Web Supplement for "The Role of Quality in Internet Service Markets"

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This web supplement is organized as follows. Section A discusses the existence of pure-strategy Bayesian Nash Equilibrium (BNE) in the model. Section B presents the proofs for the identification results in Krasnokutskaya, Song, and Tang (2017) (henceforth KST2017). These include the proofs of Propositions 1, 2 and 3 in KST2017 as well as results for the identification of sellers' costs for entry and service. The section also discusses the support conditions used in identification. Section C provides details about how we implement the nonparametric classification procedure, and reports its finite sample performance through Monte Carlo simulations. Section D provides estimation details including how the moment conditions are constructed for estimation in KST2017. This section also presents a generalized version of the expression for the conditional quality distribution for transitory sellers (equation (7) in Section 5 of KST2017) used in estimation. Section E summarizes the numerical algorithm used to solve for sellers' bidding and participation strategies in a type-specific equilibrium. Section F contains additional empirical results not reported in KST2017. In particular, at the end of the section, we investigate the social costs associated with restrictions on the international trade while emphasizing adjustment in the sellers' participation choices.

A. Existence of a Pure Strategy BNE

Throughout this section, we condition on the composition of potential sellers I_{N_l} and suppress it in the notation of σ_θ and τ_θ (type-symmetric entry and bidding strategies defined in KST2017)

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for simplicity. Let N_l^p (and N_l^t) denote the set of permanent (and transitory) sellers in N_l. We use upper case letters (e.g., $E_{i,l}, \epsilon_{i,l}$) for random variables, and lower-case letters (e.g., $e_{i,l}, \varepsilon_{i,l}$) for their realized values.

Also we suppress auction subscripts l to simplify notation, and write ($C_{i,l}$,$E_{i,l}$,$\epsilon_{i,l}$, $U_{0,l}$, α_l, β_l, N_l, N_l^p, A_l, $A_{i,l}$) as $(C_i, E_i, \epsilon_i, U_0, \alpha, \beta, N, N^p, A, A_i)$, which are considered independent draws across auctions reported in the data. In comparison, sellers' characteristics x_i, q_i are fixed for each permanent seller throughout all the auctions in the data and therefore do not need any auction subscript.

Let Θ denote the type space for seller types $\theta \equiv (\rho_i, x_i, q_i)$. Recall that $\rho_i \in \{p, t\}$ depending on whether seller i is a permanent type ("p") or a transitory type ("t"). Let $c \equiv (c_i)_{i \in N} \equiv (c_i, c_{-i})$ and $b \equiv (b_i)_{i \in N} = (b_i, b_{-i})$. For any type-symmetric entry strategy $\tau = (\tau_\theta : \theta \in \Theta)$, define

$$\pi_i(b, c_i; \tau) \equiv (b_i - c_i) p_i(b; \tau)$$

where $p_i(b;\tau)$ is the probability that an auction participant (active bidder) i wins the auction conditional on his bids b_i and that the other potential bidders follow entry strategies in τ and bid b_{-i} if participating in the second stage. That is,

$$p_i(b;\tau) \equiv \Pr\left\{ \max\left\{ \left(U_0, \max_{j \in A_i(\tau)} \alpha Q_j + x_j \beta + \epsilon_j - b_j \right\} \leq \alpha q_i + x_i \beta + \epsilon_i - b_i \right\} \right($$

where $A_i(\tau) \equiv \{j \in N \backslash \{i\} : \tau_j(E_j) = 1\}$, and the probability is with respect to the joint distribution of $(\alpha, \beta, U_0, \varepsilon)$ and E_{-i}. Let $P_i(b_i; \tau, \sigma)$ denote the probability that bidder i wins the auction by bidding b_i when all others follow the strategies prescribed in (τ, σ). By construction,

$$P_i(b_i; \tau, \sigma_{-i}) = \int\left(p_i(b_i, \sigma_{-i}(c_{-i}); \tau) dG(c_{-i}) \right.$$

where $G(c_{-i})$ denotes the distribution of C_{-i} (which does not depend on c_i because of the assumed independence).

Recall that both p_i and P_i are conditional on the composition of potential sellers I_N, which is suppressed in the notation. It is important to note that conditioning on I_N means conditioning on $(x_j)_{i \in N}$, $(q_i)_{i \in N^p}$ and the number of the transitory sellers associated with each possible value of observed characteristics.

Given a profile of type-symmetric entry strategies τ, an active bidder i's expected payoff conditional on its own action and construction cost is:

$$\begin{aligned} \Pi_i(b_i, c_i; \sigma_{-i}, \tau) &\equiv (b_i - c_i) P_i(b_i; \tau, \sigma_{-i}) \\ &= (b_i - c_i) \int\left(p_i(b_i, \sigma_{-i}(c_{-i}); \tau) dG(c_{-i}) \right. \\ &\equiv \int\left(\pi_i(b_i, \sigma_{-i}(c_{-i}), c_i; \tau) dG(c_{-i}). \right. \end{aligned} \quad (1)$$

The first-step in our proof of existence is to draw on results from Athey (2001) and establish the following proposition.

Proposition A1. *Suppose that Assumption 1 in KST2017 holds. Then for any profile of type-symmetric pure entry strategies τ, there exists a type-symmetric profile of non-decreasing pure strategies $\sigma^{(\tau)} = (\sigma_i^{(\tau)})_{i\in N}$ such that for any bidder i with type $\theta \in \Theta$, $\sigma_i^{(\tau)}$ is given by $\sigma_\theta^{(\tau)}$, where*

$$\sigma_\theta^{(\tau)}(c_i) = \arg\max_{b_i \geq 0} (b_i - c_i) P_i(b_i; \tau, \sigma_{-i}^{(\tau)}) \ \forall c_i.$$

We prove Proposition A1 by showing that the conditions in Corollary 2.1 in Athey (2001) are satisfied.

Lemma A1 (Continuity of Ex Post Payoff) *Suppose that Assumption 1 in KST2017 holds. For any profile of type-symmetric entry strategies τ, $\pi_i(b, c_i; \tau)$ is continuous in b for any c_i.*

Proof of Lemma A1. Fix a profile of type-symmetric entry strategies τ. We need to show that $p_i(b; \tau)$ is continuous in b. Let

$$A_{i,\theta}(\tau) = \{j \in N\backslash\{i\} : \theta_j = \theta \ \textit{and} \ \tau_j(e_j) = 1\}$$

denote the random set of entrants with type $\theta = (p, x, q)$. For each $i \in N$ let a_i denote a realized set of entrants competing with i should i enter. Let a_i be partitioned into subsets $\{a_{i,\theta} : \theta \in \Theta\}$ such that $\theta_j = \theta$ for all $j \in a_{i,\theta}$. Then

$$p_i(b; \tau) = \sum\nolimits_{a_i} \Pr\{A_i = a_i; \tau\} \times \tilde{p}_i(b, a_i; \tau) \tag{2}$$

where $\tilde{p}_i(b, a_i; \tau)$ denotes the probability that an active bidder i wins the auction conditional on the set of the other entrants being a_i, the vector of bids b and construction costs c; and the summation $\sum_{a_i}$ is over all the possible sets of entrants from $N\backslash\{i\}$.

For example, if a_i is such that x_j are identical for all $j \in a_i$ and if i has the lowest quality index $\bar{q}_1$,

$$\begin{aligned}
\tilde{p}_i(b, a_i; \tau) &= \Pr\left\{ \begin{array}{c} \alpha(q_l - \bar{q}_1) + (x_l - x_i)\beta + \epsilon_l - \epsilon_i \leq b_l - b_i \ \forall l \text{ s.t. } q_l \neq \bar{q}_1 \\ (x_j - x_i)\beta + \epsilon_j - \epsilon_i \leq b_j - b_i \ \forall j \text{ s.t. } q_j = \bar{q}_1 \\ U_0 - \alpha\bar{q}_1 - x_i\beta - \epsilon_i \leq -b_i \end{array} \quad A_i(\tau) = a_i \right\} \\
&= \Pr\left\{ \begin{array}{c} \alpha(q_l - \bar{q}_1) + (x_l - x_i)\beta + \epsilon_l - \epsilon_i \leq b_l - b_i \ \forall l \text{ s.t. } q_l \neq \bar{q}_1, \\ (x_j - x_i)\beta + \epsilon_j - \epsilon_i \leq b_j - b_i \ \forall j \text{ s.t. } q_j = \bar{q}_1 \\ U_0 - \alpha\bar{q}_1 - x_i\beta - \epsilon_i \leq -b_i \end{array} \right\}
\end{aligned} \tag{3}$$

where the second equality is due to the independence of entry costs from $(\alpha, \beta, U_0, \epsilon)$ and C. The probability is with respect to the joint distribution of $(\alpha, \beta, U_0, \epsilon)$. This implies that conditioning on a realized set of the other entrants a_i, the conditional probability $\tilde{p}_i$ does not depend on τ, and only depends on a subvector of $b_{-i} \equiv (b_j : j \in a_i)$. A similar expression for $\tilde{p}_i(b, a_i; \tau)$ can be derived for seller i with other types via a symmetric argument.

