Inventories, Employment and Hours[⊗]

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Abstract

This paper develops a model that integrates inventory and labor decisions. We extend a model of inventory behavior to include a detailed specification of the role of labor input in the production process, distinguishing between employment, hours and effort per worker. We estimate jointly the Euler equations for inventories and employment, a labor compensation schedule, and an hours requirement function with the cross-equation restrictions imposed. The econometric results shed light on several important topics, including the shape of the marginal cost of output, the role of labor hoarding as an explanation of pro-cyclical productivity, and the persistence of inventory stocks.

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1. Introduction

In the last two decades extensive research has been done on inventory movements. This research has focused on an analysis of several issues that are important for business cycle research.[1] The issues have been provoked by several stylized facts. One provocative fact is that in aggregate manufacturing and in most industries the variance of production exceeds the variance of sales, which contradicts the basic idea that inventories are held by firms primarily to smooth production relative to sales. This fact raised questions about, and stimulated research on, whether the short-run marginal cost of producing output is upward-sloping, which is a rather important assumption in much of macroeconomics research. A second fact is that estimates of adjustment speeds, which govern the adjustment of actual to "desired" inventory stocks, tend to be very low, or, to put the matter in more contemporary parlance, inventory stocks tend to be quite persistent. This fact spawned research on the question of whether there are important costs to changing the level of output. This is because a shock that perturbs desired inventory stocks will cause inventories to return slowly to desired levels if firms adjust production gradually.[2]

To analyze these issues, the bulk of the literature has utilized a linear-quadratic model. The model assumes that the typical firm minimizes the discounted value of expected costs. Costs consist of production costs that depend on the level of output, convex adjustment costs associated with changes in the level of output, and inventory

[1] See Blinder and Maccini (1991a,b) and Ramey and West (1999) for surveys of the literature and the presentation of relevant stylized facts.

[2] Research intended to explain one or both of the stylized facts has been done with models that emphasize stock-out avoidance motives (Blanchard, 1983; West, 1986; Kahn, 1992; Bils and Kahn, 2000), observable cost shocks in the form of real input prices (Miron and Zeldes, 1988; Durlauf and Maccini, 1995), unobservable technology shocks (Eichenbaum, 1989; Kollintzas, 1995), and declining marginal production costs (Ramey, 1991).

holding costs. To rationalize the shape of short-run production costs, an appeal is typically made to the short-run returns to labor, with diminishing (increasing) returns to labor signifying a rising (decreasing) marginal cost of output. Similarly, to rationalize adjustment costs, an appeal is made to costs attached to changing the labor force in the form of, for example, hiring and firing costs.[3] A difficulty with the model, however, is that the specification of production costs and adjustment costs is not directly derived from the role of labor input in the production process and from the nature of the labor cost structure the firm faces.

Similarly, extensive research has been done on labor demand over the last couple of decades. One of the aims of this research is to explain pro-cyclical movements in labor productivity.[4] Potential explanations hinge on technology shocks, labor hoarding or increasing returns. The empirical work on labor demand and pro-cyclical labor productivity has been conducted largely with data for manufacturing industries. These industries, however, hold inventories. Yet, the empirical work that has been done has generally ignored the fact that labor demand decisions by business firms are typically made in an environment in which inventory decisions are made as well.[5]

The issues that have been the focus of both debates--such as, why output appears to fluctuate more than sales, why inventory stocks are so persistent, and why labor

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[3] Adjustment costs are of course also associated with changes in the capital stock, but given the high frequency nature of inventory decisions, it is natural to put more emphasis on changes in the labor input.

[4] See Hamermesh (1993) and Rotemberg and Woodford (1999) for surveys. Relevant contributions have been made by Basu (1996), Basu and Kimball (1997), Bernanke and Parkinson (1991), Bils and Chang (1999), Bils and Cho (1994), Burnside, Eichenbaum and Rebelo (1993), Caballero and Lyons (1992), Chirinko (1995), Fair (1985), Fay and Medoff (1985), Morrison and Berndt (1981), Rotemberg and Summers (1990), and Sbordone (1996).

[5] Note that the early literature, mostly employing flexible accelerator models, recognized the interaction between inventories, employment and hours: see Maccini and Rossana (1984) and Rossana (1990). Haltiwanger and Maccini (1989) study empirically the interaction of inventories, hours, new hires and temporary and permanent layoffs.

productivity is pro-cyclical--hinge on the parameter estimates that emerge from the models that have been used in empirical work. Despite considerable empirical research on inventories alone or on movements in labor productivity alone, the issues under debate essentially remain open.

The purpose of this paper is to develop a model that integrates inventory and labor decisions. We extend a model of inventory behavior to include a detailed specification of the production process, the role of labor input in the production process, the structure of labor costs and the nature of adjustment costs for labor. In particular, we distinguish between altering labor input along the extensive and intensive margins, and accordingly decompose labor input into an employment decision—the extensive margin—and hours worked per worker and effort decisions—the intensive margins.[6] We allow for adjustment costs associated with employment changes and for a labor compensation schedule that is nonlinear in hours worked. The model of course includes as well the benefits and costs of holding inventories. Furthermore, we depart from the standard linear-quadratic inventory model by using a translog approximation to production costs. Alternatively, the model can be thought of as an extension of the models used in the labor demand and labor productivity literature to allow for inventory decisions.

These extensions yield several potential advantages to empirical work on inventories and labor demand. Our model yields a system of Euler equations for inventories and employment together with an hours requirement function and a compensation equation. We estimate this system of equations jointly, making full use of the cross equation restrictions implied by the theoretical model. The joint estimation

[6] Here, we follow the literature beginning with Feldstein (1967) that distinguishes between hours and employment decisions. See Hammermesh (1993) for a survey.

procedure has two advantages. The first is econometric: it gives rise to a potential gain in efficiency, as more information regarding the structure of the model is used in estimation than in econometric work done with models in the literature on inventories alone or in the literature on labor demand alone. The second is economic: joint estimation of the Euler equations together with the hours requirement function and the compensation equation yields estimates of important structural parameters that may differ from those obtained in empirical work with the inventory model alone or the labor demand model alone. This can be helpful in understanding some of the puzzles that are plaguing empirical work on inventory movements and pro-cyclical movements in labor productivity.

The model is estimated with monthly data on the nondurable aggregate and the two-digit industries of U.S. manufacturing that produce to stock. The empirical results reveal the advantages of estimating jointly the Euler equations for inventories and employment together with the compensation equation and the hours requirement function as they allow us to draw important conclusions on critical issues under debate in the inventory literature and in the labor productivity literature. Four in particular standout: First, using a new procedure for estimating the slope of marginal cost, we find evidence that the marginal cost of output is indeed generally upward-sloping, which suggests that other forces are needed to explain why production fluctuates more than sales and why labor productivity is pro-cyclical. Second, the results provide evidence of adjustment costs to labor, which is a necessary ingredient of an explanation for the pro-cyclical behavior of measured labor productivity based on labor hording. It can also help to explain persistent movements in inventories. Third, we find evidence in support of strict convexity of the labor compensation schedule in hours worked. This contributes to the rising marginal cost of production. Finally, we find that a Cobb-Douglas approximation of the production process, a special case of our model, is rejected by the data. This suggests that the wide use of such an approximation in empirical work, especially in the labor productivity literature, may be leading to incorrect inferences in some contexts.

The next section of the paper presents the model of inventories, employment, and effective hours that we work with. Succeeding sections report estimates of the parameters of our model and tests of the model with data from the nondurable sector and selected two-digit industries of US manufacturing. A concluding section summarizes the paper and suggests areas for further research.

2. A Model of Inventories and Employment

We begin with the firm's technology, which is represented by the following standard short-run neoclassical production function. Allowing for internal adjustment costs defined in terms of net changes in the quasi-fixed input, we define the production function in terms of value added:

$$VA_t = Y_t - M_t = \hat{F}(L_t, Z_t, \Delta L_t / L_{t-1}, T_t) \tag{1-a}$$

$$\hat{F}_L, \hat{F}_Z, \hat{F}_T > 0 \qquad \hat{F}_{\frac{\Delta L}{L_{-1}}} \underset{>}{\overset{<}{-}} 0 \quad \text{as} \quad \frac{\Delta L}{L_{-1}} \underset{<}{\overset{>}{-}} 0 \tag{1-b}$$

where VA_t is value added,[7] Y_t is gross output, M_t is the amount of materials and supplies purchased and used in the production process, L_t is the number of workers employed by the firm, Z_t is effective hours per worker, and $\Delta L_t / L_{t-1}$ is the net growth rate in the

[7] The definition of value added assumes for simplicity that the ratio of materials prices to output prices is constant. An interesting extension of the model is to explore the implications of variable real materials prices and the materials input decision, which we leave for future work.

firm's workforce. Effective hours worked per worker are defined as $Z_t = H_t X_t$ where H_t is the number of hours worked by each worker and X_t is work intensity, i.e. the effort level expended by a worker in each hour.[8] Note that we treat employment, which is the extensive margin by which labor input can be varied, as a separate input from effective hours, which is the intensive margin.

