

# Dynamic Auction Environment with Subcontracting

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## Abstract

This article provides evidence on the role of subcontracting in the auction-based procurement setting with private cost variability and capacity constraints. We demonstrate that subcontracting allows bidders to modify their costs realizations in a given auction as well as to control their future costs by reducing backlog accumulation. Restricting access to subcontracting raises procurement costs for an individual project by 12% and reduces the number of projects completed in equilibrium by 20%. The article explains methodological and market design implications of subcontracting availability.

**Keywords:** dynamic games, capacity constraints, generalized all-pay auction, subcontracting, procurement, auctions with interdependent valuations, auctions with re-sale

**JEL Classification:** C73, C63, D44, H57, L20

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## Introduction

The classic literature on the boundaries of the firm suggests capacity constraints as one of the reasons for outsourcing production rather than completing it in-house. In these studies the need for outsourcing is generated by stochastic demand and stochastic productivity shocks; however, the analysis is frequently confined to the perfect information setting. In this article we elaborate on the insights from this literature in the context of government infrastructure maintenance, a large market in which both capacity constraints and asymmetric information about cost are significant.<sup>1</sup> Due to the asymmetric information about costs variability, this market is organized around an auction-based allocation mechanism, and because the capacity constraints are frequently binding firms are allowed to subcontract (outsource) part of their work. We analyze the impact of such subcontracting availability on the performance of the market and inquire into its methodological and policy-related consequences. Our analysis is based on the set of calibrated parameters, such that the outcomes predicted by our model match the data for the California highway procurement market. This allows us to assess realistic magnitudes of the investigated effects.

Recent developments in auctions literature are characterized by an enhanced appreciation of the impact of capacity constraints on the performance of procurement markets. For example, Jofre-Bonet and Pesendorfer (2003) and Balat (2012) estimate that an increase in capacity utilization from one standard deviation below the average to one standard deviation above the average results in a 24% cost increase. These studies, however, do not account for the ability of firms to outsource part of their work in the subcontracting market. We argue that this omission has an important impact on the quantitative findings in the literature and on their interpretation. We modify a framework for the dynamic auction setting developed by Jofre-Bonet and Pesendorfer (2003) in which the work in a given period is allocated through first price auction and unfinished work is carried over to the next period as backlog. Such backlog subsequently increases future costs in the manner of first-order stochastic dominance. We focus on the dynamic incentives provided by the ability to subcontract in the primary market, thus our modeling of the subcontracting market itself is deliberately simple. We assume that the contractors can outsource part of the work using the secondary market, which

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<sup>1</sup>About 50% of funds spent on government procurement are allocated to construction and maintenance, environments that are traditionally associated with capacity constraints. (See, for example, the Federal Procurement Report 2007, available at <https://www.fpds.gov/>.)

is composed of a large number of small firms that undertake the amount of work co-measurable with their capacity. Each period the contractors decide whether to participate in the auction, and upon participation decide how much to bid and how much to subcontract. We assume that participants have to commit to the subcontracting policy at the time of submitting their bids. This is consistent with the rules adopted in many procurement markets.

Computing equilibria in dynamic auction games with subcontracting is a non-trivial numerical exercise, thus developing an algorithm that solves such class of games is one of the contributions of this article. Our numerical approach extends that of Saini (2013), who solves the dynamic game with capacity constraints, but without subcontracting. He shows that the equilibrium bidding strategies can be obtained by solving a standard auction with asymmetric bidders and with reparametrized cost distributions. This observation enables him to choose a specification where a closed-form solution of the auction game is available. Analysis of the dynamic game with subcontracting is more challenging because it involves deriving two interrelated policy functions for subcontracting and bidding. In addition, in the game with subcontracting, the continuation payoff after losing an auction depends on the current losing bid. As a result, bidding functions are determined by a generalized version of an “all-pay” auction with asymmetric bidders, which does not have closed-form solution for any of the known cost distributions. Instead, the bidding strategies have to be obtained as a numerical solution to the system of differential equations with boundary conditions. In contrast to Saini (2013), we compute equilibrium in our game as a limit of Markov Perfect Equilibria of finite horizon games. This alleviates concerns about the multiplicity of equilibria by providing a consistent and robust equilibrium selection rule that enables us to compare equilibrium outcomes for different models and parameter values. We embed the numerical algorithm into a routine which calibrates parameters of our model, so that model outcomes match those in the data from the California procurement market. We subsequently use these parameters to study procurement outcomes and policy effects associated with subcontracting availability.

The central feature of our setting is that bidders’ (effective) costs, which underlie the prices, are endogenously determined. Specifically, subcontracting reduces current costs by allowing to modify unfavorable within-period draws, and lowers future costs by mitigating the accumulation of backlog. Beyond reducing costs, the availability of subcontracting has consequences for equilibrium pricing.

In particular, it induces ex-ante symmetrization of cost distributions and ex-post symmetrization of specific cost draws, which intensifies competition and lowers bidders' mark-ups. The within-project cost modification also reduces the importance of private information and lowers informational rents. Moreover, an impact of backlog on future costs is reduced, therefore the dynamic considerations become less important, which further lowers equilibrium prices. These effects decrease the cost of procurement for an individual project. Additionally, lower prices allow for the allocation and completion of a greater number of projects because lower winning bids meet the reserve price more frequently. The higher rate of allocation also enables and is in part facilitated by the higher rate of participation. We estimate that in the California market the availability of subcontracting leads to 12% decrease in the average procurement costs for an individual project and in a 20% increase in the number of projects completed relative to the case without subcontracting.

Subcontracting availability has methodological ramifications. Specifically, in the markets with substantial subcontracting activity, omitting such subcontracting when estimating the distribution of private costs and the parameters associated with the impact of capacity constraints results in a downward bias. The bias from using a misspecified model without subcontracting is caused by an incorrect attribution of low equilibrium prices to low baseline costs and low importance of capacity constraints. For California data these effects are substantial: the mean of the cost distribution is biased downward by 8%, 23% and 33%, and standard deviation by 29%, 50% and 67%, under various representative levels of backlog. Similarly, the parameters capturing the effect of capacity utilization on the mean and the standard deviation of private costs are biased by on average 100%. Interestingly, the cost distributions recovered using the model without subcontracting correspond neither to the distribution of modified (effective) costs nor to the static component of modified costs. This is because the biases are mostly driven by the incorrect option value component imposed by the model without subcontracting in estimation.

The presence of subcontracting has important implications for the market design. We find that in an environment without subcontracting the equilibrium outcomes differ along several dimensions depending on using first or the second price auction. Specifically, the second price auction delivers 6% greater allocative efficiency and results in 10% higher number of projects allocated. However, it is also characterized by 14.6% higher procurement costs per individual project, which is caused by

higher cost resulting from greater backlog accumulation and by higher mark-ups charged in equilibrium. Formally, the difference in procurement costs across auction formats arises because of the cost asymmetry inherent in the setting with capacity constraints, and because of the interdependence in bidders' effective costs generated by the continuation value. The latter effect is similar to that documented in the auction models with re-sale by Haile (2001) and Bikhchandani and Huang (1989). The availability of subcontracting allows bidders to endogenously modify both the cost asymmetries and the interdependence. We find that in the setting with subcontracting the difference in the procurement cost across two formats are reduced to 1%. At the same time, the differences in allocative efficiency and in the number of allocated projects between formats remain important and amount to 4.8% and 6.3%, respectively. Thus, the choice of the auction format in the setting without subcontracting involves important trade-offs whereas in the setting with subcontracting this choice is less ambiguous.

To summarize, the article makes four contributions. First, we analyze the mechanism through which subcontracting works in the markets similar to California procurement market and measure the impact of subcontracting on procurement outcomes. Second, we study the implications of subcontracting availability for the choice between first price and second price allocative mechanisms. Third, we demonstrate methodological consequences of subcontracting availability and measure biases that would arise under a misspecified model. Finally, we develop a numerical algorithm that enables computing equilibria of the class dynamic auction games with subcontracting.

The rest of the article is organized as follows. In Section 1 we summarize the related literature. Section 2 describes the model. In Section 3 we characterize the equilibrium with subcontracting. The calibration exercise is summarized in Section 4. We analyze the properties of computed equilibrium in Section 5, study the implication of subcontracting for the choice of an auction format in Section 6, and discuss the consequences of using the misspecified model without subcontracting in estimation in Section 7. Section 8 discusses empirically relevant extensions. Section 8 concludes.

# 1 Related Literature

Our article is related to the literature on boundary of a firm which is represented by Coase (1937), Coase (1988), Williamson (1975), Jensen and Mechling (1976), Alchian and Demsetz (1972) and other studies. This literature analyzes the factors that determine what components of firms' production should be outsourced rather than performed in-house. Some of the factors they mention are dynamic (capacity) constraints, quality control, and the difficulty of creating appropriate incentives for outside workers. Our analysis abstracts from most of these issues and focuses only on the gains from subcontracting in the presence of asymmetric stochastic costs as well as capacity constraints.

We are more closely related to the literature that studies the effect of subcontracting on the performance of static auctions. For example, Wambach (2009) investigates the benefits of committing to subcontracting strategy at the time of bidding. Gale, Hausch, and Stegeman (2000) investigate subcontracting in sequential auctions. They are interested in questions similar to the ones we pose in this article. However, they focus on the environment with perfect information, where projects are allocated through a second price auction. They find as we do that firms subcontract higher amounts subsequent to recent winning. This literature also includes a considerable number of empirical articles such as Miller (2012), De Silva, Kosmopoulou, and Lamarche (2011), Moretti and Valbonesi (2011), and an experimental analysis by Nakabayashi and Watanabe (2010). Empirical research focuses on the effect of long-term relationships on subcontracting, and preferential treatment in the subcontracting market as well as the effect of uncertainty on the amount of subcontracting.

Finally, we build on the empirical literature, represented by Jofre-Bonet and Pesendorfer (2003), Groeger (2012), and Balat (2012), that measures the importance of capacity constraints in the procurement markets organized as a sequence of first-price sealed-bid auctions. Our article extends the models used by these studies. As we mentioned earlier, our numerical strategy is an extension of the method proposed in Saini (2013), who studies a dynamic procurement environment with capacity constraints but without subcontracting.

## 2 Model

This section describes a model of a dynamic procurement auction with endogenous subcontracting. The model is developed in the context of construction procurement but could be adjusted to describe other similar markets.

### Setting

We consider an infinite horizon environment where a buyer (for example, a government) seeks to allocate a project of size  $x$  to a contractor every period. We assume that projects consist of providing a certain amount of homogeneous service. Projects are allocated one at a time among two infinitely lived firms via first-price sealed-bid auctions. The contractors, upon winning a project, may engage to do all the work in-house or may decide to re-sell a part of the project to subcontractors operating in the secondary market.

**Subcontracting Market.** To simplify the exposition we assume that the subcontracting market is summarized by a (possibly) increasing supply curve,  $P(\cdot)$ , which is constant over time. In this we abstract from any possible contractor-subcontractor alliances, contractor-specific bargaining, or the possibility of capacity constraints arising in the subcontracting market. In the setting we have in mind, the subcontracting market consists of a large number of small firms that could be very heterogeneous in their costs. In addition, the project can be sub-divided into small tasks that could be completed by a subcontractor in one period. Several subcontracting firms may be hired to fulfill subcontracting demand on a given project. Under these circumstances the subcontracting supply curve would remain nearly static. This also accounts for the possibility that the subcontracting price may be increasing in quantity. We believe that these features characterize many real-life subcontracting markets. Our setting accounts for the fact (in a degenerate way) that the contractors are likely to draw correlated subcontracting costs because they are shopping in the same market.

**Productivity and Backlog.** Contractors that operate in this market are endowed with capacity,  $K_i$ ,  $i = 1, 2$ . A contractor's productivity, i.e., the amount of work he completes within a given period of time, may depend on several factors (such as weather) that are outside his control. Following the literature, we model contractor  $i$ 's within-period productivity as a random variable,  $\epsilon_{it}$ , that

takes its values from an interval  $[0, K_i]$  and is distributed according to  $F_{\epsilon, i}$ . In this market, project size  $x$  is usually large relative to contractors' capacities. This regularity re-enforced by stochastic productivity implies that a certain amount of outstanding obligations may be carried from period to period. The work that contractor  $i$  has undertaken to complete but which remains unfinished at the beginning of period  $t$  is summarized by contractor  $i$ 's backlog in period  $t$ ,  $\omega_{i,t}$ . We assume that the contractor's backlog levels are known to all market participants. Further, in our environment the issue of sequencing jobs does not arise because projects are homogeneous. The contractor works on them in the same order in which they arrive.

**Backlog and Contractors' Costs.** We assume that project costs are given by  $c_{i,t}x$  where the marginal cost  $c_{i,t}$  is the private information of contractor  $i$ . Marginal cost is drawn from the distribution  $F_c(\cdot|\omega_{it})$ , which depends on the contractor's current capacity utilization defined as  $R_{i,t} = \frac{\omega_{i,t}}{K_i}$ . Capacity utilization essentially is equal to the number of periods before the contractor would be able to start work on any new load under the best possible scenario. We assume that higher capacity utilization has an adverse effect on the project costs distribution in the sense of first-order stochastic dominance. This assumption captures the possible effects of deadlines or any other potential cost effects associated with working at full capacity over a substantial amount of time. It is easy to get a sense for the effect of this variable and its relationship to so-called capacity constraints if one imagines that the contractor has to complete the project within a certain number of periods. Then the closer capacity utilization is to the allocated number of periods the less likely it is that the project will be completed on time. If the cost of missing the deadline is positive (and proportional to the project's size), then the marginal cost of the project will be increasing in capacity utilization. We do not explicitly assume any restrictions on the duration of the project to avoid unnecessary complications in solving the model.

## Time line

Each period in the game is divided into two stages. In the first stage the new projects are allocated and in the second stage the work on the projects is performed. At the beginning of the period the state of the world is characterized by a vector of contractors' backlogs,  $\omega_t = (\omega_{1,t}, \omega_{2,t})$ . This vector determines contractors' capacity utilizations and, hence, the distribution of their costs.



**Project Allocation.** In the first stage contractors first observe realization of own entry costs,  $\kappa$ . Then they decide whether to participate in the auction or not. After participation decisions are made they are observed by all competitors. Next, those who have decided to participate simultaneously observe realizations of their marginal project costs,  $c$ , and choose their bids,  $b$ , and subcontracting amounts,  $h$ , for the new project.<sup>2</sup> Both entry and marginal project costs are contractors' private information and are independent across contractors and projects. We assume that entry costs are drawn from distribution  $F_\kappa(\cdot)$  and marginal project costs are summarized by distributions  $F_c(\cdot | R_{it})$  as explained in the previous section. Formally, contractors' participating strategies are functions of own entry cost and state,  $d(\cdot | \omega_t)$ , whereas bidding and subcontracting strategies are two-dimensional functions of marginal project cost and state:  $b = \{b^{(1)}(\cdot | \omega_t), b^{(2)}(\cdot | \omega_t)\}$ ,  $h = \{h^{(1)}(\cdot | \omega_t), h^{(2)}(\cdot | \omega_t)\}$ . The first component in both vectors corresponds to the case when the contractor is a single participant; the second component is for the case when both contractors are participating in an auction.

Following the literature (see, for example, Jofre-Bonet and Pesendorfer (2003), Li and Zheng (2009) and others) we assume that the government uses a secret reserve price,  $R$ , distributed according to  $F_R(\cdot)$  with realizations in period  $t$  denoted  $r_t$ . If the lowest bid is below reserve price the respective contractor is awarded the project and is paid his bid. The new project adjusted for subcontracting is added to winner's backlog. We assume that the contractor is required to commit to a subcontracting strategy prior to the auction and cannot renege on his commitment later. This assumption is based on the rules followed in most real-life procurement markets, though alternative specification could be of interest as well.

In the analysis that follows we assume that total payment is paid and that all of the cost is incurred right after the auction. This simplifying assumption is made for analytical convenience. However, it is not very far from reality. In real markets contractors are usually paid at the end of the job, whereas they are required to post a bond that is used to pay their suppliers and subcontractors before the auction. This implies that the problem of sub-dividing the costs or the payments over periods when work on the project lasts does not arise.

**Backlog Depreciation.** After the auction stage is concluded, the contractors observe their productivity draw,  $\epsilon$ , and reduce their backlogs by corresponding amounts.

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<sup>2</sup>More details on this type of participation model can be found in Krasnokutskaya and Seim (2011) or Athey, Levin, and Seira (2013).

**State Transition.** The state evolves according to the following equation:

$$\sigma_i(1 - h_{i,t}, \omega_t, \epsilon_{i,t}) = \max\{\omega_{i,t} + (1 - h_{i,t})x - \epsilon_{i,t}, 0\} \quad (1)$$

Note that, in the case when player  $i$  does not enter, or loses the auction, or bids above the reserve price, the state of player  $i$  transitions according to

$$\sigma_i(0, \omega_t, \epsilon_{i,t}) = \max\{\omega_{i,t} - \epsilon_{i,t}, 0\}$$

In the remainder of the article, where the arguments of  $\sigma_i$  function are straightforward, we use a simplified notation that omits  $\omega_t$  and  $\epsilon_t$ . Similar to the previous literature our specification of the transition of the states ensures that the evolution of contractors' backlogs is stochastic. In simulations, we assume that the backlog amount is limited from above by some large positive constant  $M$ . Therefore, the state space in our game is given by  $\Omega = [0, M] \times [0, M]$ .

## Markov Perfect Equilibrium

The contractors in our model are forward-looking: as in the environment without subcontracting they take into account how winning a project today impacts their competitiveness and profitability in the future. Winning has a dichotomous effect: on the one hand, the winner collects a profit in the current period; on the other hand, winning increases backlog and, due to capacity constraints, implies higher costs in the near future. Similarly, losing increases the competitor's backlog and, therefore, provides a competitive edge in the next few periods. The contractor chooses his optimal strategy by weighting current profit against the difference between the continuation values of losing and winning. In the environment with subcontracting these considerations become even more subtle. First, in the model with subcontracting the amount of work that the contractor commits to complete himself and which is added to his backlog depends on his cost realization. In addition, contractor  $i$ 's bid, which is a function of current costs, determines the range of competitor's costs for which contractor  $i$  loses, and thus affects competitor's realized subcontracting amount and his backlog accumulation. In short, dynamic incentives in the environment with subcontracting depend on current costs realization, and thus they

impact optimal strategies differentially across cost levels. This is in contrast to the environment without subcontracting, where dynamic considerations affect contractors' behavior (their pricing) uniformly across cost levels.

