

Ex-ante and Ex-post Subcontracting in Highway Procurement Markets

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PRELIMINARY AND INCOMPLETE. DO NOT CIRCULATE.

Abstract

This paper provides a novel evidence (based on a new dataset) on bidding and subcontracting behavior of primary contractors participating in California highway procurement market. We develop a model of procurement auction with subcontracting stage which is capable of rationalizing the patterns documented in the data. Next, we use this framework to assess the implications of ex-ante subcontracting rule which is frequently imposed in government procurement.

Keywords: highway procurement, subcontracting

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1 Introduction

Many infrastructure procurement markets have a two-tier structure with contractors routinely subcontracting parts of the projects they win in the primary auctions. This feature, however, remains relatively understudied in the literature. In this paper we present novel evidence on bidding and subcontracting activity in a large procurement market (California highway procurement market). We then construct a model consistent with the patterns documented in the data and use this model to study implications of ex-ante subcontracting requirement maintained in most government procurement markets in US.

The analysis in this paper is based on a new dataset which provides detailed information about bidding and subcontracting activity of individual contractors for a large number of projects auctioned in California highway procurement market. Specifically, for each project (represented as a list of items compiled by CalTrans engineers) and for each participating contractor, we observe the list of items this contractor subcontracts, with the names of associated subcontractors, and the itemized list of subcontractors' prices.

The data analysis reveals that subcontracting is very prevalent in this market. Indeed, 95% of participating contractors subcontract at least one item on an average project. Interestingly, for most tasks contractors tend to subcontract items associated with this task on a fraction of projects. This indicates that subcontracting decisions do not reflect firms' capability to perform a given task. Instead, contractors are likely to use subcontracting in order to modify their costs and improve their competitiveness in a given auction. Therefore, subcontracting decisions are likely to be influenced by the contractors' own cost of performing the task as well as competitive conditions in the primary and subcontracting markets.

Further, we document that contractors competing for a given project frequently employ overlapping sets of subcontractors. In addition, while own subcontracting decision is not predictive of the own bid level (presumably contractor may subcontract because his own cost of completing the task is high or because he has been offered a low subcontracting price), the fact that two contractors share the same subcontractor is associated with lower bids submitted by these contractors. This indicates that subcontractors compete for the right to be listed on contractors' bids. On the other hand, contractors bidding on the same project frequently hire different subcontractors, and the prices charged by a subcontractor tend to differ across contractors within the same project. This leads us to conclude that factors other than price must influence contractors' choice of subcontractors and that these factors have to be contractor-subcontractor-specific.

We use the patterns documented in the descriptive analysis to guide our modeling choices.

Specifically, we develop a model of procurement auction with subcontracting which consists of two stages. In accordance with the rules of government procurement, in the first stage contractors develop a plan of how the work would be completed should they win the project. During this stage they run a subcontracting auction for each task listed on the project and on the basis of subcontractors' quotes decide whether the task should be subcontracted and to whom. In the second stage, contractors submit their bids and the winner is determined. In accordance with the findings described in the previous paragraph, we assume that contractors rely on a discriminatory subcontracting auctions where they may engage a preferred subcontractor even if his price exceeds the lowest quote (but not more than by a certain margin). A contractor may give such a preference to subcontractor because of previous interaction, or due to reputation for quality – contractors may differ in their preference for quality which leads to contractor-subcontractor-specific preferences. There are, of course, other mechanism that may rationalize the patterns observed in the data. For example, subcontractor may have different costs for working with different contractors. While this is possible, such mechanism is not very appealing since, independent of who wins the main contract, subcontractor, if listed on the winning bid, will be facing the same scope of work. Further, the model where subcontractor's costs differences associated with various contractors are private is intractable; whereas the model where these differences are public predicts behavior which is similar to the one implied by our model.

The model we consider in this paper differs from a standard model of procurement auction along several dimensions. First, participating contractors have an opportunity to modify their costs and thus improve their competitiveness in a given auction. Further, an access to subcontracting market gives contractors' an opportunity to acquire valuable information about rivals' costs. Second, subcontractors have to interact with multiple contractors. Given the differences in contractors' preferences and possibly costs, subcontractors use different pricing strategies with different contractors. Nevertheless, the prices submitted by a given subcontractor to different contractors are related since they are based on the same underlying realization of subcontractor's costs. In this game subcontractors internalize the fact that winning an engagement with a contractor results in them working on the project only with some probability. They also recognize that their price quotes are influencing the probabilities of primary contractors winning the project.

From the equilibrium characterization point of view, this model presents a number of challenges. First, the subcontractors' pricing strategies have to account for simultaneity of subcontracting auctions as well as to take into account the dynamic consequences of subcontractors'

pricing decisions on allocation in the second stage. Discriminatory feature incorporated in subcontracting stage complicates subcontractors' incentives at the high cost realizations since at high cost levels a subcontractor may prefer to target only the auctions where he has an advantage (due to contractor's preference) rather than attempting to win a higher overall number of auctions. We believe that analysis of this feature is novel and constitutes a separate contribution of the paper.

Finally, we use this framework to assess the impact of the ex-ante subcontracting rule. We note that ex-ante subcontracting eliminates part of the inherent uncertainty about rivals' costs since just obtaining subcontractors' quotes enables contractors to narrow down their assessment of competitors' costs which induces contractors to behave more aggressively. This is in contrast to the ex-post markets where the possibility of future subcontracting activity introduces additional common uncertainty which in turn promotes less aggressive behavior on the part of bidders. On the other hand, under ex-ante subcontracting rule, subcontracting prices are determined at the bidder rather than the winner level which diffuses risk of losing from the subcontractors' point of view and thus allows them to behave less aggressively. The balance of these two effects determines the impact of ex-ante subcontracting rule.

Literature review. [INCOMPLETE] Wambach (2009), Jeziorski and Krasnokutskaya (2014), Bajari, Houghton, and Tadelis (2014), Gil and Marion (2012), Marion (2013), Miller (2014), Moretti and Valbonesi (2012), Branzoli and Decarolis (2014), Haile (2001).