The production function of course depends also on the state of technology. Further, in accordance with the concept of a short-run production function, the capital stock is taken as given at each point in time, giving rise to strict concavity of the production function.[9] We denote the variable, T_t, as the "state of technology", but it is intended to capture both the usual state of technology and the capital stock, which shift the short-run production function over time. We assume these forces change relatively smoothly, and can thus be captured by trend movements.[10]

Suppose further that materials usage is proportional to output so that $M_t = \lambda Y_t$ and substituting into (1-a) yields a production function for gross output

$$Y_t = \frac{1}{1-\lambda}\hat{F}(L_t, Z_t, \Delta L_t / L_{t-1}, T_t) = \tilde{F}(L_t, Z_t, \Delta L_t / L_{t-1}, T_t) \tag{2-a}$$

which has the same properties as stated in (1-b), namely

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[8] See the recent studies by Basu (1996), Basu and Kimball (1997), Bils and Chang (1999) and Sbordone (1996) for investigations on the role of effort in explaining the procycilcality of measured productivity. Bils and Khan (2000) also allow for a role for effort in a model that includes inventories but abstracts from the cost of adjusting factors of production.

[9] We also abstract from changes in the utilization of capital. On this issue see Shapiro (1986a), Shapiro (1996), and Basu, Fernald and Shapiro (2001).

[10] Clearly an interesting extension of the model and empirical work is to add investment in the capital stock as a decision variable. See Galeotti, Guiso, Sack and Schiantarelli (1997) for a model with capital as a fixed factor and labor as a variable factor. A key difficulty here, however, is the lack of monthly data on the capital stock, rendering empirical work difficult. Further, intuition suggests that labor is apt to be a closer substitute for inventory stocks than the capital stock. We note that, even if adjustment costs for capital are present but are not interrelated with those of labor, the first-order condition for labor we derive still holds.

$$\tilde{F}_L, \tilde{F}_Z, \tilde{F}_T > 0 \qquad\qquad \tilde{F}_{\frac{\Delta L}{L_{-1}}} \overset{<}{\underset{>}{-}} 0 \quad \text{as} \quad \frac{\Delta L}{L_{-1}} \overset{>}{\underset{<}{-}} 0 \tag{2-b}$$

Note that strict concavity of the value added production function, (1-a), of course implies strict concavity of the gross output production function, (2-a). We focus on the production function for gross output because, when inventories are introduced below, it is gross output that constitutes additions to finished goods inventories.[11]

The stock of workers is a quasi-fixed factor of production, and thus generates adjustment costs to the firm when it changes its labor force. They reflect the hiring, training and firing costs the firm incurs when it devotes resources to the process of changing its labor force. For simplicity, at this stage, adjustment costs are assumed to be internal to the firm and thus take the form of foregone output.[12] Further, we assume that adjustment costs are convex and associated only with changing employment.[13] Finally, observe that, in line with existing empirical evidence, we are assuming that the firm incurs no costs of adjusting hours worked per worker.[14]

We assume that real labor costs are defined by

$$\omega_t L_t = \omega(H_t, X_t) L_t \tag{3-a}$$

with

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[11] In this paper we not only assume that materials usage is proportional to output, we also abstract from the holding of materials and supplies and works-in-progress inventories. See Humphreys, Maccini and Schuh (2001) for an analysis of the materials usage decision and the interaction of inventory holdings at different stages of fabrication. These are extensions that we leave for future work.

[12] We note that we experimented with a version of the model with external adjustment costs for labor. This produced essentially the same results.

[13] We are well aware that there is an extensive literature that suggests the need to depart from symmetric quadratic adjustment costs. See Hamermesh and Pfann (1996) for a review of the literature. In this paper, however, we decided to maintain this hypothesis in order not to depart in too many directions from the standard linear quadratic inventory model. Moreover, even with convex adjustment costs, the model is already quite complex. Accounting for non-convexities is thus left for future work.

[14] See, for example, Shapiro (1986b), Bils (1987), Hamermesh (1993).

$$\omega_E > 0 \qquad \omega_X > 0 \tag{3-b}$$

$$\omega_{EE} \geq 0 \qquad \omega_{XX} \geq 0 \qquad \omega_{EE}\omega_{XX} - \omega_{EX}^2 \geq 0 \tag{3-c}$$

$\omega_t = \omega(H_t, X_t)$ is real labor cost or real labor compensation per worker. We assume that labor costs are increasing in the number of hours worked and effort and are convex.[15] The remuneration of effort can be explicit (as in the case of piece rates payment systems or of bonuses linked to the achievement of production targets) or implicit (as in the case of promotions linked to performance).

Hours worked per worker and effort are both variable factors of production. Further, we assume that, when the firm makes decisions on H_t and X_t at time t, it knows the exogenous variables for time t, including those that determine the labor compensation schedule. Once the labor compensation schedule is specified below, these will be the straight-time hourly wage rate and benefits per worker. Hence, the firm chooses H_t and X_t to minimize real labor costs $\omega(H_t, X_t)L_t$ subject to the production function, (2-a), and the definition of effective hours per worker, $Z_t = H_t X_t$. The optimality condition reduces to

$$\frac{\omega_H(H_t, X_t)H_t}{\omega(H_t, X_t)} = \frac{\omega_X(H_t, X_t)X_t}{\omega(H_t, X_t)} \tag{4}$$

Condition (4) relates hours worked and effort along the optimal path. It states that the elasticity of the labor cost function with respect to hours must equal the elasticity with respect to effort, as in Basu and Kimball (1997). Under appropriate conditions, (4) implies that effort is an increasing function of the number of hours worked, that is

[15] See Bils and Cho (1994) for a general equilibrium model that relates implicitly the convexity of labor cost in hours and effort to the underlying preferences of households.

$$X_t = X(H_t) \tag{5}$$

with $X'(H_t) > 0$.[16] This condition is very useful for estimation purposes because it allows one to eliminate from the problem the unobservable effort variable.

Substituting (5) into (2-a) yields the effective production function

$$Y_t = \tilde{F}(L_t, X(H_t)H_t, \Delta L_t / L_{t-1}, T_t)$$

$$= F(L_t, H_t, \Delta L_t / L_{t-1}, T_t) \tag{6}$$

where

$$F_H = \tilde{F}_Z \frac{\partial Z_t}{\partial H_t} = \tilde{F}_Z \left[X(H_t) + H_t X'(H_t) \right] > 0$$

and where $F_L = \tilde{F}_L$, $F_{\frac{\Delta L}{L_{-1}}} = \tilde{F}_{\frac{\Delta L}{L_{-1}}}$, and $F_T = \tilde{F}_T$. We also assume that the effective production function is strictly concave. The effective production function has the advantage that it eliminates explicitly the unobservable effort variable and variation in hours now carries with it variation in effort.

Now, inverting (6) yields an hours requirement function

$$H_t = H(Y_t, L_t, \Delta L_t / L_{t-1}, T_t) \tag{7}$$

with

$$H_Y > 0 \qquad H_L, H_T < 0 \qquad H_{\frac{\Delta L}{L_{-1}}} \gtrless 0 \quad \text{as} \quad \frac{\Delta L}{L_{-1}} \gtrless 0$$

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[16] More specifically

$$\frac{dX_t}{dH_t} = \frac{\omega_H + \omega_{HH} H_t - \omega_{HX} X_t}{\omega_X + \omega_{XX} X_t - \omega_{HX} H_t} > 0$$

This follows from our assumptions that labor cost per employee is increasing in H_t and X_t so that $\omega_H > 0, \omega_X > 0$, is convex so that $\omega_{HH} \geq 0, \omega_{XX} \geq 0$, and the additional assumptions that $\omega_{HH} H_t - \omega_{HX} X_t \geq 0$ and $\omega_{XX} X_t - \omega_{HX} H_t \geq 0$.

The hours requirement function is strictly convex, given the strict concavity of the effective production function and some additional mild restrictions on the production function.[17]

Further, substituting (5) into (3-a), real labor costs can now be written as

$$\omega(H_t, X_t)L_t = \omega[H_t, X(H_t)]L_t = \hat{\omega}(H_t)L_t \tag{8}$$

with $\hat{\omega}' = \omega_H + \omega_X X'(H_t) > 0$. We further assume that $\hat{\omega}(H_t)$ is convex in hours worked.