We analyze Markov Perfect Equilibria of the dynamic auction game as defined in Maskin and Tirole (1988). In particular, we consider strategies that depend only on payoff-relevant histories. In our case, payoff-relevant information is summarized by a vector of contractors' backlogs. Indeed, own backlog fully determines the distributions of the contractor's cost and productivity in period  $t$ . Thus, current backlog variables determine the contractor's profitability in the current period as well as his backlogs in future periods. Hence, we can summarize the state of the market at time  $t$  by the vector of contractors' backlogs at the beginning of period  $t$ . Contractor  $i$  decides on state-dependent optimal action consisting of participation,  $\mathbf{d}_i(\cdot; \omega)$ , bidding,  $\{\mathbf{b}_i^{(1)}(\cdot; \omega), \mathbf{b}_i^{(2)}(\cdot; \omega)\}$ , and subcontracting,  $\{\mathbf{h}_i^{(1)}(\cdot; \omega), \mathbf{h}_i^{(2)}(\cdot; \omega)\}$ , functions that for every realization of his private costs,  $\kappa$  and  $c$  respectively, determine whether he participates in an auction or not, the bid he submits if he participates and the portion of work he commits to completing in-house upon winning. We define a stage payoff of our game as the expected profit that could be collected in a given period. Note that this stage payoff is stationary, that is, it does not change over time conditional on state (backlogs) and actions. This fact enables us to restrict our attention to stationary strategies. As standard theory suggests, for each strategy profile  $\mathbf{g} = \{(\mathbf{d}_i(\cdot; \omega), \mathbf{b}_i^{(1)}(\cdot; \omega), \mathbf{b}_i^{(2)}(\cdot; \omega), \mathbf{h}_i^{(1)}(\cdot; \omega), \mathbf{h}_i^{(2)}(\cdot; \omega))\}_{i=1,2; \omega \in \Omega}$ , and a starting state  $\omega^0$  there exists an (almost) unique Markov process that determines the joint distribution of private costs  $\kappa_{i,t}, c_{i,t}$ , states  $\omega_t$ , and actions  $d_{i,t} = \mathbf{d}_i(\kappa_{i,t}, \omega_t)$ ,  $b_{i,t}^{(n)} = \mathbf{b}_i^{(n)}(c_{i,t}, \omega_t)$ ,  $h_{i,t}^{(n)} = \mathbf{h}_i^{(n)}(c_{i,t}, \omega_t)$  for each  $t = 0, \dots, \infty$ . For a given strategy profile we define a value function of contractor  $i$  as a sum of discounted future expected profits where the expectation is taken with respect to the stochastic process. Formally,

$$V_i(\omega_0; \mathbf{g}_i, \mathbf{g}_{-i}) = E \left\{ \sum_{t=0}^{\infty} \delta^t d_{i,t} \left[ (1 - d_{-i,t}) \mathbf{1}_{\{b_{i,t}^{(1)} < r_t\}} \left( b_{i,t}^{(1)} - (1 - h_{i,t}^{(1)}) c_{i,t} x - P(h_{i,t}^{(1)} x) h_{i,t}^{(1)} x \right) + \right. \right. \quad (2)$$

$$\left. \left. d_{-i,t} \mathbf{1}_{\{b_{i,t}^{(2)} < b_{-i,t}^{(2)}, b_{i,t}^{(2)} < r_t\}} \left( b_{i,t}^{(2)} - (1 - h_{i,t}^{(2)}) c_{i,t} x - P(h_{i,t}^{(2)} x) h_{i,t}^{(2)} x \right) - \kappa_{i,t} \right] \omega_0 \right\},$$

where  $\delta$  denotes the discount rate common to all contractors, and the expression in the first brackets

$$\mathbf{1}_{\{b_{i,t}^{(1)} < r_t\}} \left( b_{i,t}^{(1)} - (1 - h_{i,t}^{(1)})c_{i,t}x - P(h_{i,t}^{(1)}x)h_{i,t}^{(1)}x \right)$$

denotes the expected profit of the contractor if he is a single participant whereas the expression in the second brackets

$$\mathbf{1}_{\{b_{i,t}^{(2)} < b_{-i,t}^{(2)}, b_{i,t}^{(2)} < r_t\}} \left( b_{i,t}^{(2)} - (1 - h_{i,t}^{(2)})c_{i,t}x - P(h_{i,t}^{(2)}x)h_{i,t}^{(2)}x \right)$$

summarizes the period  $t$  profit of contractor  $i$  when two contractors are competing in the auction. Notice that contractor's  $i$  period payoff is zero if he does not participate or meet the reserve price. We refer to this value function as the *ex-ante value function*, because it describes the value to the contractor before he acquires private information about his costs in the current period.

We consider Markov Perfect Equilibria  $\mathbf{g}^* = (\mathbf{g}_1^*, \mathbf{g}_2^*)$ , such that  $V_i(\omega_0 | \mathbf{g}_i^*, \mathbf{g}_{-i}^*) \geq V_i(\omega_0 | \mathbf{g}_i, \mathbf{g}_{-i}^*)$  for all  $\mathbf{g}_i$ ,  $i = 1, 2$ , for all  $\omega_0 \in \Omega$ , and given that contractors have correct beliefs about the distribution of their competitors' private costs.

### 3 Equilibrium Characterization

#### Bellman Equation

Under standard assumptions contractors' optimal behavior in this environment can be summarized by a Bellman equation. To simplify the presentation we develop the relevant Bellman equation in steps.

In what follows we suppress dependence of the value function on  $(\mathbf{g}_i, \mathbf{g}_{-i})$  for the brevity of notation. We also consider the interim continuation value of player  $i$ , conditional on drawing the entry costs  $\kappa_i$  as  $\tilde{V}_i(\omega, \kappa_i)$ . Then ex-ante value function is given by  $V_i(\omega) = \int \tilde{V}_i(\omega, \kappa_i) dF_\kappa(\kappa_i)$ . The

Bellman equation for player  $i$  could be written in the following way:

$$\tilde{V}_i(\omega, \kappa_i) = \max_{d_i} \int_{\kappa_{-i}} \left\{ d_i \left[ \mathbf{d}_{-i}(\kappa_{-i}) U_i(\omega; A = \{i, -i\}) + (1 - \mathbf{d}_{-i}(\kappa_{-i})) U_i(\omega, A = \{i\}) - \kappa_i \right] + (1 - d_i) \left[ \mathbf{d}_{-i}(\kappa_{-i}) U_i(\omega, A = \{-i\}) + (1 - \mathbf{d}_{-i}(\kappa_{-i})) U_i(\omega, A = \quad) \right] \right\} \quad (3)$$

Here  $A$  denotes the set of auction participants and  $U_i(\omega, A)$  denotes continuation value of bidder  $i$  for a given configuration of auction participants,  $A$ . Next we derive the expression for  $U_i(\omega, A = \{i, -i\})$  in detail and briefly comment on other cases.

### Continuation Value when both Contractors Participate

In the case when both contractors participate in an auction, contractor  $i$ 's dynamic payoff from winning conditional on the realization of private costs  $c_i$ , submitting bid  $b_i^{(2)}$ , and using subcontracting action  $h_i^{(2)}$  is given by

$$(b_i^{(2)} - (1 - h_i^{(2)})c_i)x - P(h_i^{(2)}x)h_i^{(2)}x + \delta E_\epsilon V_i(\sigma_i(1 - h_i^{(2)}), \sigma_{-i}(0)) \quad (4)$$

where the expectation is taken with respect to the distribution of within period realizations of productivity  $\epsilon$ .

Contractor  $i$ 's dynamic payoff from losing is given by

$$\delta E_{\epsilon, c_{-i}} [V_i(\sigma_i(0), \sigma_{-i}(1 - \mathbf{h}_{-i}^{(2)}(c_{-i}))) \mathbf{b}_{-i}^{(2)} < b_i^{(2)}] \quad (5)$$

In the remainder of the article we frequently drop the dependence of bidding and subcontracting strategies as well as cost distribution on the state  $\omega$  to keep the notation simple. Notice that if contractor  $i$  loses the auction, his competitor's backlog increases. However, as opposed to an environment without subcontracting, the future competitor's backlog, that is  $\sigma_{-i}(1 - \mathbf{h}_{-i}^{(2)}(c_{-i}))$ , depends on the competitor's costs  $c_{-i}$  through the subcontracting strategy. In turn, contractor  $i$ 's bid determines the set of the competitor's cost to which he may lose. Thus, contractor  $i$ 's payoff from losing depends on his own bid.

We put together the above pieces to obtain

$$\begin{aligned}
U_i(\omega; A = \{i, -i\}) = \int_{c_i} \max_{b,h} \left\{ W_i(b) \left[ (1 - F_R(b)) \left( b - (1-h)c_i x - P(hx)hx + \right. \right. \right. & (6) \\
\left. \left. \left. \delta E_\epsilon V_i(\sigma_i(1-h), \sigma_{-i}(0)) \right) + F_R(b) \delta E_\epsilon V_i(\sigma_i(0), \sigma_{-i}(0)) \right] + \right. \\
(1 - W_i(b)) \delta E_{c_{-i}} \left[ (1 - F_R(\mathbf{b}_{-i}^{(2)}(c_{-i}))) E_\epsilon V_i(\sigma_i(0), \sigma_{-i}(1 - \mathbf{h}_{-i}^{(2)}(c_{-i}))) + \right. \\
\left. \left. F_R(\mathbf{b}_{-i}^{(2)}(c_{-i})) E_\epsilon V_i(\sigma_i(0), \sigma_{-i}(0)) \mathbf{b}_{-i}^{(2)} < b \right] \right\} dF_i(c_i).
\end{aligned}$$

Here,  $W_i(b) = (1 - F_{-i}(\mathbf{b}_{-i}^{(2)})^{-1}(b))$  denotes the probability that bidder  $i$  submits the lowest in the auction.

### Other cases

The continuation value to contractor  $i$  when he is the only participant is similar in structure to the case of two participants except the probability of winning is determined by the distribution of the secret reserve price rather than the distribution of competitor's bids and the continuation value when losing does not depend on the contractor's own bid:

$$\begin{aligned}
U_i(\omega, A = \{i\}) = \int_{c_i} \left\{ \max_{b,h} (1 - F_R(b)) \left[ b - (1-h)c_i x - P(hx)hx + \right. \right. & (7) \\
\left. \left. \delta E_\epsilon V_i(\sigma_i(1-h), \sigma_{-i}(0)) \right] + F_R(b) \delta E_\epsilon V_i(\sigma_i(0), \sigma_{-i}(0)) \right\} dF_i(c_i).
\end{aligned}$$

The continuation value to contractor  $i$  when only contractor  $-i$  enters is similar to the continuation value of losing in the case with two bidders except that competitor's probability of winning is determined by the secret reserve price:

$$\begin{aligned}
U_i(\omega, A = \{-i\}) = \delta E_{c_{-i}} \left[ (1 - F_R(\mathbf{b}_{-i}^{(1)}(c_{-i}))) E_\epsilon V_i(\sigma_i(0), \sigma_{-i}(1 - \mathbf{h}_{-i}^{(1)}(c_{-i}))) + \right. & (8) \\
\left. F_R(\mathbf{b}_{-i}^{(1)}(c_{-i})) E_\epsilon V_i(\sigma_i(0), \sigma_{-i}(0)) \right]
\end{aligned}$$

Finally, if nobody enters the continuation value to contractor  $i$  is determined by expected depletion

of competitors' backlogs:

$$U_i(\omega, A = \emptyset) = \delta E_\epsilon V_i(\sigma_i(0), \sigma_{-i}(0)) \quad (9)$$

## Participation Strategies

Given the structure of the game a contractor should participate in the auction if an interim value from participation net of entry costs exceeds interim value from not participation. That is, when

$$\begin{aligned} \kappa_i \leq & p_{-i}(\omega)[U_i(\omega, A = \{i, -i\}) - U_i(\omega, A = \{-i\})] + \\ & (1 - p_{-i}(\omega))[U_i(\omega, A = \{i\}) - U_i(\omega, A = \emptyset)] \end{aligned} \quad (10)$$

for a given competitor's probability of entry  $p_{-i}$ . The right hand side of the above condition does not depend on  $\kappa_i$  therefore participation behavior is determined by a threshold strategy with a threshold given by

$$\begin{aligned} K_i(\omega) = & p_{-i}(\omega)[U_i(\omega, A = \{i, -i\}) - U_i(\omega, A = \{-i\})] + \\ & (1 - p_{-i}(\omega))[U_i(\omega, A = \{i\}) - U_i(\omega, A = \emptyset)] \end{aligned} \quad (11)$$

such that

$$\mathbf{d}_i(\kappa_i) = \begin{cases} 1 & \text{if } \kappa_i \leq \bar{K}_i(\omega) \\ 0 & \text{otherwise} \end{cases}$$

The entry probabilities  $(p_i, p_{-i})$  are thus given by

$$\begin{aligned} p_i(\omega) &= F_\kappa(\bar{K}_i(\omega)) \\ p_{-i}(\omega) &= F_\kappa(\bar{K}_{-i}(\omega)) \end{aligned} \quad (12)$$

Conditions (10) and (12) define the equilibrium entry probabilities and thus threshold participation strategies of this game. The threshold strategies may not be unique. We follow the literature in verifying uniqueness for a vector of parameters we use in our policy analysis.

## Optimal Subcontracting Strategies

Notice that the payoff from losing and the probability of winning do not depend on bidder  $i$ 's subcontracting strategy, both in the case with two participants and when  $i$  is the only contractor participating. This allows us to solve for contractor  $i$ 's optimal subcontracting strategy using his continuation value from winning, which has the same structure in both cases. It implies that  $\mathbf{h}_i^{(1)} = \mathbf{h}_i^{(2)}$ , thus superscripts are omitted in subsequent exposition. Formally,

$$\mathbf{h}_i(c_i; \omega) = \arg \max_h (-(1-h)c_i x - P(hx)hx + \delta E_\epsilon V_i(\sigma_i(1-h), \sigma_{-i}(0))).$$

Notice that the second derivative of the payoff with respect to the subcontracting strategy is negative,

$$-P''(hx)hx^2 - 2P'(hx)x + x\delta E_\epsilon V_{i,11}''(\sigma_i(1-h), \sigma_{-i}(0)) < 0,$$

if one of the following conditions is satisfied.

(A1) *The subcontracting supply schedule,  $P(\cdot)$ , is convex in quantity supplied and the expected future value function,  $E_\epsilon V_i(\cdot, \cdot)$ , is concave in own state.*

(A1')  *$E_{\epsilon'} V_i(\cdot, \cdot)$  is not concave but  $E_\epsilon V_{i,11}''(\sigma_i(1-h), \sigma_{-i}(0))$  is small relative to  $-P''(hx)hx^2 - 2P'(hx)x$  and  $P(\cdot)$  is convex or vice versa if  $P(\cdot)$  is not convex but  $-P''(hx)hx^2 - 2P'(hx)x$  is small relative to  $x\delta E_\epsilon V_{i,11}''(\sigma_i(1-h), \sigma_{-i}(0))$  and  $E_\epsilon V_i(\cdot, \cdot)$  is concave in own state.*

It would be challenging to establish the concavity of the expected value function in this very general setting. Therefore we verify this property in simulations.

**Proposition 1** *If condition (A1) or (A1') holds, then the optimal subcontracting action  $h_i^*$ , exists, is unique, and is determined by the following equations:*

$$\begin{aligned} h_i^* &= 1 && \text{if } c_i - P'(x)x - P(x) - \delta E_{\epsilon'} V_{i,1}'(\sigma_i(0), \sigma_{-i}(0)) > 0 \\ h_i^* &= 0 && \text{if } c_i - P(0) - \delta E_{\epsilon'} V_{i,1}'(\sigma_i(1), \sigma_{-i}(0)) < 0 \\ 0 < h_i^* &< 1 && \text{if } c_i - P'(h_i^*x)h_i^*x - P(h_i^*x) - \delta E_\epsilon V_{i,1}'(\sigma_i(1-h_i^*), \sigma_{-i}(0)) = 0. \end{aligned}$$



As can be seen above, an optimal  $h_i^*$  does indeed depend on the state and on contractor  $i$ 's current cost realization, i.e.,  $h_i^* = \mathbf{h}_i(c_i; \omega)$ . Notice that the contractor subcontracts only at cost levels that are sufficiently high relative to  $P(0) + \delta E_\epsilon V'_{i,1}(\sigma_i(1), \sigma_{-i}(0))$ , which is the marginal cost of the first unit purchased in the subcontracting market net of the dynamic cost of completing the whole project. In general, when deciding on the level of subcontracting the bidder weighs the marginal cost (both static and dynamic) of completing this unit in-house,  $c_i - \delta E_\epsilon V'_{i,1}(\sigma_i(1 - h_i^*), \sigma_{-i}(0))$  against the cost of purchasing this unit in the subcontracting market,  $P'(h_i^* x) h_i^* x - P(h_i^* x)$ , which accounts for the fact that the price in the subcontracting market may potentially grow as he attempts to purchase more units.

**Corollary 1** *If condition (A1) or (A1') holds, then the optimal subcontracting policy is weakly increasing in the realization of static marginal cost.*

This property obtains by differentiating the first-order condition for  $0 < h_i < 1$  with respect to  $c_i$ . Details are provided in the Appendix.

To summarize, several useful properties of subcontracting functions arise in the areas of the state space where the expected value function is concave in own state and has positive cross-partial derivatives. However, theoretically there are no guarantees that these properties of the expected value function hold everywhere or even at some subset of the state space. We verify these properties for the parameter values we use in our policy analysis.

### Flat supply curve

It is instructive to consider the case of a flat subcontracting supply curve,  $P(z) = p_0$ . In a static game, contractors choose not to subcontract if  $p_0 > c_i$  and to subcontract everything if  $p_0 < c_i$ . Subcontracting is only used to “improve” high cost realizations. In contrast, in a dynamic game, non-zero amounts of subcontracting are optimal as long as

$$c_i > \delta E_\epsilon V'_{i,1}(\sigma_i(1), \sigma_{-i}(0)) + p_0.$$

Notice that  $p_0 > \delta E_\epsilon V'_{i,1}(\sigma_i(1), \sigma_{-i}(0)) + p_0$  if  $E_\epsilon V'_{i,1} < 0$ , a condition we would expect to hold in equilibrium, is satisfied. Contractors outsource more in a dynamic equilibrium because they

additionally use subcontracting to alleviate future capacity constraints, sometimes at the expense of short-term efficiency. It is still optimal to subcontract to the limit if  $p_0 < c_i$ . However, full subcontracting remains optimal if  $p_0 > c_i > \delta E_\epsilon V'_{i,1}(\sigma_i(0), \sigma_{-i}(0)) + p_0$ . Also, in contrast to the static game, intermediate levels of subcontracting occur on a non-degenerate interval,

$$[\delta x E_\epsilon V'_{i,1}(\sigma_i(1), \sigma_{-i}(0)) + p_0, \delta x E_\epsilon V'_{i,1}(\sigma_i(0), \sigma_{-i}(0)) + p_0].$$

The width of this interval depends on the curvature of the value function, the discount factor and the size of the contract.

## Bidding Strategies

Next, we characterize bidding strategies in the environment with subcontracting. We characterize the case with two bidders in details and then comment on the bidding strategies when only a single bidder is present. Everywhere in this section we assume that bidder  $i$  is participating in the auction.

### Auction with Multiple Participants

In this section we show that after optimal subcontracting strategies are determined and given a vector of value functions, the contractors' optimization problem in the auctions with more than one bidder can be re-arranged to resemble a static asymmetric procurement auction with an "all-pay" feature. An "all-pay" feature arises because in the environment with subcontracting bidder  $i$ 's continuation value of losing an auction depends on his bid.

For the purpose of this section we assume that the subcontracting supply schedule is such that contractors choose to subcontract at all cost realizations and it is never optimal to subcontract the whole project. We make this assumption in order to maintain the smoothness of the bidding problem, which in turn facilitates the existence and computation of bidding strategies. We believe that such an assumption is without loss of generality for the purpose of our analysis: indeed, an optimal subcontracting policy could be approximated by a strategy which implies that a bidder subcontracts a very small portion of the project where the optimal policy prescribes to subcontract zero amount and such that the subcontracted share is slightly less than one if the optimal policy

prescribes subcontracting the whole project.

We further assume that the ex-ante value function is decreasing in own state and is increasing in the state of the competitor, i.e.,  $V'_{i,1}(\omega_i, \omega_{-i}) < 0$ , and  $V'_{i,2}(\omega_i, \omega_{-i}) > 0$ , and either condition (A1) or condition (A1') holds (and therefore the optimal subcontracting strategy is increasing in the bidder's own cost,  $\mathbf{h}'_{i,1} > 0$ ). We expect the first two properties to arise due to the presence of capacity constraints and the limited availability of subcontracting. Indeed, as in the game without subcontracting, the high level of own backlog increases the risk of the high current cost realization. In addition, the relation between the project's size and the contractor's capacity ensure that high backlogs persist into the future. Limited subcontracting means that these concerns could not be completely eliminated. Similarly, the high levels of the competitor's backlog implies a higher chance of the competitor having high costs both in the current and in the next few future periods. While these properties are intuitively justified, it would be difficult to establish them formally. We verify these properties numerically.