The structure of the paper is as follows. In Section 2 we summarize the highway procurement process in California. Section 3 describes construction of the data set and characterizes subcontracting patterns reflected in the data. Section 4 describes a simple procurement auction model with ex-ante subcontracting stage which is capable of rationalizing patterns documented in the previous section. In Section 5 we discuss the identification and estimation of the structural model.

2 Procurement Process

We study the market for highway procurement in the state of California which is supervised by California Department of Transportation (CalTrans). The projects transacted in this market deal with highway repairs, highway construction and associated work such as signing, striping and landscaping. These projects are allocated through a first-price sealed bid auction mechanism.

Procurement process proceeds through the following stages. First, the project is announced.

At this point only a short description of work with the time and location details is provided. The interested contractors may request a detailed project description compiled by government engineers which lists all the items included in the project with the engineer’s estimate of the size and the cost of each item. The list of tasks is fixed for the purpose of auction. Each contractor has to summarize his bid in the form which states the price for each item on this list. Second, contractors work on the plan for project completion, finding out their costs broken down by item. During this time contractors are approached by subcontractors who quote their prices for the items they are qualified to perform. CalTrans requires that all the details of project implementation including subcontracting agreements have to be settled before the bid is submitted. The finalized plan of work (who does what and at what price) has to be reflected in the bid documents. Third, at the previously announced date submitted bid documents are opened and the winner is determined on the basis of the total bid which is equal to the sum of the item-specific prices.

The subcontracting of work is strictly regulated. Specifically, the government imposes an upper bound of 40% of the project value for the amount that contractor is allowed to subcontract. The subcontractors have to be certified to do work for the government. Further, CalTrans pays subcontractors directly to ensure that they receive their pay on time and possibly to enforce the rule that the subcontractor listed on the bid documents is the one who does the work.

3 Data

3.1 Data Construction

We have assembled a novel dataset which allows us to gain insights into the subcontracting activities in the California highway procurement market. Existing empirical research on highway procurement traditionally relies on the data collected from bid summaries which include the names and the bids of all participating contractors as well as engineers estimate, contract duration, location and the type of work.¹ In contrast, our data set is constructed on the basis of the full bid documents which for each project contain the list of items involved, the item-specific bids for each participating contractor, and, for each participating contractor, the list of

¹See, for example, Hong and Shum (2002), Porter and Zona (1993), Jofre-Bonet and Pesendorfer (2003), Krasnokutskaya (2011), etc. More recently, the researchers also used information on the identities of contractors who purchased bid documents for a specific project and thus seriously contemplated submitting a bid in a corresponding auction (see, for example, Krasnokutskaya and Seim (2011) and Balat (2014)); as well as more detailed information on the list of tasks involved, negotiated bid adjustments subsequent to the auction, and the details of the project supervision (as in Bajari, Houghton, and Tadelis (2014), Bajari and Lewis (2011)).

the subcontractors this firm plans to use and the items each subcontractor is hired to perform. We assembled this information for all the projects auctioned by CalTrans between January 2002 and December 2016.

In constructing this dataset we had to overcome several challenges. First, we had to match subcontractors to the items and thus the prices they were promised in connection with a given project. Specifically, subcontractors' assignments in the bid documents are occasionally described verbally (instead of referring to the item number) and the verbal description was not always identical to the description used in the official CalTrans list of items. In order to reconcile the two pieces of data we designed and implemented a word recognition algorithm. The final dataset includes only the projects for which we were able to match the subcontracting assignments for all items and for all contractors with high degree of confidence. The matching algorithm is applied mostly to the earlier years in our data. In the later years (after 2010) contractors mostly provided item numbers rather than verbal descriptions to summarize the scope of subcontractors' assignments. This feature additionally allowed us to verify performance of the matching algorithm by checking that the subcontractors' areas of specialization as inferred from the data do not change over the years, and that the patterns of behavior documented on the basis of the whole dataset are similar to those documented from the more recent data.

The next issue we had to tackle was related to the fact that the number of distinct items recorded in our dataset is very large. This does not allow us to study patterns in subcontracting activity with any degree of statistical confidence. We used the state-issued document summarizing the state-approved cost for each item to aggregate items into larger classes (tasks). This document effectively lists every item which appeared on the bid documents in the previous ten years. The items are arranged by specialization and each item is assigned a six digit number. We aggregated these items into groups (tasks) such that all items associated with the same task share the first two digits in the number assigned to the item by CalTrans.

Focusing on the projects associated with road or bridge work, we identified 13 distinct tasks (types of work) which appear in our data. We performed various checks to verify that such grouping indeed makes sense. For example, we find that for any two items which appear on the same bid document, and which belong to the same task according to our grouping, the probability that both tasks are subcontracted (by the same contractor) if one of them is subcontracted is 89%. This number is only slightly lower from the one which obtains when the groups of tasks are formed on the basis of the first three digits of the task numbers (93%).

We use this final dataset where the bid information is arranged by project, by contractor

within project, and by item for each contractor and project, and where items are further characterized by the type of work defined as explained above to study patterns in the subcontracting behavior. We summarize our finding in the next section.

3.2 Descriptive Data Analysis

In this section we provide a number of statistics summarizing the data in general and the subcontracting activity reflected in the data specifically.

3.2.1 General Summary Statistics

Table 1 provides general summary statistics. It indicates that the projects vary in size quite substantially: from \$190,000 on a lower end to \$10,670,000 on the upper end of the size distribution with the median project's value equal to \$700,000. Projects further differ in the allowed duration which ranges from one to eight months with the median duration equal to 50 days. Engineering description breaks the project into items with the number of items ranging from 14 to 57 and the median number of items given by 22. Our algorithm for aggregating items into tasks which is described above indicates that the project on average consists of four tasks (the average number of tasks is 3.77 and the median number is 4). Also, an auction on average attracts 5 bidders (the median number of bidders is 4 with smaller projects attracting higher number of participants).

The table further reports statistics traditionally considered in auction markets. The variable "money-left-on-the table" is constructed as the difference between the lowest and second lowest bid divided by the lowest bid: $\frac{(\text{rank}2 - \text{rank}1)}{\text{rank}1}$. It reflects the level of uncertainty about contractors' costs or informational asymmetries in the market. In our data, the second lowest bid is on average 9% higher than the winning bid. This is comparable to other datasets used to study highway procurement market. Information asymmetries can also be seen from the relative difference of the winning bid to the engineer's estimate. The winning bid is on average 7% below the engineer's estimate.