The firm is assumed to minimize the discounted value of its expected real costs, which consist of real labor costs and real inventory holding costs. That is, the firm minimizes

$$E_t \sum_{j=0}^{\infty} \beta^j [\hat{\omega}(H_{t+j})L_{t+j} + \Phi(N_{t+j-1}, S_{t+j})] \tag{9}$$

where $\hat{\omega}(H_t)L_t$ is real labor costs to the firm, $\Phi(N_{t-1}, S_t)$ is the real cost of holding inventories, N_t is the stock of real finished goods inventories at the end of period t, S_t is real sales, β is a constant real discount factor, and E_t is an expectation operator conditional on information available at time t. We will assume that the firm takes sales as given,[18] which is common in the inventory literature, and to be a price-taker in input markets and thus takes real input prices as given.

[17] The restrictions are either $F_{HL} > 0$, or $F_H F_{LL} - F_L F_{LH} < 0$ and $F_L F_{HH} - F_H F_{LH} < 0$.

[18] This assumption is stronger than necessary. An alternative is to assume that the firm has price setting power, in which case price or sales is an endogenous variable and the optimality conditions require equality between marginal revenue and marginal cost. In principle one could estimate the condition for sales jointly with the cost minimization conditions. This however, would require making specific assumptions about the structure of demand and the determination the markup of prices over marginal costs. In this paper, we focus only on the cost minimization conditions, and leave extensions of the model to deal with the price/sales decision for future work.

Inventory holding costs, $\Phi(N_{t-1}, S_t)$, are assumed to be U-shaped in the stock of inventories, given sales. Inventory holding costs balance two forces. They tend to rise with the stock of inventories, reflecting increased storage costs, insurance costs, etc. On the other hand, they tend to fall with inventory stocks, reflecting the idea that higher inventory stocks, given sales, enable the firm to avoid "stock-outs" and therefore avoid suffering lost sales. The presence of sales in the inventory holding cost function captures the idea of an "accelerationist" motive to holding inventories, as higher expected sales induce the firm to add to inventories to avoid stockouts.

The minimization of the present value of expected total costs, (9), is subject to the hours requirement function, (7), and to the inventory accumulation equation

$$N_t - N_{t-1} = Y_t - S_t \tag{10}$$

where N_{t-1}, the initial stock of inventories, is taken as given by the firm.

After some manipulation, the optimality conditions for inventories and employment reduce to

$$\begin{aligned} E_t \{ & \hat{\omega}'(H_t) L_t H_Y(Y_t, L_t, \Delta L_t / L_{t-1}, T_t) + \beta \Phi_N(N_t, S_{t+1}) \\ & -\beta \hat{\omega}'(H_{t+1}) L_{t+1} H_Y(Y_{t+1}, L_{t+1}, \Delta L_{t+1} / L_t, T_{t+1}) \} = 0 \end{aligned} \tag{11}$$

$$\begin{aligned} E_t \{ & \hat{\omega}(H_t) + \hat{\omega}'(H_t) L_t H_L(Y_t, L_t, \Delta L_t / L_{t-1}, T_t) + \hat{\omega}'(H_t) \left(L_t / L_{t-1} \right) H_{\frac{\Delta L}{L_{-1}}}(Y_t, L_t, \Delta L_t / L_{t-1}, T_t) \\ & -\beta \hat{\omega}'(H_{t+1}) \left(L_{t+1} / L_t \right)^2 H_{\frac{\Delta L}{L_{-1}}}(Y_{t+1}, L_{t+1}, \Delta L_{t+1} / L_t, T_{t+1}) \Big\} = 0 \end{aligned} \tag{12}$$

where, using (10), output is defined by $Y_t = S_t + N_t - N_{t-1}$, and hours per worker are defined by (7). We assume that firm makes decisions at time t on inventories and employment before it knows sales, the variables that determine the labor compensation schedule (the straight-time hourly wage rate and benefits per worker), and the stochastic component of technology.[19]

Condition (11) is the optimality condition on inventories. It requires the firm to balance the marginal cost of producing a unit of output this period and holding it in inventories with the marginal cost of producing it next period.

Condition (12) is the optimality condition on employment. It requires the firm to equate the marginal gain to acquiring an additional worker with the marginal cost of acquiring an additional worker. The former includes the reduction in cost due to the decrease in the number of hours or effort level required to produce a given level of output when the stock of workers increases (for given adjustment costs). The latter includes both the increase in the remuneration per employee and the net marginal hiring and training costs.

Finally, substituting the hours requirement function into real labor costs yields short-run production cost function implied by the model

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[19]Recall from above that we assumed that the decisions on H_t and X_t at time t are made after the exogenous variables dated t are known. We thus assume that the firm makes decisions on N_t and L_t on the basis of an information set that is smaller (since it excludes sales, the variables that determine labor compensation, and the technology shock at time t) than the one that it uses when it makes decisions on H_t and X_t. Essentially, at time t, the firm sets expected employment and expected inventories before the exogenous variables are revealed. The firm is assumed to carry out its plan for employment. Then, after the exogenous variables are revealed, the firm adjusts actual hours and effort and actual inventories to absorb shocks. This structure of decision-making is analogous to that used in the papers on labor-hoarding and factor-hoarding by Burnside, Eichenbaum and Rebelo (1993) and Burnside and Eichenbaum (1996).

$$\hat{\omega}(H_t)L_t = \hat{\omega}\left[H(Y_t, L_t, \Delta L_t / L_{t-1}, T_t)\right]L_t$$

$$= C(Y_t, L_t, \Delta L_t / L_{t-1}, T_t), \qquad \text{(13-a)}$$

with

$$C_Y = \hat{\omega}' H_Y L_t > 0 \qquad C_T = \hat{\omega}' H_T L_t < 0 \qquad \text{(13-b)}$$

$$C_L = \hat{\omega}\left[1 + \left(\frac{H}{\hat{\omega}}\hat{\omega}'\right)\left(\frac{L}{H}H_L\right)\right] > 0 \qquad \text{(13-c)}$$

$$C_{\frac{\Delta L}{L}} = \hat{\omega}' H_{\frac{\Delta L}{L}} L_t \gtrless 0 \quad \text{as} \quad \frac{\Delta L_t}{L_t} \gtrless 0 \qquad \text{(13-d)}$$

where we have assumed that $\left(\frac{H}{\omega}\hat{\omega}'\right)\left(\frac{L}{H}H_L\right) > -1$ to insure that costs rise with higher employment. Observe that the short-run production function implied by the model depends on the level of output, the stock of employed workers, adjustment costs associated with changes in the stock of employed workers, and the state of technology. Further, straightforward calculations reveal that short-run production costs are convex, given the convexity of the hours requirement function, the convexity of labor costs, and mild restrictions on the elasticities of the hours requirement function and the structure of labor costs.[20]

Before proceeding, it is useful to compare the structure of our model with that of the standard inventory and labor demand models. In contrast with the standard inventory

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[20] Sufficient conditions for convexity are $\left(\frac{H}{\omega}\hat{\omega}''\right)\left(\frac{L}{H}H_L\right) < -1$ and $\frac{Y}{H_Y}H_{YY} < \frac{Y}{L}\frac{H_Y}{H_L}[1 + \frac{L}{H_Y}H_{YL}]$. Further, a sufficient condition to ensure that $C_{YL} < 0$ is that $\left(\frac{L}{H_Y}H_{YL}\right) < -1$

model, we have allowed for an explicit specification of the production process with a distinction between employment, hours worked, and effort per worker, with labor costs reflecting the need to remunerate these different dimensions of the labor input. In addition, observe that labor costs, which are the analogue to short-run production costs in the standard inventory model, depend, on the stock of the quasi-fixed factor—the number of workers employed—as well as on the level of output. Finally, labor costs incorporate adjustment costs, which are captured by the change in employment and not by the change in output, as it is assumed in a rather *ad hoc* fashion in the standard model.

In contrast with the standard labor demand model, we have allowed for inventory decisions. In particular, changes in demand or costs, which produce changes in employment, hours worked or effort in the standard model, may be satisfied at least in part by inventory movements in our model. This significantly expands the margins along which the firm may respond to exogenous shifts in demand or costs.

3. The Translog Model

3.1. General Translogarithmic Specification

Our empirical objective is to estimate (11) and (12) jointly with the labor costs function and the hours requirement function. To accomplish this requires a parameterization of the hours requirement function, inventory holding costs and labor costs per worker, to which we now turn.