Let $|a_{i,\theta}|$ denote the cardinality of $a_{i,\theta}$ for $\theta \in \Theta$, which partition a_i based on seller types. For any realized $a_i = \cup_{\theta \in \Theta} a_{i,\theta}$ and a profile of type-symmetric entry strategies τ,

$$\Pr\{A_i(\tau) = a_i\} = \prod_{\theta \in \Theta} \lambda_\theta(\tau)^{|a_{i,\theta}|}[1 - \lambda_\theta(\tau)]^{n_\theta - |a_{i,\theta}|}$$

where $\lambda_\theta(\tau) \equiv \mathbf{P}\{\tau_\theta(E_i) = 1\}$. Note the equality holds because entry costs are independent of $(\alpha, \beta, U_0, \epsilon)$ and C, and are independent across the bidders. We have also used that $\Pr\{\tau_i(E_i) = 1\} = \lambda_\theta(\tau)$ for any seller i with type θ in a type-symmetric equilibrium. Hence $\{A_{i,\theta}(\tau) : \theta \in \Theta\}$ are independent from C. Under Assumption 1, the right-hand side of (3) must be continuous in b for any a_i. It then follows from (2) that $p_i(b; \tau)$ is continuous in b. $\square$

Lemma A2 (Single-Crossing Property of Interim Payoff) *Suppose Assumption 1 in KST2017 holds and potential sellers follow a profile of type-symmetric entry strategies τ. For each i, $\Pi_i(b_i, c_i;\ \tau, \sigma)$ satisfies the (Milgrom-Shannon) single-crossing property of incremental returns in (b_i, c_i) whenever σ consists of non-decreasing pure strategies.*

Proof of Lemma A2. Note that under independence between C and $(\alpha, \beta, U_0, \epsilon)$, $\tilde{p}_i(b, a_i; \tau)$ in (3) does not depend on c, which in turn implies $p_i(b; \tau)$ also does not depend on c under Assumption 1. Thus

$$\begin{aligned} P_i(b_i; \tau, \sigma) &= \int p_i(b_i, \sigma_{-i}(c_{-i}); \tau) dG(c_{-i}) \\ &= \int \sum_{a_i} \Pr\{A_i(\tau) = a_i\} \tilde{p}_i(b_i, \sigma_{-i}(c_{-i}), a_i) dG(c_{-i}) \\ &= \sum_{a_i} \Pr\{A_i(\tau) = a_i\} \tilde{P}_i(b_i, a_i) \end{aligned} \tag{4}$$

where

$$\begin{aligned} \tilde{P}_i(b_i, a_i) &\equiv \int \tilde{p}_i(b_i, \sigma_{-i}(c_{-i}), a_i) dG(c_{-i}) \\ &= \Pr\left\{ \begin{array}{c} \sigma_j(C_j) - \alpha(q_j - q_i) - (x_j - x_i)\beta + \epsilon_i - \epsilon_j \geq b_i \ \forall j \in a_i \\ \text{and } \alpha q_i + x_i\beta + \epsilon_i - U_0 \geq b_i \end{array} \right\} \end{aligned}$$

with the probability being with respect to the distribution of $(\alpha, \beta, U_0, \varepsilon)$ and C_{-i}. (Recall that q_i's are constant parameters anchored to the identity of potential bidders). Under Assumption 1 in KST2017, C_i is independent from C_{-i} and $(\alpha, \beta, U_0, \epsilon)$, and $\tilde{P}_i(b_i, a_i)$ does not depend on c_i.

Substitute (4) into (1) to get

$$\Pi_i(b_i, c_i; \sigma_{-i}, \tau) = (b_i - c_i) \sum_{a_i} \Pr\{A_i(\tau) = a_i\} \tilde{P}_i(b_i, a_i).$$

Thus we can write

$$\frac{\partial^2}{\partial b_i \partial c_i}\Pi_i(b_i, c_i; \sigma_{-i}, \tau) = -\sum\nolimits_{a_i} \left(\Pr\{A_i(\tau) = a_i\} \frac{\partial \tilde{P}_i(b_i, a_i)}{\partial b_i}\right.$$

Under Assumption 1, $\partial \tilde{P}_i(b_i, a_i)/\partial b_i \leq 0$. This proves the lemma. □

Proposition A1 then follows from Lemmas A1 and A2 by applying Corollary 2.1 in Athey (2001). It then remains to show that a type-symmetric monotone p.s.BNE (pure strategy Bayesian Nash equilibrium) exists in the overall game where entry decisions are endogenous. First, notice that under our independence assumptions in Assumption 1, a profile of type-symmetric *monotone* pure entry strategy is characterized by a profile of type-specific cutoffs $t \equiv (t_\theta : \theta \in \Theta)$ such that for all i with type θ the entry strategy is $\tau_\theta(e_i) = 1\{e_i \leq t_\theta\}$.

In what follows, we replace $\Pi_i(.,.;\tau,\sigma)$ with $\Pi_i(.,.;t,\sigma)$ in notation as entry strategies are fully characterized by t; and we replace then notation $\sigma^{(\tau)}$ by $\sigma^{(t)}$ in Proposition A1 to highlight the fact that in equilibrium bidding strategies must be related to cutoffs t in the entry stage.

Next, we show that under additional conditions there exists t such that

$$t = \mathbf{E}\left[\Pi_i\left(\left(\sigma_\theta^{(t)}(C_i), C_i; \sigma_i^{(t)}, t\right)\right]\left(\equiv \Psi_\theta(t)\right. \quad (5)$$

for all i with type θ.

Condition A1. *(i) The joint density of $(\alpha, \beta, U_0, \varepsilon)$ is bounded above. (ii) For all seller type θ and $\eta > 0$ and type-symmetric cutoffs t there exists some $\delta_0 > 0$ (which may depend on η and t such that $|\sigma_\theta^{(s)}(c_i) - \sigma_\theta^{(t)}(c_i)| \leq \eta$ for all c_i and all s with $\|s - t\| \leq \delta_0$.*

Part (ii) of the assumption is a high-level condition on the profile of strategies $\sigma^{(t)}$ that is BNE in the bidding stage *for a given profile of entry strategy τ*. The condition states that if we consider $\sigma^{(t)}(c_i)$ as a class of functions in t that are indexed by c_i then this class of functions is equicontinuous. This condition requires that the changes in seller strategies in the bidding stage move continuously with entry thresholds at all possible levels of project costs. We expect this condition to hold under mild primitive conditions such as positive low bounds on the density of $(\alpha, \beta, U_0, \varepsilon)$, C and E.

In order to illustrate why this condition is most likely to hold in our context, consider a related model where the procurement auction in the bidding stage is standard first-price (lowest-price). With a smooth distribution of entry costs, the changes in the entry thresholds have continuous effects on the distribution of the set of entrants, which is integrated out as the bidders maximize their ex ante payoffs. Thus the equilibrium strategies can be shown to vary continuously in entry threshold by typical arguments based on the Theorem of Maximum. In our case the presence of buyers' random tastes in $(\alpha, \beta, \epsilon, U_0)$ introduces an additional smoothing effect as the sellers calculate their ex ante payoffs. As a result we expect Condition A1 to hold under mild, standard regularity conditions.

Lemma A3. *Under Assumption 1 in KST2017 and Condition A1 above, $\Psi_\theta(t)$ is continuous in t for all $\theta \in \Theta$.*

Proof of Lemma A3. By construction

$$\Psi_\theta(t) \equiv \int \left[\phi_\theta^{(t)}(c_i) - c_i\right] \left[\sum_{a_i} \tilde{Q}_i(a_i, t)\tilde{P}_i\left(\sigma_\theta^{(t)}(c_i), a_i\right)\right] dF_\theta(c_i)$$

where F_θ is the cost distribution for a type-θ seller; and $\tilde{Q}_i(a_i, t)$ is short-hand for $\Pr\{A_i(\tau) = a_i\}$ when a profile of type-symmetric monotone pure entry strategies τ are characterized by t. It then follows from Condition A1 that for all $\eta > 0$ and type-symmetric t, there exists $\delta_0 > 0$ such that $\|s - t\| \leq \delta_0$ implies the difference between the integrand evaluated at (c_i, s) and at (c_i, t) has an absolute value smaller than η for all c_i. $\square$

With the support of E_i being closed intervals, the support of the cutoffs t is convex and compact. Hence the existence of the type-symmetric monotone pure-strategy BNE in the overall game follows from Brower's Fixed-Point Theorem.

B. Identification Proofs in KST2017

Throughout the proof, we suppress the auction subscripts l to simplify notation, and write $(C_{i,l}, E_{i,l}, \epsilon_{i,l}, B_{i,l}, U_{0,l}, \alpha_l, \beta_l, N_l, A_l, A_{i,l}, I_{N_l}, I_{A_l})$ as $(C_i, E_i, \epsilon_i, B_i, U_0, \alpha, \beta, N, A, A_i, I_N, I_A)$, which are considered i.i.d. draws across auctions in the data. Note that sellers' characteristics x_i, q_i are fixed for each seller throughout all auctions in the data and do not need any auction subscript.

B1. Proof of Proposition 1

In what follows, we suppress the dependence of sellers' strategies and winning probabilities on the set of potential bidders N and its composition I_N. Partition the set of entrants A according to whether a seller is weakly preferred to the outside option or not. That is $A = A^{(1)} \cup A^{(0)}$, where

$$A^{(1)} \equiv \{k \in A : U_k \geq U_0\} \text{ and } A^{(0)} \equiv A \backslash A^{(1)}.$$

In general, both $A^{(1)}$ and $A^{(0)}$ contain permanent and transitory sellers with various levels of quality and observed characteristics.

For any pair of *permanent* potential sellers i, j, let $\mathcal{A}_{i,j}$ denote the support of $(A^{(1)}, A^{(0)})$ *after excluding i and j*. That is, $\mathcal{A}_{i,j} \equiv \{(a, a') : a, a' \subseteq N\backslash\{i, j\} \text{ with } a \cap a' = \varnothing\}$. For any $(a, a') \in \mathcal{A}_{i,j}$, define

$$\mathcal{P}_{i,j}(b; a, a') \equiv \Pr\left\{ i \text{ wins} \;\middle|\; \begin{array}{l} i \in A, j \notin A, B_i = b, i, j \in N \\ A^{(1)}\backslash\{i\} = a, A^{(0)}\backslash\{i\} = a' \end{array} \right\}$$

for any $b \in \mathcal{B}_i$.