3.2.2 Subcontracting Activity

Table 2 summarizes subcontracting activity recorded in our data. The table indicates that subcontracting is very prevalent in this market. Indeed, if we consider an indicator variable which

Table 1: General Summary Statistics

		Project Size			
		All	Small	Medium	Large
Engineer's estimate (in mln)	mean	4.83	0.19	0.54	10.67
	median	0.69	0.19	0.53	3.25
	s.d.	16.57	0.05	0.16	24.16
Working days	mean	128.25	32.41	57.10	231.27
	median	50.00	30.00	35.00	125.00
	s.d.	192.94	19.16	69.01	251.97
# items	mean	43.18	15.51	21.83	73.75
	median	22.00	14.00	17.00	57.00
	s.d.	50.05	7.81	12.84	63.52
# tasks	mean	3.77	3.59	3.72	3.88
	median	4.00	4.00	4.00	4.00
	s.d.	0.67	0.77	0.74	0.53
# contractors	mean	4.94	5.18	4.83	4.97
	median	4.00	4.00	4.00	5.00
	s.d.	2.68	3.18	2.68	2.52
Money-left-on-the-table (%)	mean	8.94	14.05	10.29	5.96
	median	6.04	9.31	7.05	4.59
	s.d.	12.11	18.56	10.91	9.74
(estimate-bid1)/estimate (%)	mean	-6.93	-10.86	-5.94	-6.69
	median	-7.85	-12.47	-6.25	-8.15
	s.d.	20.64	23.98	21.28	18.68

Notes: This table reports statistics summarizing general features of the data.

Table 2: Summary Statistics on Subcontracting

		Project Size			
		All	Small	Medium	Large
# contractors (per project)	mean	4.94	5.18	4.83	4.97
	median	4.00	4.00	4.00	5.00
	s.d.	2.68	3.18	2.68	2.52
Instance of subcontracting (per contractor×project)	mean	0.95	0.88	0.94	0.98
	median	1.00	1.00	1.00	1.00
	s.d.	0.21	0.33	0.23	0.12
Instance of subcontracting (per contractor×task)	mean	0.35	0.36	0.34	0.38
	median	0.39	0.40	0.38	0.41
	s.d.	0.11	0.13	0.13	0.12
% subcontracted items (per contractor×project)	mean	0.38	0.38	0.40	0.35
	median	0.35	0.35	0.40	0.33
	s.d.	0.21	0.25	0.22	0.18
% value subcontracted (per contractor×project)	mean	0.29	0.24	0.28	0.31
	median	0.22	0.20	0.19	0.24
	s.d.	0.27	0.23	0.26	0.28
% subcontracted tasks (per contractor×project)	mean	0.43	0.41	0.44	0.43
	median	0.50	0.50	0.50	0.50
	s.d.	0.31	0.34	0.31	0.29
% value subcontracted (task) (per contractor×project)	mean	0.08	0.12	0.10	0.06
	median	0.04	0.06	0.05	0.02
	s.d.	0.12	0.15	0.13	0.10
# subcontractors (per project)	mean	12.60	7.95	9.56	17.17
	median	10.00	7.00	9.00	16.00
	s.d.	7.84	4.01	4.35	9.08
# subcontractors (per contractor×project)	mean	4.68	2.77	3.38	6.51
	median	4.00	3.00	3.00	6.00
	s.d.	3.07	1.39	1.68	3.46
# contractors (per subcontractor×project)	mean	1.90	1.83	1.82	1.95
	median	1.00	1.00	1.00	1.00
	s.d.	1.40	1.58	1.32	1.41

Notes: This table reports statistics characterizing subcontracting activity in this market.

Table 3: Subcontracting Activity by Task

		Electrical	Striping and Marking	Area Signs	Traffic Control System	Water Pollution Control
# unique items		813	335	4	130	6
# unique items (per project)	mean	2.76	6.63	1.00	1.16	1.00
	s.d.	3.82	3.90	0.04	0.76	0.09
% subcontracted	mean	0.31	0.48	0.52	0.30	0.06
(per contractor)	s.d.	0.46	0.50	0.50	0.46	0.25

Notes: This table summarizes several specialized tasks which we construct using our item aggregation method.

is equal to one if a given contractor subcontracts at least one item for a given project then an average value of this variable across contractors and across projects is 0.95 whereas a median value of within project average of such variable is one. This means that in a median project every contractor subcontracts at least one item. Further, the analysis indicates that contractors tend to subcontract close to 40% of items (30% of project value) when bidding for a project on average.

Interestingly, when we compute, for a given task, a fraction of projects with this task that a given contractor subcontracts, and then study the value of this variable across contractors, we find that an average contractor subcontracts a given task roughly 40% of the time. This indicates that subcontracting decisions are not driven by firm’s capability to perform this task. Rather, it appears to be a more flexible decision which reflects contractors’ incentives to modify their costs given the economic conditions associated with a given project. Table 3 investigates subcontracting activity for several specialized tasks. The results indicate that the intensity of subcontracting activity varies across task with some tasks subcontracted more frequently than others.

Returning to Table 2. The table indicates that the average number of distinct subcontractors appearing on the bid documents for a given project is on average equal to 13 (median number is 10) whereas an average number of contractors on a given project is 5 (median number is 4). This indicates that some contractors employ the same subcontractor. Indeed, such occurrence is very prevalent in the data. As the table shows, on average a given subcontractor appears on the bid documents of the two distinct contractors participating in the same auction.