Denote  $E_\epsilon V_i(\sigma_i(0), \sigma_{-i}(1 - \mathbf{h}_{-i}(\bar{c}_{-i})))$  by  $\underline{V}_i(\omega)$ . This is the lowest possible payoff from losing given  $\omega$  because  $\mathbf{h}_{-i}(\cdot)$  is at its highest possible level.

For the purpose of further exposition we introduce a new object,  $\phi_i(c_i, \omega)$ , which we refer to as *effective costs* as opposed to *original* (or *current*) costs,  $c_i$ ,

$$\phi_i(c_i; \omega) = (1 - h_i)c_i x + P(h_i x)h_i x - \delta(E_\epsilon V_i(\sigma_i(1 - h_i), \sigma_{-i}(0)) - \underline{V}_i(\omega)). \quad (13)$$

The first part of  $\phi$  is a *static effective cost*, which captures the immediate impact of subcontracting on markups. The second part represents a dynamic opportunity cost of winning the auction against the least efficient opponent. Note that effective cost  $\phi$  does not depend on the number of active bidders.

We assume that  $V_i$  is decreasing in the own backlog, and increasing in the backlog of the opponent, therefore, we know that

$$-\delta(E_\epsilon V_i(\sigma_i(1 - h_i), \sigma_{-i}(0)) - \underline{V}_i(\omega)) > 0,$$

i.e., the dynamic cost component is always positive and therefore raises the cost of the project.

Further, notice that

$$\phi'_{i,1}(c_i; \omega) = (1 - h_i)x + \{P'(h_i x)h_i x + P(h_i x) + \delta E_\epsilon V'_{i,1} - c_i\}h'_i x > 0, \quad (14)$$

where the non-negativity of the term in the brackets follows from the necessary first-order conditions for the optimality of the subcontracting function and from our assumption that  $\mathbf{h}'_i > 0$ . Therefore, we can re-write the contractor's optimization problem in terms of  $\phi_i$ . More specifically, through the change of variables we will view the bidding function as a function of effective costs,  $\phi_i(c)$ , rather than  $c$ , i.e.,

$$\mathbf{b}_i^{(2)}(\cdot; \omega) : [\underline{\phi}_i(\omega), \bar{\phi}_i(\omega)] \rightarrow [\underline{b}^{(2)}(\omega), \bar{b}^{(2)}(\omega)].$$

Similarly, we define the inverse bid function, which maps bids into effective costs rather than into real costs:

$$\xi_i(\cdot; \omega) : [\underline{b}^{(2)}(\omega), \bar{b}^{(2)}(\omega)] \rightarrow [\underline{\phi}_i(\omega), \bar{\phi}_i(\omega)].$$

In a similar way we can express subcontracting schedule  $\mathbf{h}_i$  as a function of  $\phi_i$ . The equilibrium of the bidding game is characterized by a system of first-order differential equations in  $\xi_i$  and  $\xi_{-i}$ , which represent the necessary first-order conditions associated with the bidding problem for  $i = 1, 2$ ,

$$\begin{aligned} (1 - F_{\phi, -i}(\xi_{-i}(b))) \left\{ f_R(b)(b - \phi_i) + F_R(b) - f_R(b) \delta [EV(\sigma_i(0), \sigma_{-i}(0)) - \underline{V}_i] \right\} - \\ f_{\phi, -i}(\xi_{-i}(b)) \xi'_{-i}(b) F_R(b) \left\{ b - \phi_i - \delta [EV(\sigma_i(0), \sigma_{-i}(1 - \mathbf{h}_{-i}(\xi_{-i}(b)))) - \underline{V}_i] \right\} = 0 \end{aligned} \quad (15)$$

and boundary conditions. Notice that, in our setting, the supports of effective cost distributions are naturally different for contractors with different backlog levels. We adjust the standard argument (see Kaplan and Zamir (2012)) accounting for the all-pay component in order to obtain the boundary condition for our optimization program. More specifically, without loss of generality, assume that

$\bar{\phi}_1 \leq \bar{\phi}_2$ .<sup>3</sup> Then,

$$\begin{aligned}\xi_1(\bar{b}, \omega) &= \bar{\phi}_1 \\ \xi_2(\bar{b}, \omega) &= \bar{b} \\ \xi_1(\underline{b}, \omega) &= \underline{\phi}_1 \\ \xi_2(\underline{b}, \omega) &= \underline{\phi}_2,\end{aligned}\tag{16}$$

where  $\bar{b}$  is the highest equilibrium bid.

Next, let  $b_0$  be implicitly defined by

$$\begin{aligned}\{(1 - F_R(b_0)) - f_R(b_0)(b_0 - \bar{\phi}_1) + \delta f_R(b)[E_\epsilon V_1(\sigma_1(0), \sigma_2(0)) - \underline{V}_2]\}(1 - F_{\phi,2}(b_0)) - \\ \{b_0 - \bar{\phi}_1 - \delta[E_\epsilon V_1(\sigma_1(0), \sigma_2(\mathbf{h}_2)) - \underline{V}_2]\}(1 - F_R(b_0))f_{\phi,2}(b_0) = 0\end{aligned}\tag{17}$$

The proposition below summarizes conditions that determine the value of  $\bar{b}$ .

**Proposition 2** *If  $b_0$  is defined as in (17) then  $\bar{b} = \min\{b_0, \bar{\phi}_2\}$ .*

The proof is in the Appendix.

The problem in (15) with boundary conditions defined in (16) and Proposition 2 satisfies all of the usual conditions sufficient to guarantee the existence and uniqueness of the pair of equilibrium bidding functions (see Reny and Zamir (2004) and Athey (2001)).

### Auction with a Single Bidder

If contractor  $i$  is a single bidder in an auction his choice of bid solves the optimization problem below given an optimal subcontracting strategy  $\mathbf{h}_i$  that determines the effective cost  $\phi_i$ :

$$\max_b (1 - F_R(b))(b - \phi_i + \delta \underline{V}_i) + F_R(b) \delta E_\epsilon V_i(\sigma_i(0), \sigma_{-i}(0)).\tag{18}$$

It is a one dimensional and monotonic optimization problem which can be solved using bisection method.

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<sup>3</sup>Notice that the ranking of  $\bar{\phi}_1$  and  $\bar{\phi}_2$  does not necessarily reflect the ranking of  $\omega_1$  and  $\omega_2$  due to the dynamic cost component.

## Dynamic Option Effect

The optimal pricing behavior in the environment with capacity constraints is driven in part by option value considerations because winning or losing an auction has implications beyond collecting expected within-period profit. Notice that in the environment *without* subcontracting and *without* secret reserve, the contractor's optimization problem can be represented as

$$\max_{b_i} (1 - F_{-i}(\xi_{-i}(b_i))) \left( b_i - c_i x + \delta \left[ E_\epsilon V_i(\sigma_i(1), \sigma_{-i}(0)) - E_\epsilon V_i(\sigma_i(0), \sigma_{-i}(1)) \right] \right) + \delta E_\epsilon V_i(\sigma_i(0), \sigma_{-i}(1)). \quad (19)$$

Therefore, the option value impact on optimal pricing could be conveniently summarized by the constant shift to bidder's costs in the problem above, i.e., the difference between the continuation value conditional on losing and the continuation value conditional on winning,

$$-\delta \left[ E_\epsilon V_i(\sigma_i(1), \sigma_{-i}(0)) - E_\epsilon V_i(\sigma_i(0), \sigma_{-i}(1)) \right].$$

In this case, the option value effect translates into a uniform upward shift of static bidding strategies.

Similar property holds in the environment *with* subcontracting when only one bidder is present. The situation is more complex in the environment *with* subcontracting when two bidders participate in an auction. The optimal pricing behavior in (15) is affected by dynamic option value considerations through three terms  $E_\epsilon V(\sigma_i(0), \sigma_{-i}(1 - \mathbf{h}_{-i}(\xi_{-i}(b))))$ ,  $E_\epsilon V(\sigma_i(1 - \mathbf{h}_i(\phi_i)), \sigma_{-i}(0))$ , and  $E_\epsilon V(\sigma_i(0), \sigma_{-i}(0))$ . The first term enters the necessary first-order conditions directly and represents bidder  $i$ 's marginal continuation value conditional on losing. The second term enters optimality conditions through the effective costs term,  $\phi_i$ , and represents bidder  $i$ 's continuation value conditional on winning. The third term represents the case in which the contract is not awarded because both contractors bid above the secret reserve.

In contrast to the model without subcontracting, the terms involving the value function enter the optimality conditions in separate places and cannot be conveniently localized. Further, the dynamic considerations in the model with subcontracting depend on the current cost realization through the subcontracting function and therefore affect the shape (slope and curvature) as well as the level of

the bidding function. In an auction environment with asymmetric bidders this impact could not be easily derived because no closed-form solution exists.

We can obtain an insight into the effect of option value considerations on pricing by analyzing the difference in bidding behavior between static and dynamic auction. We compute the option value component as the difference between the bid function in the fully dynamic model and the bid function that would be used in a static auction with the distributions of costs given by the distributions of static effective costs from the dynamic environment (keeping the subcontracting unchanged), i.e.,

$$\tilde{c}_i = (1 - \mathbf{h}_i)c_ix + P(\mathbf{h}_ix)\mathbf{h}_ix.$$

The results are presented in Section 5.

## 4 Calibration to Highway Procurement Data

In this section we calibrate parameters of our model so that contractors' decisions and auction outcomes derived from the model match those computed from the the California highway procurement data.

### California Highway Procurement Market

We use the data from California Department of Transportation (CalTrans) covering years 2002 through 2013.<sup>4</sup> CalTrans is responsible for construction and maintenance of roads and highways within California. The services for the related projects are procured by means of first-price sealed-bid auctions. The projects are formulated as lists of tasks such as escalation, resurfacing, replacing the base or filling in cracks. The companies participating in the market tend to subcontract a subset of tasks to smaller firms specializing in the corresponding type of work. Generally, the size of a project exceeds monthly firm productivity (even after subcontracting). Thus, in this setting firms regularly carry over a backlog of work.

For every project our dataset includes an estimate of costs constructed by CalTrans' engineers

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<sup>4</sup>Jofre-Bonet and Pesendorfer (2003) also use the data from CalTrans but covering a different time period.

from past winning bids for similar projects, the deadline imposed by CalTrans for project completion, the type and description of work and the list of subcontracted tasks. We restrict our attention to large paving projects and focus on large paving companies regularly participating in this market. Pertinent features of this environment are summarized in Table 1.

According to Table 1 an average large project takes 9 months to complete. Contractors tend to subcontract one third of the project on average. Further, an overwhelming majority of large paving projects attracts at most two large bidders. Participation decisions are important features of our environment because large firms do not participate in every auction.<sup>5</sup> As Table 1 indicates a large company wins a large project in a given month with probability close to 1/4 on average. To make our model consistent with the empirical environment we set a period length equal to two months in this calibration exercise.

While the formal reserve price is not announced the CalTrans authorities may reject a bid at their discretion if it is deemed “irregular”. This is reflected in Table 1 which reports that a project is allocated with probability 48% if a single large bid is submitted. We capture this feature of the environment by assuming that CalTrans uses a secret reserve price in the allocation process. Unfortunately, we have very little guidance in our modeling of the reserve price because the exact rules that apply when bids are labeled as “irregular” are not known. In order to minimize arbitrariness in our analysis we assume that the distribution of the reserve price is constant across states. The set of auction participants often includes so-called fringe bidders, i.e., smaller companies that participate in the market infrequently. These firms are less likely to carry backlog and generally are not impacted by capacity constraints. The probability that a project is not allocated reported above includes the probability that a project is allocated to one of the fringe bidders (which is equal to about 10% for the projects we use in calibration analysis). The random reserve price thus in part reflects the competitive pressure imposed by these participants. We hold the distribution of reserve price fixed in counterfactual analysis. However, it is also possible to recompute the bidding strategies of fringe bidders under the assumption that their subcontracting decisions are motivated by static considerations.

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<sup>5</sup>Anecdotal evidence suggests that the potential participants and those who decided to submit bids are known in advance to most firms in the market. This is even more likely to be true in the case of large firms. Therefore, the assumption of our model that bids depend on the actual number of the active bidders is consistent with this setting.



## Calibration Methodology

We parameterize our model as follows. The distribution of unit costs,  $c$ , (the total costs obtain as a product of unit costs and project size) are assumed to be normal with the mean and variance that depend on firm's backlog of work,  $\mu_{i,t}^c = \alpha_0 + \alpha_1 * \omega_{i,t}$  and  $\sigma_{i,t}^c = \beta_0 + \beta_1 * \omega_{i,t}$ . Further, we truncate the support of the costs distribution corresponding to zero backlog at 5% (or zero whichever is higher during the current round of optimization) and 90% quantiles of the corresponding normal distribution respectively and allow the support to be shifted at the same rate as the mean of the cost distribution as backlog increases, that is,  $\underline{c}_{i,t} = \underline{c}_{i,t}^0 + \alpha_1 * \omega_{i,t}$  and  $\bar{c}_{i,t} = \bar{c}_{i,t}^0 + \alpha_1 * \omega_{i,t}$ . The secret reserve price,  $R$ , is assumed to be distributed according to the normal distribution, the mean and the variance of which ( $\mu_R$  and  $\sigma_R$ ) we aim to recover in this calibration exercise. We truncate this distribution as well so that the low boundary of the support coincides with the low boundary of the costs distribution with zero backlog and the upper bound is set sufficiently high so that it is never below the upper bound of the cost distribution. Firms' entry costs for a given auction are assumed to be drawn from an exponential distribution with a scalar parameter,  $\kappa$ .

We further assume that subcontracting supply function belongs to 3-parameter family of linear-hyperbolic functions given by  $P(z) = \gamma_0 + \gamma_1 z + \gamma_2 \frac{z}{1-z}$ . The hyperbolic term is added out of technical considerations to ensure that bidders never subcontract the whole project. Such concerns do not arise under realistic values of the parameters but they have to be addressed to guarantee smooth execution of the calibration program. We thus fix parameter  $\gamma_2$  in such a way that hyperbolic term only becomes important very near the full project size.

We aim to pin down nine parameters through this calibration exercise:  $\alpha_0$ ,  $\alpha_1$ ,  $\beta_0$ ,  $\beta_1$ ,  $\mu_R$ ,  $\sigma_R$ ,  $\kappa$ ,  $\gamma_0$ ,  $\gamma_1$  and  $K$  (an upper bound on the support of firms' productivity  $\epsilon$ ). We set the size of the project to be equal to the size of an average large paving project in our data<sup>6</sup> (\$4.37mln). The length of the period is fixed to be equal to two months as is explained in the previous section. The parameters values are chosen so that the distance between the statistics computed from our model and those in the data is minimized. In the inner loop of this optimization process a full solution of the model is computed for each set of parameters under consideration. This computation is based on the numerical algorithm we develop for this article which is summarized in the Appendix. We use

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<sup>6</sup>The size of the project is approximated by an estimate of project's costs constructed by CalTrans engineers.

the following statistics:

- (1) the probability that a single firm entered
- (2) the probability that project is allocated under monopoly
- (3) the mean and the variance of the normalized bids<sup>7</sup> under monopoly and an average backlog
- (4) the means and the variances of the normalized bids under duopoly and under various backlog vectors of participating firms such as (empty,full), (full,empty) and (empty,empty)
- (5) the average of the subcontracted share of a project
- (6) an average time to complete the current backlog and the newly won project as represented by the average duration allocated for project (adjusted for project size)

In the computation of the average bids the vector of backlogs  $(\omega_1, \omega_2)$  should be interpreted to mean that the own backlog of the firm for which an average is computed is given by  $\omega_1$  and the competitor's backlog is given by  $\omega_2$ . Further, in the notations above, *empty* and *full* indicate that a contractor's backlog belongs to [25%, 50%] and [50%, 75%] quantile ranges of the stationary backlog distribution respectively. The backlog statistics computed from the model are set to correspond to the moments of the empirical distribution of augmented backlog which reflects the full amount of obligations undertaken by the firm (that is, it includes subcontracted amounts). In computing such augmented backlog both within the model and from the data we assume that projects are completed sequentially in the historical order and that the subcontracting work is performed at the same time as the firm works on the projects and is performed proportionally so as to be completed at the same time. This is a purely technical assumption made in order to make the backlog computed from the model comparable to that given in the data.

In this exercise statistic (1) identifies the parameter of the entry cost distribution ( $\kappa$ ); statistics in (4) identify parameters of the costs distribution and the impact of the backlog on the costs distribution ( $\alpha_0, \alpha_1, \beta_0, \beta_1$ ); these statistics together with (5) also identify the parameters of the subcontracting supply curve ( $\gamma_0, \gamma_1$ ); whereas statistics (2) and (3) identify parameters of the reserve price distribution ( $\mu_R, \sigma_R$ ). Finally, statistics in (6) identify the support of firms' productivities ( $K$ ).

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<sup>7</sup>The bids are scaled by the project size.

## Calibrated Parameters

Table 2 reports calibrated parameters of the model with subcontracting. All parameters appear to have expected signs and are of reasonable magnitude. The parameters associated with the capacity utilization indicate that important capacity constraints may arise in this market. Specifically, the mean, standard deviation, and upper support bound of the distribution of project costs increase with capacity utilization. At the average backlog level (\$2.21mln or capacity utilization of 1.15) these parameters of the cost distribution are 56%, 33% and 27% higher than corresponding parameters of the backlog-free cost distribution. Further, we find that the upper bound on the firms' productivity is approximately equal to 43% of the project size so that a firm working at full capacity should be able to complete the project in 2.1 periods. The average cost of entry conditional on participation is about 7% of the average project costs, which is broadly consistent with alternative estimates for entry costs previously reported in the literature.<sup>8</sup> The parameters of the subcontracting supply schedule indicate that it is relatively flat over the reasonable range of amounts.

We summarize the fit of our model to the data in Table 3. The model appears to fit data quite well despite significant simplification of the reality adopted in the article. Calibrated moments related to the probability of a single company entering and the mean of the bid distribution of a single bidder are somewhat further away from the data counterparts than other moments. The discrepancy most likely arises because of the restrictiveness of our specification of the reserve price. Recall that the distribution of the reserve price is assumed to be constant across states. This may not be the case in reality. Unfortunately, we have no information on the criteria CalTrans uses to qualify some high bids as unsuitable. Nevertheless, the calibrated moments' values are quite close to data counterparts even for these moments.

## 5 Properties of the Model with Subcontracting

In this section we summarize the properties of Markov Perfect Equilibrium of the dynamic game with subcontracting corresponding to calibrated parameters' values. We contrast this equilibrium with the one which obtains in the game without subcontracting.

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<sup>8</sup>See, for example, Li and Zheng (2009), Krasnokutskaya and Seim (2011) or Groeger (2012).

## Properties of the Stationary Equilibrium

We begin by investigating the properties of the stationary equilibria of the games with and without subcontracting.

Figure 1 shows three-dimensional graphs of the joint stationary distribution of backlog vectors, the stationary distribution of the individual contractor's backlog and the distribution of differences in backlogs for the games with and without subcontracting. These graphs indicate that the distribution of the contractor's backlog and the distribution of the differences in contractors' backlogs in the setting without subcontracting stochastically dominate the corresponding distributions that arise in the model with subcontracting. Specifically, as Table 4 shows, the average backlog of an individual contractor in the model without subcontracting is about 38.7% higher than the average backlog in the model with subcontracting, whereas the difference in contractors' backlogs is 22% higher. Thus, subcontracting reduces both backlog accumulation and cost asymmetries to an important degree.