Lemma B1 *Suppose Assumptions 1 and 2 in KST2017 hold. Fix a pair of permanent bidders* (i, j) *with* $x_i = x_j$.
(a) For any $(a, a') \in \mathcal{A}_{i,j}$ *and* $b \in \mathcal{B}_i \cap \mathcal{B}_j$,

$$q_i \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} q_j \Rightarrow \mathcal{P}_{i,j}(b; a, a') \left\{ \begin{array}{c} \geq \\ = \\ \leq \end{array} \right\} \mathcal{P}_{j,i}(b; a, a'). \tag{6}$$

(b) For some $(a, a') \in \mathcal{A}_{i,j}$, *the two weak inequalities implied in (6) hold strictly with positive probability for all* b *in a subset of* $\mathcal{B}_i \cap \mathcal{B}_j$ *that has positive Lebesgue measure.*

Proof of Lemma B1. Part (a). Recall in our model each seller $i \in N$ draws a private entry cost E_i independently from $F_{E|x_i,q_i}$ and makes an entry decision based on E_i and the composition of N. As a result, sellers' entry decisions are independent. Besides, Assumption 1 in KST2017 implies that in equilibrium sellers' bidding strategies are functions of their private costs for the project and do not depend on the buyer's preference taste $(\alpha, \beta, \epsilon, U_0)$. Given any $(a, a') \in \mathcal{A}_{i,j}$, let $\mathcal{E}(a, a')$ be a shorthand for the event "$U_s \geq U_0$ $\forall s \in a$ and $U_{s'} < U_0$ $\forall s' \in a'$". Let $\Delta\epsilon_{s,i} \equiv \epsilon_s - \epsilon_i$, $\Delta x_{i,s} \equiv x_i - x_s$ and $\Delta q_{i,s} \equiv q_i - q_s$. Then

$$\begin{aligned} \mathcal{P}_{i,j}(b; a, a') \;=\; & \int \Pr\left\{ \begin{array}{l} -\epsilon_i \leq x_i\beta + \alpha q_i - b - u_0 \text{ and} \\ \Delta\epsilon_{s,i} - B_s \leq \Delta x_{i,s}\beta + \alpha\Delta q_{i,s} - b \;\forall s \in a \end{array} \middle| \; \alpha, \beta, u_0, \mathcal{E}(a, a') \right\} \\ & \times dF\left(\alpha, \beta, u_0 | \mathcal{E}(a, a')\right). \end{aligned} \tag{7}$$

The independence between sellers' private entry costs, project costs and the buyer's tastes α, β and outside option U_0 implies that the event that "$i \in A, j \notin A, B_i = b$" can be excluded from the events conditioned on in the conditional distribution of (α, β, U_0) on the right-hand side of (7). Furthermore, that the entry decisions are independent from (α, β, U_0), ϵ and C, that the project costs are independent across the sellers, and that these project costs are independent from (α, β, U_0) and ϵ imply the event that "$i \in A, j \notin A, B_i = b$" can be excluded from the conditioning set in the integrand once we condition on α, β, u_0 and $\mathcal{E}(a, a')$. By similar arguments, $\mathcal{P}_{j,i}(b; a, a')$ takes a form that is identical to $\mathcal{P}_{i,j}$ in (7), only with i replaced by j.

By Assumption 1, the joint distribution of ϵ_i and $(\epsilon_s, B_s)_{s \in a}$ is identical to that of ϵ_j and $(\epsilon_s, B_s)_{s \in a}$ once conditioned on α, β, u_0 and $\mathcal{E}(a, a')$. Besides, the conditional distribution $F\left(\alpha, \beta, u_0 | \mathcal{E}(a, a')\right)$ does not depend on the identity of sellers i and j. It then follows that the claim in part (a) of the lemma holds for all pairs of permanent sellers i, j with $x_i = x_j$.

Part (b). Consider a simple case where $a = \{s\}, a' = \{t\}$ where $q_s = q_i$ and $x_s = x_i$ but q_t is unrestricted. This case happens with positive probability under our maintained assumptions about the entry stage. Let U_i, U_s, U_t denote the payoff for the buyer from sellers i, s, t respectively.

By definition,

$$\mathcal{P}_{i,j}(b;a,a') = \frac{\Pr\{U_i > U_s, U_s \geq U_0 > U_t \mid B_i = b, i \in A, j \notin A, i,j \in N\}}{\Pr\{U_s \geq U_0 > U_t \mid B_i = b, i \in A, j \notin A, i,j \in N\}}. \tag{8}$$

Under Assumptions 1, the denominator on the right-hand side of (8) does not depend on events in the

conditioning set, and the denominator is $\Pr\{U_s \geq U_0 > U_t\}$ and does not depend on the identities i or j.

On the other hand, the numerator, by the law of total probability and the maintained independence assumptions, is

$$\begin{aligned}
& \int \Pr\{U_i > U_s, U_s \geq U_0 > U_t \mid \alpha, \beta, u_0, B_i = b\}\, dF(\alpha, \beta, u_0) \\
= & \int \Pr\left\{ \begin{array}{c} B_s - \epsilon_s \leq x_s\beta + \alpha q_s - u_0 \ , \ B_t - \epsilon_t > x_t\beta + \alpha q_t - u_0 \\ \epsilon_i + B_s - \epsilon_s > (x_s - x_i)\beta + \alpha(q_s - q_i) + b \end{array} \quad \alpha, \beta, u_0 \right\} dF(\alpha, \beta, u_0) \\
= & \int \left(\begin{array}{c} \Pr\left\{ \begin{array}{c} B_s - \epsilon_s \leq x_s\beta + \alpha q_s - u_0, \\ B_s + \epsilon_i - \epsilon_s > (x_s - x_i)\beta + \alpha(q_s - q_i) + b \end{array} \right\} \times \\ \Pr\{B_t - \epsilon_t > x_t\beta + \alpha q_t - u_0\} \end{array} \right) dF(\alpha, \beta, u_0)
\end{aligned} \tag{9}$$

where the second equality follows from the independence between C, ϵ and (α, β, U_0). A similar expression holds for $\mathcal{P}_{j,i}$, only with the subscripts i in (9) replaced by j.

Without loss of generality, suppose $q_i > q_j$. This implies $q_s - q_i = 0 < q_s - q_j$ (because $q_s = q_i$ by our supposition). Also, by construction $x_i - x_s = x_j - x_s = 0$. Under Assumption 1, the joint distribution of $(\epsilon_i, B_s - \epsilon_s)$ is identical to that of $(\epsilon_j, B_s - \epsilon_s)$. Furthermore, B_s is independent from B_i and $(\epsilon_i, \epsilon_s, \alpha, \beta, U_0)$; and both $(\epsilon_i - \epsilon_s)$ and $(\epsilon_j - \epsilon_s)$ are continuously distributed with positive density over a connected support.

It then follows from Assumption 2 in KST2017 that there exists a subset of $\mathcal{B}_i \cap \mathcal{B}_j$ with positive Lebesgue measure so that for all b in this subset, there is positive probability that $(B_s - b)$ is close enough to 0. Also recall the distribution of α is independent from the sellers' bids. Hence there is positive probability that α is sufficiently small so that for all b in this afore-mentioned subset of $\mathcal{B}_i \cap \mathcal{B}_j$, we have $\mathcal{P}_{i,j}(b;a,a') > (<)\ \mathcal{P}_{j,i}(b;a,a')$ whenever $q_i > (<)\ q_j$. $\square$

Proof of Proposition 1. By definition and the law of total probability, we can write $r_{i,j}(b)$ as:

$$\sum_{(a,a')} \left(\begin{array}{l} \Pr\{i \text{ wins} \mid A^{(1)}\backslash\{i\} = a, A^{(0)}\backslash\{i\} = a', B_i = b, i \in A, j \notin A, i,j \in N\} \times \\ \Pr\{A^{(1)}\backslash\{i\} = a, A^{(0)}\backslash\{i\} = a' \mid B_i = b, i \in A,\ j \notin A, i,j \in N\} \end{array} \right).$$

Under Assumption 1, the first conditional probability in the summand above is $\mathcal{P}_{i,j}(b;a,a')$. Besides, the second conditional probability in the summand does not depend on the event "$B_i =$

b_i" and

$$\begin{aligned} & \Pr\{A^{(1)}\backslash\{i\} = a, A^{(0)}\backslash\{i\} = a' | \, i \in A, j \notin A, i, j \in N\} \\ = \quad & \Pr\{A^{(1)}\backslash\{j\} = a, A^{(0)}\backslash\{j\} = a' | \, j \in A, i \notin A, i, j \in N\} \end{aligned}$$

under the same assumptions. The proposition then follows from Lemma B1. ■

B2. Proofs of Propositions 2 and 3

Proof of Proposition 2. Consider a pair of permanent sellers i, j with $(x_i, q_i) = (x_j, q_j)$ and a composition of *other* entrants I_a with $a \subset N\backslash\{i, j\}$ which satisfy Assumption 3 for some price vector b_a. Let $b \equiv (b_i, b_j, b_a)$. Let $\varphi_{i,j}(a, b, I_a)$ denote the probability that i wins conditional on "$\{i, j\} \in A$, $B_i = b_i$, $B_j = b_j$ and the composition of the other entrants in A is I_a and quotes b_a". Under Assumptions 1,

$$\varphi_{i,j}(a, b, I_a) = \Pr\{\epsilon_j - \epsilon_i \leq b_j - b_i, Y_i(b_a, I_a) - \epsilon_i \leq -b_i | I_a\}.$$

Note the event conditional on the right-hand side is not "$\{i, j\} \in A$, $B_i = b_i$, $B_j = b_j$ and the composition of the other entrants in A is I_a and quotes b_a" any more. Instead, the right-hand side is only conditioning on the composition of the other entrants than i, j. This is because sellers' private entry and project costs and the quality of transitory sellers are independent from $(U_0, \alpha, \beta, \epsilon)$. In fact $\varphi_{i,j}$ is a function of b and I_a only. Also, under these maintained assumptions, ϵ_i, ϵ_j and $Y_i(b_a, I_a)$ are mutually independent once conditional on b_a and I_a. With B_j and B_i being independent from $(U_0, \alpha, \beta, \epsilon)$, evaluating $\varphi_{i,j}(a, b, I_a)$ at different quoted prices b_i, b_j amounts to evaluating a fixed joint distribution of $\epsilon_j - \epsilon_i$ and $Y_i(b_a, I_a) - \epsilon_i$ conditional on I_a and b_a at different points on the support. By Assumption 3, the joint distribution of $(\epsilon_j - \epsilon_i, Y_i(b_a, I_a) - \epsilon_i)$ conditional on I_a and b_a is identified from $\varphi_{i,j}$ over its full support. By the mutual independence between ϵ_i, ϵ_j and $Y_i(b_a, I_a)$ given I_a and b_a and the non-vanishing characteristic functions of ϵ_i's, the proposition follows from the Kotlarski's Theorem (or Theorem 2.1.1 in Rao (1992)).