We now look at the differences in item-level bids across bidders. Table 4 shows summary statistics for the difference in item-level bids (in absolute value) across pair of bidders who are

Table 4: Item-Level Bid Difference

	all	both do not subcontract	only one subcontracts	both subcontract		
				all	different subc.	same subc.
Observations	46330	27657	9836	8837	3621	5216
mean	0.58	0.66	0.54	0.38	0.49	0.31
s.d.	0.98	1.06	0.90	0.72	0.80	0.66
25th percentile	0.11	0.14	0.10	0.06	0.11	0.04
50th percentile	0.32	0.38	0.30	0.18	0.28	0.12
75th percentile	0.65	0.71	0.61	0.44	0.55	0.33

Notes: The difference is computed as the difference in item-level bids (in absolute value) across pair of bidders who are randomly chosen from a set of participants for each auction. The difference is reported as a fraction of the lowest among the two item-level bids.

randomly chosen from the set of participants for each auction. We observe that median difference between the item-level bids from two different bidders is 32%. The difference is the biggest when the two bidders do not subcontract the item (median equal to 38%), and the smallest when the two subcontract the item using the same subcontractor (median is equal to 12%). Conditional on both contractors subcontracting the item, 60% share the same subcontractor. Of those who subcontract the item using the same subcontractor, 14% have a difference of 0 and 45% have a difference of less than 10%.

3.3 Preliminary Regression Analysis

Here we explore the relationship between the bids and subcontracting decisions in a regression analysis. The data analysis summarized in the previous sections indicates that contractors are likely to use subcontracting in order to improve competitiveness of their bids. While we expect the presence of subcontracting opportunities to lower the contractor’s cost, and therefore his bid, the correlation between the prime contractor’s final bid and the decision to subcontract part of the project, in theory, can go either way. Indeed, a contractor may subcontract because his own cost realization is high or because he is presented with an attractive subcontracting offer. To fix ideas, consider the following simple example. Say the project involves only one task and that bidders have the option to subcontract it. Bidders subcontract the task

Table 5: Bids and Subcontracting

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
constant	1.098*** (0.015)	1.110*** (0.012)	1.101*** (0.015)	1.175*** (0.028)	1.191*** (0.026)	1.180*** (0.028)	1.274*** (0.123)	1.277*** (0.122)	1.264*** (0.123)
eng. estimate				0.189 (0.473)	0.167 (0.472)	0.178 (0.472)	-0.011 (0.501)	-0.047 (0.500)	-0.040 (0.500)
days				0.148*** (0.054)	0.141*** (0.054)	0.144*** (0.054)	0.189** (0.075)	0.195*** (0.075)	0.196*** (0.075)
num. items				-0.001** (0.000)	-0.001* (0.000)	-0.001* (0.000)	-0.001 (0.000)	-0.000 (0.000)	-0.001 (0.000)
num. bidders				-0.015*** (0.004)	-0.016*** (0.004)	-0.016*** (0.004)	-0.012*** (0.004)	-0.013*** (0.004)	-0.013*** (0.004)
% value subcontracted	0.001 (0.044)		0.049 (0.049)	0.001 (0.044)		0.051 (0.049)	0.000 (0.044)		0.044 (0.049)
% value same subcontractor		-0.157* (0.082)	-0.199** (0.092)		-0.170** (0.082)	-0.211** (0.092)		-0.158* (0.083)	-0.193** (0.092)
time FE	no	no	no	no	no	no	yes	yes	yes
region FE	no	no	no	no	no	no	yes	yes	yes
work type FE	no	no	no	no	no	no	yes	yes	yes
observations	1160	1160	1160	1160	1160	1160	1160	1160	1160

Notes: The dependent variable is the normalized bid: bid/estimate. ***, **, * denote significance at the 1%, 5% and 10% level.

if the subcontracting price is below the bidders' own cost of performing the task. Furthermore, suppose there are two contractors and only one subcontractor. Let $i = A, B$ denote contractors. Let c_c^i and c_s be independent random variables denoting contractor i 's own cost and the subcontractor's quote for the task, respectively. Let $\text{subc}^i = 1\{c_s < c_c^i\}$ denote contractor i 's subcontracting decision. Then, bidder i 's project cost is given by

$$c^i = \text{subc}^i \cdot c_s + (1 - \text{subc}^i) \cdot c_c^i.$$

It is straightforward to show that the sign (and magnitude) of the correlation between the final cost, c^i , and the subcontracting decision, subc^i , depends on the shapes of distributions of c_c^i and c_s . As an illustration consider the following three cases: (i) $c_c^i \sim U[0, 1]$, $c_s \sim U[0, 1]$; (ii) $c_c^i \sim U[0, 1]$, $c_s \sim U[0, 2]$; and, (iii) $c_c^i \sim U[0, 2]$, $c_s \sim U[0, 1]$. We can then show that $\text{corr}(c^i, \text{subc}^i) = 0$ under case (i); $\text{corr}(c^i, \text{subc}^i) < 0$ under (ii); and $\text{corr}(c^i, \text{subc}^i) > 0$ under (iii). The intuition is simple. In the data, it is difficult to distinguish between i 's subcontracting decision resulting from i receiving a high cost draw or a low subcontracting quote.

While, on the one hand, we cannot make a prediction on the sign of $\text{corr}(c^i, \text{subc}^i)$, on the other, the correlation between bidder i 's final cost, c^i , and the subcontracting decision of i 's rival who is using the same subcontractor is expected to be negative if subcontractors compete for the right to appear on the bid document.² The intuition, again, is simple. Indeed, the fact that i 's rival is subcontracting and is using this specific subcontractor increases the likelihood that i is facing a low subcontracting quote.

To investigate the relationship between bids and subcontracting decisions, we run reduced-form regressions in which the dependent variable is the bid normalized by the engineer's estimate (i.e., bid/estimate) and our measure of subcontracting is the fraction of the contractor's bid that is subcontracted.³ Additionally, to capture the fact that bidders sometimes share the same subcontractor we construct a variable that measures the fraction of the bid that is being subcontracted with subcontractors also listed by the bidder's rivals.⁴ We report the results in Table 5. We see that subcontracting has a positive, but not significant, effect on bids. On the other hand, as expected, a contractor's bid is negatively related to the fraction of the project's value that subcontracted with shared subcontractors. The previous results are robust across different specifications in which we incrementally control for project characteristics, the number of

²In the three cases considered above, it is straightforward to show that $\text{corr}(c^i, \text{subc}^{-i}) < 0$.

³In the regressions we only use one (randomly selected) bid per auction and thus, the sample size equals the number auctions. Results don't change when we use all bids.

⁴The match is done on an item-level basis, and the value then is aggregated at the project-bidder level.