Several variables of interest are summarized in Table 4. All the variables with the exception of the moments of the backlog distribution are reported on the per project level and conditional on participation and allocation wherever appropriate. Notice that in our setting the impact of subcontracting on procurement process has two margins: the cost of procurement per project and the number (or fraction) of projects that are allocated and thus completed in the equilibrium.<sup>9</sup>

Table 4 indicates the procurement cost of an allocated project is 12.3% higher if subcontracting is not available. In the sections below we review in detail several features of the model that generate this effect. Now we provide a brief summary. The reduction in procurement cost arises because the contractors' costs of completing the project and the markups they charge in equilibrium under subcontracting are lower. The contractors' costs are lower predominantly as a result of a lower backlog accumulation documented above. Beyond the lower backlog accumulation, subcontracting enables contractors to endogenously modify unfavorable cost draws for a specific project. The markups are reduced because cost modification both within period and over time leads to symmetrization of the cost distributions and reduction in the importance of private information. It is also worth noting that the price effects of associated with subcontracting are limited by the presence of the

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<sup>9</sup>We investigate these margins separately without attempting to combine them into a single welfare index because we lack a measure for the welfare loss associated with a delayed or canceled project.

secret reserve price. For example, the secret reserve does not allow the inefficient bidders to win the auction sufficiently often, which limits the pass-through of higher contractors' costs. Similarly, the contractors with high option value of losing (which arises when they have low backlog and low realization of original costs - see Figure 7) are not able to win frequently if they pass their option value into prices.

Table 4 shows that a fraction of projects allocated and thus completed in equilibrium is higher in the setting with subcontracting (the difference in frequencies is 20%).<sup>10</sup>, which provides the first measurement of the impact of subcontracting capacity on the operation of California procurement market. This result is not entirely surprising because the market with subcontracting has access to larger working capacity.<sup>11</sup> The rate of allocation in the market with subcontracting increases because lower prices enable contractors to beat the reserve price more often. Further, the higher allocation rate results in higher expected profits conditional on participation, despite that contractors' profits conditional on allocation are 11.5% lower (as we would expect because markups decline). Higher expected profits encourage more participation, which further increases the allocation rate. We investigated how restrictiveness of the reserve price affects these findings by simulating equilibrium outcomes under alternative distributions of the reserve price. We find that the results are moderately sensitive to the levels of reserve price: specifically, the fraction of the projects allocated in equilibrium increases by 24% (instead of by 20%) if the mean of the distribution of the reserve price is 15% higher than the level recovered in the calibration exercise. Similarly, the fraction of allocated projects increases only by 14% in the environment with more restrictive reserve price (the mean is 15% lower than the calibrated level). These findings are reported in the on-line Appendix.

To summarize, in California procurement market, subcontracting availability results in the reduction of per-project procurement costs, greater contractors' participation and in the higher fraction of completed projects.

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<sup>10</sup>We hold the distribution of reserve prices fixed in this exercise.

<sup>11</sup>The increase in allocation rate reflects in part transfer of profit from the set of fringe bidders towards the set of large bidders. However, this effect is likely to be small.

## The Value Function

Figures 2 and 3 depict the computed ex-ante value functions for the games with and without subcontracting. They show both the three-dimensional graphs of the ex-ante value function and the cross-sections of the value functions for different values of the contractor's own state.

The value function in the game with subcontracting is higher than that of the game without subcontracting. This is because in the equilibrium of the game with subcontracting the rate of project allocation exceeds the rate of allocation in the game without subcontracting, which generates both higher profit conditional on participation and generally higher unconditional one-shot profit.

We find that the value functions of both games are decreasing in the contractor's own state and are increasing in the competitor's state. In particular, the value function from the game with subcontracting declines by 22% when contractor's own backlog increases from none to the size of one project; it increases by 15% under similar changes in the competitor's backlog. In contrast, the value function from the game without subcontracting declines by 20% for comparable changes in contractor's own backlog and increases by 13% in response to changes in the competitor's backlog. These numbers indicate that the value function of the game with subcontracting is steeper both in the contractor's own and the competitor's state. In other words, the difference in contractor's states is more important in the game with subcontracting relative to the game without subcontracting. This effect is generated by the following mechanism. In a setting without subcontracting the reserve price prevents firms with low backlog values (and thus high option value of losing) from winning at prices that reflect their option values. Because of this, low-backlog firms lose quite frequently so that their probabilities of winning are quite similar to those of high-backlog contractors as demonstrated in Figure 9. In contrast, subcontracting provides a channel through which option value could be controlled /modified. This means that low-backlog contractors are able to win more often than high-backlog contractors and at price levels that reflect their option value of losing. This makes states more distinct when subcontracting is available. Note that this mechanism works against the effect induced by the alternative role of subcontracting, i.e. the modification of unfavorable costs realizations, which reduces the differences across states. Our results indicate that in California procurement market the mechanism which enhances the differences in expected profits across states slightly dominates the symmetrization mechanisms mentioned earlier.

Finally, notice that the computed value function indeed satisfies monotonicity conditions under which the optimal subcontracting and bidding strategies are derived in earlier sections.

## Subcontracting Strategy

Figure 4 plots subcontracting policy as a function of the contractor's private marginal cost,  $c$ , whereas Table 5 reports expected subcontracting levels across different states. The subcontracted portion of the project is close to zero for very low values of contractor's private costs, monotonically increases over the range of costs, and becomes relatively flat at the high cost realizations.

The panels demonstrate how the subcontracting strategy changes in contractor's own as well as the competitor's state. The level of subcontracting conditional on cost realization increases only slightly as the competitor's backlog increases. In contrast, contractor's own state has a more significant impact. As Figure 4 indicates, the slope of subcontracting strategy with respect to cost realizations is increasing in the contractor's own backlog. That is, a contractor with a low backlog level subcontracts higher fraction of the project than a contractor with high backlog level at low cost realizations. However, this ranking is reversed at high cost realizations. These regularities reflect the multiple roles of subcontracting. Specifically, subcontracting allows bidders to endogenously determine the distribution of their future costs, to modify the option value of losing instead of winning, and to improve the current realization of costs and hence their competitiveness in a current auction. The former is more important for the contractors with low backlog and for low current cost realizations when their option value of losing is high. The latter is more important for contractors with high backlogs (especially for high cost realizations), who are disadvantaged in a current auction due to unfavorable cost distribution.

The contractors with high backlog levels tend to subcontract more on average as Table 5 shows. This is because the support of the distribution of private costs shifts to the right as own states increases and because the subcontracting levels increase in the cost realization. This effect is quite substantial and dominates subcontracting differences at lower cost realizations. Specifically, the fraction of work which is subcontracted increases from 9% to 17% and to 25% of the project size as contractor's own state moves from 0 to 0.5 and to 1, respectively (the competitor's state is fixed at 0).

## The Effect of Subcontracting on Contractors' Costs

In this section we document the impact of subcontracting on contractor's costs for a given project. More specifically, we compare the distribution of the original costs to the distribution of the ex-post or static effective costs.

Table 6 shows that in the states with low backlog the subcontracting is primarily motivated by dynamic considerations, i.e., the subcontracting is used to avoid high costs in the future. For example, in the case when both contractors have zero backlogs the mean and the standard deviation of the distribution of ex-post costs are 9.4% and 4.3% higher than the mean and the standard deviation of the distribution of original costs. Conversely, in the states where own backlog is high, the subcontracting is used to reduce unfavorable cost draws. The magnitude of this effect increases in the contractor's own state. More specifically, the means of the ex-post cost distributions are 5%, and 13% lower than the means of the original cost distributions, and the standard deviations of ex-post cost distributions are 20% and 55% lower than the standard deviations of the original cost distributions for backlog levels 1.5 and 3, respectively.

These properties of the model have two important implications. First, the availability of subcontracting impacts within-state pricing competition due to the symmetrization of cost distributions as well as it causes the reduction in the importance of private information (and thus informational rents). Second, the subcontracting mitigates the impact of backlog on the distribution of original costs, because it allows bidders to modify their high cost realizations. This effect becomes more important at high levels of capacity utilization. Bids are based on these modified costs, hence, the reduced form analysis that studies correlation between the bids and capacity utilization may not correctly reflect the importance of capacity constraints in this market.

## Properties of Pricing (Bidding) Strategies

In our setting the bidding schedules are derived as functions of effective costs. Figure 5 plots the distribution of original and effective costs for several states. The graphs show that the effective cost is monotone in original cost. This sum of the static and dynamic part of effective costs is summarized by the distribution the high realizations "bunched" together, and have the variance that is lower



than the variance of the original cost distribution. Figures 6, 8 and Table 7 summarize the bidding strategies in the environment with subcontracting and compare them to the bidding strategies that arise in the environment without subcontracting.

The bidding strategies are characterized by several features that typically arise in the auctions with asymmetric bidders:<sup>12</sup> the more efficient contractor bids less aggressively at every cost realization, exploiting his advantage in terms of the distribution of costs; however, the bids submitted by the more efficient contractor are on average lower than the bids submitted by the less efficient contractor. This regularity arises because the less efficient contractor's distribution of costs allocates more mass towards higher cost realizations in contrast to the distribution of costs of a more efficient contractor. The difference in contractors' bidding strategies increases with the difference in their backlogs. Further, capacity constraints shift the support of the original cost distribution to the right as backlog levels increase. For the effective costs, the supports are further shifted by dynamic option value. As a result, the competing contractors may be associated with the distributions of effective costs with different supports. In equilibrium, the least efficient (in terms of the effective costs) contractor may be priced out of the market at the upper end of the support, i.e., his probability of winning an auction at any feasible price is zero. In these circumstances the contractor is indifferent between bids that result in zero profit. Without loss of generality we assume that he submits a bid equal to his effective costs. Notice that the contractor with a lower backlog is not necessarily more efficient in terms of effective costs, i.e., his support may be further to the right relative to the contractor with higher backlog. This is because the option value for the contractor with lower backlog may be higher than that of the contractor with higher backlog and this effect may, in principle, dominate the direct impact of the backlog on the support of original costs. However, in all of the figures shown in the article, the contractor with a lower backlog is also more efficient in terms of effective costs.

Next, we discuss the impact of subcontracting availability on bidding functions. We begin by analyzing the project-specific bidding schedules in the model with subcontracting. Figure 6 shows that these schedules as functions of the original cost realizations are lower and flatter than those from the model without subcontracting. This regularity arises because of cost modification that we described in the previous section and the reduction in the importance of the option value considerations. We

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<sup>12</sup>Asymmetry is in terms of the distributions of effective costs and is induced by the differences in the backlog levels.

first comment on the latter effect and then demonstrate how the two effects combine to produce the properties documented in Figure 6.

Figure 7 plots the option value component of prices for several states and for both the models with and without subcontracting. It shows that option value in the model with subcontracting tends to be lower than in the model without subcontracting and is mostly declining in the state and original cost realization.

Figure 8 demonstrates how the cost modification and option value effects are combined to result in the bid schedules that are lower and flatter than the bidding schedules in the model without subcontracting. Specifically, this figure plots (a) the bidding function that would arise in a static auction based on the distribution of the original costs; (b) the static bidding function based on the distribution of the static effective costs,<sup>13</sup> and (c) the bidding function from the dynamic model with subcontracting. The difference between the strategies in (a) and (b) reflects the effect of cost modification. The resulting distribution of static effective costs usually has both lower mean and lower variance than the distribution of the original costs. Thus, not only the contractor's ex-post costs are lower on average than the original cost draws, but the mark-ups based on the static effective costs are also lower than those based on the original costs due to the within-period symmetrization and the reduction in the importance of private information. The difference between the schedules in (b) and (c) reflects the option value component which is declining in original cost. This component thus contributes to flattening of the bid functions. The bidding functions under the "no subcontracting" regime obtain by adding option value component to the static bidding functions based on the distribution of original costs (as in (a)). This explains why the bidding functions under subcontracting are lower: the option value component is added to the schedule in (b) rather in (a) and it is smaller in magnitude relative to the "no subcontracting" case.

Table 7 documents the average bid profile as a function of own state. It shows that bids, on average, increase with own capacity utilization. In addition, the average bid profile is much lower and flatter in the model with subcontracting relative to the model without subcontracting. More specifically, in the California procurement market the bids under subcontracting would, on average, be 6%, 10% and 12% lower than those in the environment without subcontracting for states  $(0, 0)$ ,  $(0.5, 0)$

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<sup>13</sup>These are based on subcontracting functions from the dynamic model.

and  $(1, 0)$  respectively. As above, this regularity arises because subcontracting permits contractors to modify unfavorable cost draws and thus mitigate the impact of higher capacity utilization. The use of subcontracting for costs modification leads to stronger within-project symmetrization of contractor's costs and reduction in the importance of private information. Both of these effects grow in magnitude as the capacity utilization increases and work to lower bidders' mark-ups. Further, the importance of option value considerations is lower in the model with subcontracting and the disparity in importance of this component increases in own capacity utilization.

The differences in pricing strategies between the settings with and without the subcontracting translate into the difference in winning patterns. As Figure 9 and Table 8 demonstrate, the probability that a firm with low capacity utilization wins the auction is higher in the environment with subcontracting relative to that without subcontracting (the difference is two and three percentage points for the states with  $(0.5, 0)$  and  $(1, 0)$  respectively). Hence subcontracting improves the efficiency of the allocative mechanism.

## Robustness Analysis

We explore the sensitivity of contractors' strategies and of equilibrium outcomes to the availability of subcontracting opportunities as summarized by the flatness of the subcontracting supply schedule. The results of this analysis are documented in the on-line Appendix. We consider two schedules in addition to the one obtained in calibration exercise (baseline schedule): a schedule which is steeper and a schedule which is flatter than the baseline schedule. We find that all the properties we documented above are preserved under these permutations. The magnitudes of the effects are generally increasing in the flatness of the subcontracting supply curves.

An on-line Appendix also contains the results of the analysis exploring the effects of the policies which regulate access to subcontracting: (a) imposing an upper limit on the fraction of the work that can be subcontracted; (b) requiring that a certain amount of work should be subcontracted to so-called disadvantaged businesses. We find that results for the first policy are similar to those we obtain in the previous section while comparing the equilibria with and without subcontracting. Generally, the cost of procurement for an individual project increases as the policy becomes more restrictive, and the fraction of projects allocated and completed in the equilibrium decreases. For

the affirmative action program we find that it actually works to decrease the procurement cost for an individual project because subcontracting facilitates symmetrization and thus intensifies competition. The competitive effect dominates the increase in production costs associated with sub-optimal subcontracting levels. This effect, however, is not very large because the policy usually requires only a small increase in the low bound on the fraction of project subcontracted.

## 6 Auction Format and Procurement Cost Ranking

In this section we compare two auction mechanisms that are widely used in practice: the first price and the second price sealed bid auctions. We demonstrate that government preference for one mechanism versus another may differ in the environments with and without subcontracting. We begin by providing theoretical background for this analysis. Next, we characterize contractors' strategies in the environment where allocation is implemented through the second price auction. Then, we compare outcomes across the settings using simulations based on calibrated parameter values.

### Theoretical Motivation

The revenue equivalence of standard auction mechanisms (such as a first-price and a second-price auctions) that has been shown to hold for simple settings<sup>14</sup> usually breaks down in more complex environments such as the ones we consider in this article.

In a dynamic setting with capacity constraints but without subcontracting the revenue equivalence is likely to break for two reasons. First, this auction environment is inherently asymmetric because the distribution of bidder's costs depends on his backlog and auction participants usually differ in their backlog levels. Second, in a dynamic environment contractors' bids are based on effective costs that incorporate the continuation values under winning and losing. Thus the effective costs are endogenously determined by the outcome of the current auction. The continuation values depend on the outcomes of the future auctions. Through this channel they introduce interdependence of bidders' current effective costs and may potentially cause revenue equivalence to be violated. This mechanism is similar to the one documented for the markets with re-sale analyzed in Bikhchandani and Huang

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<sup>14</sup>See Myerson (1981).

(1989) or Haile (2001). (In the model with re-sale bidders' final values are endogenously determined by ex-post re-sale opportunities. Re-sale opportunities arise because bidders observe only a signal correlated with their use (baseline) values at the time of the primary auction. In the second stage, when these values are realized, opportunities for trade may exist. This creates interdependence and potentially introduces common values in the effective valuations at the time of the primary auction. In the model with re-sale the continuation payoffs conditional on winning and losing differ and depend on bidders' current signals that are observed after primary auction. However, the continuation value under losing does not depend on the bid submitted in a primary auction. The continuation values of the dynamic game with capacity constraints but without subcontracting have the same properties.)

Introducing subcontracting in the dynamic game with capacity constraints affects both channels described above. Subcontracting reduces cost asymmetries and mitigates the impact of intertemporal linkages. We would expect it to work to restore the revenue equivalence. The government preference between the two auction mechanisms is further complicated by endogenous participation that is likely to differ across the two formats and across the environments with and without subcontracting. The rate of participation would impact both the expected procurement costs and the number of projects allocated in the equilibrium.

## Equilibrium in the Environment with the Second-Price Auction

In this section we summarize contractor's optimization problem in the environment where projects' allocation is implemented through the second price sealed bid auctions. In a subsequent exposition we use notations  $h_i^{SP}$ ,  $\beta_i^{SP}(\cdot)$ ,  $\phi_i^{SP}(\cdot)$  and  $V_i^{SP}(\cdot)$  to emphasize that these objects are associated with the game where projects are allocated by means of second price auction.

As in the case with the first price auction we consider  $V_i^{SP}(\omega) = \int \tilde{V}_i^{SP}(\omega, \kappa_i) dF_\kappa(\kappa_i)$  where the corresponding Bellman equation is as in (3) except that it incorporates continuation values  $U_i^{SP}(\omega; A)$  that correspond to the second price auction environment. In the case when both contractors participate in an auction contractor 1's dynamic payoff conditional on the realization of private costs  $c$ , submitting bid  $b$  and subcontracting action  $h$ , and provided that his competitor uses strategies  $\beta_2^{SP}$

and  $h_2^{SP}$ , is given by

$$\begin{aligned}
U_1^{SP}(\omega; A = \{1, 2\}) = & \int_{c_1} \max_{b, h} \left\{ W_1^{SP}(b) \left[ \int_{\xi_2^{SP}(b)}^{\bar{\phi}_2} (1 - F_R(\beta_2^{SP}(\phi_2))) [\beta_2^{SP}(\phi_2) - \phi_1 + \delta \underline{V}_1^{SP}] f_{\phi_2}(\phi_2) d\phi_2 + \right. \right. \\
& \left. \int_{\xi_2^{SP}(b)}^{\bar{\phi}_2} \int_b^{\beta_2^{SP}(\phi_2)} [r - \phi_1 + \delta \underline{V}_1^{SP}] f_R(r) f_{\phi_2}(\phi_2) dr d\phi_2 + \delta F_R(b) (1 - F_{\phi_2}(\xi_2^{SP}(b))) EV(\sigma_1(0), \sigma_2(0)) \right] + \\
& \delta (1 - W_1^{SP}(b)) \left[ \int_{\underline{\phi}_2}^{\xi_2^{SP}(b)} (1 - F_R(\beta_2^{SP}(\phi_2))) EV(\sigma_1(0), \sigma_2(h_2^{SP}(\phi_2))) f_{\phi_2}(\phi_2) d\phi_2 + \right. \\
& \left. \left. \int_{\underline{\phi}_2}^{\xi_2^{SP}(b)} F_R(\beta_2^{SP}(\phi_2)) EV(\omega'_1, \omega'_2) f_{\phi_2}(\phi_2) d\phi_2 \right] \right\}.
\end{aligned}$$

Here as before,  $W_1^{SP}(b) = (1 - F_2(\beta_2^{SP})^{-1}(b))$  denotes the probability that bidder  $i$  submits the lowest bid in the auction; whereas

$$\phi_i^{SP}(c_i; \omega) = (1 - h_i^{SP})c_i x + P(h_i^{SP} x) h_i^{SP} x - \delta (E_\epsilon V_i^{SP}(\sigma_i(h_i^{SP}), \sigma_{-i}(0)) - \underline{V}_i^{SP})$$

denotes the effective costs. The cases with a single bidder could be characterized in a similar way and are omitted for brevity.

An optimal subcontracting strategies in this setting are derived in the same way as in the case of the environment with first-price auction except that they depend on the continuation value function from the setting with the second price auction.