Next, without loss of generality, consider a pair of permanent sellers i, j such that $x_i = x_j$ and $q_i > q_j$. Fix another composition I_a for other entrants $a \subset N\backslash\{i, j\}$ and a bid vector b_a that satisfy Assumption 3 for such a pair of permanent sellers i, j. By the same argument as that used above, the joint distribution of $(\alpha\Delta q_{j,i} + \Delta\epsilon_{j,i}, Y_i(b_a, I_a) - \epsilon_i)$ conditional on I_a and b_a is fully identified over its support for the fixed (b_a, I_a) under Assumption 3. Because the marginal distribution of match components is identified as above, so is the distribution of the difference $\Delta\epsilon_{j,i}$. With $\alpha\Delta q_{j,i}$ independent from $\Delta\epsilon_{j,i}$, the distribution of $\alpha\Delta q_{j,i}$ is identified from the marginal distribution of $\alpha\Delta q_{j,i} + \Delta\epsilon_{j,i}$ (which does not depend on I_a or b_a under independence conditions in Assumption 1), using the non-vanishingness of the characteristic functions of $\epsilon_j - \epsilon_i$. It then follows that the constant parameter $\Delta q_{j,i}$ (recall i, j are permanent sellers) and the distribution of α are jointly identified up to a scale normalization. ■

Proposition 2 shows the quality difference between all pairs of permanent sellers sharing the same observed characteristics is identified. Under the condition that the lowest quality level for a permanent seller is the same across groups with different observed characteristics x_i, we can recover the level of quality for sellers with various observed characteristics up to a location normalization that sets the (constant) lowest quality to zero.

Next, we introduce the additional rank condition to be used in Proposition 3. Suppose Assumption 3 holds for a pair of permanent sellers $\{i, j\}$ with $q_i \neq q_j$ and $x_i = x_j$, and let I_a, b_a denote a composition for a set of other entrants than i, j and a bid vector associated with this set, which satisfy Assumption 3 for such i, j. Let N^p (and N^t) denote the set of permanent (and transitory) potential bidders in an auction. Partition a set $a \subset N\backslash\{i, j\}$ into a set that consists of transitory sellers only $a^t \equiv a \cap N^t$ and one that consists of permanent sellers only $a^p \equiv a \cap N^p$. Let I_{a^t} denote the composition of a^t, and let $Q_{a^t} \equiv (Q_s : s \in a^t)$. Let $b_{a^t} \equiv (b_s : s \in a^t)$ denote the vector of bids submitted by entrants who are transitory sellers; and likewise define b_{a^p}. By construction, $b_a \equiv (b_{a^p}, b_{a^t})$ and

$$\begin{aligned} \psi_i(y, \alpha, b_a, I_a) &\equiv \Pr\{Y_i(b_a, I_a) + \alpha q_i \leq y \mid \alpha, I_a\} \\ &= \Pr\{U_0 \leq y|\alpha\} \sum\nolimits_{q_{a^t}} \lambda(q_{a^t}; b_{a^t}, I_{a^t}) \Lambda_i(y, q_a; \alpha, b_a, I_a) \end{aligned} \tag{10}$$

where $\lambda(q_{a^t}; b_{a^t}, I_{a^t}) \equiv \Pr\{Q_{a_t} = q_{a^t}|\alpha, b_{a^t}, I_{a^t}\}$ (which does not depend on α under independence conditions in Assumption 1) and

$$\Lambda_i(y, q_a; \alpha, b_a, I_a) \equiv \Pr\left\{ \begin{array}{l} (x_s - x_i)\beta + \alpha q_s - b_s + \epsilon_s \leq y \ \forall s \in a^p \\ (x_{s'} - x_i)\beta + \alpha q_{s'} - b_{s'} + \epsilon_{s'} \leq y \ \forall s' \in a^t \end{array} \ \middle| \ \alpha, I_a \right\} \tag{11}$$

and the summation $\sum_{q_{a^t}}$ is over all possible quality profile for transitory sellers in a^t.[1]

In what follows, we fix I_a and b_{a^p} and suppress them in the notation for ψ_i, λ and Λ_i. Likewise, we can construct a similar equation for the same y, α with $\Pr\{U_0 \leq y \mid \alpha\} > 0$ but different vector $\tilde{b}_{a^t} \neq b_{a^t}$ such that $(b_{a^p}, \tilde{b}_{a^t})$ also satisfy Assumption 3. Taking the ratio of these two equations associated with b_{a_t} and $\tilde{b}_{a_t}$, and re-arranging terms, we get

$$\psi_i(y, \alpha, \tilde{b}_{a^t}) \sum\nolimits_{q_{a^t}} \lambda(q_{a^t}; b_{a^t}) \Lambda_i(y, q_{a^t}; \alpha, b_{a^t}) = \psi_i(y, \alpha, b_{a^t}) \sum\nolimits_{q_{a^t}} \lambda(q_{a^t}; \tilde{b}_{a^t}) \Lambda_i(y, q_{a^t}; \alpha, \tilde{b}_{a^t}), \tag{12}$$

[1] The last equality in (10) follows from the law of total probability, the independence between (α, U_0) and ϵ, and the fact that for all $s' \in a^t$, $Q_{s'}$ is an independent draw from a distribution conditional on $x_{s'}$. That the distribution of U_0 given α is independent from I_a is due to the independence between sellers' entry costs and (U_0, α); and that $\lambda(q_{a^t}; b_{a^t}, I_{a^t})$ is not a function of α is because the price quoted are functions of project costs and privately observed quality q_{a^t}, which independent from α once I_{a^t} is controlled for.

which is an equations in $2K^*$ unknown weights, or probability masses,

$$\{\lambda(b_{a^t}), \lambda(\tilde{b}_{a^t})\} \equiv \{\lambda(q_{a^t}; b_{a^t}), \lambda(q_{a^t}; \tilde{b}_{a^t})\}_{q_{a^t}},$$

with K^* being the cardinality of the support of Q_{a^t} given x_{a^t}. In addition each group of the K^* weights need to sum to one:

$$\sum\nolimits_{q_{a_t}} \lambda(q_{a^t}; b_{a^t}) = \sum\nolimits_{q_{a^t}} \lambda(q_{a^t}; \tilde{b}_{a^t}) = 1. \tag{13}$$

We need the follow condition on the support of (U_0, α).

Assumption 4. *There exists a set $\mathcal{U} \equiv \{(y^{(r)}, \alpha^{(r)}) : r = 1, 2, ..., R\}$ from the joint support of (U_0, α) with $R \geq 2(K^* - 1)$ such that the matrix of coefficients in the linear system of $(R + 2)$ equations in $\{\lambda(b_{a^t}), \lambda(\tilde{b}_{a^t})\}$, which is constructed by stacking (13) with R equations of (12) evaluated at the pairs in $\mathcal{U}$, has full rank.*

With (U_0, α) continuously distributed, this full rank condition can be expected to hold for a set $\mathcal{U}$ with positive measure, provided there is sufficient variation in ψ_i, Λ_i over the support of (U_0, α). We also need an extended version of the support condition in Assumption 3 in KST2017.

Assumption 5. *There exists a composition I of permanent sellers such that for some composition of the other sellers I' and a vector of their bids b', the support of the random vector $((\alpha\Delta q_{s,i} + \Delta x_{s,i}\beta + \Delta\epsilon_{s,i})_{s\in n\backslash\{i\}}, Y_i(b', I') - \epsilon_i)$ conditional on (b', I') is a subset of the support of $(B_j - B_i, -B_i)$ for an $i \in n$ whenever n is a set of permanent sellers that has composition I with $|n| \geq \dim(\beta) + 1$ and $(x_s - x_i)_{s\in n\backslash\{i\}}$ having full rank. The distribution of the random vector conditional on (b', I') has a non-vanishing characteristic function.*

Assumption 5 requires that there exists a composition I for a set of permanent sellers such that for some composition of the other sellers I' and a vector of their bids b', the support condition similar to part (i) in Assumption 3 holds for some $i \in a$ whenever the set of permanent sellers a has composition I, with $|a| \geq \dim(\beta) + 1$ and $(x_s - x_i)_{s\in a\backslash\{i\}}$ being non-singular.

Proof of Proposition 3. Without loss of generality, consider a composition I for some permanent sellers $\{i, j\}$ and a composition I_a for other entrants $a \subset N\backslash\{i, j\}$ and an associated bid vector b_a that satisfy Assumption 5. Then by the same argument in Proposition 2, the distribution of $(\alpha\Delta q_{j,i} + \epsilon_j - \epsilon_i, Y_i(b_a, I_a) - \epsilon_i)$ for the given I_a, b_a is identified over the full support from the impact of independent variation in B_i and B_j on the probability that i wins when $A = a \cup \{i, j\}$, $B_i = b_i$, $B_j = b_j$ and the other entrants quote b_a.