Table 6: Item-Level Bids and Subcontracting

	(1)	(2)	(3)	(4)	(5)
constant	0.906*** (0.072)	0.905*** (0.072)	0.904*** (0.072)	0.902*** (0.073)	0.902*** (0.073)
eng. estimate	0.095 (0.131)	0.097 (0.131)	0.096 (0.131)	0.110 (0.132)	0.109 (0.132)
days	0.019 (0.030)	0.019 (0.029)	0.019 (0.030)	0.019 (0.029)	0.019 (0.029)
num. items	-0.000 (0.000)	-0.000 (0.000)	-0.000 (0.000)	-0.000 (0.000)	-0.000 (0.000)
num. bidders	0.005** (0.002)	0.005** (0.002)	0.005** (0.002)	0.005* (0.002)	0.005* (0.002)
subcontracted	-0.002 (0.006)		0.005 (0.007)		0.002 (0.006)
same subcontractor		-0.014* (0.007)	-0.018** (0.008)		
% items same subcontractor				-0.079** (0.034)	-0.081** (0.034)
time FE	yes	yes	yes	yes	yes
region FE	yes	yes	yes	yes	yes
work type FE	yes	yes	yes	yes	yes
observations	46756	46756	46756	46756	46756

Notes: Dependent variable is normalized item-level bids. Standard errors in parenthesis. Standard errors clustered at the auction level. ***, **, * denote significance at the 1%, 5% and 10% level.

bidders, and time, region, and work type fixed effects.

We also explore the relationship between bids and subcontracting decisions at a item level. The results are shown in Table 6. The left-hand-side variable is the item-level normalized bid, where the normalization is done on an item-by-item basis. The variable ‘subcontracted’ is a dummy variable that equals 1 if the contractor has chosen to subcontract the item. To capture common subcontractors across bidders we use a dummy variable that equals 1 if there is another bidder in the auction that is listing the same subcontractor for the same item. Alternatively, we also use the fraction of items for which the bidder lists a subcontractor also listed by his rivals. In all the regressions we cluster the standard errors at the auction-bidder level. Similarly to what we found in the previous regressions, there is no effect of the bidder’s choice to subcontract an item on the submitted bid for that item; but when the subcontractor is also listed by one of the contractor’s rivals, we find that the effect is negative and significant, as expected. We find similar results if we instead use the fraction of items that feature a common subcontractor.

3.4 Summary and Modeling Implications

The descriptive analysis summarized above highlights several patterns which guide our modeling of subcontracting process in this market. Specifically, on the basis of this analysis we conclude that

1. For each task, contractors choose to subcontract or not on the project-by-project basis. The decision most likely depends on contractor’s own cost of performing the task in house as well as economic conditions associated with a given project and a subcontracting market for a given task.
2. Subcontractors compete for the opportunity to work with a given contractor (this is indicated by regression analysis). We therefore model subcontracting stage as a secondary auction run by each contractor and for each task. At this stage we abstract away from subcontractors’ participation decisions and assume that all subcontractors qualified for a given task (available during the time of project for which project is scheduled) participate in all subcontracting auctions for this task.
3. The subcontracting decisions are not based on straightforward comparison of price quotes such as we would see in the first price auction. If they were, we would observe (a) the same subcontractor being employed by all subcontracting contractors. Also, we would tend to see (b) the same subcontractor charging the same price to different contractors (for this

we would also need main contractors to be symmetric in their cost distributions). The regularities (a) and (b) do not hold in the data. Different prices could potentially be rationalized by asymmetries between primary contractors. However, even in this case the same contractor would be engaged by all subcontracting contractors. Notice that this regularity cannot be driven by subcontractor’s capacity constraints since only one contractor will eventually win the project. To account for this regularity we model subcontracting auction as an auction with preferential treatment where primary contractor may accept a bid which is higher by a certain margin than the lowest bid in the subcontracting auction if this bid is submitted by a preferred subcontractor (more details on this are provided in the model section). Such a preference may be given if a subcontractor has better reputation or if contractor has previous experience of working with this subcontractor. Of course, the features we observe in the data could be rationalized in a different way. For example, it is possible that the subcontractor’s cost for the task differs across contractors. However, if the model where contractor-subcontractor differences in costs are private information is intractable whereas the model where such differences are commonly known is equivalent to our setting. We therefore believe that our approach represent a reasonable first approximation of this environment.

4 Auction Model with Subcontracting

In this section we present a first-price (procurement) auction model in which bidders are allowed to subcontract part of the project. We analyze both the optimal bidding strategies for contractors in the primary market, and the optimal pricing strategies for subcontractors in the secondary market. The auction consists of two stages. In the first stage the contractors draw their costs for task one and simultaneously hold subcontracting auctions for task two. At the end of this stage contractors’ costs are realized. Then in stage two contractors construct their bids, submit them to auctioneer and the winner of the primary auction is determined. Notice that here we maintain the rule imposed in most US government procurement auctions that the subcontracting decisions have to finalized prior to the main auction.

The interrelationship between the primary and subcontracting markets adds several key features that differentiate our model from a standard first-price auction model. On the one hand, subcontractors: (i) internalize the fact that their quotes affect the contractors’ likelihood of winning the project; and (ii) take into account that winning an engagement with a given contractor

does not necessary result in them working on the project. On the other hand, primary contractors: (i) use subcontracting to modify their costs realizations, and (ii) take into account the information about rivals' costs contained in subcontractors' prices.

We begin with a simplified setting (two tasks, two main contractors, two subcontractors) in order to highlight the new issues introduced by allowing for subcontracting. The simple model can be generalized in a straightforward way to allow for the number of tasks, the number of primary contractors and/or the number of subcontractors to be greater than two.

4.1 A Simple Model

To fix ideas, consider a special case where task one is always performed in house whereas task two is always subcontracted. The equilibrium characterization we present below generalizes to the case when contractor may choose to subcontract or not for both tasks. We'll present the details for this more general setting in the future drafts of the paper.

Two ex-ante symmetric contractors (A and B) compete for the project. The letting process consists of two stages. In the first stage contractors prepare their bids. As part of this process they solicit quotes from subcontractors and decide which subcontractor to hire. We assume that both contractors interact with the same set of subcontractors (subcontractor R and T in this simplified example) and that contractors choose among subcontractors by implementing a discriminatory auction (details below). At this stage contractors also learn the realization of their own cost of implementing task one. In the second stage primary contractors prepare and submit their bids which reflect their subcontracting decisions, and the winner is determined.