As before, the equilibrium bid strategies,  $(\beta_1^{SP}, \beta_2^{SP})$ , and the inverse bid strategies,  $(\xi_1^{SP}, \xi_2^{SP})$ , are treated as functions of effective costs,  $\phi_1$  and  $\phi_2$ . The solution to the bidding problem is characterized by the necessary first-order conditions in the form of a system of differential equations in  $\xi_1^{SP}$  and  $\xi_2^{SP}$ .

$$\begin{aligned}
& [\xi_1^{SP}(b) - b + \delta EV_1^{SP}(\sigma_1(0), \sigma_2(0)) - \delta \underline{V}_2^{SP}] f_R(b) (1 - F_{\phi_2}(\xi_2^{SP}(b))) + \tag{20} \\
& [\xi_1^{SP}(b) - b + \delta EV_1^{SP}(\sigma_1(0), \sigma_1(h^{SP}(\xi_2^{SP}(b)))) - \delta \underline{V}_2^{SP}] (1 - F_R(b)) f_{\phi_2}(\xi_2^{SP}(b)) (\xi_2^{SP})'(b) = 0.
\end{aligned}$$

Further, let  $b_0$  be implicitly defined by

$$\begin{aligned} & [\bar{\phi}_1 - b_0 + \delta EV_1^{SP}(\sigma_1(0), \sigma_2(0)) - \delta \underline{V}_2^{SP}] f_R(b)(1 - F_{\phi_2}(b_0)) + \\ & [\bar{\phi}_1 - b_0 + \delta EV_1^{SP}(\sigma_1(0), \sigma_2(h_2^{SP}(\xi_2(b_0)))) - \delta \underline{V}_2^{SP}] (1 - F_R(b_0)) f_{\phi_2}(b_0) = 0. \end{aligned} \quad (21)$$

Then the boundary conditions to the differential equations in (20) are summarized in the Proposition below.

**Proposition 3** *Let  $(\beta_1^*, \beta_2^*)$  be a vector of equilibrium bidding functions and  $(\beta_1, \beta_2)$  are such that their inverse functions,  $(\xi_1, \xi_2)$ , solve (20). Without loss of generality assume that  $\underline{\phi}_1 \leq \underline{\phi}_2$  and  $\bar{\phi}_1 \leq \bar{\phi}_2$ . Define  $\underline{b} = \beta_2^*(\underline{\phi}_2)$ ,  $\hat{\phi}_1$  so that  $\beta_1^*(\hat{\phi}_1) = \underline{b}$ ,  $\bar{b} = \beta_1^*(\bar{\phi}_1)$ , and  $\hat{\phi}_2$  so that  $\beta_2^*(\hat{\phi}_2) = \bar{b}$ . Then,*

$$\begin{aligned} \beta_1^*(\phi) &= \left\{ \begin{array}{ll} \beta_1(\phi) & \text{if } \phi \geq \hat{\phi}_1 \\ \phi + \delta(EV_1^{SP}(\sigma_1(0), \sigma_2(0)) - \underline{V}_2^{SP}) & \text{if } \phi < \hat{\phi}_1 \end{array} \right\} \\ \beta_2^*(\phi) &= \left\{ \begin{array}{ll} \beta_2(\phi) & \text{if } \phi \leq \hat{\phi}_2 \\ \text{any } f_2(\phi) \geq \bar{b} & \text{if } \phi > \hat{\phi}_2 \end{array} \right\}. \end{aligned} \quad (22)$$

Further,  $\bar{b} = \min\{b_0, \bar{\phi}_2\}$ ,  $\hat{\phi}_2 = \bar{b}$ .

The proof of this proposition is very similar to the proof of Proposition 2. It is omitted for brevity. Notice that bidder 1 in the Proposition above always wins against bidder 2 when  $\phi_1 \leq \hat{\phi}_1$ . Therefore, his bid is chosen so that he can beat the reserve price while taking into account that the price he is paid is sometimes set by the reserve price. Notice that the bidding strategy of bidder 1 is discontinuous at  $\hat{\phi}_1$ .

## The Results of Format Comparison

The results of the simulation study that compares performance of the first-price and second-price auctions in the environment with subcontracting are summarized in Table 9. We document the average cost of procurement for an individual project, the average probability of project to be allocated, and allocative efficiency in a stationary equilibria for the two format levels.

We find that there is substantial difference between the auction formats when subcontracting is not available. Despite the improvement in allocative efficiency (by 6%) the procurement costs for an individual project is 14.6% higher under the second price auction relative to the setting with the first price auction. This increase in procurement costs arises due to higher levels of backlog accumulation (14.1% higher) under second price auction and due to higher mark-up levels that are charged by bidders in this environment (per project profit margin increases by 18.3%). The increased profitability results in higher participation rates which in turn increases the probability of project allocation by 10.9%. The later effect is responsible for higher levels of backlog accumulation. The government choice between the two formats will thus importantly depend on the relative weights it assigns to the per project cost of procurement versus the fraction of projects allocated (cost of delaying the project) versus social (allocative) efficiency.

In contrast, in the environment with subcontracting the differences between the two formats in the positive dimensions (allocative efficiency and the rate of allocation) are still large: the second price auction is more efficient (by 4.8%) and is characterized by higher frequency of allocation (6.3% higher). At the same time the difference in the procurement costs for an individual project is much smaller both in absolute and percentage terms (1.7%). The choice of the auction format is thus less ambiguous. The second price auction is likely to be preferred under a large number of circumstances. Interestingly, in the environment with the second price auction bidders tend to subcontract more which indicates that competition is more intense.

Our analysis indicates that out of the two channels we described in Section 6 the cost asymmetries play a more important role in this comparison. To establish this we solve individual auctions in both formats under a symmetric state and using the value function computed for the environment with the first price auction. Such an analysis would focus on the differences in auction formats generated by the value interdependence channel because the cost asymmetry channel is eliminated. We find that under these circumstances the difference between the two formats is rather small even in the case without subcontracting. The procurement costs tend to actually be lower under the second price auction (by 2-3%).



## 7 Methodological Implications of Subcontracting

In this section we investigate consequences of failing to account for subcontracting activity in empirical analysis based on the data generated by the environment where subcontracting is available. We also propose a structural test for the importance of subcontracting in a given setting that does not explicitly rely on the data for subcontracting decisions of primary contractors.

### Implication for Estimation Bias

The analysis in the previous sections demonstrates that in the environment with subcontracting and for a given vector of backlogs, contractors submit bids based on bidding functions that are lower and flatter than those used by contractors in the environment without subcontracting. They also tend to participate in bidding more frequently. We would expect, therefore, that empirical methodologies that rely on the bid and participation data<sup>15</sup> may obtain biased estimates of the cost distribution and of the parameters capturing the importance of capacity constraints if they rely on the assumption of no subcontracting while using data generated by the environment with subcontracting. We verify this conjecture below.

In particular, we use the distributions of bids and participation frequencies conditional on state that we observe in the data generated by the model with subcontracting to recover the value function and the distribution of private costs that would be consistent with these objects under the dynamic model without subcontracting. To do this we modify the methodology proposed in Jofre-Bonet and Pesendorfer (2003)<sup>16</sup> to allow for stochastic backlog depreciation. We then compare the distribution of private costs recovered under the misspecified model to the underlying distribution of private costs. In this exercise the distribution of bids is nonparametrically estimated from simulated data. The number of observations is set to be so large that the sample variation in our estimates is negligible. The discrepancy between the primitive and the recovered distribution of costs under the correct specification is due to numerical errors.

The results of the analysis are summarized in Table 10 and are depicted in Figure 10. The first panel of the table establishes a benchmark for the performance of the estimation procedure.

<sup>15</sup>Such as those proposed in Jofre-Bonet and Pesendorfer (2003) or Balat (2012).

<sup>16</sup>Tirerova (2014) extends this methodology to account for strategic participation decisions.

It demonstrates that when the estimation procedure is applied to the data generated by the model without subcontracting it correctly recovers parameters of the distribution of private costs.

In contrast, the methodology recovers the cost distribution with the mean and variance that are lower than those of the primitive distribution when applied to the bid distributions generated by the model with subcontracting. The downward bias is substantial at 8% for the mean and 29% for the standard deviation of the costs distribution under the state  $(0, 0)$ . The bias is increasing in the bidder's own backlog level: 23% and 33% for the means and 50% and 67% for the standard deviations when the cost distribution is recovered from the bid distributions corresponding to states  $(1.5, 1.5)$  and  $(3, 3)$  respectively. Thus, this analysis confirms that failing to account for subcontracting in estimation is likely to result in biased estimates for the distribution of private costs.

Further, the difference in the means and standard deviations of the cost distributions that correspond to different backlog values inform us of the strength of capacity constraints.<sup>17</sup> Again, we find that when this methodology is applied to the correct model, it produces reasonably accurate estimates of the capacity constraint parameter (estimated 0.14 and 0.045 versus the true values of 0.142 and 0.046). On the other hand the estimation under the misspecified model results in 100% downward bias for both the mean and the standard deviation shift parameters.

Notice that the estimated means and standard deviations are substantially lower than the mean and the standard deviations of the distribution of the static effective cost component (or ex-post costs) as can be seen by comparing the estimated parameters for the cost distribution at zero backlog to the first row of Table 6. The mean of the effective static cost component is 1.44 and the standard deviation is 0.7 as opposed to the estimated mean of 1.2 and standard deviation of 0.5. This is because the biases arise largely due to imputing wrong dynamic option effects from the model without the subcontracting rather using the model with subcontracting.

It is worth noting that these calculations abstract from backlog mismeasurement issues that are likely to arise in many realistic settings. The mismeasurement occurs when the backlog is computed as the sum of loads awarded to a given contractor without accounting for subcontracted portions. Such mismeasurement would additionally induce us to underestimate technological capacity con-

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<sup>17</sup>To use the estimates reported in the article notice that they are given for the distribution of the full costs, i.e.  $E[C|\omega_i] = (\alpha_0 + \alpha_1 * \omega_i)\bar{x}$ , and  $\text{Std.Dev}(C|\omega_i) = (\beta_0 + \beta_1\omega_i)\bar{x}$ . To recover  $\alpha_1$  we need to subtract  $E[C|\omega^0]$  from  $E[C|\omega^1]$  and divide the difference by  $(\omega^1 - \omega^0)\bar{x}$ . The calculation is similar for  $\beta_1$ .

straints, and by impacting the option value, measurement is likely to further bias our estimates of the distribution of costs.

## Testing for the Importance of Subcontracting

In procurement markets detailed information on bidding and participation is usually available. In contrast, the data on subcontracting are usually either not collected or, if collected, are of very poor quality. As a result many studies of procurement markets are not able to take into account the effects of subcontracting. For this reason, it would be useful to have means to assess the economic significance of subcontracting without data thereon.

Unfortunately, there is no obvious descriptive test that may be used to make such assessment. As our analysis indicates, the effect of backlog on costs and therefore bids may not be completely eliminated even if subcontracting is available in the market. The degree to which subcontracting relaxes capacity constraints depends on the price of subcontracting relative to the contractor's costs. Hence, the statistically significant relationship between the bids and the backlog does not necessarily rule out subcontracting. On the other hand, we document that even when subcontracting is relatively expensive and thus is not sufficiently extensive to eliminate capacity constraints, it still may have an important effect on bids, and thus result in a sizable estimation bias for its omission in estimation.

The analysis of the model with subcontracting suggests, however, that it should be possible to test for the importance of subcontracting within the framework of structural analysis. Specifically, the option value component in the model without subcontracting does not depend on a current cost realization. In contrast, the option value component in the model with subcontracting is decreasing in cost. The slope of this component for a given level of the bidder's own backlog depends both on the number of active competitors (i.e., those who win when this bidder loses) and on the backlog(s) of the active competitor(s) if this number is positive. Thus, if the inversion procedure from the previous section is correctly applied to the data generated by the model without subcontracting, then the estimated distribution of costs recovered conditional on the bidder's own backlog does not depend on the number of his active competitors or their backlogs. On the other hand, if this procedure is applied to the data generated by the model with subcontracting, then the inversion procedure will not correctly account for the variation in option value component and thus the distribution of

bidders' costs recovered from a misspecified model will vary in the number of active competitors and their backlogs. This suggests a structural test for the null of no subcontracting. Specifically, a researcher should recover the distribution of costs for a given level of bidder's own backlog but for several different numbers of active competitors or different vectors of competitors' backlogs. After that he should test for the equality of recovered costs distributions. If the equality can be rejected so can be the null of no subcontracting.

**Details of the Test.** The equality of two function (representing the densities of cost distributions, for example),  $H_0 : f_1(c) = f_2(c)$ , can be tested using the test statistics  $\hat{T}_n = \sum_{i=1}^{i=N} d_n^2(\hat{f}_{1,n}(t_i) - \hat{f}_{2,n}(t_i))^2$ , where  $\{t_i\}_{i=1,\dots,N}$  is the finite set of points on the real line and  $\hat{f}_{1,n}$  and  $\hat{f}_{2,n}$  are corresponding estimates of the values of these functions based on a sample consisting of  $n$  observations. The asymptotic distribution of this statistic can be constructed using subsampling procedure (specifics for implementing subsampling procedure can be found in Politis, Romano, and Wolf (1999)). To ensure the power of the test, it may be preferable to use re-centered test statistic following Hall and Horowitz (1996):  $\tilde{T}_n = \sum_{i=1}^{i=N} d_n^2(\hat{f}_{0,b}(t_i) - \hat{f}_{0,n}(t_i))^2$  where  $\hat{f}_{0,b} = \hat{f}_{1,b} - \hat{f}_{2,b}$  is computed in a subsampling procedure and  $\hat{f}_{0,n} = \hat{f}_{1,n} - \hat{f}_{2,n}$  is computed from the data.

## 8 Extensions and Additional Empirical Considerations

In this analysis we focus on a stylized model of a procurement market with subcontracting that emphasizes the features relevant for the discussion of the dynamic aspects of the operation of this market. In different applications other factors that typically arise in real-life procurement markets with subcontracting may be important. In this section we provide an (incomplete) summary of such factors and in some cases suggest how these factors may be incorporated in our framework.

**Heterogeneous Units.** In some markets, such as construction or maintenance procurement, the projects do not consist of a homogeneous amount of work; rather, they are composed of heterogeneous units such that each unit represents a homogeneous amount of work. The implication of this is that the costs of work may not be evenly allocated across these units and, in fact, a contractor may have a relatively low cost of completing some units and high cost of completing others. In addition, the work from different units may have to be subcontracted in different markets. Under these circumstances

a separate backlog may have to be considered for each type of work that could be included in the project. A contractor would have to decide on a separate subcontracting schedule for each submarket and then aggregate these decisions in a measure of effective costs reminiscent of the one we use in this article. The analysis of such an environment may be challenging due to the dimensionality issue.

**Contractor-Specific Price of Subcontracting.** In our analysis we assume that all contractors have access to subcontracting at the same price. In reality subcontracting prices are often negotiated and as a result are contractor-specific. If negotiated prices are public information, this feature does not substantially complicate the analysis except that otherwise symmetric contractors choose different subcontracting and bidding strategies in this environment.

If subcontracting prices are the private information of the contractor-subcontractor pair, then the model changes. Formally, in this environment contractors will have to form expectations about the subcontracting price and, therefore, about subcontracted levels of their competitors conditional on the cost draw. As a result the competitive effect of subcontracting will be softened. If subcontracting prices are correlated as they are likely to be if contractors are working with the same set of subcontractors, then subcontracting will induce correlation in the effective costs of bidders. This will result in the reduction of the variance of competitor's costs conditional on the contractor's own draw and will result in more aggressive bidding. However, this effect will be softer relative to the model we present in this article.

**Horizontal Subcontracting.** In some markets the primary and subcontracting markets may not be clearly separated. More specifically, firms participating in the subcontracting market may occasionally submit bids in the primary market. If this is the case, the subcontracting market cannot be summarized by the subcontracting supply curve. It would be important to model the decision to subcontract work to a specific firm as well as to account for the correlation in bids by the company that participates in both markets and the companies that use this subcontractor. Such an environment is potentially much more complicated than the one we consider in this article. Fortunately, horizontal subcontracting does not occur very often in the markets that we had in mind when writing this article. Marion (2011) investigates the issue of horizontal subcontracting using auction data from the California highway procurement market. He finds that during the period between 1996 and 2005, about 10% of projects received a bid from a company that was also listed as a subcontractor on

another bid. This feature affected 7% of the bids submitted during this time period. These figures, while not negligible, indicate that horizontal subcontracting is not a main concern that needs to be addressed when investigating the effect of subcontracting availability on the functioning of such markets.

**Capacity Constraint in Subcontracting Market.** In some markets subcontractors may undertake large assignments that may take several periods to complete. In such settings a researcher may be concerned about capacity constraints potentially affecting operation of such markets. However, introducing capacity constraints into the model of subcontracting market would complicate the analysis. In particular, this would require modeling the subcontracting market at the level of an individual subcontracting firm which would increase the dimensionality of the state space as well as impose much higher (in many cases unrealistic) standards on the data that could be used in the analysis of such a market. Fortunately, in many procurement markets (including California procurement) contractors tend to split the work between many subcontractors so that the share of an individual firm is small relative to the size of that firm. It is thus likely that capacity constraints issues are less important in these subcontracting markets.

**Timing of Subcontracting.** We assume that the decision to subcontract is made at the time of bidding. In some markets these decisions could be made as part of the ongoing work on the project. Formally, it means that a contractor can adjust the subcontracting levels in every period in which he works on the project if we keep subcontracting project-specific. The subcontracting strategy will depend on productivity realization in addition to state. Such a contingent strategy would have to be “integrated back” to compute the expected costs of the project at the time of bidding.

**Alternative Reasons to Subcontract.** This article focuses on capacity constraints as the main motivation for contractors to subcontract part of the project work. The literature on the boundaries of a firm enumerates multiple reasons such as increasing marginal costs, lack of production capability, and others. We find that in the environments we study, the capacity constraints channel generates important methodological and policy implications. Undoubtedly, these other channels only add to the importance of accounting for the availability of subcontracting opportunities in empirical research.

## Conclusion

This article provides evidence based on California highway procurement market on the role of subcontracting in the auction-based procurement in the settings with private cost variability and capacity constraints. We measure impact on procurement costs paid by the government as well as on the amount of work allocated and thus completed in the equilibrium while accounting for the effects on production costs, pricing, and contractor's participation. We find that availability of subcontracting results in lower procurement costs per project which arise because of the reduction in the production costs as well as in margins charged by contractors in equilibrium. The lower prices work to increase the number of projects that are allocated in equilibrium. The latter effect simultaneously enables and is facilitated by a higher rate of auction participation.

We find that availability of subcontracting has an important mechanism design implications. The environment with capacity constraints is inherently characterized by cost asymmetries. That is why the revenue equivalence of standard auction mechanisms (such as first-price and second-price auctions) breaks down in this setting. Additionally, in a dynamic setting strategic pricing is used to control own as well as competitor's backlog accumulation that in turn determine players' continuation values and thus make their "effective private costs" interdependent. This property is reminiscent of the re-sale models where the presence of re-sale market created interdependence in continuation values for the bidders participating in the primary market. These effects are present in the environments both with and without subcontracting. The latter effect is more subtle in the setting with subcontracting because continuation value of losing as well winning depends on contractor's bid. As expected, we find that the revenue equivalence does not hold in both environments. In the case when subcontracting is not available the choice between the auction formats involves non-trivial trade-offs. Specifically, the second price auction is characterized by higher allocative (social) efficiency and also by a higher number of projects allocated in equilibrium. On the other hand, it also results in a substantially higher procurement costs per individual project. In contrast, when subcontracting is available the allocative efficiency and increase in the number of allocated projects still arise under the second price auction but the procurement costs for an individual project are importantly reduced. The auction format choice is thus less ambiguous in the presence of subcontracting.

Finally, the availability of subcontracting has methodological implications. We find that if the model which ignores subcontracting is used to analyze the data generated by the market where subcontracting is available the estimates of the costs and capacity constraints will be highly biased. We also propose a structural test that could be used to assess the economic importance of subcontracting in a given environment without an explicit use of subcontracting data.