We utilize a translog approximation for the firm's hours requirement function. Specifically

$$\ln H_t = \alpha_0 + \alpha_Y \ln Y_t + \alpha_L \ln L_t + \alpha_T \ln T_t + 0.5\gamma_{YY} (\ln Y_t)^2$$

$$+0.5\gamma_{LL} (\ln L_t)^2 + 0.5\gamma_{TT} (\ln T_t)^2 + \gamma_{LY} \ln L_t \ln Y_t \tag{14}$$

$$+\gamma_{YT} \ln T_t \ln Y_t + \gamma_{LT} \ln T_t \ln L_t + 0.5\psi \left(\frac{\Delta L_t}{L_{t-1}} \right)^2$$

where the α_i's are first-order translog parameters and the γ_{ij}'s are second-order translog parameters of the production process and ψ is an adjustment cost parameter. Note that to achieve a specification that is parsimonious in the parameters to be estimated we have assumed that hours requirement function is multiplicatively separable in adjustment costs.[21] We assume that the state of technology contains a deterministic component that grows at a constant rate so that $\ln T_t = \varsigma t$. We normalize ς to unity so that hereafter $\varsigma = 1$.[22] The state of technology also contains a stochastic component, which will be introduced below.

Inventory holding costs are assumed to be given by

$$\Phi\left(N_{t-1}, S_t\right) = 0.5\delta \left(\frac{N_{t-1}}{S_t} - \theta \right)^2 S_t \tag{15}$$

with $\delta, \theta > 0$. Here, inventory holding costs are assumed to be proportional to sales, implying that marginal inventory holding costs depend on the inventory-sales ratio.

Finally, following Shapiro (1986b), Bils (1987), and Cooper (2002), labor costs are assumed to take the form

[21] See, for example, Chirinko (1995) for a similar parameterization of the hours requirement function in a model without inventories. See also Considine (1997) for the use of a translog cost function, albeit with a different specification of adjustment costs, in a model of a multi-output firm with inventories.

[22] A normalization is needed because ς cannot be separately identified from the parameters of the hours requirement function that involve the state of technology.

$$\hat{\omega}(H_t)L_t = \left\{ b_t + w_t^s \left[\bar{H} + \mu_o + \mu_1 \left(H_t - \bar{H} \right) + \frac{\mu_2}{2} \left(H_t - \bar{H} \right)^2 + \kappa_t + \xi_{1t} \right] \right\} L_t \qquad (16)$$

Labor costs per employee, $\hat{\omega}(H_t)$, are essentially the sum of three components. The first component, b_t, is the portion of real non-wage compensation per worker that is unrelated to hours. The second component is a payment for normal hours worked, and is a product of the straight-time hourly wage rate, w_t^s, and the normal level of hours worked, $\bar{H}$. The final component assumes that overtime wages are approximately quadratic in hours worked per employee. The variable, κ_t, captures linear and quadratic time trends. Finally, ξ_{1t} is a random component that can be thought of as an approximation/measurement error.[23] This formulation allows for rising marginal labor costs to increasing hours, as long as $\mu_2 > 0$, and captures implicitly the effects of expanding both hours and effort. Moreover, when combined with adjustment costs on employment, the cost structure implies that the firm faces rising marginal cost to expanding both the labor force and hours worked per worker.

Assuming rational expectations and thus replacing expected values with actual values, the Euler equations for inventories and employment are

$$w_t^s \left[\mu_1 + \mu_2 \left(H_t - \bar{H} \right) \right] \left(\frac{L_t H_t}{Y_t} \right) \left[\alpha_Y + \gamma_{YY} \ln Y_t + \gamma_{YL} \ln L_t + \gamma_{YT} t \right] + \beta\delta \left(\frac{N_t}{S_{t+1}} - \theta \right) \qquad (17)$$

$$-\beta w_{t+1}^s \left[\mu_1 + \mu_2 \left(H_{t+1} - \bar{H} \right) \right] \left(\frac{L_{t+1} H_{t+1}}{Y_{t+1}} \right) \left[\alpha_Y + \gamma_{YY} \ln Y_{t+1} + \gamma_{YL} \ln L_{t+1} + \gamma_{YT} (t+1) \right] = \xi_{2t+1}$$

[23] See footnote 30 for a further discussion of the nature of the error terms.

$$b_t + w_t^s \left[\bar{H} + \mu_o + \mu_1 \left(H_t - \bar{H} \right) + \frac{\mu_2}{2} \left(H_t - \bar{H} \right)^2 + \kappa_t \right] \tag{18}$$

$$+ w_t^s \left[\mu_1 + \mu_2 \left(H_t - \bar{H} \right) \right] H_t \left\{ \alpha_L + \gamma_{LL} \ln L_t + \gamma_{YL} \ln Y_t + \gamma_{LT} t + \psi \left(\frac{\Delta L_t}{L_{t-1}} \right) \left(\frac{L_t}{L_{t-1}} \right) \right\}$$

$$- \beta \psi w_{t+1}^s \left[\mu_1 + \mu_2 \left(H_{t+1} - \bar{H} \right) \right] H_{t+1} \left(\frac{\Delta L_{t+1}}{L_t} \right) \left(\frac{L_{t+1}}{L_t} \right)^2 = \xi_{3t+1}$$

where ξ_{2t+1} is a forecast error and ξ_{3t+1} is composite error that consists of a forecast error and the error in the labor compensation schedule.[24]

Observe that (17) and (18) contain cross-equation restrictions in that γ_{YL} and the parameters of labor costs appear in both equations. An advantage of estimating the inventory and employment Euler equations jointly is that no special normalization is needed to identify and interpret parameters. This is in contrast to the standard linear-quadratic inventory model. The reason is that labor costs are measured in labor cost per worker, so that the parameters of the employment Euler equation are identified and are measured in terms of units of labor costs.[25] Given that there are cross-equation restrictions, namely, for example, that γ_{YL} must be the same in both equations, and that the inventory and employment equations are estimated jointly, the parameters of the inventory equation are identified as well and are measured in terms of units of labor

[24] We assume that the error in the labor compensation schedule at time t is uncorrelated with the forecast errors for time t.

[25] Observe in particular that in (16) labor cost per worker

$$\hat{\omega}(H_t) = b_t + w_t^s \left[\bar{H} + \mu_o + \mu_1 \left(H_t - \bar{H} \right) + \frac{\mu_2}{2} \left(H_t - \bar{H} \right)^2 + \kappa_t + \xi_{1t} \right]$$

appears as a variable without a parameter attached to it. Hence, the parameters of the labor Euler equation are measured in terms of units of labor costs.

costs. This is a real advantage as it enables us to avoid issues regarding appropriate normalizations that are needed to interpret parameter estimates and that have plagued debates in the inventory literature.

Finally, using our assumptions about technology, and assuming that $\gamma_{TT} = 0$, we obtain a requirement function for hours worked, which is

$$\ln H_t = \alpha_0 + \alpha_Y \ln Y_t + \alpha_L \ln L_t + \alpha_T t + 0.5\gamma_{YY}(\ln Y_t)^2 + 0.5\gamma_{LL}(\ln L_t)^2 \qquad (19)$$

$$+\gamma_{LY} \ln L_t \ln Y_t + \gamma_{YT} t \ln Y_t + \gamma_{LT} t \ln L_t + 0.5\psi\left(\frac{\Delta L_t}{L_{t-1}}\right)^2 + \xi_{4t}$$

where ξ_{4t} is the stochastic component to the state of technology.[26] Equation (19) can be estimated jointly with the Euler equations (17) and (18) and the labor cost function, (16), yielding a system of four equations with a rich set of cross-equations restrictions.

3.2. Empirical Results: General Translog Model

We now present the empirical results obtained when the Euler equations for inventory and employment (17) and (18), the hours requirement function, (19), and the compensation equation (16) are estimated jointly by GMM.[27] We use monthly data on inventories, sales, hours and employment from the nondurable sector of the manufacturing sector of the US economy for the period 1959-1994. We present detailed results both for the nondurable aggregate and the two-digit industries within the

[26] Note that the stochastic shock to technology enters the log hours requirement function additively and thus will not appear in (17) and (18).

[27] The Euler equations are heavily nonlinear in the variables, rendering it impossible to solve the Euler equations for the decision rules and to estimate the parameters of the model with the decision rules.

nondurables sector that produce to stock.[28] The inventory and sales data are published by the Bureau of Economic Analysis. They are seasonally adjusted and expressed in constant 1987 dollars. The employment, hours and wage data are from the Bureau of Labor Statistics establishment survey, and refer to production workers, and the non-wage costs are data for fringe benefits from the U.S. Chamber of Commerce at annual frequencies.[29] Throughout the empirical work, we set the discount factor to .995, which is equivalent to an annualized discount rate of 6 percent.

The instrument set used in the GMM estimation includes lagged values (or functions of lagged values) of output, employment, hours, the straight-time wage rate, the inventory-sales ratio, and the growth rate of employment. (See the notes to Table 1 for details.) We have experimented with using different lags for the instruments and we present here the results obtained when the instrument set contains variables lagged twice, three times, four times, and five times. Moreover, we have allowed for heteroskedasticity, an MA(1) error structure, and contemporaneous correlation of the errors.[30]

Joint estimation of the two Euler equations for inventories and employment, the hours requirement function and the compensation equation imposes several cross

[28] Empirical work with durable goods industries requires extending the model to allow for the firm to hold work-in-progress and materials and supplies inventories. As Humphreys, Maccini and Schuh (2001) show, such input inventories are relatively more important in durable goods industries.