We use $c_{1,A}$ and $c_{1,B}$ to denote contractors' private costs for completing task one. We assume that $c_{1,A}$, $c_{1,B}$ are drawn from the distribution $F_1(\cdot)$ (with the associated density $f_1 > 0$ and the support of $[\underline{c}_1, \bar{c}_1]$). Further, $c_{s,R}$ and $c_{s,T}$ denote private costs of subcontractors R and T for completing task two which are drawn from $F_s(\cdot)$ (with associated density $f_s > 0$ and the support of $[\underline{c}_s, \bar{c}_s]$). We assume that the cost draws are independent across tasks, contractors, and subcontractors and that the distributions $F_1(\cdot)$ and $F_s(\cdot)$ are common knowledge to both contractors and subcontractors.

An equilibrium of this auction game is summarized by two pairs of subcontracting pricing functions, $(Q_{AR}(\cdot), Q_{BR}(\cdot))$ and $(Q_{AT}(\cdot), Q_{BT}(\cdot))$, and two families of contractors' bidding functions that are indexed by realized subcontracting prices, $\{\beta(\cdot|q_A, q_B), \beta(\cdot, |q_B, q_A)\}_{(q_A, q_B)}$. Here, $Q_{i,j}(\cdot) : [\underline{c}_s, \bar{c}_s] \rightarrow R_+$ for $j \in \{R, T\}$ and $i \in \{A, B\}$ are the functions which map subcontracting costs realizations into the price a subcontractor chooses to submit to a given contractor;

$\beta_i(\cdot) = \beta_i(\cdot | q_i, q_{-i}) : [\underline{c}, \bar{c}] \rightarrow \mathbb{R}_+$ for $i \in \{A, B\}$ are the function which map the points from the support of contractor i 's cost distribution into positive real numbers. We use q_i to denote the price quoted by the subcontractor that contractor i choses to hire. Bidding functions of contractors are indexed by the vector of subcontracting prices since own subcontracting price is an important determinant of contractor's costs and since all subcontracting prices are known to all contractors.

Primary Auction. At the time of the primary auction the contractors' costs of completing the project are realized. Specifically, contractor A's cost of completing task one is $c_{1,A}$; the costs of task two is given by q_A which is the price quoted by the subcontractor that contractor B chose to hire. Similarly, contractor B's cost of completing task one is $c_{1,B}$; the costs of task two is given by q_B which is the price quoted by the subcontractor that contractor B chose to hire. We assume that both contractors know q_A and q_B – we will clarify this point below. Due to this feature and because the realized cost of task two is potentially different across contractors, the contractors have asymmetric costs in the primary auction stage: $c_A = c_{1,A} + q_A$ with $c_A \propto F_1(c_A - q_A)$ and $c_B = c_{1,B} + q_B$ with $c_B \propto F_1(c_B - q_B)$.

Similar to the case of a first price auction, a contractor chooses his bidding strategy in such a way that for every possible cost draw from the support of F_1 , $\tilde{c} \in [\underline{c}_1, \bar{c}_1]$, he chooses $\beta_i(\tilde{c}) = \tilde{b}$ where

$$\begin{aligned} \tilde{b} &= \arg \max_b (b - \tilde{c} - q_i) \Pr(b \leq \beta_{-i}(c_1 + q_{-i})) \quad \text{or} \\ \tilde{b} &= \arg \max_b (b - \tilde{c} - q_i) \int_c (1 - F_1(\beta_{-i}^{-1}(b) - q_{-i})) dF_1(c_1). \end{aligned} \tag{1}$$

The expression in (1) clarifies how own and rival subcontracting prices affect contractors' bidding strategies. Specifically, for a given own bid \tilde{b} the rival's subcontracting price shifts the contractor's probability of winning. In contrast, own subcontracting price shifts own cost of completing the job and thus ex-post profitability conditional on winning and on own bid.

For the purpose of characterizing subcontracting pricing decisions we introduce two separate notations for the probability of contractor i winning primary auction for a given vector of realized subcontracting prices (q_i, q_{-i}) . Specifically, $p_1(q_A, q_B)$ is the probability that contractor A wins when $q_A \leq q_B$ and $p_2(q_A, q_B)$ is the probability that contractor A wins when $q_A > q_B$. The usefulness of this notation will become clear in the next section.

Subcontracting Stage. The search for subcontractor is implemented through a discriminatory auction by each contractor. Specifically, we allow for contractor A to have some preference for subcontractor T – at this point, we do not clarify the reasons for such preference but it may arise due to reputation or quality considerations, or because of prior interactions with a given subcontractor. We assume that the preference of contractor A for subcontractor T is captured by the discount δ_{AT} such that when considering two quotes, q_{AT} and q_{AR} , contractor A will choose subcontractor T if q_{AT} (the quote from subcontractor T) does not exceed q_{AR} (the quote from subcontractor R) by more than δ_{AT} , i.e. $q_{AT} \leq q_{AR} + \delta_{AT}$. Notice that contractor’s preferences can be summarized by a single number δ_{AT} since δ_{AR} should be equal to $-\delta_{AT}$ given the rule described above. In this example, we further simplify the situation by assuming that contractor A prefers subcontractor T whereas contractor B prefers subcontractor R and that $\delta_{AT} = \delta_{BR} = \delta$.

Further, as was mentioned before we maintain that all qualified subcontractors submit quotes to all primary contractors’ secondary auctions. The subcontracting stage is thus consists of two simultaneous discriminatory auctions (run by contractors A and B correspondingly). In this stage each subcontractor prepares two price quotes: one for the auction run by contractor A, $(Q_{A,j}(\cdot))$, and the other one for the auction run by contractor B, $(Q_{B,j}(\cdot))$. Quotes submitted in these auctions by any given subcontractor j are based on the same cost realization $c_{s,j}$ which he draws from the distribution $F_S(\cdot)$, $(q_{A,j} = Q_{A,j}(c_{s,j}), q_{B,j} = Q_{B,j}(c_{s,j}))$. These quotes are therefore related. However, they are not necessarily identical due to the difference in main contractors’ preferences. Recall that q_i denotes the subcontracting quotes chosen by contractor A. Therefore, $q_A = q_{AT}$ if $q_{AT} \leq q_{AR} + \delta_{AT}$ and $q_A = q_{AR}$ otherwise. Similarly, $q_B = q_{BR}$ if $q_{BR} \leq q_{BT} + \delta_{BR}$ and $q_B = q_{BT}$ otherwise.