In short, subcontracting is an integral part of the procurement market. Through endogenous costs determination it importantly impacts contractors' participation and pricing decisions and thus shapes the competition in such markets. This means that most policy decisions have to take into account potential consequences of subcontracting availability on policy outcomes.

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## Appendix A

*Proof of the Corrolary 1.* This follows by differentiating the first-order condition for  $0 < \mathbf{h}_i < 1$  with respect to  $\omega_{-i}$ :

$$1 - P''(\mathbf{h}_i(c_i)x)\mathbf{h}_i'(c_i)\mathbf{h}_i(c_i)x^2 - 2P'(\mathbf{h}_i(c_i)x)\mathbf{h}_i'(c_i)x - \delta x E_\epsilon V''_{i,1}(\omega_i - \epsilon_i + x(1 - \mathbf{h}_i(c_i)), \omega_{-i} - \epsilon_{-i})(-\mathbf{h}_i'(c_i)) = 0 \quad (23)$$

and

$$\mathbf{h}_i'(c_i) = (P''(\mathbf{h}_i(c_i)x)\mathbf{h}_i(c_i)x^2 + 2P'(\mathbf{h}_i(c_i)x)x - \delta x E_\epsilon V''_{i,1}(\omega_i - \epsilon_i + x(1 - \mathbf{h}_i(c_i)), \omega_{-i} - \epsilon_{-i}))^{-1},$$

where  $\mathbf{h}_i'$  denotes the derivative of the subcontracting function with respect to the current realization of per unit project costs.

***Proof of Proposition 2.*** The expected profit of bidder 1 with effective cost realization  $\bar{\phi}_1$ ,  $\bar{\pi}_1(b, \bar{\phi}_1)$ , is maximized at  $\bar{b}$ , i.e.

$$\begin{aligned} & \{(1 - F_R(\bar{b}))(\bar{b} - \bar{\phi}_1) + \delta F_R(\bar{b})[E_\epsilon V_1(\sigma_1(0), \sigma_2(0)) - \underline{V}_2]\}(1 - F_{\phi,2}(\mathbf{b}_2^{-1}(\bar{b}))) \\ & + \delta \int_{\underline{\phi}_2}^{\mathbf{b}_2^{-1}(\bar{b})} [(1 - F_R(\mathbf{b}_2))E_\epsilon V_1(\sigma_1(0), \sigma_2(\mathbf{h}_2)) + F_R(\mathbf{b}_2)V_1(\sigma_1(0), \sigma_2(0)) - \underline{V}_2]f_{\phi,2}(\phi)d\phi \geq \\ & \{(1 - F_R(b))(b - \bar{\phi}_1) + \delta F_R(b)[E_\epsilon V_1(\sigma_1(0), \sigma_2(0)) - \underline{V}_2]\}(1 - F_{\phi,2}(\mathbf{b}_2^{-1}(b))) \\ & + \delta \int_{\underline{\phi}_2}^{\mathbf{b}_2^{-1}(b)} [(1 - F_R(\mathbf{b}_2(\phi)))E_\epsilon V_1(\sigma_1(0), \sigma_2(\mathbf{h}_2)) + F_R(\mathbf{b}_2(\phi))E_\epsilon V_1(\sigma_1(0), \sigma_2(0)) - \underline{V}_2]f_{\phi,2}(\phi)d\phi \end{aligned}$$

or

$$\begin{aligned} & \{(1 - F_R(\bar{b}))(\bar{b} - \bar{\phi}_1) + \delta F_R(\bar{b})[E_\epsilon V_1(\sigma_1(0), \sigma_2(0)) - \underline{V}_2]\}(1 - F_{\phi,2}(\xi_2(\bar{b}))) \\ & + \delta \int_{\underline{\phi}_2}^{\xi_2(\bar{b})} [(1 - F_R(\mathbf{b}_2))E_\epsilon V_1(\sigma_1(0), \sigma_2(\mathbf{h}_2)) + F_R(\mathbf{b}_2)V_1(\sigma_1(0), \sigma_2(0)) - \underline{V}_2]f_{\phi,2}(\phi)d\phi \geq \\ & \{(1 - F_R(b))(b - \bar{\phi}_1) + \delta F_R(b)[E_\epsilon V_1(\sigma_1(0), \sigma_2(0)) - \underline{V}_2]\}(1 - F_{\phi,2}(\xi_2(b))) \\ & + \delta \int_{\underline{\phi}_2}^{\xi_2(b)} [(1 - F_R(\mathbf{b}_2(\phi)))E_\epsilon V_1(\sigma_1(0), \sigma_2(\mathbf{h}_2)) + F_R(\mathbf{b}_2(\phi))E_\epsilon V_1(\sigma_1(0), \sigma_2(0)) - \underline{V}_2]f_{\phi,2}(\phi)d\phi \end{aligned}$$

Next, the derivative of  $\bar{\pi}_1(b, \bar{\phi}_1)$  with respect to  $\xi_2$  is given by

$$\bar{\pi}'_{1,\xi_2} = [- (b - \bar{\phi}_1) + \delta[E_\epsilon V_1(\sigma_1(0), \sigma_2(\mathbf{h}_2)) - \underline{V}_2]](1 - F_R(b))f_{\phi,2}(\xi_2(b)).$$

Notice that  $\bar{\pi}'_{1,\xi_2}(b = \bar{\phi}_1) > 0$  and is decreasing in  $b$ , so there must exist  $\tilde{b} \in [\bar{\phi}_1, \bar{\phi}_2]$  such that  $\bar{\pi}'_{1,\xi_2} \geq 0$  for  $b \leq \tilde{b}$  and  $\bar{\pi}'_{1,\xi_2} \leq 0$  for  $b \geq \tilde{b}$  (i.e.  $\bar{\pi}'_{1,\xi_2}(\tilde{b}) = 0$  or  $\tilde{b} = \bar{\phi}_2$ ).

Then  $\bar{\pi}_1(b, \bar{\phi}_1)$  is maximized at  $\tilde{b}$  for  $b \leq \tilde{b}$  because

$$\bar{\pi}'_{1,b} = (1 - F_R(b))(1 - F_{\phi,2}(\xi_2(b))) + \bar{\pi}'_{1,\xi_2}\xi'_2(b) \geq 0.$$

Notice that  $\bar{\pi}'_{1,b} \geq 0$  for some range of  $b$  such that  $b \geq \tilde{b}$ .

Further,  $\bar{\pi}_1(b, \bar{\phi}_1)$  is decreasing in  $\xi_2$  for  $b \geq \tilde{b}$ . Recall that in equilibrium  $\xi_2(b) \leq b$  then

$$\begin{aligned} & \{(1 - F_R(b))(b - \bar{\phi}_1) + \delta F_R(b)[E_\epsilon V_1(\sigma_1(0), \sigma_2(0)) - \underline{V}_2]\}(1 - F_{\phi,2}(\xi_2(b))) \\ & + \delta \int_{\phi_2}^{\xi_2(b)} [(1 - F_R(\mathbf{b}_2(\phi)))E_\epsilon V_1(\sigma_1(0), \sigma_2(\mathbf{h}_2)) + F_R(\mathbf{b}_2(\phi))E_\epsilon V_1(\sigma_1(0), \sigma_2(0)) - \underline{V}_2]f_{\phi,2}(\phi)d\phi \geq \\ & \{(1 - F_R(b))(b - \bar{\phi}_1) + \delta F_R(b)[E_\epsilon V_1(\sigma_1(0), \sigma_2(0)) - \underline{V}_2]\}(1 - F_{\phi,2}(b)) \\ & + \delta \int_{\phi_2}^b [(1 - F_R(\mathbf{b}_2(\phi)))E_\epsilon V_1(\sigma_1(0), \sigma_2(\mathbf{h}_2)) + F_R(\mathbf{b}_2(\phi))E_\epsilon V_1(\sigma_1(0), \sigma_2(0)) - \underline{V}_2]f_{\phi,2}(\phi)d\phi \end{aligned}$$

The right-hand side of the inequality above is maximized at  $b_0$  such that

$$\begin{aligned} & \{(1 - F_R(b_0)) - f_R(b_0)(b_0 - \bar{\phi}_1) + \delta f_R(b)[E_\epsilon V_1(\sigma_1(0), \sigma_2(0)) - \underline{V}_2]\}(1 - F_{\phi,2}(b_0)) - \\ & \{b_0 - \bar{\phi}_1 - \delta[E_\epsilon V_1(\sigma_1(0), \sigma_2(\mathbf{h}_2)) - \underline{V}_2]\}(1 - F_R(b_0))f_{\phi,2}(b_0) = 0 \end{aligned}$$

Thus we obtain that two cases are possible

(a)  $\bar{b} = \tilde{b}$  if  $\tilde{b} = \bar{\phi}_2$

(b)  $\bar{b} = \min\{b_0, \bar{\phi}_2\}$  where  $b_0$  as defined above.

# Appendix B

## Numerical Algorithm

Our algorithm is an extension of the method used in Chen, Doraszelski, and Harrington Jr (2009) to dynamic auctions. It involves computing a limit on Markov Perfect Equilibria in the finite horizon games, which alleviates multiplicity of equilibria by providing a consistent and robust equilibrium selection rule.<sup>18</sup> The algorithm is composed of two parts: (i) an inner-loop computing optimal subcontracting and bidding strategies, as well as the value function of the game with  $n$  periods, and (ii) an outer-loop computing an equilibrium of an infinite horizon game. In an effort to simplify the exposition we present the numerical algorithm only for the game without participation decision. Solving the game with participation involves replacing Bellman equation (24) with equation (3). It also requires adding an intermediate step of finding a fixed point in the entry game, which can be achieved by solving the system of equations (12) by a Newton method.

In the remainder of this section the value function of the  $n$ -stage game is denoted as  $V^{(n)}$ . The value function of the game with  $n+1$  stages can be obtained from  $V^{(n)}$  using the following Bellman equation:

$$\begin{aligned}
 V_i^{(n+1)}(\omega) = & \int_{c_i} \left[ \max_{b_i, h_i} W_i(b_i, \omega; \mathbf{g}_{-i}) \times \right. \\
 & \times (b_i - (1 - h_i)c_i x - P(h_i x)h_i x + \delta E_\epsilon V_i^{(n)}(\omega_i - \epsilon_i + (1 - h_i)x, \omega_{-i} - \epsilon_{-i})) \\
 & \left. + \delta \int_{\underline{c}_{-i}(\omega)}^{\mathbf{b}_{-i}^{-1}(b_i, \omega)} E_\epsilon V_i^{(n)}(\omega_i - \epsilon_i, \omega_{-i} - \epsilon_{-i} + (1 - \mathbf{h}_{-i}(c_{-i}))x) dF_{-i}(c_{-i}) \right] dF_i(c_i).
 \end{aligned} \tag{24}$$

We follow the *parametric value function iteration* procedure suggested in Judd (1998) to parametrically approximate  $V^{(n)}(\cdot)$ ,  $\hat{V}^{(n)}(\cdot|\theta^n) \approx V^{(n)}(\cdot)$ . More specifically, we define a 2-dimensional grid on the state space,  $\mathbf{\Omega}^D = \{(\omega_1^d, \omega_2^d) : d = 1, \dots, D\}$ . We use 7 grid points on each dimension, which, taking into account the symmetry of the environment, results in 28 grid points. We use the parametric approximation of the value function from the  $n$ -stage game,  $\hat{V}^{(n)}$ , to obtain data pairs  $(\omega^d, v^d)$  for  $\hat{V}^{(n+1)}$  by evaluating the right-hand side of equation (24) on the grid. The parametric approximation of  $\hat{V}^{(n+1)}$  is then obtained through a 4th-order Chebyshev regression. We stop when  $\|\hat{V}^{(n)} - \hat{V}^{(n+1)}\|$

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<sup>18</sup>This approach was first proposed in Maskin and Tirole (1987).

is small.

The most computationally intensive part of the algorithm involves solving the right hand-side of the Bellman equation to produce the interpolation data  $(\omega^d, v^d)$  – an inner-loop of our algorithm. The procedure involves multiple steps. First, we note that we can pre-compute the expected future value function

$$\hat{V}_i^{(n)}(\omega) = E_\epsilon \hat{V}_i^{(n)}(\bar{\omega}_i - \epsilon_i, \bar{\omega}_{-i} - \epsilon_{-i}),$$

where  $\omega$  is an interim backlog after adding the current auction results but before subtracting the utilization. An interim backlog is given by  $\bar{\omega}_i = \omega_i - (1 - h_i)x$  in case player  $i$  won the auction and is given by  $\bar{\omega}_i = \omega_i$  in case player  $i$  loses the auction. This expectation is numerically computed on the grid  $\Omega^{\mathbf{D}}$  using an adaptive Simpson quadrature, and interpolated using a 4<sup>th</sup>-order Chebyshev regression. Without pre-computing this expectation the algorithm would be numerically infeasible. The value of the Bellman equation  $v^d$  at the grid point  $\omega^d$  can now be obtained by using

$$v_i^d = \int_{c_i} \left\{ \max_{b_i, h_i} W_i(b_i, \omega^d; \mathbf{g}_{-i}) \left[ b_i - (1 - h_i)c_i x - P(h_i x) h_i x + \delta \hat{V}_i(\omega_i^d + (1 - h_i)x, \omega_{-i}^d)^{(n)} \right] \right. \\ \left. + \delta \int_{\underline{c}_{-i}(\omega^d)}^{\mathbf{b}_{-i}^{-1}(b_i, \omega^d)} \hat{V}_i^{(n)}(\omega_i^d, \omega_{-i}^d + (1 - \mathbf{h}_{-i}(c_{-i}))x) dF_{-i}(c_{-i}, \omega^d) \right\} dF_i(c_i). \quad (25)$$

Next, we compute the optimal subcontracting functions and then use the subcontracting functions to compute the optimal bidding functions. This is the most challenging part of the process. For every state grid point  $\omega^d \in \Omega^{\mathbf{D}}$  we define a grid on the support of the distribution of original costs,  $\mathbf{C}_i(\omega) = (\underline{c}(\omega_i), \dots, c_i^r, \dots, \bar{c}(\omega_i))$ . Having solved for the optimal subcontracting level at each cost grid point, we then obtain the subcontracting strategy, and the effective-cost functions as well as the distribution functions of the effective costs through cubic spline interpolation.

Having computed all of the components, namely  $V(\hat{\cdot}), h(\cdot), \phi(\cdot)$  (as well as  $F_\phi(\cdot)$  and  $f_\phi(\cdot)$ ), we proceed to solve for inverse bid strategies  $(\xi_1(\cdot, \omega^d), \xi_2(\cdot, \omega^d))$  using the system of differential equations (15) with boundary conditions given by Proposition 2. We use a shape-preserving projection method with the Chebyshev basis to guarantee the monotonicity of the inverse bid functions. Our basis consists of 4<sup>th</sup>-order complete Chebyshev polynomials and is defined on a Chebyshev grid.<sup>19</sup> We

<sup>19</sup>A Chebyshev grid is composed of the roots of a Chebyshev polynomial of the first kind. Using the roots of the Chebyshev polynomial instead of an equidistant grid makes the numerical procedure more stable for ill-conditioned

reduce our task to the following constrained optimization problem:

$$\begin{aligned}
& \min_{\underline{b}, \theta_1^\xi, \theta_2^\xi} \sum_{b^k \in \mathbf{B}} [R_1(b^k | \theta_1^\xi)]^2 + [R_2(b^k | \theta_2^\xi)]^2 \\
& \text{s.t. } \forall k; \hat{\xi}'_1(b^k | \theta_1^\xi) > 0, \quad \hat{\xi}'_2(b^k | \theta_2^\xi) > 0 \\
& \quad \forall k; \hat{\xi}_1(b^k | \theta_1^\xi) < b^k, \quad \hat{\xi}_2(b^k | \theta_2^\xi) < b^k \\
& \quad \underline{b} < \bar{b} + \tau \\
& \quad \underline{\phi}_1 = \hat{\xi}_1(\underline{b} | \theta_1^\xi), \quad \underline{\phi}_2 = \hat{\xi}_2(\underline{b} | \theta_2^\xi) \\
& \quad \bar{\phi}_2 = \hat{\xi}_1(\bar{b} | \theta_1^\xi), \quad \bar{\phi}_2 = \hat{\xi}_2(\bar{b} | \theta_2^\xi)
\end{aligned} \tag{26}$$

where  $R_i$  is the residual from evaluating first-order conditions (15) using the approximation of the inverse bid function  $\hat{\xi}$  instead of the true inverse bid function  $\xi$ . Note that one inequality contains a bandwidth parameter  $\tau$ , which controls the flatness of the bid strategies. It is set to a very small non-binding number and is used to improve the stability of the numerical iterations.

To summarize, we provide a flow description of the algorithm:

- (I) Fix the terminal value  $V_i^{(0)} \equiv 0$ . Fix a D-point grid of the state space  $\Omega^{\mathbf{D}} = \{(\omega_1^d, \omega_2^d) : d = 1, \dots, D\}$ .
- (1) For every point  $\omega^d$  and given  $n$ 
  - (a) For both players, given the value functions in the n-stage game  $V_i^{(n)}$ , solve for an optimal subcontracting strategy  $h_i^{(n)}(\cdot, \omega^d)$ , effective-cost functions  $\phi_i^{(n)}(\cdot, \omega^d)$  and determine the CDF and PDF of the pseudo cost  $\phi_i^{(n)}$ .
  - (b) Solve the Boundary Value Problem for inverse bidding strategies  $\xi_i^{(n)}(\cdot, \omega^d)$
  - (c) Perform an iteration on the Bellman equation (24) to compute  $V_i^{(n+1)}(\omega^d)$
- (2) Use a projection method to fit a parametric approximation  $\hat{V}_i^{(n+1)}(\omega | \theta^{(n+1)})$  outside of the grid  $\Omega^{\mathbf{D}}$
- (3) For each point on the grid  $\Omega^{\mathbf{D}}$ , perform an integration of  $\epsilon$  to obtain  $E_\epsilon \hat{V}_i^{(n+1)}(\omega^d - \epsilon)$ , where  $\omega^d$  is an interim backlog

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problems. For more discussion see Judd (1998).

- (4) Use a projection method to fit a parametric approximation  $\hat{V}_i^{(n+1)}(\omega|\theta_2^{(n+1)})$  outside of the grid  $\Omega^D$
- (S) Stop if  $\|\hat{V}^{(n)} - \hat{V}^{(n+1)}\| < \epsilon$  or goto (1).

## Simulation Details

The computational algorithm described above provides an approximation of the equilibrium value function  $\hat{V}^{(n+1)}$  as well as the equilibrium bidding and subcontracting strategies on the cost and backlog grid,  $\{(c_i^r, \omega^d) : c_i^r \in \mathbf{C}_i(\omega^d), \omega^d \in \Omega^D\}$ . To simulate the equilibrium path we need to know the strategies outside of the cost and backlog grid. One option is to re-solve for optimal strategies as needed for a given  $\omega$  on the path (see Saini (2013)). However, this option is computationally infeasible because the strategies do not have closed-form solutions. We solve this issue by interpolating  $b_i(c_i, \omega)$ , and  $h_i(c_i, \omega)$  outside the cost and backlog grid.

Note that our three-dimensional grid is non-rectangular because the support of  $C_i(\omega^d)$  depends on  $\omega^d$ . That is why we perform the interpolation in several steps. First, a cubic spline interpolation for the upper bound  $\bar{b}$  is constructed. It is later used to determine if the contractor is priced out of the market. Next,  $\omega^d$ -specific linear transformation is used to project a uniform grid  $C_i(\omega^d)$  onto a  $[0, 1]$  interval. This procedure converts our grid into a rectangle one and enables fitting the 3-dimensional cubic splines to obtain  $\hat{h}_i$  and  $\hat{b}_i$ .