Next, recover the distribution of $(\alpha\Delta q_{j,i}, Y_i(b_a, I_a))$ given I_a, b_a using the distribution of $(\alpha\Delta q_{j,i} + \epsilon_j - \epsilon_i, Y_i(b_a, I_a) - \epsilon_i)$ given I_a, b_a and the distribution of $(\epsilon_j - \epsilon_i, \epsilon_i)$, with the latter joint distribution identified due to Proposition 2. To see how, note $(\epsilon_j - \epsilon_i, \epsilon_i)$ is independent from $(\alpha\Delta q_{j,i}, Y_i(b_a, I_a))$ for any fixed I_a and b_a due to Assumption 1 and the characteristic function of (ϵ_i, ϵ_j) is non-vanishing. With the quality levels q_i and q_j identified following Proposition 2, we recover the distribution of $Y_i(b_a, I_a)+\alpha q_i$ conditional on α, I_a, b_a using the joint distribution of $(\alpha\Delta q_{j,i}, Y_i(b_a, I_a))$ given I_a, b_a. This means $\psi_i(y, \alpha, b_a, I_a)$ is identified for all y.

Evaluate the linear system mentioned in Assumption 4 at the set of other entrants a with $x_s = x_i$ for all $s \in a$. Again, fix I_a, b_{a^p} and a^p (and therefore the quality and characteristics of sellers in a^p), and suppress them in the notation for ψ_i, λ and Λ_i. Then the right-hand side of (11) is simplified to

$$\Pr\left\{\epsilon_s \leq y + b_s - \alpha q_s \ \forall s \in a \middle| \alpha\right\} \tag{14}$$

which is identified because the distribution of match components is already recovered as above. Thus, by evaluating (12) at the same $(I_a, b_{a^t}, \tilde{b}_{a^t})$ but different values of (α, y) with $\Pr\{U_0 \leq y \mid \alpha\} > 0$ provides us with the linear system of $R+2$ equations in $2K^*$ unknown weights $\{\lambda(b_{a^t}), \lambda(\tilde{b}_{a^t})\}$. Under the full rank condition in Assumption 4, $\{\lambda(b_{a^t}), \lambda(\tilde{b}_{a^t})\}$ is identified. Finally, with the $2K^*$ probability masses $\{\lambda(b_{a^t}), \lambda(\tilde{b}_{a^t})\}$ recovered, we can identify the conditional distribution $F_{U_0|\alpha}$ using the equality in (10).

Next, without loss of generality, suppose Assumption 5 holds for $i \in n$ (where n is a set of permanet sellers with the composition I) and a set of transitory sellers a with the composition $I' = I_a$ and associated bid vectors $b' \equiv b_a$. Let b_n denote the vector of bids from n. Under our assumptions,

$$\begin{aligned} & \Pr\left\{i \text{ wins}\middle| A = n \cup a,\, b_n\right\} \\ = \ & \Pr\left\{\begin{array}{c} \Delta\epsilon_{s,i} + \alpha\Delta q_{s,i} + \Delta x_{s,i}\beta \leq b_s - b_i \ \forall s \in n\backslash\{i\} \\ Y_i(b_a, I_a) - \epsilon_i \leq -b_i \end{array}\right\}\Bigg(\end{aligned}$$

where as before $Y_i(b_a, I_a)$ denotes the maximum of $U_0 - \alpha q_i - x_i\beta$ and $\Delta x_{k,i}\beta + \alpha(Q_k - q_i) - b_k + \epsilon_k$ for $k \in a \subset N_t$. The support condition of the proposition implies the marginal distribution of $(\Delta\epsilon_{s,i} + \alpha\Delta q_{s,i} + \Delta x_{s,i}\beta)_{s\in n\backslash\{i\}}$ is identified. With the distribution of ϵ, α and the quality differences already identified, it follows from the independence between β and (α, U_0) that the joint distribution of $(\Delta x_{s,i}\beta)_{s\in n\backslash\{i\}}$ is identified over its full support. With $(x_s - x_i)_{s\in n\backslash\{i\}}$ having full-rank, the distribution of β is identified from that of $(\Delta x_{s,i}\beta)_{s\in n\backslash\{i\}}$ using Jacobian transformation (by multiplying the joint density of $(\Delta x_{s,i}\beta)_{s\in n\backslash\{i\}}$ with the absolute value of the determinant of $(x_s - x_i)_{s\in n\backslash\{i\}}$). ■

If the distribution of β is degenerate at a constant vector, the model is identified under weaker restrictions. To see this, recall that the first step in the proof of the second claim in Proposition 3 is to identify the joint distribution of $(\Delta\epsilon_{s,i} + \alpha\Delta q_{s,i} + \Delta x_{s,i}\beta)_{s\in n\backslash\{i\}}$. With ϵ independent from (α, β) and with the distribution of $(\Delta\epsilon_{s,i})_{s\in n\backslash\{i\}}$ identified, this implies means the joint distribution of $(\alpha\Delta q_{s,i} + \Delta x_{s,i}\beta)_{s\in n\backslash\{i\}}$ is identified. If β is a constant vector, then there are only $\dim(\beta) + |n| - 1$ parameters to recover from the continuous distribution of such a $(|n|-1)$-vector

$(\alpha \Delta q_{s,i} + \Delta x_{s,i}\beta)_{s\in n\backslash\{i\}}$. Recall that $\{x_i : i \in n\}$ is a constant vector of observed characteristics that do not vary throughout the data, and that the distribution of α is identified in Proposition 2. Hence we can use the second moments of $(\alpha \Delta q_{s,i} + \Delta x_{s,i}\beta)_{s\in n\backslash\{i\}}$ to identify $(\Delta q_{s,i})_{s\in n\backslash\{i\}}$, which in turn can be used to recover β under the rank condition maintained in the proposition. Note that this implies that, in order to recover the quality index of each permanent seller when β is constant, we may only need to normalize the location of the lowest quality level for sellers in any one (as opposed to all) of the groups defined by observed characteristics.

B3. Identification of Entry and Project Costs

The identification of the project cost distribution follows from an argument similar to that in Guerre, Perrigne and Vuong (2000). Let $G_\theta(b) \equiv \Pr(j \text{ wins } | j \in A_l, B_{j,l} = b)$, when the type of seller j is summarized by $\theta \equiv (\rho, x, q)$.

Proposition 4. *Suppose that the conditions of Proposition 3 hold, and that G_θ is differentiable with non-zero derivatives over its domain for each θ. Then the distribution of C_j conditional on the seller type θ is identified.*

Proof of Proposition 4. For a given seller i with type $\theta = (p, x, q)$, define

$$G_\theta(b) \equiv \Pr\{i \text{ wins } | i \in A, B_i = b\} = \Pr\{\varpi_i \leq -b\}$$

where ϖ_i denotes the maximum of $\max_{j\in A\backslash\{i\}}[\alpha(Q_j - q_i) + \Delta x_{j,i}\beta + \Delta\epsilon_{j,i} - B_j]$ and $U_0 - \alpha q_i - x_i\beta - \epsilon_i$. Note the randomness in ϖ_i comes from $\alpha, \beta, U_0, \epsilon$ and A as well as the bids $B_j, j \in A\backslash\{i\}$. (Note the definition of ϖ_i differs from that of $Y_i(b_a, I_a)$ because the former does not conditional on any vector of bids b_a or the composition of competing entrants I_a.) Provided sellers' equilibrium bidding strategies are differentiable, the smoothness conditions maintained in Assumptions 1 imply that G_θ is differentiable almost everywhere. Hence the first-order condition for bidder i choosing price b_i in equilibrium is:

$$(b_i - c_i)\, G'_\theta(b_i) + G_\theta(b_i) = 0. \tag{15}$$

Note that the conditional winning probability as a function of b_i, $G_\theta(b_i)$ is identified. Hence so is its derivative $G'_\theta(b_i)$. This then implies that the inverse bidding strategy (and consequently the distribution of private project cost C_i which depends on x_i, q_i but not ρ_i under Assumption 1 is identified for sellers with type $\theta = (p, x, q)$ as long as $G'_\theta(b_i) \neq 0$. ∎

If the sellers' equilibrium bidding strategies are invertible and differentiable in their costs, Assumption 1 implies that the distribution of equilibrium bids is differentiable almost everywhere. Using a change-of-variable argument in Guerre, Perrigne and Vuong (2000), we identify the sellers' inverse bidding strategies and the distribution of project costs.

For identification of the entry cost distribution, we use exogenous variation in an observed variable, say, Z_l, which does not affect the entry cost distribution but in general enters the seller's ex ante payoff prior to entry decisions. For example, Z_l could be the number of sellers in N_l which affects sellers' ex ante payoff due to the competition effect. Alternatively, Z_l could be the average of observed characteristics x among all sellers that are irrelevant to entry costs. In this case, the average observed characteristics may affect ex ante payoff through its correlation with project costs.

Proposition 5. *Suppose that the conditions of Proposition 4 hold, and that there exists auction-level heterogeneity Z_l reported in the data such that Z_l and $E_l = \{E_{j,l}\}_{j \in N_l}$ are independent. Then for each z in the support of Z_l, the $p^*_\theta(z)$-quantile of the entry cost distribution for each seller with type θ is identified, where $p^*_\theta(z)$ denotes the equilibrium entry probability for a seller of type θ when $Z_l = z$.*

Proof of Proposition 5. The equilibrium entry threshold for a seller with type θ (which is a cutoff in the support entry signals below which the seller decides to enter), in the presence of exogenous project cost shifters $Z = z$, is characterized by:

$$t_\theta(z) = \mathbf{E}\left[\Pi_i\left(\sigma^*_\theta(C_i, Z), C_i, Z; p^*, \sigma^*\right) \middle| Z = z\right] \tag{16}$$

where p^* is the vector of type-symmetric equilibrium entry probabilities, which are directly identifiable from the data. (There is a slight abuse of notation in that we now write Π_i as a function of p^* as opposed to τ^*. Also recall that we are suppressing the dependence on the set of potential bidders N and its composition I_N in the current section.) The inverse equilibrium bidding strategies are recovered as Proposition 4 and the distribution of bidders are identified; and the distribution of buyer tastes $(\alpha, \beta, U_0, \epsilon)$ is also identified. Hence the right-hand side of (16) is identified for all z. The left-hand side by definition is the $p^*_\theta(z)$ quantile of the distribution of entry costs for a seller with type θ.[2] ■

In the empirical section, we parametrize the distribution of entry costs and use GMM for estimation, exploiting the variation in the observed characteristics of potential entrants as a source of exogenous cost shifters.