Characterizing Equilibrium Subcontracting Price Functions. In this analysis we assume that contractors’ preferences are common knowledge. This means that in monotone pricing equilibrium each contractor is informed about the subcontracting quotes that his rival receives since he can invert the quotes which are submitted to him to learn subcontractors’ costs and then use the costs to compute subcontracting quotes submitted to his rival.

Notice that contractors and subcontractors are ex-ante symmetric in this setting (given the assumption $\delta_{AT} = \delta_{BR} = \delta$). Thus, subcontractors are using symmetric pricing strategies in the equilibrium. We use $Q_+(\cdot)$ and $Q_-(\cdot)$ to denote subcontractor’s pricing strategies in the auction where he is a preferred and where is not a preferred participant correspondingly, so that $Q_{AT}(\cdot) = Q_+(\cdot)$, $Q_{BT}(\cdot) = Q_-(\cdot)$ and $Q_{AR}(\cdot) = Q_-(\cdot)$, $Q_{BR}(\cdot) = Q_+(\cdot)$.

It is easy to show that subcontractor will not submit quotes such that he would win an

auction where he is not preferred and loose an auction where he is preferred (indeed, he can do improve the outcome by submitting in the auction where he is preferred by contractor the quote he submitted in the auction where he is not preferred by contractor plus δ .) For example, it easy to show that in equilibrium $q_{AR} \leq q_{BR}$ and $q_{BT} \leq q_{AT}$.

We describe the interim profit function and the necessary first order conditions for subcontractor R below. The profit function for subcontractor T could be characterized in a similar way.

$$\begin{aligned} \Pi(c_{s,R}, q_{AR}, q_{BR}) &= (q_{AR} - c_{s,R})Pr(R \text{ wins } A \text{ and } B; A \text{ wins over } B) \\ &\quad + (q_{BR} - c_{s,R})Pr(R \text{ wins } A \text{ and } B; B \text{ wins over } A) \\ &\quad + (q_{BR} - c_{s,R})Pr(R \text{ wins } B \text{ and loses } A; B \text{ wins over } A). \end{aligned}$$

This can be re-written as

$$\begin{aligned} \Pi(c_{s,R}, q_{AR}, q_{BR}) &= \pi_1(c_{s,R}, q_{AR}, q_{BR}) (1 - F_s(\xi_{AT}(q_{AR} + \delta))) \\ &\quad + (q_{BR} - c_{s,R}) \cdot \int_{\xi_{BT}(q_{BR} - \delta)}^{\xi_{AT}(q_{BR})} p_2(q_{BR}, Q_{AT}(c_{s,T})) f_s(c_{s,T}) dc_{s,T} \\ &\quad + (q_{BR} - c_{s,R}) \int_{\xi_{AT}(q_{BR})}^{\xi_{AT}(q_{AR} + \delta)} p_1(q_{BR}, Q_{AT}(c_{s,T})) f_s(c_{s,T}) dc_{s,T}. \end{aligned}$$

Here $\xi_{A,i}(\cdot)$ and $\xi_{B,i}(\cdot)$ denote inverse of pricing functions used by i ; $\pi_1(c_{s,R}, q_{AR}, q_{BR})$ is the expected profit of R conditional on winning both subcontracting auctions with quotes q_{AR} and q_{BR} : $\pi_1(c_{s,R}, q_{AR}, q_{BR}) = (q_{AR} - c_{s,R})p_1(q_{AR}, q_{BR}) + (q_{BR} - c_{s,R})p_2(q_{BR}, q_{AR})$. The term $(1 - F_s(\xi_{AT}(q_{AR} + \delta)))$ reflects the probability that A wins the auction where he is less preferred which means that in this case he wins both auctions (see the discussion above). The second and the third terms represent the case when subcontractor R only wins secondary auction B where he is preferred. Under these circumstances two cases are possible: (a) subcontractor R quote in auction B is lower than the winning quote of subcontractor T in auction A; subcontractor R quote in auction B is higher than the winning quote of subcontractor T in auction A. Both cases are possible since both subcontractors are preferred in the secondary auctions which they win. The second and the third terms in the expression above reflect these two possibilities.

The price quotes chosen by subcontractor R should satisfy first order necessary conditions. Maintaining that $\bar{c}_s \geq \xi_{AT}(q_{AR} + \delta) > \xi_{BT}(q_{BR} - \delta) \geq \underline{c}_s$, and differentiating $\Pi(c_{s,R}, q_{AR}, q_{BR})$

with respect to q_{BR} obtains the first F.O.C. equation:

$$\begin{aligned}
& \frac{\partial \pi_1(c_{s,R}, q_{AR}, q_{BR})}{\partial q_{BR}} (1 - F_s(\xi_{AT}(q_{AR} + \delta))) \tag{2} \\
& + \int_{\xi_{BT}(q_{BR}-\delta)}^{\xi_{AT}(q_{BR})} p_2(q_{BR}, Q_{AT}(c_{s,T})) f_s(c_{s,T}) dc_{s,T} + \int_{\xi_{AT}(q_{BR})}^{\xi_{AT}(q_{AR}+\delta)} p_1(q_{BR}, Q_{AT}(c_{s,T})) f_s(c_{s,T}) dc_{s,T} \\
& + (q_{BR} - c_{s,R}) \int_{\xi_{BT}(q_{BR}-\delta)}^{\xi_{AT}(q_{BR})} \frac{\partial p_2(q_{BR}, Q_{AT}(c_{s,T}))}{\partial q_1} f_s(c_{s,T}) dc_{s,T} \\
& + (q_{BR} - c_{s,R}) \cdot \int_{\xi_{AT}(q_{BR})}^{\xi_{AT}(q_{AR}+\delta)} \frac{\partial p_1(q_{BR}, Q_{AT}(c_{s,T}))}{\partial q_1} f_s(c_{s,T}) dc_{s,T} \\
& - (q_{BR} - c_{s,R}) \cdot p_2(q_{BR}, Q_{AT}(\xi_{BT}(q_{BR} - \delta))) \cdot f_s(\xi_{BT}(q_{BR} - \delta)) \cdot \xi'_{BT}(q_{BR} - \delta) = 0.
\end{aligned}$$