The strategies can now be evaluated at an arbitrary point  $(c_i, \omega)$  in the following way. First,  $c_i$  is projected onto a  $[0, 1]$  interval using a correct  $\omega$ -dependent linear transformation, then  $\hat{h}_i$  or  $\hat{b}_i$  is evaluated at a corresponding point. Once the bids are known, the winner is determined and backlogs are adjusted. We record that player  $i$  lost the auction if  $c_i$  is greater than the cut-off point  $\bar{b}(\omega)$ .

We use the interpolated strategies to simulate the stationary distribution and long-run industry path. The stationary distribution is used to obtain average industry statistics, while the discounted procurement cost is computed along the equilibrium path. In order to obtain a stationary distribution we perform  $10^4$  warm-up draws and average subsequent  $10^5$  draws. To obtain the long-run industry path we simulate  $10^3$  draws of 80 consecutive periods and assume that the  $81^{st}$  state persists forever. Note that the contribution of the  $81^{st}$  period is equal to  $\delta^{81} = 0.002$ .



## Methodological Details from the “Estimation Bias” Section

We use the bid distributions computed for the model with subcontracting to recover the cost distribution that would be consistent with these distributions under the misspecified model without subcontracting. For this purpose we use the method proposed by Jofre-Bonet and Pesendorfer (2003) and Tirerova (2014). We extend this method to allow for stochastic backlog depreciation and secret reserve.

Let  $f_{b,i}$  and  $F_{b,i}$  be the density and cumulative distribution function of the equilibrium bid distribution for contractor  $i$  under state  $\omega$ , respectively. Similarly, let  $f_R$  and  $F_R$  be the density and distribution of the secret reserve. Define  $h_i(b; \omega) = \frac{f_{b,i}(b; \omega)}{1 - F_{b,i}(b; \omega)}$ ,  $h_i(b) = \frac{f_R(b)}{1 - F_R(b)}$  to be the respective hazard rates. We use the necessary first order conditions from the contractor’s bidding problem to recover the inverse bid function consistent with the observed bid distribution and the model without subcontracting:

$$cx = b - \delta[E_\epsilon V_i(\omega_i - \epsilon_i, \omega_j + x - \epsilon_j) - E_\epsilon V(\omega_i - \epsilon_i + x, \omega_j - \epsilon_j)] + \frac{1}{h_{-i}(b; \omega) + h_R(b)},$$

where  $h_i(b; \omega) = \frac{f_{b,i}(b; \omega)}{1 - F_{b,i}(b; \omega)}$  is a hazard rate, whereas  $f_{b,i}$  and  $F_{b,i}$  are the density and cumulative distribution function of the equilibrium bid distribution for contractor  $i$  under state  $\omega$  respectively. In the first stage we recover these densities from the bootstrap data using fully non-parametric kernel estimator. As shown later, we need to recover these distributions only at finite set of states.

Jofre-Bonet and Pesendorfer (2003) show that the value function used in the equation above can be inferred from the distribution of bids. We modify their argument in a similar way as Tirerova (2014) to obtain the following representations. The continuation value conditional on the duopoly can be expressed as:

$$U_i(\omega; A = \{i, -i\}) = \int_b \frac{(1 - F_{-i}(b))(1 - F_R(b))f_i(b)}{h_{-i}(b) + h_R(b)} db +$$

$$\left[ P(-i \text{ wins} | \omega) + \int_b \frac{h_{-i}(b)(1 - F_i(b))(1 - F_R(b))f_{-i}(b)}{h_{-i}(b) + h_R(b)} db \right] \delta EV(\sigma_i(0), \sigma_{-i}(1)) +$$

$$\left[ P(\text{no allocation} | \omega) + \int_b \frac{h_R(b)(1 - F_i(b))(1 - F_{-i}(b))f_R(b)}{h_{-i}(b) + h_R(b)} db \right] \delta EV(\sigma_i(0), \sigma_{-i}(0))$$

The continuation value conditional on being the only bidder can be expressed as:

$$U_i(\omega, A = \{i\}) = \int_b \frac{(1 - F_R(b))f_i^{(1)}(b)}{h_R(b)} db + \delta E_\epsilon V_i(\sigma_i(0), \sigma_{-i}(0)).$$

The continuation value conditional on the competitor being the only bidder can be expressed as:

$$U_i(\omega, A = \{-i\}) = P(-i \text{ wins} | \omega, \text{monopoly}) \delta E_\epsilon V_i(\sigma_i(0), \sigma_{-i}(1)) + \\ P(\text{no allocation} | \omega, \text{monopoly}) E_\epsilon V_i(\sigma_i(0), \sigma_{-i}(0))$$

The continuation value conditional on no bidders can be expressed as:

$$U_i(\omega, A = \{\emptyset\}) = \delta E_\epsilon V_i(\sigma_i(0), \sigma_{-i}(0))$$

Tirerova (2014) shows that conditional expectation of the entry cost can be expressed as

$$p_i(\omega) E[\kappa < \bar{K}_i(\omega) | p] = -K_i(\omega) \left[ \frac{p_i(\omega)}{\log(1 - p_i(\omega))} + (1 - p_i(\omega)) \right],$$

where

$$K_i(\omega) = p_{-i}(\omega) [U_i(\omega, A = \{i, -i\}) - U_i(\omega, A = \{-i\})] + \\ (1 - p_{-i}(\omega)) [U_i(\omega, A = \{i\}) - U_i(\omega, A = \emptyset)]$$

Combining the above equations we obtain

$$V_i(\omega) = p_i(\omega) E[\kappa < \bar{K}_i(\omega) | p] + p_i(\omega) p_{-i}(\omega) U_i(\omega; A = \{i, -i\}) + \\ p_i(\omega) (1 - p_{-i}(\omega)) U_i(\omega, A = \{i\}) + (1 - p_i(\omega)) p_{-i}(\omega) U_i(\omega, A = \{-i\}) + \\ (1 - p_i(\omega)) (1 - p_{-i}(\omega)) U_i(\omega, A = \{\emptyset\}) \quad (27)$$

Jofre-Bonet and Pesendorfer (2003) solve the corresponding equation by an approximate interpolation that assumes that the backlog process never leaves the grid. In such a case, the value function can be obtained by solving a system of linear equations. In our case the integration with respect to

the productivity shock creates non-linearities. Instead, we solve the equation (27) by a projection method which uses two-dimensional, second degree, complete Chebyshev polynomial basis functions. Specifically, we approximate the value function with  $\hat{V}(\cdot|\theta)$ , where  $\theta$  is a vector of polynomial coefficients. The approximation  $\hat{V}(\cdot|\theta)$  can be numerically integrated and in combination with the estimates of the conditional bid distributions implies the approximation of the continuation values  $\hat{U}(\cdot|\theta)$ . We define a residual function for the projection method  $R(\omega^d|\theta)$  using the equation (27). We choose  $\theta^*$  that minimizes sum of squares of residuals on the finite grid states,  $\Omega^{\mathbf{D}}$ . Note that the residuals have to be computed only at finite set of grid points  $\Omega^{\mathbf{D}}$ , thus, we need to recover the conditional bid distributions and entry probabilities for a finite number of states.

# Figures and Tables

## A. Calibration Details

Table 1: Summary Statistics for California Procurement Market

Variable	Mean	Std.Dev
Project size, large projects (mln)	4.37	2.1
Duration, large projects (months)	9.5	4.51
Subcontracted share of project	0.33	0.11
Fraction of projects with a single large bidder	0.47	0.22
Fraction of projects with two large bidders	0.34	0.19
Fraction of projects with more than two large bidders	0.09	0.13
Number of large projects won (per period, per large firm)	0.24	0.12
Pr(project is allocated if monopoly)	0.48	0.24

This table reports summary statistics for the set of California highway procurement projects used in calibration exercise. It consists of 856 projects that are between \$2mln and \$10mln auctioned in California market between years 2005 and 2010.

Table 2: Calibrated Parameters

Cost distribution	Baseline lower bound ( $\underline{C}^0$ )	0.00
	Baseline upper bound ( $\bar{C}^0$ )	1.16
	Mean intercept ( $\alpha_0$ )	0.29
	Backlog mean and support shift ( $\alpha_1$ )	0.14
	Baseline standard deviation ( $\beta_0$ )	0.16
	Backlog standard deviation shift ( $\beta_1$ )	0.046
Backlog process	Project size ( $\bar{x}$ )	\$4.37M
	Maximum productivity - bimonthly ( $K$ )	\$1.92M
Subcontracting schedule	Intercept ( $\gamma_1$ )	0.54
	Linear part ( $\gamma_2$ )	0.07
	Hyperbolic part ( $\gamma_3$ )	0.03
Entry cost	( $\kappa$ )	\$0.66M

Table 3: Statistics Used in Calibration

Moment	Model	Data
Prob. of duopoly	0.32	0.34
Prob. of monopoly	0.51	0.47
Allocated with monopoly	0.47	0.48
Duration (months)	10.2	9.5
Avg. duopoly bids (empty, full)	0.85	0.83
Avg. duopoly bids (full, empty)	0.89	0.89
Avg. duopoly bids (empty, empty)	0.95	0.99
Avg. monopoly bid	1.31	1.27
Std. dev. of duopoly bid	0.10	0.12
Avg. work done by the bidder	0.65	0.67

This table summarizes performance of the calibration procedure. Specifically, it reports the value of the statistics used in calibration that are computed from the data and from the model at the calibrated parameter values. In the computation of the average bids the vector of backlogs  $(\omega_1, \omega_2)$  should be interpreted to mean that the own backlog of the firm for which an average is computed is given by  $\omega_1$  and the competitor's backlog is given by  $\omega_2$ . Further, in the notations above, *empty* and *full* indicate that contractor's backlog is within the [25%, 50%] and [50%, 75%] quantile ranges of the stationary backlog distribution respectively.

## B. Computed MPE

Figure 1: Stationary Distribution of Backlogs

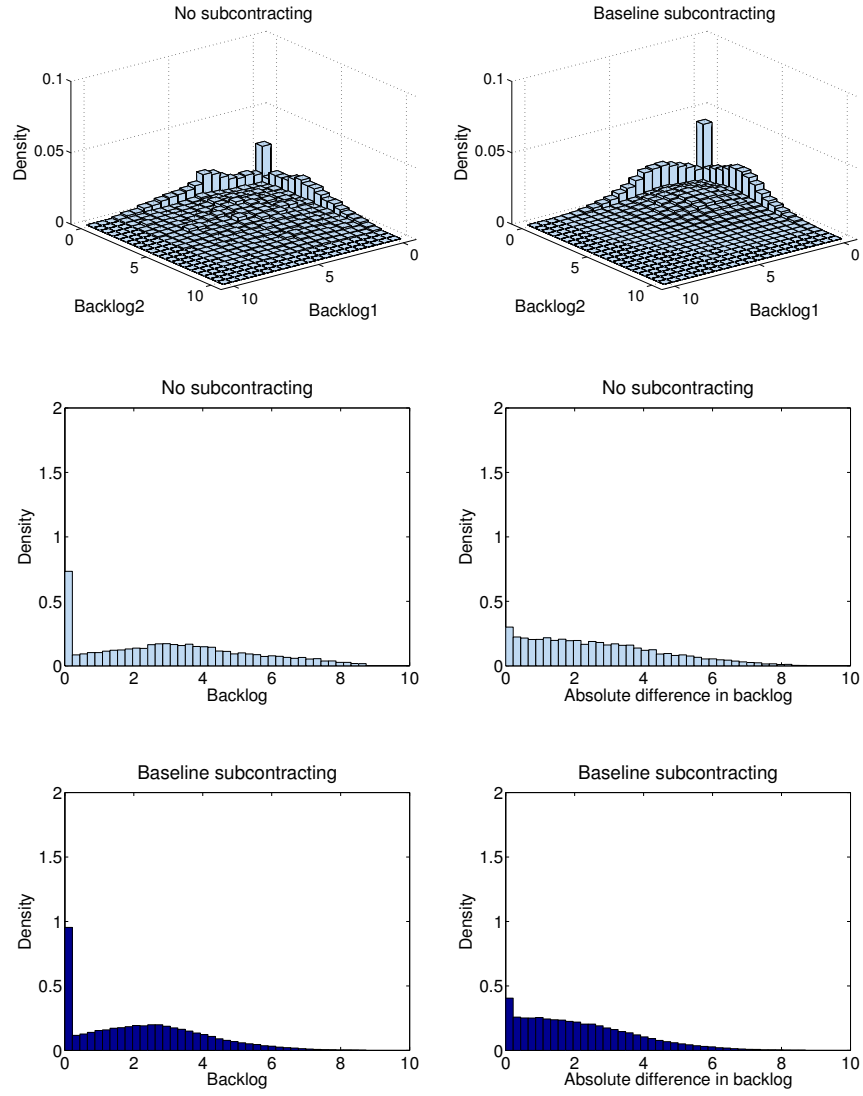
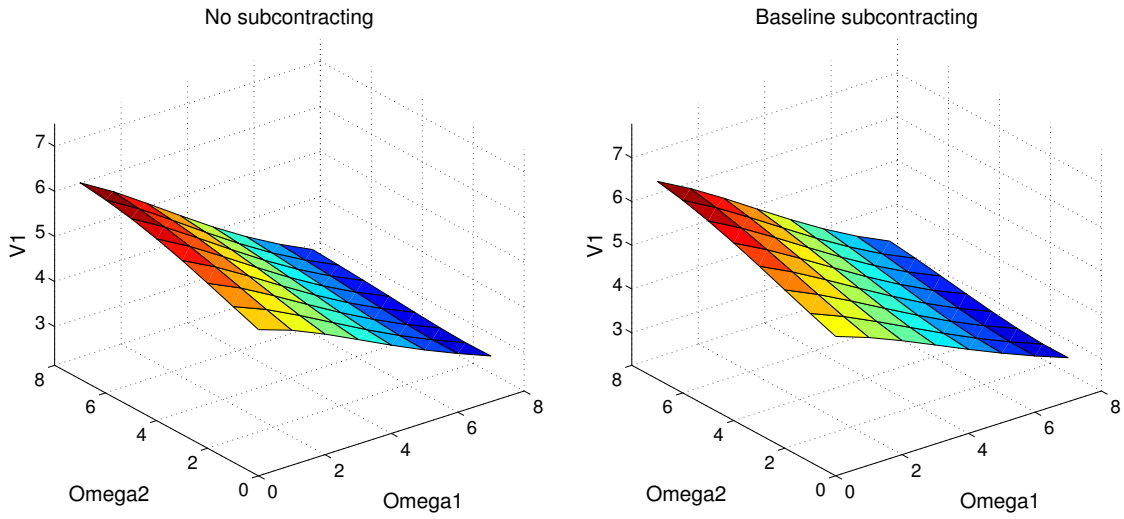
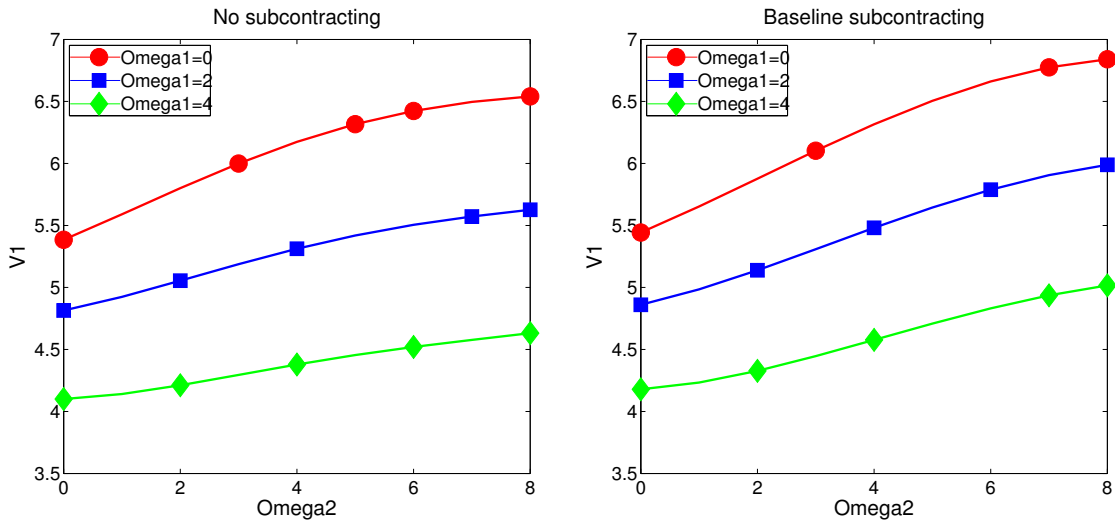


Figure 2: Value Function 3D



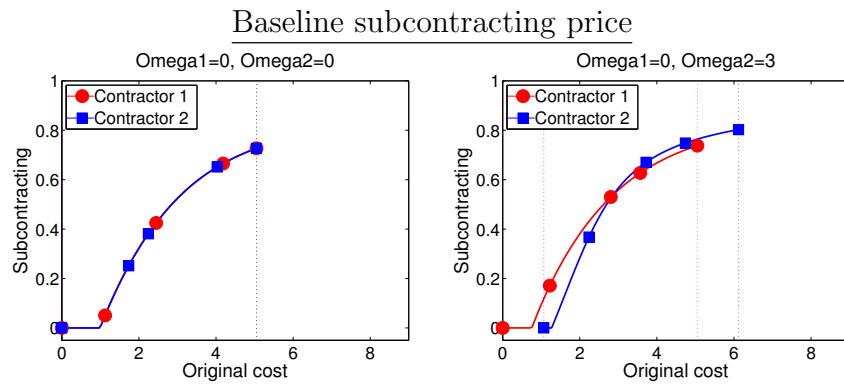
This figure shows the value function of the models with and without subcontracting computed under the calibrated parameter values.

Figure 3: Value Function



This figure shows the sections of the value function that correspond to the different levels of own backlog. It graphs the value function against the level of competitor's backlog.

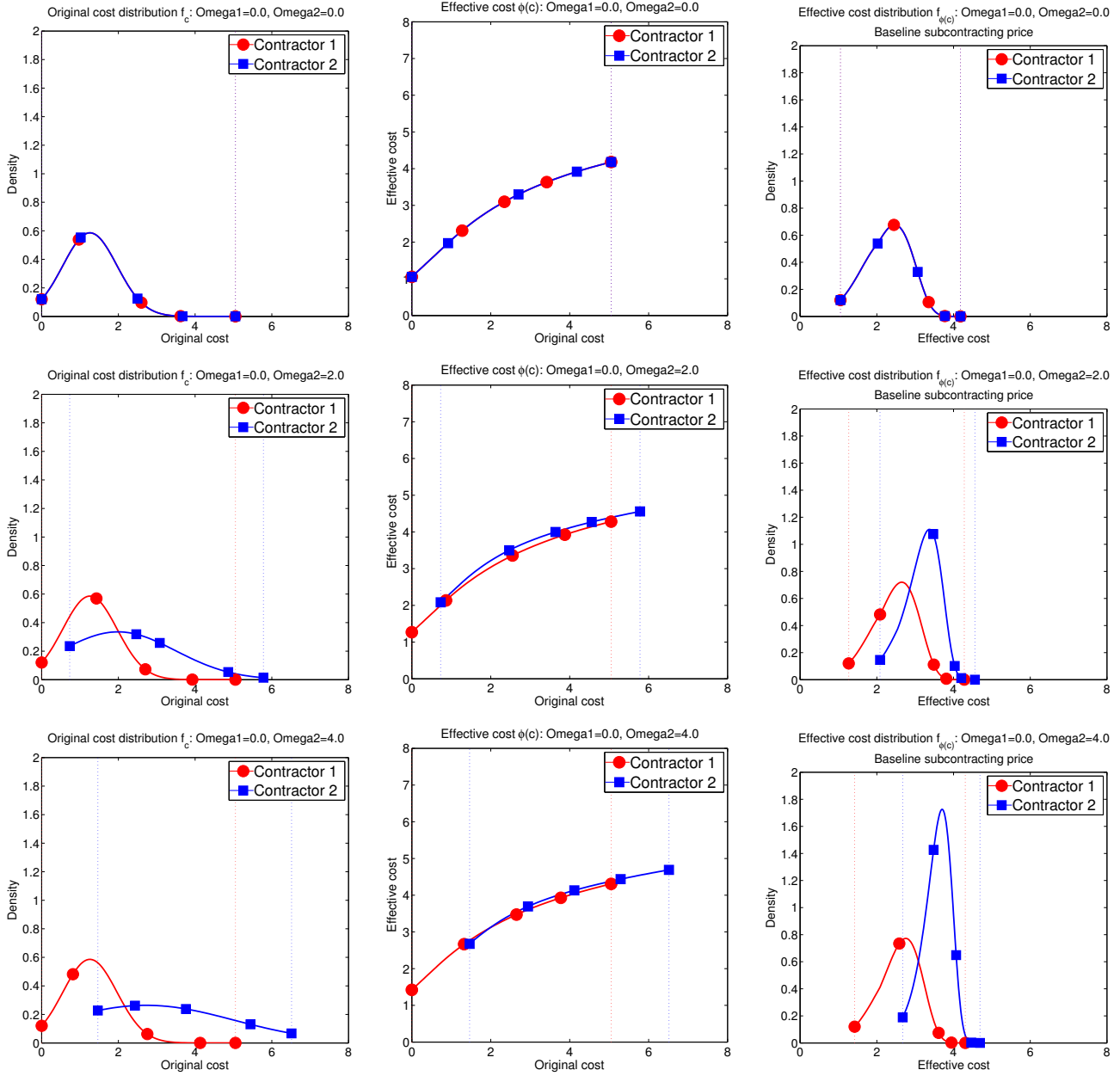
Figure 4: Subcontracting Strategy



This figure shows the subcontracting strategy under several backlog configurations.