B4. Support Conditions

We discuss how our model generates the price variation in the support condition used in identification. We do so in the context of a stylized model that abstracts away from observed characteristics x_i, seller endogenous participation and presence of transitory sellers, but that has an allocation rule involving unobserved quality indices and stochastic match components.

[2] The proof allows for dependence between Z and I_N. For example, Z could be the number of potential bidders in an auction, i.e., the cardinality of N.

Adding endogenous entry and transitory sellers would complicate the algebra without adding much insight.

Suppose the set of entrants in the stylized model is $A = \{i, j\}$, which is known to both participants. In this case, the support condition for identifying the distribution of match components is reduced to: "there exist i, j with $q_i = q_j$ such that the support of $(B_j - B_i, -B_i)$ includes that of $(\epsilon_j - \epsilon_i, \tilde{U}_0 - \epsilon_i)$", where we let $\tilde{U}_0$ denote the difference of outside option U_0 and the weighted quality index αq_i. Let $[\underline{\varepsilon}, \overline{\varepsilon}]$ denote the marginal support of ϵ_i. Note that, in a type-symmetric equilibrium which we consider here, the support of bids from i, j are identical, and denoted by $[\underline{b_i}, \overline{b}_i]$.

A set of sufficient conditions for the support restrictions is: "there exist i, j with $q_i = q_j$ s.t. $\overline{b} > \overline{\varepsilon} - \underline{u}_0$ and $\underline{b} < \underline{\varepsilon} - \overline{u}_0$," which can be satisfied if both (a) $\overline{b} > \overline{\varepsilon} - \underline{u}_0$ and (b) $\overline{b} - \underline{b} > (\overline{u}_0 - \underline{u}_0) + (\overline{\varepsilon} - \underline{\varepsilon})$ hold. Condition (a) holds provided $\overline{b} \geq \overline{c_i} > \overline{\varepsilon} - \underline{u}_0$, where $\overline{c_i}$ is the supreme of the support of private costs for i and j. Condition (b) essentially requires the support of bids to be large relative to that of outside utility and match components. Intuitively, (b) also holds when the support of sellers' private costs is sufficiently large. We now provide an argument for this intuition.

The idea is to show that the bidding strategy is continuous in the length of the support of match component $\overline{\varepsilon} - \underline{\varepsilon}$ and the support of outside option. Under type-symmetric pure-strategy BNE the bidders' strategies solve the maximization problem:

$$\sigma_i(c) \equiv \arg\max_b (b - c) \Pr\left(\tilde{U}_0 - \epsilon_i \leq -b\right) \Pr\left(-B_j + \epsilon_j - \epsilon_i \leq -b\right) \tag{17}$$

The second probability in (17) represents i's belief, which is formed from i's knowledge of the distribution of private costs, and the distribution of quality indexes in the population of sellers.

Suppose ϵ_i are i.i.d. uniform over $[\underline{\varepsilon}, \overline{\varepsilon}]$. Applying the law of total probability, we can write the objective function for seller i with costs c as

$$(b - c) \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \left[\left(\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \frac{1 - F_{B_j}(b - \Delta\varepsilon_{i,j})}{\overline{\varepsilon} - \underline{\varepsilon}} d\varepsilon_j \right) \frac{F_{\tilde{U}_0}(-b + \varepsilon_i)}{\overline{\varepsilon} - \underline{\varepsilon}} \right] d\varepsilon_i. \tag{18}$$

Changing variables between ε_r and $\tau_r \equiv \frac{\varepsilon_r - \underline{\varepsilon}}{\overline{\varepsilon} - \underline{\varepsilon}}$ for $r = i, j$, we can write (18) as

$$(b - c) \int_0^1 \left(\int_0^1 F_{\tilde{U}_0}(\underline{\varepsilon} - b + \delta\tau_i) \left[1 - F_{B_j}(b - \delta\Delta\tau_{i,j})\right] d\tau_j \right) d\tau_i \tag{19}$$

where $\delta \equiv \overline{\varepsilon} - \underline{\varepsilon}$ is the length of support of match components. Note (19) is continuous in both δ and the length of support for $\tilde{U}_0$. It then follows from an application of the Theorem of Maximum that the support of bids is continuous in the size of the support of match component and outside options. Provided private costs vary sufficiently, the support of bids in a standard auction model with no match components (i.e. ϵ is degenerate at 0) and no outside option is an interval with non-degenerate interior. It follows from the implication of the Theorem of Maximum that condition (b) holds whenever the support of ϵ and $\tilde{U}_0$ is small enough. By the same token, we can provide similar structural justifications for the support conditions for recovering quality levels using i, j with $q_i \neq q_j$, as long as variation in private costs and quality differences are sufficiently large relative to that of buyers' tastes in $(\alpha, \epsilon, \tilde{U}_0)$. In the current example, adding transitory sellers would not require any qualitative change in the argument for the support conditions.

C. Details of the Classification Procedure

This section provides the details of the nonparametric classification methodology. A complete treatment of this methodology and formal results are found in Krasnokutskaya, Song, and Tang (2016).

C1. Estimation of Classifications with Known Number of Groups

We consider the case where X_i takes values from a finite set $\{x_1, ..., x_m\}$. Thus the set of the permanent sellers can be partitioned into groups, $\bar{N}_x, x = x_1, ..., x_m$, so that for each x, and for any $i, j \in \bar{N}_x$, $x_i = x_j = x$. The analysis in this section is performed conditional on x. For brevity, we omit conditioning on x in the exposition below, so that we simply write $\bar{N}^p$ instead of $\bar{N}_x$. Define K_0 to be the number of distinct quality levels among the permanent sellers.

For ease of exposition, we first present the case with the number of quality levels K_0 equal to 2 so that $q_i \in \{\bar{q}_h, \bar{q}_l\}$ for a pair of unknown numbers $\bar{q}_h$ and $\bar{q}_l$. We explain how the algorithm generalizes to the case with $K_0 > 2$ later. Let $\bar{N}_h \subset \bar{N}^p$ be the collection of high quality sellers within group λ and $\bar{N}_l \subset \bar{N}^p$ be the collection of low quality sellers within group λ. Our object of interest is the following ordered group structure

$$T = (\bar{N}_h, \bar{N}_l)$$

of $\bar{N}^p$. We estimate T in three steps. First, for each $i \in \bar{N}^p$, we estimate two ordered partitions: one partition consists of the group of the sellers with higher or equal quality than that of i (denoted by $\bar{N}_1(i)$) and the rest (denoted by $\bar{N}^p \setminus \bar{N}_1(i)$), and the other partition consists of the group of sellers with lower or equal quality than that of i (denoted by $\bar{N}_2(i)$) and the rest. Second, among the two ordered partitions, we choose the one that is mostly likely to coincide with $(\bar{N}_h, \bar{N}_l)$. Third, we choose i such that the estimated partition associated with this i is most strongly supported by the data.

Suppose that we have obtained the bootstrap p-values $\hat{p}_{ij}^{+}$, $\hat{p}_{ij}^{-}$, and $\hat{p}_{ij}^{0}$ as explained in the main text of the paper. Then we recover the group structure in three steps as follows.

Step 1: Define

$$\begin{aligned}\hat{N}_1(i) &= \{j \in \bar{N}^p \backslash \{i\} : \hat{p}_{ij}^+ < \hat{p}_{ij}^-\} \text{ and} \\ \hat{N}_2(i) &= \{j \in \bar{N}^p \backslash \{i\} : \hat{p}_{ij}^+ > \hat{p}_{ij}^-\}.\end{aligned}$$

Step 2: For each $i \in \bar{N}^p$, we define[3]

$$\begin{aligned}s_1(i) &= \frac{1}{|\hat{N}_1(i)|} \sum_{j \in \hat{N}_1(i)} \log \hat{p}_{ij}^+ \text{ and} \\ s_2(i) &= \frac{1}{|\hat{N}_2(i)|} \sum_{j \in \hat{N}_2(i)} \log \hat{p}_{ij}^-,\end{aligned}$$

and let

$$s(i) = \begin{cases} s_1(i), \text{ if } i \in \bar{N}^p \backslash \hat{N}_1(i) \\ s_2(i), \text{ otherwise} \end{cases},$$

where $|\hat{N}_2(i)|$ and $|\hat{N}_1(i)|$ denote the cardinalities of the sets $\hat{N}_1(i)$ and $\hat{N}_2(i)$. For each index i, we have two classifications, $(\hat{N}_1(i), \bar{N}^p \backslash \hat{N}_1(i))$ or $(\bar{N}^p \backslash \hat{N}_2(i), \hat{N}_2(i))$. The first classification regards i as high type and the second classification regards i as low type. The index $s(i)$ measures the degree of misclassification caused by each of the two cases. When most agents are correctly classified, $s(i)$ becomes severely negative.