Differentiating $\Pi_R(c_{s,R}, q_{AR}, q_{BR})$ with respect to q_{AR} obtains the second F.O.C. equation:

$$\begin{aligned}
& \left(p_1(q_{AR}, q_{BR}) + (q_{AR} - c_{s,R}) \frac{\partial p_1(q_{AR}, q_{BR})}{\partial q_1} + (q_{BR} - c_{s,R}) \frac{\partial p_2(q_{BR}, q_{AR})}{\partial q_2} \right) \times \tag{3} \\
& \quad = \frac{\partial \pi_1(c_{s,R}, q_{AR}, q_{BR})}{\partial q_{AR}} \\
& \quad \times (1 - F_s(\xi_{AT}(q_{AR} + \delta))) - \\
& - \underbrace{((q_{AR} - c_{s,R}) \cdot p_1(q_{AR}, q_{BR}) + (q_{BR} - c_{s,R}) \cdot p_2(q_{BR}, q_{AR}))}_{=\pi_1(c_{s,R}, q_{AR}, q_{BR})} \cdot f_s(\xi_{AT}(q_{AR} + \delta)) \cdot \xi'_{AT}(q_{AR} + \delta) \\
& + (q_{BR} - c_{s,R}) \cdot p_1(q_{BR}, q_{AR} + \delta) \cdot f_s(\xi_{AT}(q_{AR} + \delta)) \cdot \xi'_{AT}(q_{AR} + \delta) = 0.
\end{aligned}$$

Further, in the case of no simultaneity and no dynamics, it is known (see Mares and Swinkels (2014)) that there is an equilibrium in strictly monotone strategies such that the contractor who is less preferred cannot win for costs high enough and at the threshold point the bid is equal to the cost. In our setting the situation is somewhat different. Suppose subcontractor R submits two quotes, (q_{AR}, q_{BR}) , in auction A (where R is less preferred) and B (where R is more preferred), respectively; and suppose that the quotes are such that given the strategies of the rivals the situation when R loses B but wins A never happens and that $q_{BR} > q_{AR}$. What if quotes are such that R wins B but loses A . To subcontractor R , this is both a bad and a good news: (a) this is a bad news because R will work on the project only if B wins the main auction; (b) this is a good news because, first, if R does end up working on the project, it is at a higher price and second, because A with the price $Q_{AT}(c_{s,T})$ is a weaker competitor part of the time, relative to the situation when he would have chosen quote q_{AR} , because of the preferential treatment.

It is our conjecture that, as a result of the good news, the subcontractors bid less aggressively in the auctions where they are less preferred (relative to the case of no simultaneity and no

dynamics) and, thus, in the equilibrium the quotes q_{AR} and q_{BR} are closer to each other than in the case with no dynamics and no simultaneity. Another consequence of the good news bit is that R 's bid at the cost value where he stops winning in auction A (threshold costs) is *strictly higher than the cost*. The threshold cost level and the corresponding price are such that R is indifferent between the bad news and the good news described above. With cost realization above the threshold R only can win subcontracting auction B . These findings are summarized in the Theorem presented below.

Theorem 1 *There is a symmetric BNE in pure monotone strategies with the following properties:*

- (a) *Strategy functions Q_{BR} and Q_{AT} are the same. Let us denote them as $Q_+(\cdot)$ (strategy when preferred).*
- (b) *Strategy functions Q_{BT} and Q_{AR} are the same. Let us denote them as $Q_-(\cdot)$ (strategy when not preferred).*
- (c) *For $\delta > 0$ small enough we have $c^* \in (\underline{c}_s, \bar{c}_s)$ such that $Q_-(c^*) = Q_+(\bar{c}_s) - \delta$. Thus, with cost realizations above c^* the less preferred bidder has no chance of winning the subcontracting auction.*
- (d) *$\bar{q} = Q_+(\bar{c}_s)$ is the solution to the following optimization problem:*

$$\max_q (q - \bar{c}_s) \cdot \Pr(c_{-j}^S > c^*, \text{contractor with quote } q \text{ wins over contractor with quote } Q_+(c_{-j}^S)),$$

which can be equivalently written as

$$\max_q (q - \bar{c}_s) \cdot \int_{c^*}^{\bar{c}_s} p_2(q, Q_+(c_{-j}^S)) dF_S(c_{-j}^S),$$

- (e) *The values c^* , $Q_+(c^*)$ and $Q_-(c^*)$ are such that*

$$(Q_-(c^*) - c^*) \cdot p_1(Q_-(c^*), Q_+(c^*)) + (Q_+(c^*) - c^*) \cdot (p_2(Q_+(c^*), Q_-(c^*))) = (Q_+(c^*) - c^*) p_1(Q_+(c^*), \bar{q})$$

- (f) *$0 \leq Q_+(c) - Q_-(c) \leq \delta$ for any $c \in [\underline{c}_s, \bar{c}_s]$.*

5 Identification of Model Primitives

The model primitives that need to be recovered in the case of the simple model are the distribution of contractors' costs for task one, F_1 , the distribution of subcontractors' costs for task two, F_s , and the factor reflecting preferential treatment, δ . We exploit the structure on our data which reports the bids by task, the identity of subcontractor and the price at which the task two is subcontracted by each of the contractors.

The identification is straightforward when $\delta_{i,j}$ are observed at the level of contractor-subcontractor pair or when many observations per pair are observed (alternatively, δ can be parameterized). Indeed, the standard GPV argument can be used to recover the distribution of costs for task one; similarly, the first order conditions associated with subcontractors' pricing problem could be used to recover the distribution of subcontractors' costs.

The identification is somewhat more complicated when contractors may decide whether or not to subcontract a given task or not. The identification in that case resemble identification argument for Roy's model.

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