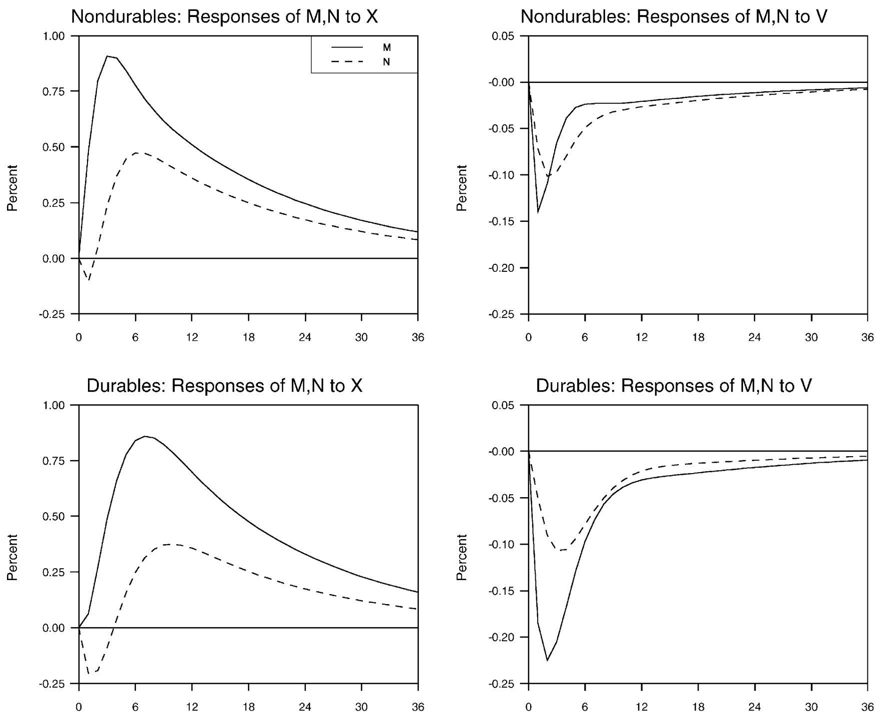


Fig. 3. Impulse response functions for the value added model.

faster than that of output inventories largely because adjustment costs on deliveries are smaller than adjustment costs on labor ($\varphi > \phi$).

Input and output inventories decline in response to a positive materials shock. The temporary price increase causes the firm to postpone deliveries and reduce input inventories, but the reduction in output inventories is more subtle. A negative input inventory gap emerges, which the firm wants to eliminate. With deliveries too dear, the firm must cut materials usage and hence production. With sales unchanged, output inventories decline too. Recall that convexity and balancing of marginal costs in the Euler equations make it optimal for the firm to endure two moderate inventory gaps rather than two disparate ones.

These dynamic patterns are broadly consistent with the data. Input inventories respond more, and more quickly, than output inventories to both shocks—behavior consistent with the stylized fact that input inventory investment is more variable than output inventory investment. Moreover, the relatively greater variance of input inventory investment is more pronounced in the durable goods industry, also consistent with the stylized facts.

Finally, the responses of both inventory stocks are quite persistent. For example, it take at least half a year for stocks to reach their peak response before declining gradually. Although this behavior seems consistent with the aggregate data, the micro foundations of such sluggish adjustment by firms continues to be a puzzle for the inventory literature.

## 6. Summary

This paper takes a step toward redressing the inventory literature's general neglect of input inventories, which are more important empirically than output inventories. It offers a viable new stage-of-fabrication model that extends the traditional linear-quadratic inventory model for output inventories to include the delivery, usage, and stocking of input materials. On balance, the econometric evidence suggests that the stage-of-fabrication model does a reasonable job of matching the data. The evidence is particularly striking in light of the very tight restrictions imposed by the joint estimation of input and output inventory decision rules.

Overall, the results clearly indicate that material inputs play an important role in understanding producer behavior, both theoretically and empirically. Producers' decisions of how much materials to order and how much materials to use in production affect—and are affected by—all aspects of production through dynamic stage-of-fabrication linkages. Failure to impose these linkages appears to be inconsistent with the data. The value added specification outperforms the gross production specification, and adjustment costs on the change in materials usage are critical to fitting the data.

The model should be viewed as a first step toward a more general stage-of-fabrication theory because several simplifications need to be relaxed. First, order backlogging (unfilled orders) should be introduced. Second, input inventories should be disaggregated into materials and work-in-process components, and the production process should be generalized to yield production of intermediate goods. Third, it would be desirable to incorporate general equilibrium linkages by explicitly modeling both sides of the upstream (materials) and downstream (finished goods) markets. Finally, further complexity of the model will put increasing stress on the heavily parameterized linear-quadratic framework, so it will probably be necessary to take a direct approach to specifying production and cost functions.

## Appendix A. Data appendix

The real inventory and shipments (sales) data are from the Census Bureau's Manufacturers' Shipments, Inventories, and Orders (M3) survey. The M3 data are seasonally adjusted and deflated in constant $1987 by the Bureau of Economic Analysis (BEA), as described by Hinrichs and Eckman (1981). Also, we marked up the inventory data from cost basis to market basis using the procedure outlined by West (1983). An implicit price index for shipments (final goods) is obtained from the ratio of real shipments to nominal shipments.

The nominal wage data are average hourly earnings of production or non-supervisory workers from the Bureau of Labor Statistics' (BLS) establishment survey. The wage data are seasonally adjusted. Real wages are obtained by deflating with the shipments implicit price index.

We constructed new materials price indexes for disaggregated industries because the BLS's Producer Price Program contains only aggregate manufacturing materials price indexes. Our materials price indexes are constructed from highly detailed commodity Producer Price Indexes (PPI) aggregated to the 2-digit SIC industry level using the information on the manufacturing industrial input–output structure from the *1982 Benchmark Input–Output Tables of the United States* (1982) (U.S. Department of Commerce (1991)). These disaggregated materials price indexes are available upon request. See Humphreys et al. (1997) for more details.

In the annual NBER data, material deliveries are obtained by adding materials usage, less energy, to materials and supplies inventory investment. Thus, the definition of input inventories include only materials and supplies stocks in these data, and not work-in-progress stocks as is in the remainder of the paper.

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