[29] We use the number of production workers for consistency with the available monthly wage data. All data are seasonally adjusted, with the exception of the straight wage data that does not display any pronounced seasonal pattern.

[30] Note that sales, the straight-time wage rate, and benefits per worker are not known when the employment and inventory decisions are made, so that the Euler equations for inventories and employment, (17) and (18), have MA(1) errors, given our assumption in footnote 25. If we also allow the error term in the labor compensation schedule and the stochastic shock to technology to have an MA(1) structures, then these assumptions imply that all the four equations in the system will have MA(1) errors. Moreover, we also allow the four error terms to be contemporaneously correlated. Finally, we have experimented with different assumptions about the MA structure of the error terms, in particular, MA(2) errors, obtaining similar estimates of the parameters and of their standard errors.

equation restrictions. Many of the technological parameters appear both in the Euler equations and in the hours requirement function. For example, an important parameter capturing the interaction between the inventory and employment decisions in our model is the parameter γ_{YL}, which is common to both Euler equations, multiplying current output in the equation for employment and current and future employment in the equation for inventories. Moreover, the parameters of the compensation equation, the μ_i's, also appear in the Euler equations. The inclusion of the compensation equation in the system helps to pin down parameter values, since the μ_i's appear in both the Euler equations and the compensation equation.

(Place Table 1 about here.)

In Table 1, we report results for the aggregate nondurable sector (in column 1) and for five two-digit industries within the nondurables sector that are commonly referred to as industries that produce to stock (in columns 2 through 6). These include Tobacco (SIC 21), Apparel and Textiles (SIC 23), Chemicals (SIC 28), Petroleum and Coal (SIC 29), and Rubber and Plastics (SIC 30).

The parameter estimates are very supportive of the model in both its inventory and labor demand dimensions. The target inventory-sales ratio, θ, is precisely estimated and the estimates suggest that firms hold finished goods inventories equal to as much as four weeks of sales in the Chemical sector and to as little as one week of sales in the Tobacco sector. Target inventories equal approximately three weeks of sales for the aggregate of nondurables. The slope of marginal inventory holding costs, δ, is positive, as expected, and significant at conventional levels for four sectors out of five (SIC 21, 23, 29, 30) and for the nondurable aggregate. However, it is negative and significant in SIC

28. In previous work that focuses exclusively on the inventory Euler equation, δ is not usually estimated, since the model requires a normalization and often δ is the parameter that is normalized to one. These results provide evidence for an "accelerator" motive for holding inventories for the vast majority of sectors.

The labor adjustment cost parameter estimate, ψ, is positive and highly significant in all cases but one (SIC 29). This is in accordance with our assumptions of strictly convex adjustment costs for labor. The existence of adjustment costs is a necessary condition for labor hording providing one of the explanations of the pro-cyclicality of measured labor productivity. In our model, since it is costly to change the number of workers, firms will react to favorable shocks, partly by increasing hours worked per worker and effort required by each worker. Obviously, labor hording may not be the only or the main explanation of pro-cyclical labor productivity in the presence of technology shocks. Further, adjustment costs for labor are also useful for rationalizing the persistence of inventory movements. Adjustment costs associated with changing the number of workers employed give rise to adjustment costs to changing output. In the face of adjustment costs to changing output, inventories will return slowly to desired levels when the latter are perturbed by shocks.

A word is in order regarding the quantitative importance of adjustment costs implied by our econometric estimates. The results reported in Table 1 suggest that adjustment costs, when correctly signed can be sizeable, but only if the firm tries to adjust fast on the extensive margin. For instance, using the parameter estimate for the nondurables aggregate, when the firm is expanding its workforce by 10 percent in a month, adding one more worker generates adjustment costs that are approximately half of the monthly wage bill for that worker. Further, since marginal adjustment costs are linear, the cost of adding one worker is half as large when employment is expanding by 5 percent. These figures suggest that adjustment costs are economically important for large changes in the labor force. For small changes, adjustment costs are less important.[31] This provides an incentive for firms to spread employment adjustment over several months.

The parameters of the compensation schedule, μ_1 and μ_2, are very tightly estimated and imply that labor compensation is increasing and strictly convex in hours worked, again with only the exception of SIC 29.[32] This is quite interesting because it means that firms face increasing marginal costs on both the extensive and the intensive margin, either in the form of increasing marginal adjustment cost when they expand the workforce, or in the form of rising marginal wages that they need to pay in order to elicit the desired hour or effort response. Furthermore, the estimates of the μ_i's are quite plausible in size. In particular, the estimate of μ_1 varies between 1.2 and 1.7 and the ones for μ_2 (with one exception) vary between .006 and .013. To give an idea of the impact of overtime on labor costs, consider the estimated parameters for the nondurables

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[31] Industry 28 is characterized by larger adjustment costs than the other industries, that display, instead, smaller adjustment costs than the nondurable aggregate. A comparison of our results on adjustment costs with those in the literature is not easy, since most empirical studies focusing on labor are done with lower frequency data. Moreover, there is no unique way of assessing the magnitude of adjustment costs, and the results depend crucially upon the rate of change of employment that is assumed in the calculations. That said, using quarterly data for manufacturing, Shapiro (1986b) finds that adjustment costs for production workers are not significantly different from zero, while adjustment costs for non-production workers are significant and quantitatively important. In Chirinko (1995), using annual data for the non-financial business sector, the own adjustment cost parameter for total employment is not significant but the one capturing interrelated adjustment cost with capital is significant. The size of adjustment costs is quantitatively small. The adjustment costs parameter for changes in total hours is statistically significant in Sbordone (1996) in a model with variable effort. The empirical literature, taken together, seems to suggest that labor adjustment costs are not large (see Hamermesh and Pfann (1996)).

[32] Recall that in our model variation in hours captures variation in effort so that convexity in hours picks up any convexity in the relationship between hours and effort or any convexity between effort and compensation.

aggregate. They imply that the overtime premium when hours worked are one higher than normal hours of forty hours per week is approximately half of the straight wage.[33] Moreover, the marginal cost of increasing hours starting from 41 hours per week is approximately 4 percent higher than it would be starting from normal hours. Our qualitative results on the convexity of the compensation schedule are consistent with the evidence presented in Bils (1987).

A further insight into the role of adjustment costs can be obtained by comparing the cost of adding one worker, working normal hours, to adding an equivalent number of hours by introducing one hour of overtime (going from 40 to 41 hours). The direct cost of an additional worker includes the wage, benefits and adjustment costs. Were it not for adjustment costs, it would be cheaper to hire one additional worker, since the overtime premium is approximately half of the straight wage and it exceeds the additional benefit costs, at their sample average value of 22 percent of wage cost. The presence of hiring costs and possible future firing costs increases the cost of hiring one additional worker. For instance, considering only the contemporaneous adjustment costs involved in hiring one worker, the cost of increasing the labor input along the extensive and intensive margin are equalized when employment expands by 5 percent monthly.

The parameters of the hours requirement function are also very precisely estimated. However, the signs and magnitudes of the first and second order parameters of the effective hours requirement function are by themselves not especially informative. Nonetheless, the parameter estimates can be used to check the convexity of labor costs employment and output. Since labor costs are in effect short-run production costs in our

[33] As used here, the "overtime premium" includes also the remuneration for the increased effort associated with increased hours (the two are positively correlated).

model, these are much more informative statistics. The formulae for convexity of labor costs, for a given level of $\Delta L_t / L_{t-1}$, are straightforward to calculate.[34] Using the parameter estimates from each column of Table 1, and evaluating the formulae at sample means, we report the results at the bottom of Table 1. Convexity requires C_{YY}, C_{LL} and $DET = C_{YY}C_{LL} - C_{YL}^2$ to be positive. Subscripts denote partial derivatives and *DET* is the second order principal minor.

C_{YY} is positive and significant in four sectors (SIC 23, 28, 29, 30) and for the nondurable aggregate, and negative and insignificant in another (SIC 21). This evidence implies that the marginal cost of output is upward-sloping, given the stock of the quasi-fixed factor, in a majority of sectors. C_{LL} is positive and significantly different from zero in all cases, with the exception of sector 29, where it is negative and significant. In the remaining one, it is fairly flat. *DET* is positive and significant in all cases, but for sectors 21 and 29, where is negative, but not significant.