Step 3: We choose i^* that minimizes $s(i)$ over $i \in \bar{N}^p$, i.e.,

$$i^* = \operatorname{argmin}_{i \in \bar{N}^p} s(i). \tag{20}$$

Now we take

$$(\hat{N}_l, \hat{N}_h) = \begin{cases} (\bar{N}^p \backslash \hat{N}_2(i^*), \hat{N}_2(i^*)), & \text{if } s_1(i^*) \geq s_2(i^*) \\ (\hat{N}_1(i^*), \bar{N}^p \backslash \hat{N}_1(i^*)), & \text{if } s_1(i^*) < s_2(i^*). \end{cases}$$

We take the estimator of T as

$$\hat{T} = (\hat{N}_l, \hat{N}_h).$$

The quantity $s(i)$ indicates the weakness of the likelihood that i is classified into her right quality group. Then we choose i^* that minimizes $s(i)$ over $i \in \bar{N}$, and let $\hat{N}_h = \hat{N}_h(i^*)$ and $\hat{N}_l = \hat{N}_l(i^*)$. We take $\hat{\mathcal{C}} = (\hat{N}_h(i^*), \hat{N}_l(i^*))$ as an estimated classification of players into two quality groups.

The generalization of the procedure to the case of $K_0 > 2$ with K_0 known can proceed as follows. First, we split $\bar{N}$ into $\hat{N}_h$ and $\hat{N}_l$ using the algorithm for $K_0 = 2$. Then we find a

[3] Alternatively, one could replace $\hat{p}_{ij}^+$ in the definition of $s_1(i)$ and $\hat{p}_{ij}^-$ in the definition of $s_2(i)$ by $\hat{p}_{ij}^0$. The consistency results of this paper are not affected by this modification.

minimum value (denoted by $\hat{p}_h$) of $\log \hat{p}_{ij}^0$ among the pairs (i,j) such that $i \neq j$, and $i,j \in \hat{N}_h$, and a minimum value (denoted by $\hat{p}_l$) of $\log \hat{p}_{ij}^0$ among the pairs (i,j) such that $i \neq j$, and $i,j \in \hat{N}_l$. If $\hat{p}_h < \hat{p}_l$, we split $\hat{N}_h$ into $\hat{N}_{hh}$ and $\hat{N}_{hl}$ using the same algorithm for $K_0 = 2$, and otherwise, we split $\hat{N}_l$ into $\hat{N}_{lh}$ and $\hat{N}_{ll}$ using the same algorithm for $K_0 = 2$. We repeat the procedure. For example, suppose that we have classifications $\hat{N}_1, ..., \hat{N}_{k-1}$ obtained. For each $r = 1, ..., k-1$, we compute the minimum value (say, $\hat{p}_r$) of $\log \hat{p}_{ij}^0$ among the pairs (i,j) such that $i \neq j$ and $i,j \in \hat{N}_r$, and then select its minimum (say, $\hat{p}_{r^*}$) over $r = 1, ..., k-1$. We split $\hat{N}_{r^*}$ into $\hat{N}_{r^*h}$ and $\hat{N}_{r^*l}$ using the algorithm for $K_0 = 2$ to obtain a classification of S into k groups. We continue until the groups become as many as K_0.

C2. Selection of the Number of Groups

The methodology outlined above assumes that we know the exact number of groups. To accommodate the situation with real life data without knowledge of the number of the groups, we offer a method of consistent selection of the number of groups. We suggest that the number of groups should be selected to minimize the criterion function that balances a measure of goodness-of-fit that captures a misspecification bias versus a penalty term that penalizes overfitting. The goodness-of-fit measure is based on the variance test approach.

Given an estimated classification $\bar{N}^p = \cup_{k=1}^K \hat{N}_k$ with K groups, let

$$\hat{V}(K) = \frac{1}{K} \sum_{k=1}^{K} \min_{i,j \in \hat{N}_k} \log \hat{p}_{ij}^0 \ ,$$

for each $k = 1, ..., K$.

Suppose that K_0 is the true number of groups. Let $g(L) \to \infty$ be such that $g(L)/\sqrt{L} \to 0$ as $L \to \infty$. Then, define

$$\hat{Q}(K) \equiv \hat{V}(K) + Kg(L).$$

We select K as follows:

$$\hat{K} = \text{argmin}_{1 \leq K} \hat{Q}(K).$$

The idea of this selection is based on the following intuition. For simplicity, let us assume that $|\bar{N}^p|$ is fixed and L increases to infinity. (Krasnokutskaya, Song, and Tang (2016) considered a more general case where $|\bar{N}^p|$ is allowed to increase with L at a slow speed.) First, $\hat{V}(K) = O_P(1)$, as $L \to \infty$, if $K \geq K_0$, because there is no misspecification bias in this case. The limit $O_P(1)$ measures the asymptotic behavior of the goodness-of-fit of the model when the classification is weakly finer than the true classification. Since $Kg(L) \to \infty$ as $L \to \infty$, the minimization of $\hat{Q}(K)$ over K leans toward a lower choice of K that is closer to K_0. On the other hand, if the classification is strictly coarser than the true classification, i.e., $K < K_0$, the quantity $\hat{V}(K)$ diverges at a rate faster than $g(L)$, as $L \to \infty$, due to misspecification. In this case, the minimization of $\hat{Q}(K)$ over K excludes K such that $K < K_0$ for large samples. Thus we obtain the consistency of $\hat{K}$ under regularity conditions. Details in a more general set-up are found in Krasnokutskaya, Song, and Tang (2016).

C3. Details of Implementation

Here we give details of the choice of the kernel function, bandwidths and the penalization scheme in nonparametric classification. In the empirical analysis, we used a triweight kernel function: $K(u) = 1\{|u| \leq 1\}(35/32)(1-u^2)^3$. The bandwidth selection followed the usual Silverman's rule of thumb. The bootstrap Monte Carlo number used to construct the bootstrap p-values was 200.

Finally, as for the penalization scheme, we have chosen simply:

$$Kg(L) = K \log \log L.$$

Our simulation studies unreported here confirm that this choice of penalty scheme works well in various simulation designs.

C4. Confidence Sets for Quality Classifications

Suppose that we have K_0 groups and the set of permanent bidders N has size n. Since we can estimate K_0 consistently, we assume we know it. We fix $k = 1, ..., K_0$ and construct a confidence set for the k-th quality group. In other words, we would like to construct a random set $\hat{\mathcal{C}}_k \subset \bar{N}^p$ such that

$$\text{liminf}_{L \to \infty} P\{\bar{N}_k \subset \hat{\mathcal{C}}_k\} \geq 1 - \alpha,$$

where $\bar{N}_k$ is the set of permanent bidders with quality $\bar{q}_k$. For this, We need to devise a way to approximate the finite sample probabilities like $P\{\bar{N}_k \subset \hat{\mathcal{C}}^k\}$. Since we do not know the cross-sectional dependence structure among the sellers, we use a bootstrap procedure that preserves this dependence structure from the original sample. The remaining issue is to determine the space in which the random set $\hat{\mathcal{C}}_k \subset \bar{N}^p$ can take values in. It is computationally infeasible to consider all possible such sets. Instead, we proceed as follows. First we estimate $\hat{N}_k$ as prescribed above and also obtain $\hat{\tau}_{ij}^0$, the test statistic defined in the main text. Given the estimate $\hat{N}_k$, we construct a sequence of sets as follows:

Step 1: Find $i_1 \in \bar{N}^p \backslash \hat{N}_k$ that minimizes $\min_{j \in \hat{N}_k} \hat{\tau}_{i_1,j}^0$, and construct $\hat{\mathcal{C}}_k(1) = \hat{N}_k \cup \{i_1\}$.
Step 2: Find $i_2 \in \bar{N}^p \backslash \hat{\mathcal{C}}_k(1)$ that minimizes $\min_{j \in \hat{C}_k(1)} \hat{\tau}_{i_2,j}^0$, and construct $\hat{\mathcal{C}}_k(2) = \hat{\mathcal{C}}_k(1) \cup \{i_2\}$.

...

Step m: Find $i_m \in \bar{N}^p \backslash \hat{\mathcal{C}}_k(m-1)$ that minimizes $\min_{j \in \hat{C}_k(m-1)} \hat{\tau}_{i_m,j}^0$ and construct $\hat{\mathcal{C}}_k(m) = \hat{\mathcal{C}}_k(m-1) \cup \{i_m\}$.

Repeat Step m up to $n = |\bar{N}^p|$.

Now, for each bootstrap iteration $s = 1, ..., B$, we construct the sets $\hat{N}_{k,s}^*$ and $\{\hat{\mathcal{C}}_{k,s}^*(m)\}$ following the steps described above but using the bootstrap sample. (Note that this bootstrap sample should be drawn independently of the bootstrap sample used to construct bootstrap p-values $\hat{p}_{ij}$ in the classification.)

Then, we compute the following:

$$\hat{\pi}^k(m) \equiv \frac{1}{B} \sum_{s=1}^{B} 1 \left\{ N_k \subset \hat{\mathcal{C}}^*_{k,s}(m) \right\}.$$

Note that the sequence of sets $\hat{\mathcal{C}}^*_{k,s}(m)$ increases in m. Hence the number $\hat{\pi}^k(m)$ should also increase in m. An $(1-\alpha)\%$-level confidence set is given by $\hat{\mathcal{C}}^*_k(m)$ with $1 \leq m \leq n$ such that

$$\hat{\pi}^k(m-1) < 1 - \alpha \leq \hat{\pi}^k(m).$$

Note that such m always exists, because $\hat{\mathcal{C}}^*_{k,s}(n) = \bar{N}^p$.

C5. Finite Sample Performance: Monte Carlo Simulations

Here we explore properties of our classification algorithm in simulation analysis.

We choose the distributions of project and entry costs to be the same across quality levels and given by truncated normals with $N(1.5, 0.2^2)$ and $N(0.08, 0.02^2)$ correspondingly. The bidders are assumed to be heterogeneous with respect to quality only. Buyers' tastes, therefore, are represented by the distributions of α and ϵ. We fix the distribution of α to be truncated normal $N(0.4, 0.2^2)$ with support $[0, 1]$. The distribution of ϵ is also chosen to be truncated normal with mean 0 and variance σ_ϵ^2. We vary σ_ϵ in experiments below to explore the sensitivity of our methodology to the noise in buyers' tastes. We truncate the support of epsilon at $[-\sigma_\epsilon, \sigma_\epsilon]$. Further, we assume that the distribution of the outside option coincides with the cost distribution of high-quality bidders. Finally, we assume that the set of suppliers consists of 30 programmers and is split equally between high- and low-quality suppliers.