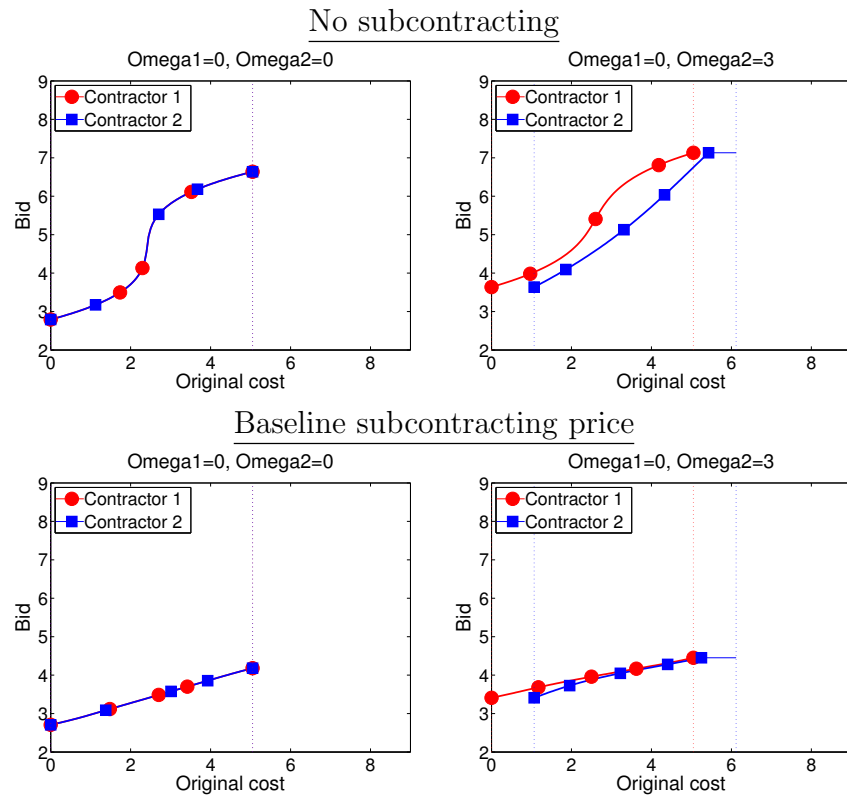


Figure 5: Distribution Density Functions of Original and Effective Cost



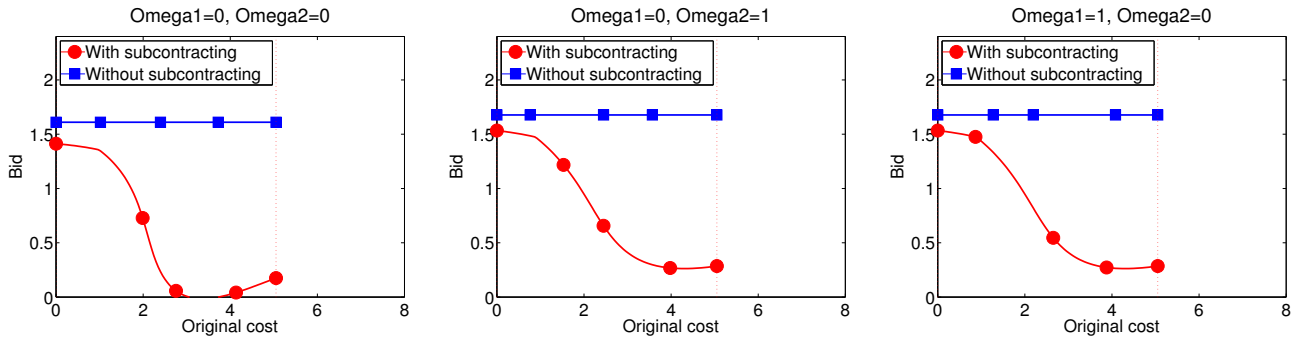
This figure compares the distributions of the original and the effective costs for a calibrated subcontracting supply schedule and across different backlog configurations.

Figure 6: Bidding Strategies



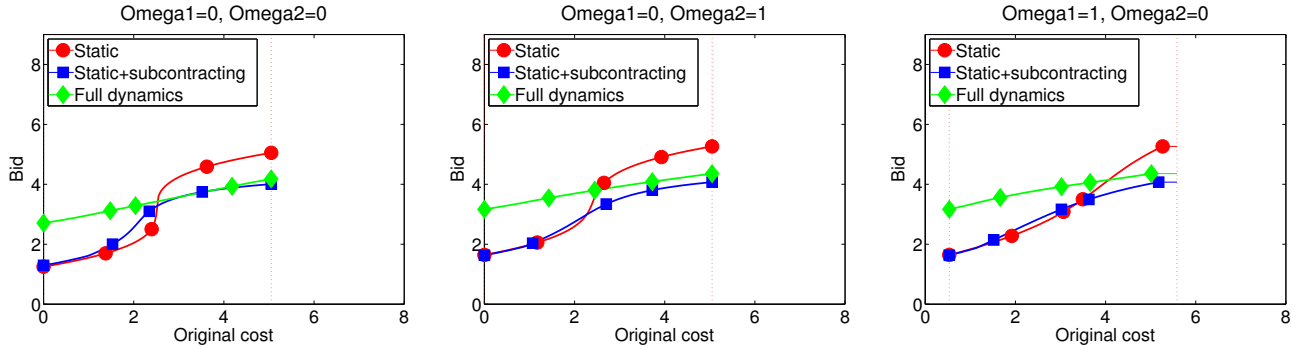
This figure shows the bid functions under several backlog configurations.

Figure 7: Option Value Component of Prices (Bids)



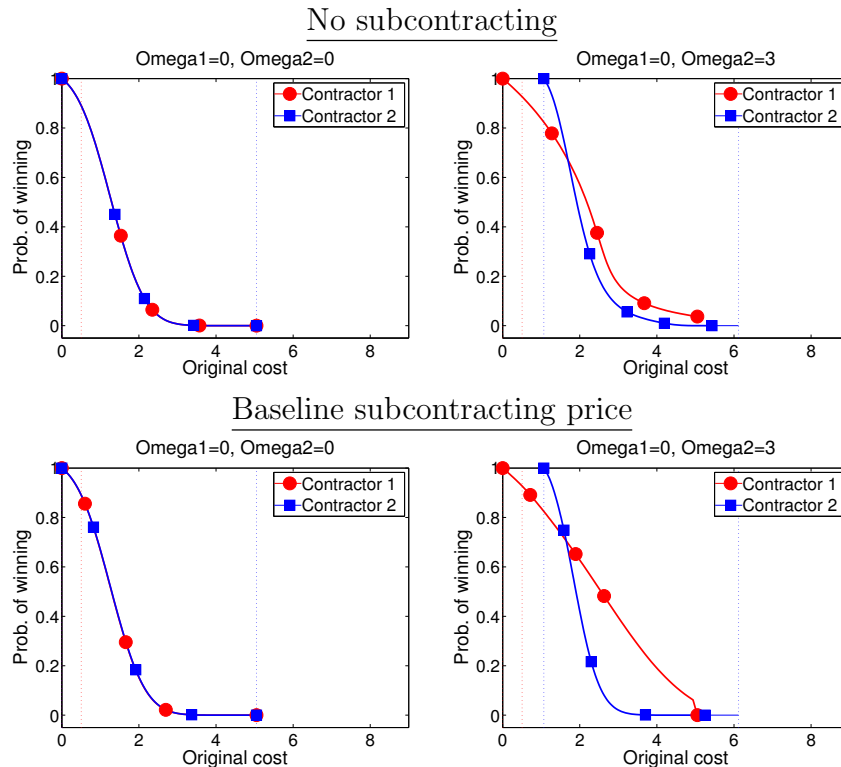
This figure plots the option value component of prices for the models with and without subcontracting for several state vectors. This component is computed as a difference between the bid function based on the full effective costs and the bid function based on the static component of effective costs.

Figure 8: Decomposition of the Bidding Function



This figure isolates several effects that characterize bidding behavior in the dynamic model with subcontracting. All the bidding functions are computed on the basis of the equilibrium subcontracting strategy from the fully dynamic model. The red line corresponds to the bidding strategy that would be optimal in a static auction given the distributions of original costs implied by the corresponding backlog configuration. The blue line corresponds to the bidding strategy that would be optimal in a static auction given the cost distributions that coincide with the distributions of the static part of the effective costs (implied by the subcontracting function from the dynamic model). The green line corresponds to the bidding strategy from a fully dynamic model. It adds option value considerations to the bidding behavior described by the blue line.

Figure 9: Probability of winning



This figure shows the probability of winning under several backlog configurations.

Table 4: Summary of Equilibrium Variables

	Allocated projects (fraction)	Expected number of bidders	Conditional on allocation			Backlog distribution (mean)	Difference in backlogs (mean)
			Firm's profit	Procurement cost	Work done		
Subcontracting	0.51	1.19	\$1.44M	\$4.76M	0.73	\$2.21M	\$2.16M
No subcontracting	0.41	1.07	\$1.76M	\$5.35M	1.00	\$3.06M	\$2.64M
	-20.2%	-9.6%	21.8%	12.3%	36.3%	38.7%	22.1%

This table reports the average values of several variables in the stationary equilibria of the environments with and without subcontracting.

Table 5: Expected Subcontracting Levels

Subcontracting regime	$\omega_1 = 0$	$\omega_1 = 0.5$	$\omega_1 = 1$	$\omega_1 = 0$	$\omega_1 = 0$
	$\omega_2 = 0$	$\omega_2 = 0$	$\omega_2 = 0$	$\omega_2 = 0.5$	$\omega_2 = 1$
No subcontracting	-0.00	-0.00	-0.00	-0.00	-0.00
Subcontracting	0.09	0.17	0.25	0.10	0.10

This table reports the expected subcontracting levels conditional on the state and for the environments with and without the subcontracting.

Table 6: Moments of the Distributions of Private Costs

$\omega_1$	$E(c\bar{x})$	$\text{std}(c\bar{x})$	$E(\phi(c))$	$\text{std}(\phi(c))$	$E(\phi(c)_S)$	$\text{std}(\phi(c)_S)$	$E(\phi(c)_D)$	$\text{std}(\phi(c)_D)$
0.00	1.32	0.65	2.29	0.54	1.44	0.73	0.857	0.200
0.50	1.59	0.79	2.64	0.60	1.73	0.85	0.910	0.257
1.00	1.89	0.89	2.94	0.61	2.03	0.87	0.916	0.295
2.00	2.49	1.11	3.37	0.61	2.52	0.93	0.849	0.327
4.00	3.56	1.32	3.83	0.52	3.23	0.77	0.593	0.247

The table summarizes variation in private costs of contractor 1 for three configurations of backlogs, keeping the backlog of the opponent at 0. The second and third columns show the mean and the standard deviation of the distribution of original private costs (i.e. costs before subcontracting is taken into account) whereas the fourth and the fifth columns present the mean and the standard deviation of the static part of the effective private costs. Finally, the sixth and the seventh columns contain the mean and the standard deviation of the full effective private costs (i.e. costs after subcontracting is taken into account that also include the continuation value of winning).

Table 7: Expected Bids

Subcontracting regime	$\omega_1 = 0$	$\omega_1 = 0.5$	$\omega_1 = 1$	$\omega_1 = 0$	$\omega_1 = 0$
	$\omega_2 = 0$	$\omega_2 = 0$	$\omega_2 = 0$	$\omega_2 = 0.5$	$\omega_2 = 1$
No subcontracting	0.35	0.39	0.42	0.38	0.39
Subcontracting	0.33	0.35	0.37	0.35	0.36

This table reports the expected bid levels conditional on the state and for the environments with and without the subcontracting.

Table 8: Expected Probability of Winning the Contract

Subcontracting regime	$\omega_1 = 0$	$\omega_1 = 0.5$	$\omega_1 = 1$	$\omega_1 = 0$	$\omega_1 = 0$
	$\omega_2 = 0$	$\omega_2 = 0$	$\omega_2 = 0$	$\omega_2 = 0.5$	$\omega_2 = 1$
No subcontracting	0.50	0.44	0.39	0.56	0.61
Subcontracting	0.50	0.42	0.36	0.58	0.64

This table reports the expected probability of winning the contract conditional on the state and for the environments with and without the subcontracting.

## C. Comparison of Auction Formats

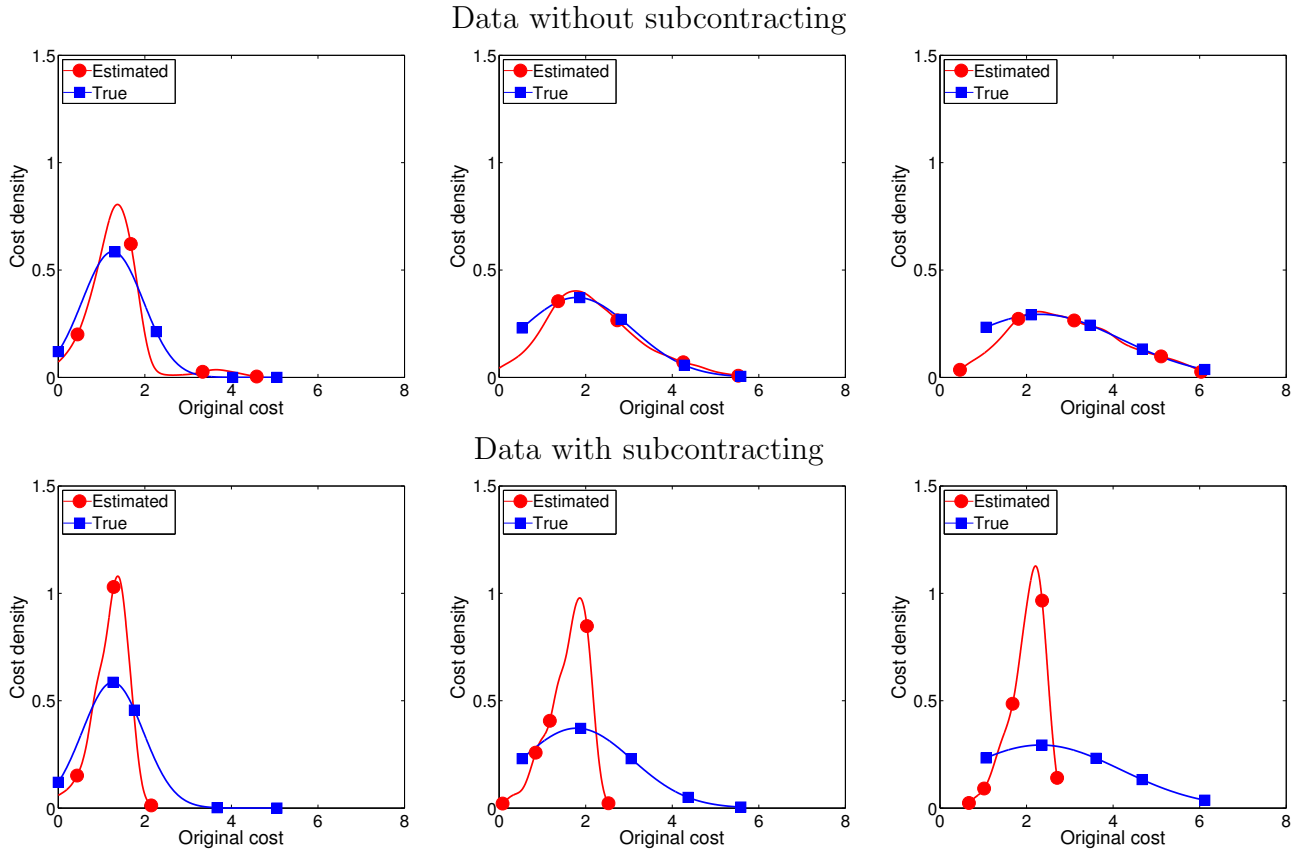
Table 9: Auction format comparison

Without subcontracting							
	Expected number of bidders	Duopoly					
		Allocated projects	Firm's profit	Procurement cost	Work done	Efficiency	Backlog
First-price	1.07	0.65	\$1.38M	\$4.55M	1.00	0.88	\$3.06M
Second-price	1.16	0.72	\$1.63M	\$5.21M	1.00	0.94	\$3.50M
	8.4%	10.9%	18.3%	14.6%	0.0%	6.8%	14.1%
With subcontracting							
	Expected number of bidders	Duopoly					
		Allocated projects	Firm's profit	Procurement cost	Work done	Efficiency	Backlog
First-price	1.17	0.77	\$1.00M	\$3.83M	0.75	0.93	\$2.21M
Second-price	1.22	0.82	\$1.01M	\$3.87M	0.73	0.98	\$2.31M
	4.3%	6.5%	1.5%	1.04%	-2.8%	5.4%	4.7%

This table reports the results of the analysis comparing the average values of several variables in stationary equilibria of the environments with and without subcontracting where allocation is implemented through the first price and the second price auctions respectively.

# D. Methodological Implications of Subcontracting

Figure 10: Estimation Bias



This figure shows the true and the estimated densities of the distribution of private costs recovered through an estimation procedure based on the assumption of no subcontracting and on the basis of the data with and without the subcontracting for the state vectors  $(0.0, 0.0)$ ,  $(1.5, 1.5)$ , and  $(2.9, 2.9)$ .

Table 10: Estimation Bias

State	True mean	Estimated mean	Difference (%)	True std. dev.	Estimated std. dev.	Difference (%)
Data without subcontracting						
$\omega = (0.0, 0.0)$	1.3	1.29	-0.7%	0.7	0.7	0.0%
$\omega = (1.5, 1.5)$	2.2	2.18	-0.9%	1.0	1.03	3.0%
$\omega = (2.9, 2.9)$	3.0	2.99	-0.3%	1.3	1.32	1.5%
Data with subcontracting						
$\omega = (0.0, 0.0)$	1.3	1.2	-7.7%	0.7	0.54	-23%
$\omega = (1.5, 1.5)$	2.1	1.6	-24%	1.0	0.5	-50%
$\omega = (2.9, 2.9)$	3.0	2.0	-33%	1.3	0.4	-67%

This table reports the estimated means and standard deviations of the distributions of private costs recovered under the assumption of no subcontracting from the data generated by the model without subcontracting as well as the model with subcontracting. All the estimates are given for the distribution of full costs such that  $C_i = (\alpha_0 + \alpha_1 \times \omega_i) \times \bar{x}$ .