These results imply that labor costs and therefore short-run production costs are indeed strictly convex in Y_t and L_t in most sectors and for the aggregate, given adjustment costs. The Petroleum and Coal sector (SIC 29) stands out as a problematic case. Further, it can easily be proven that, for positive values of ψ, δ and θ, convexity

[34] Specifically, the formulae needed to check the convexity of labor costs under the translog specification are:

$$C_{YY} = (LH/Y^2)[H\hat{\omega}_{HH}m_Y^2 + \hat{\omega}_H(\gamma_{YY} + m_Y(m_Y - 1))] > 0,$$
$$C_{LL} = (H/L)[H\hat{\omega}_{HH}m_L^2 + \hat{\omega}_H(\gamma_{LL} + m_L(m_L + 1))] > 0,$$
$$C_{YL} = (H/Y)[H\hat{\omega}_{HH}m_Y m_L + \hat{\omega}_H(\gamma_{YL} + m_Y)$$
$$DET = C_{YY}C_{LL} - C_{YL}{}^2 > 0$$

where, $m_Y = \alpha_Y + \gamma_{YY}\ln Y + \gamma_{YL}\ln L + \gamma_{YT}t$, $m_L = \alpha_L + \gamma_{YL}\ln Y + \gamma_{LL}\ln L + \gamma_{LT}t$,
$\hat{\omega}_H = w_t^s\left[\mu_1 + \mu_2\left(H_t - \overline{H}\right)\right]$, and $\hat{\omega}_{HH} = w_t^s\mu_2$.

of labor costs in N_t and L_t, for given $\Delta L_t / L_{t-1}$, is sufficient to guarantee the overall convexity of the cost function, provided that the change in employment is not too negative.

Moreover, C_{YL} is negative and significant for the nondurable aggregate, Tobacco, Apparel and Textiles, and Petroleum and Coal. This is what one would expect, given that employment is a quasi-fixed factor, which determines the position of short-run marginal cost, and increases in employment should reduce the marginal cost of output.

Consider next the slope of marginal cost with respect to output, which has been debated at length in the inventory literature.[35] As we have stated above, standard assumptions about the production function imply that labor costs are convex in output. Our specification of the slope of marginal cost with respect to output, however, differs from that in the standard linear-quadratic inventory model for two reasons. First, here we are keeping the quasi-fixed factor, L_t, constant when varying output, which is accomplished by varying the perfectly variable factor of production—effective hours worked per worker. Second, in the standard inventory model adjustment costs depend upon the change in output, whereas in our model adjustment costs are a function of the change in the number of workers. However, a specification of the slope of marginal cost with respect to output that is closer to that in the standard model is represented by MC_{YY} in Table 1. MC_{YY} is the derivative of the marginal cost of output calculated by allowing the stock of workers as well as effective hours worked per worker to change when output

[35] See Galeotti, Guiso, Sack and Schiantarelli (1997) for a discussion of the estimation of the slope of marginal cost in the presence of quasi-fixed factors of production. In particular, they point out the need to recognize that production costs depend on the stock of the quasi-fixed factor of production, as well as its change, in calculating the slope of marginal cost.

changes, for given adjustment costs.[36] The results indicate that marginal cost is increasing in output in the majority of cases: MC_{YY} is positive and significant in the nondurable aggregate and in three industries (SIC 28, 29, 30). It is positive but not quite significant in another (SIC 23), and negative and insignificant in the remaining industry (SIC 21), implying flat marginal cost. For no industry is there strong evidence of decreasing marginal cost.[37]

Finally, as Table 1 reveals, the J-Statistic, which is used to test of the over-identifying restrictions of the model, formally rejects the model at conventional significance levels. However, the rejections are not overwhelming in that the J-Statistic is generally just above the critical value that would warrant acceptance of the over-identifying restrictions at the 1% level. Such rejections are not unusual for traditional linear-quadratic inventory models. The rejections, however, should not be too surprising for two reasons. First, our four equation model imposes numerous cross equation restrictions. This gives rise to the unavoidable trade off between obtaining more precise estimates, when such restrictions are imposed, and putting the model to greater risk of

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[36] In general, to calculate the slope of the marginal cost of output, use (11) to compute

$$MC_Y = C_Y + C_L \frac{\partial L}{\partial Y}$$

and

$$MC_{YY} = C_{YY} + 2C_{YL}\frac{\partial L}{\partial Y} + C_{LL}\left(\frac{\partial L}{\partial Y}\right)^2 + C_L \frac{\partial^2 L}{\partial Y^2}$$

The first term in MC_Y is the marginal cost of output calculated by varying hours worked per worker. The second term in MC_Y is the product of the marginal cost of output calculated by varying the number of workers and the response of the number of workers to a change in output. The latter may be compute from the Euler equation for labor, which is (8) in general, or (18) for the general trans-log model that is used in the empirical work here.

[37] Although reached by a very different procedure, our overall conclusion in favor of increasing marginal cost of production is consistent with that reached by Bils and Kahn (2000), Durlauf and Maccini (1995), and Eichenbaum (1989).

rejection. Second, some studies have shown that the J-Test may lead to over-rejecting the null hypothesis in small samples.[38]

3.3. Some Special Cases

In this section we present results for some interesting special cases of our model obtained by assuming, first, that the production function is Cobb Douglas, second, that the representative firm does not hold inventories, and, third that the compensation schedule for labor costs is linear.

In recent empirical work on productivity, the Cobb-Douglas production function has been widely used. Moreover, the Cobb-Douglas production function has *de facto* become particularly prominent with the tendency in dynamic general equilibrium models to approximate the Euler equations and the production possibility set with their log-linearized counterparts. An advantage of using a translog hours requirement function is that a Cobb-Douglas specification is a special case of our model. Specifically, a Cobb-Douglas production function yields an implied effective hours requirement function that is a special case of (19) with all the second-order parameters, the γ_{ij}, set to zero, that is:

$$\gamma_{YY} = \gamma_{LL} = \gamma_{YL} = \gamma_{YT} = \gamma_{LT} = 0 \tag{20}$$

We call these the "Cobb-Douglas restrictions".

(Place Table 2 about here.)

We use a Wald statistic to test the hypothesis that the Cobb-Douglas restrictions are satisfied. As the Wald Statistic in Part A of Table 2 indicates, these restrictions are

[38] See Kocherlakota (1990) and Mao (1990). However, there is debate on this issue. See Tauchen (1986) for evidence of under-rejection and Ogaki, Jang and Lim (2003) for a summary discussion.

resoundingly rejected. In Table 2, for the sake of brevity, we also report for the restricted model selected coefficients, the summary statistics on the curvature of the cost function, and the J Statistic. It is useful to compare the parameter estimates from the restricted model with those of the unrestricted model presented in Table 1. One difference is that ψ, the adjustment cost parameter, in the restricted model is the wrongly signed and/or insignificant from zero in SIC 21, 23, 29 and 30, and it is less precisely estimated in the nondurable aggregate. These results are quite different from the unrestricted model and lead to the implication that adjustment costs are generally unimportant, which in turn has important implications for the debate on the role of labor hoarding in explaining the pro-cyclicality of labor productivity and in rationalizing the persistence of inventory movements. Another difference is that in the restricted model C_{YY} is negative and significant in the nondurable aggregate and SIC 21 and 30, and it is less precisely estimated in SIC 29. Similarly, MC_{YY} is now negative and significant for the nondurable aggregate and for SIC 21 and 30. These results suggest that there is less evidence that marginal cost is rising than is in fact the case in the unrestricted model. In summary, imposition of the Cobb Douglas restrictions is rejected by the data and leads to potentially incorrect inferences regarding the strength of adjustment costs to labor and the shape of the cost function.

Another interesting variation of the model is to explore the effects of ignoring the inventory decision. The literature on labor demand and in particular the recent literature that is focused on explaining pro-cyclical labor productivity has ignored the impact of the fact that manufacturing firms typically hold finished goods inventories. But, if firms hold inventories, our model implies that inventory decisions should interact with labor decisions, and thus estimates of labor Euler equations will in general be affected by whether or not inventory Euler equations are taken into account.

Part B of Table 2 presents the results for a model in which inventories are absent. In this case, production equals sales so that $Y_t = S_t$, and there is no Euler equation for inventories.[39] There is of course an Euler equation for employment that is estimated jointly with the hours requirement function and the compensation equation.[40] In this case, the estimates of the curvature of the cost function are very similar to the ones obtained in our model with inventories. The main difference is that the adjustment cost parameter, ψ, is now smaller and less precisely determined in all industries. This is contrary to the estimates of the adjustment cost parameter in the complete model with inventories. Hence, joint consideration of the employment and inventory decisions helps to sharpen the estimates of the cost of adjusting employment. This in turn provides evidence for labor hoarding as a potential explanation for pro-cyclical labor productivity.

Finally, we explore the implications for the model of assuming that the compensation schedule for labor costs is linear in hours worked, which is a widely used specification of labor costs in models underlying empirical work.[41] This specification is captured in our model by the restriction that $\mu_2 = 0$. We reported above that μ_2 is positive and significant in all but one industry. We thus reject the restriction that $\mu_2 = 0$, and conclude that marginal labor costs are rising in hours worked. We now explore the

[39] Note that we continue to use the data series for production used in estimating the equations of the complete model.

[40] Note that in this special case we are only estimating jointly a three equation model, rather than a four equation model as we do in the general case. This may explain the fact that the J-Test does not reject the over-identifying restrictions at conventional levels in two sectors, SIC 21 and 30.

[41] We note that in our model variation in hours worked also captures variation in effort in the same direction.

implications of assuming $\mu_2 = 0$ for the other parameters of the model and the curvature of the cost function. The empirical results for imposing this restriction are presented in Part C of Table 2. The main implication is that the slope of the marginal cost of production is substantially affected. As the Table indicates, when this restriction is imposed, C_{YY} and MC_{YY} are negative and significant in SIC 21, 29 and 30, whereas in the unrestricted model both were positive and significant, or insignificant. Further, C_{YY} and MC_{YY} tend now to be positive but insignificant in the nondurable aggregate and SIC 28, whereas they were positive and significant in the unrestricted model. Taken together, the results for the restricted model lead to the incorrect implication that the marginal cost of production is declining or flat, while in the unrestricted model we found that marginal cost is generally upward-sloping. Furthermore, the results highlight the importance that a rising marginal compensation schedule plays as a factor contributing to the rising marginal cost of production.[42]

4. Conclusions

In this paper, we propose a model that focuses on the interaction of inventory decisions and labor input decisions. Unlike the standard inventory model, we carefully specify the role of labor in the firm's decision process, including the decomposition of labor input into separate employment and effective hours worked decisions in the

[42] To see the importance of μ_2 for the slope of marginal cost, observe that the formulae for C_{YY}, etc. presented in footnote 31 indicate that $\hat{\omega}_H$ and $\hat{\omega}_{HH}$ play an important role in C_{YY} and ultimately, as footnote 33 indicates, for MC_{YY}. Note as well that, when $\mu_2 = 0$, $\hat{\omega}_{HH} = 0$ and $\hat{\omega}_H = w_t^s \mu_1$. The restriction that $\mu_2 = 0$ thus has an important effect on the slope of marginal cost.

production process, the structure of labor costs, and the nature of adjustment costs on labor. Further, we use a quadratic labor compensation schedule and a translog approximation for labor costs and thus production costs. Unlike the labor demand literature, we allow the firm to take into account inventory positions in making employment and hours decisions. The model yields Euler equations for both inventories and employment. The Euler equations are estimated jointly with the hours requirement function and the compensation schedule for labor costs taking into account all the relevant cross-equation restrictions. The model is estimated with monthly data on the non-durable aggregate and selected two-digit industries of U.S. manufacturing.

The empirical work yields a number of interesting results and implications:

(1) We find reasonably strong evidence that the short-run marginal cost of producing output is upward-sloping in non-durable manufacturing and all but one two-digit industry. This implies that the evidence does not support the idea that declining short-run marginal cost is the reason that production varies more than sales. Nor does it support the idea that short-run increasing returns to labor is an explanation for pro-cyclical labor productivity.

(2) We find considerable evidence that adjustment costs for employment play a significant role in inventory and labor demand decisions for the non-durable aggregate and for the vast majority of the two digit industries. There is, therefore, evidence that adjustment costs and labor hoarding are a potential explanation for the pro-cyclical behavior of observed labor (or total factor) productivity and that adjustment costs are a rationale for the persistence of inventory movements.

(3) We find widespread evidence that the compensation schedule for labor costs is strictly convex in hours worked. When combined with adjustment costs on employment, this implies that firms face increasing marginal costs in changing the labor input both along the extensive and intensive margins. Further, the strict convexity of the compensation schedule plays an important role in generating a rising marginal cost of production.

(4) We find strong evidence for an "accelerator" motive for holding inventories. The target inventory-sales ratio is very precisely estimated and plausible in all industries. Further, the estimates of the slope of inventory holding costs are positive and significant in the nondurable aggregate and all but one two-digit industry.

(5) Certain simplifying assumptions that are often made in the empirical and theoretical literature are rejected by the data. Specifically, the restrictions imposed on the model by a Cobb-Douglas specification are rejected. This result affects inferences regarding the slope of the marginal cost of production and the role of adjustment costs in labor productivity movements and the persistence of inventories.

Given that the empirical work demonstrates the importance of looking at the interaction of decisions by the firm, the model and empirical work need to be extended in a number of directions. The extensions are also prompted by the fact that the over-identifying restrictions of the model are formally rejected, though not overwhelmingly so. This suggests the possibility that some of the simplifying assumptions we have made need to be relaxed. One extension is to expand the margins along which the firm makes decisions to include the capital utilization and investment decisions. However, we note again that a difficulty here is obtaining reliable monthly data on the capital stock.

Another is to relax the assumption that the materials input decision is a fixed proportion of output and to allow for inventory decisions at different stages of fabrication, including not only finished goods decisions, which is done here, but also work-in-progress and materials and supplies decisions. This would permit the model to be applied to durable goods industries where work-in-progress and materials and supplies inventories play a relatively more important role.[43] Moreover, it would be interesting, although very complex due to the aggregation difficulties, to allow for fixed components in labor adjustment costs, in addition to the convex component used here. A final extension is to model the pricing decision of firms and to estimate it jointly with the conditions for cost minimization. All this is left for future work.

[43] See Humphreys, Maccini and Schuh (2001) for a model with inventories at different stages of fabrication but without the detailed structure of labor input decisions presented here.

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Table 1: General Model

Parameter	(1) Nondurables	(2) SIC 21	(3) SIC 23	(4) SIC 28	(5) SIC 29	(6) SIC 30
μ_o	6.256*** (0.102)	3.551*** (0.192)	6.175*** (0.134)	2.554*** (0.111)	0.869*** (0.144)	5.245*** (0.116)
μ_1	1.422*** (0.017)	1.205*** (0.019)	1.200*** (0.008)	1.283*** (0.021)	1.698*** (0.028)	1.370*** (0.014)
μ_2	0.013*** (0.0002)	0.0009*** (0.0001)	0.0008*** (0.0001)	0.014*** (0.001)	-0.017*** (0.001)	0.006*** (0.0004)
κ_1	0.012*** (0.001)	0.013*** (0.001)	-0.001* (0.0006)	0.014*** (0.0006)	0.009*** (0.001)	0.009*** (0.001)
κ_2	-0.00002*** (0.0000)	-0.00002*** (0.0000)	0.000003** (0.0000)	-0.00002*** (0.0000)	0.000005* (0.0000)	-0.00002*** (0.0000)
α_0	-12.455*** (0.618)	7.313*** (0.082)	8.629*** (0.170)	-202.716*** (11.032)	18.253*** (0.581)	5.976*** (0.182)
α_Y	0.244*** (0.037)	0.0003*** (0.0008)	0.00003*** (0.000)	-0.070*** (0.013)	-0.002*** (0.0003)	-0.006*** (0.0007)
α_L	4.811*** (0.159)	0.005 (0.035)	-0.008 (0.047)	76.283*** (3.555)	-4.584*** (0.241)	0.553*** (0.063)
α_T	0.002*** (0.0002)	0.001*** (0.0001)	0.002*** (0.000)	0.134*** (0.003)	0.003*** (0.0004)	0.0009*** (0.0001)
γ_{YY}	0.019*** (0.003)	-0.0002*** (0.000)	0.000004*** (0.000)	-0.002*** (0.0007)	-0.0001 (0.0001)	-0.0004*** (0.00008)
γ_{LL}	-0.638*** (0.020)	-0.216*** (0.009)	-0.135*** (0.007)	-13.721*** (0.573)	0.783*** (0.050)	-0.233*** (0.011)
γ_{YL}	-0.036*** (0.005)	0.00006*** (0.0000)	-0.000005*** (0.000)	0.012*** (0.002)	0.0005*** (0.0001)	0.001*** (0.0001)
γ_{YT}	-0.00004*** (0.0001)	0.00000*** (0.0000)	-0.00000*** (0.0000)	-0.00004* (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
γ_{LT}	-0.0002*** (0.0000)	-0.0008*** (0.0000)	-0.0005*** (0.000)	-0.021*** (0.0006)	-0.0007*** (0.0001)	0.0001*** (0.0004)

ψ	4.839*** (1.093)	0.045*** (0.016)	0.917*** (0.211)	97.834*** (27.812)	-0.310*** (0.096)	0.588*** (0.123)
θ	0.616*** (0.033)	0.244*** (0.007)	0.894*** (0.066)	0.813*** (0.015)	0.467*** (0.005)	0.532*** (0.015)
δ	101.262** (51.542)	1.079*** (0.130)	0.021*** (0.007)	-65.654*** (9.807)	8.067*** (0.843)	12.334*** (1.482)
C_{YY}	26.164*** (7.233)	-0.262 (0.216)	0.050** (0.021)	23.934*** (9.227)	0.855*** (0.367)	10.667** (5.544)
C_{LL}	0.597*** (0.031)	4.497*** (0.937)	0.136*** (0.037)	4588.13*** (306.168)	-113.394*** (13.764)	2.904*** (0.283)
C_{YL}	-1.173*** (0.168)	-0.086*** (0.020)	-0.0006*** (0.0002)	50.211*** (8.651)	-0.507*** (0.114)	0.576*** (0.079)
DET	14.234*** (4.710)	-1.187 (1.003)	0.007** (0.003)	107291.0*** (42104.5)	-97.248** (48.629)	30.649** (15.787)
MC_{YY}	23.857*** (7.391)	-0.361 (0.230)	0.029 (0.022)	24.888*** (9.825)	0.858*** (0.368)	12.018*** (5.694)
J Test	150.449 [0.004]	152.838 [0.002	148.393 [0.005]	165.636 [0.038]	150.589 [0.002]	140.288 [0.004]

Notes:

(i) Standard errors are in parentheses, and are estimated with a covariance matrix that allows for heteroscedasticity and MA(1) errors.

(ii) Three asterisks denote significance at the1% level; two asterisks denote significance at the 5% level; and one asterisk denotes significance at the 10% level.

(iii) Number of observations: 414.

(iv) The *J* test indicates the Test of Over-identifying Restrictions. P-value in square brackets.

(v) List of instrument: $(W_t^s L_t H_t / Y_t), (N_{t-1}/S_t), \overline{W}_t, \ln Y_t, (\ln Y_t)^2, H_t, (\Delta L_t / L_{t-1})^2, t, t^2$. Instruments are lagged two, three, four, and five periods (except for time trend).

(vi) No trend square in the instrument list of sector 29; no time trends in the instrument list of sector 30.

(vii) The parameters, κ_1 and κ_2 , are the parameters attached to the linear and quadratic time trends in the labor compensation equation.

Table 2: Special Cases

A. Cobb-Douglas Model

Parameter	(1) Nondurables	(2) SIC 21	(3) SIC 23	(4) SIC 28	(5) SIC 29	(6) SIC 30
μ_1	1.431*** (0.018)	1.246*** (0.019)	1.268*** (0.040)	1.157*** (0.018)	1.706*** (0.023)	1.351*** (0.011)
μ_2	0.016*** (0.0007)	0.001*** (0.0002)	0.003*** (0.0002)	0.023*** (0.001)	-0.018*** (0.0009)	0.009*** (0.0004)
ψ	4.322*** (1.518)	-0.086 (0.078)	-0.279 (0.313)	235.536*** (59.951)	-1.010*** (0.081)	0.180 (0.155)
θ	0.642*** (0.068)	0.245*** (0.002)	0.700*** (0.004)	0.814*** (0.003)	0.495*** (0.002)	0.634*** (0.005)
δ	0.000003 (0.000)	0.976*** (0.040)	0.021*** (0.0005)	-61.588*** (2.127)	6.667*** (0.230)	8.857*** (0.425)
C_{YY}	-0.11E-05*** (0.001E-06)	-0.085*** (0.029)	0.003*** (0.001)	11.359*** (1.901)	0.191* (0.058)	-0.857* (0.437)
C_{LL}	0.974*** (0.030)	15.156*** (2.775)	0.758*** (0.043)	7658.13*** (319.514)	-131.871*** (7.161)	6.497*** (0.230)
C_{YL}	-0.32E-07*** (0.31E-08)	0.002*** (0.0007)	-0.000003*** (0.0000)	18.327*** (2.931)	-0.086* (0.024)	-0.006** (0.003)
DET	-0.10E-05*** (0.99E-07)	-1.287*** (0.445)	0.002*** (0.000)	86657.1*** (13932.2)	-25.177*** (7.179)	-5.569** (2.819)
MC_{YY}	-0.11E-05*** (0.10E-06)	-0.084*** (0.029)	0.003*** (0.001)	11.428*** (1.900)	0.202*** (0.061)	-0.856** (0.437)
Wald Test	2870.896 [0.000]	2269.487 [0.000]	3773.536 [0.000]	4334.576 [0.000]	502.277 [0.000]	1504.527 [0.000]
J Test	157.142 [0.003]	172.564 [0.000]	187.766 [0.000]	196.272 [0.000]	174.098 [0.000]	177.343 [0.000]

B. No Inventories Model

Parameter	(1) Nondurables	(2) SIC 21	(3) SIC 23	(4) SIC 28	(5) SIC 29	(6) SIC 30
μ_1	1.460*** (0.001)	1.201*** (0.021)	1.181*** (0.011)	1.295*** (0.021)	1.683*** (0.033)	1.377*** (0.014)
μ_2	0.015*** (0.001)	0.0008*** (0.0001)	0.0003 (0.0002)	0.011*** (0.0001)	-0.016*** (0.001)	0.006*** (0.0004)
ψ	4.703*** (1.351)	0.016 (0.020-.à)	0.352 (0.154)	20.532 (19.473)	-0.986*** (0.124)	0.132*** (0.064)
C_{YY}	1155.46*** (483.779)	427323.0*** (56522.4)	-178771.0*** (22823.6)	0.13E+07*** (148672.0)	20759.9*** (3396.74)	129619.0*** (14931.5)
C_{LL}	1.311*** (0.110)	5.935*** (1.093)	0.128*** (0.035)	4270.19*** (338.826)	-114.164*** (12.411)	2.162*** (0.230)
C_{YL}	-78.811*** (7.372)	383.083*** (65.123)	125.185*** (10.374)	-50101.5*** (5784.45)	102.299 (164.083)	150.569*** (27.359)
DET	-4695.87*** (739.210)	0.24E+07 (625360.0)	-38588.3*** (6022.64)	0.31E+10*** (0.41E+09)	-0.23E+07*** (488479.0)	257552.0*** (58455.5)
MC_{YY}	-3300.52*** (382.612)	454463.0 (57655.9)	-95327.1*** (25067.0)	807638.0*** (69341.3)	20464.1*** (3063.31)	187022.0*** (18687.5)
J Test	127.586 [0.000]	93.702 [0.109]	120.561 [0.001]	134.624 [0.000]	116.996 [0.001]	85.090 [0.139]

C. Linear Compensation Schedule Model

Parameter	(1) Nondurables	(2) SIC 21	(3) SIC 23	(4) SIC 28	(5) SIC 29	(6) SIC 30
μ_1	1.355*** (0.028)	1.165*** (0.014)	1.177*** (0.007)	1.488*** (0.011)	1.353*** (0.009)	1.416*** (0.012)
ψ	0.022 (0.285)	0.047*** (0.021)	0.253*** (0.102)	69.361*** (18.087)	0.002 (0.003)	0.031 (0.020)
θ	0.594*** (0.013)	0.262*** (0.010)	0.796*** (0.036)	0.452*** (0.131)	0.486*** (0.017)	1.139*** (0.457)
δ	27.092*** (8.384)	5.107*** (0.888)	0.022*** (0.005)	-266.750*** (105.047)	2.120*** (0.678)	0.0007 (0.0007)
C_{YY}	1.479 (1.128)	-3.128*** (1.408)	0.047*** (0.016)	115.406 (88.159)	-1.316*** (0.286)	-0.014*** (0.002)
C_{LL}	-0.020*** (0.008)	-1.494*** (0.667)	0.015*** (0.007)	1049.25*** (9.063)	-3.822*** (0.214)	-0.436*** (0.027)
C_{YL}	-0.024 (0.017)	-0.585*** (0.134)	0.0004*** (0.0001)	-5.462 (3.824)	-0.619*** (0.049)	0.00002*** (0.000)
DET	-0.030 (0.019)	4.332 (2.907)	0.0007* (0.0004)	121060.0 (92445.5)	4.649*** (1.134)	0.006*** (0.001)
MC_{YY}	1.890 (1.177)	-1.325 (1.496)	0.069*** (0.032)	135.118 (89.155)	-1.433*** (0.291)	-0.013*** (0.002)
J Test	144.877 [0.010]	152.663 [0.003]	165.822 [0.000]	182.634 [0.000]	153.723 [0.001]	141.659 [0.004]

Notes: See Table I. The Wald test refers to the restrictions implied by the Cobb-Douglas specification relative to the more general translog model of Table I. P-values in round brackets.