We use the same numerical algorithm as in the main paper to solve for participation and bidding strategies of our game. The data are generated through repetition of the following steps:

1. At each round, 10 randomly selected bidders from the set above are declared to be potential bidders.

2. For each potential bidder we draw an entry and project cost. We, then, apply participation and bidding strategies to these draws to determine whether a potential bidder enters the set of active participants and if he does what bids he submits.

3. Next, we take draws from the distributions of α, ϵ, and of the outside option. The winner of the project is determined by evaluating submitted bids using the realizations of the outside option and of buyer's tastes.

4. The data record the set of potential bidders with their qualities, outcomes of participation and entry decisions as well as the buyer's choice.

Table 1: Results of Simulation Study

		Probability of Correct Classification					
		$L = 300$		$L = 200$		$L = 100$	
d_d	σ_ϵ	q_H	q_L	q_H	q_L	q_H	q_L
0.3	$0.2\sigma_c$	0.9773	0.9901	0.9613	0.9547	0.9314	0.9013
0.3	$0.5\sigma_c$	0.9645	0.9858	0.9477	0.9512	0.9223	0.8998
0.3	$1.5\sigma_c$	0.9619	0.9782	0.9457	0.9401	0.9207	0.8941
0.1	$0.2\sigma_c$	0.9632	0.9774	0.9329	0.9503	0.9164	0.8904
0.1	$0.5\sigma_c$	0.9551	0.9743	0.9263	0.9421	0.9034	0.8815
0.1	$1.5\sigma_c$	0.9518	0.9701	0.9227	0.9397	0.8927	0.8623

This table reports results of the simulation study of the sensitivity of the classification procedure to the quality differences, the magnitude of the preference noise, and the data set size. The latter is measured in the average number of bids per supplier. The difference in quality levels is measured relative to the project costs spread, i.e., $d_q = \frac{q_H - q_L}{\bar{c} - \underline{c}}$. The variance of the preference noise is measured relative to the project cost variance, i.e., $\sigma_\epsilon = d_\epsilon \sigma_c$.

We use the simulated data to investigate the sensitivity of our methodology to the magnitude of the quality differences, the noise in buyer's preferences, and the number of available observations. For the first two experiments we tie the quality differences and the noise magnitude to the variance in the private project costs. That is, we consider (a) high-quality differences with $\Delta q = 0.3(\bar{c} - \underline{c})$ and (b) low-quality differences with $\Delta q = 0.1(\bar{c} - \underline{c})$. Similarly, we consider (c) low preference noise with $\sigma_\epsilon = 0.2\sigma_c$, (d) medium preference noise with $\sigma_\epsilon = 0.5\sigma_c$, (e) high preference noise with $\sigma_\epsilon = 1.2\sigma_c$. Finally, we explore how the performance of our procedure changes with sample size. Our procedure is performed at the individual level, therefore, we explore the performance of our procedure as a function of the average number of bids per supplier (L).

We run the simulation experiments as follows. For every set of parameters, we apply our procedure to 500 data sets simulated according to steps (1)-(4) described above. We then compute for every supplier the fraction of the data sets in which his type was correctly recovered. We report the average of these fractions across bidders of the same quality level in Table 1.

The results of the simulation analysis show that the classification procedure performs quite well. In particular, it is not very sensitive to the magnitude of the preference noise. We would expect the preference noise to impede the recovery of the quality level since it disguises the link between the probability of winning and the quality of participant. It would be natural to expect that the procedure should impose higher data requirements in the presence of more noise. However, the variation in prices successfully compensates for the noise in buyers' preferences at least for moderate levels of noise.

As expected, the performance of the procedure does depend on the importance of the quality differences. The estimation is more precise when quality differences are large and grows less precise as quality differences diminish. Finally, the procedure is sensitive to the size of the data set. As the number of bids drops from 300 bids per supplier to 100 bids per supplier the probability of correct classification drops from 0.96 to 0.92 for high-quality suppliers and from 0.98 to 0.89 for low-quality suppliers. The classification of low-quality suppliers is affected to a larger degree since due to the lower probability of participation, the number of bids they submit is substantially below the average.

D. Details on the Estimation Procedures

D1. A Representation Result

In this section, we derive a generalized version of the representation in (7) in the main text which is used for constructing part of the moment conditions in the estimation step. For each project l, recall that $N_l = N_l^p \cup N_l^t$ denotes the set of potential bidders for project l, where N_l^p denotes the set of potential permanent bidders and N_l^t the set of potential transitory bidders. Similarly, $A_l = A_l^p \cup A_l^t$ denotes the set of active bidders for projects l, where A_l^p and A_l^t denote the sets of permanent and transitory active bidders respectively. (From here on, we suppress the auction index l from notation for simplicity.)

Recall that the sets $\bar{N}^p$ and $\bar{N}^t$ denote the total set of permanent sellers and transitory sellers. As in the main text, we write A, A^p, A^t, etc. to denote the random set of entrants, and a,a^p,a^t, etc., to denote their particular realizations. For each set $a = a^p \cup a^t$ with $a^p \subset \bar{N}^p$ and $a^t \subset \bar{N}^t$, we recall the definitions of the compositions, I_a in Section 5.2.2.

When collecting individual observations to the project level we find it convenient to arrange observations in a certain order. More specifically, the observations for permanent and transitory sellers are allocated into separate vectors. Within each vector we enumerate observations for actual entrants first then for non-entrants. Then within the set of actual entrants, we order the observations for permanent sellers according to (x, q)-characteristics in the order of

$$(\bar{x}_1, \bar{q}_1(\bar{x}_1)), ..., (\bar{x}_1, \bar{q}_{K(\bar{x}_1)}(\bar{x}_1)), ..., (\bar{x}_m, \bar{q}_{K(\bar{x}_m)}(\bar{x}_m))$$

and then for transitory sellers according to x-characteristics in the order of $\bar{x}, ..., \bar{x}_m$. We order observations similarly for the set of non-entrants as well. Thus we maintain the order of individual bids as described above so that it is easy to match individual bids to seller characteristics using only information contained in $I = (I_A, I_N)$, i.e., the information on the sizes of (x, q)-groups of the potential and active permanent sellers and the sizes of x-groups of the potential and active transitory sellers in auction l. Indeed, since the bids are submitted by active sellers, the relevant part of I is I_{A^p} and I_{A^t}. Let us denote a vector of bids by $b = [b^{p\prime}, b^{t\prime}]'$, where b^p and b^t are shorthand for b_{a^p} and b_{a^t}, i.e., vectors of bids submitted by active permanent and transitory sellers respectively with their j-th entries denoted by b^t_j and b^p_j. If I_{A^p} reports that A^p contains two bidders from group $(\bar{x}_1, \bar{q}_1(\bar{x}_1))$ and three bidders from group $(\bar{x}_2, \bar{q}_1(\bar{x}_2))$ then we know that bids b^p_1 and b^p_2 are submitted by bidders from group $(\bar{x}_1, \bar{q}_1(\bar{x}_1))$ and bids b^p_3, b^p_4, b^p_5 are submitted by sellers from group $(\bar{x}_2, \bar{q}_1(\bar{x}_2))$.

Recall that Q_i denotes the random quality of transitory seller i, and write

$$Q_A = (Q_i)_{i \in A} \text{ and } Q_a = (Q_i)_{i \in a}.$$

Similarly, for a nonstochastic vector $q = (q_i)$ of numbers q_i, we write $q_A = (q_i)_{i \in A}$ and $q_a = (q_i)_{i \in a}$. To simplify notation, we let

$$\omega(q_a; I_{a^t}) \equiv \prod_{i=1}^{|a^t|} \Pr(Q_i = q_i | x_i),$$

where a^t denotes the realized set of active transitory bidders, and $\Pr(Q_i = q_i | x_i)$ denotes the probability of transitory bidder i's quality being q_i when his characteristic is x_i. This latter probability is a model primitive. We also let

$$g(b^t; q_a, I_{a^t}) \equiv \omega(q_a; I_{a^t}) \prod_{i=1}^{|a^t|} f(b_i | Q_i = q_i, I) \Pr(i \in A^t | Q_i = q_i, I).$$

Here $\Pr(i \in A^t | Q_i = q_i, I)$ is the conditional probability of a transitory bidder entering the auction when his quality is q_i. Note that $g(b^t; q_a, I_{a^t})$ depends on I_N though we suppress it from notation for simplicity. Let $\Pr(Q_A = q_A | b, I)$ denote the conditional probability of $Q_A = q_A$ given the bid vector b and composition I. (Note that this probability depends on A only through I.) Our representation result characterizes this conditional probability in terms of $g(b^t; q_a, I_{a^t})$'s.

Proposition 2. *Suppose that Assumptions 1-3 hold. Then,*

$$\Pr(Q_A = q_A | b, I) = \frac{g(b^t; q_A, I_{A^t})}{\sum_{q'_A \in \mathbb{Q}_A} g(b^t; q'_A, I_{A^t})}.$$

Proof: First, note that by the Bayes Rule, we can write

$$\begin{aligned} \Pr(Q_A = q_A | b, I) &= \frac{f_b(b | Q_A = q_A, I) \Pr(Q_A = q_A | I)}{f_b(b | I)} \\ &= \frac{f_b(b^t | Q_A = q_A, I) \Pr(Q_A = q_A | I)}{f_b(b^t | I)}. \end{aligned} \tag{21}$$

The second equality holds because the bids of permanent sellers are independent of bids and qualities of transitory sellers, i.e., $f_b(b^p|Q_A = q_A, I) = f_b(b^p|I)$.

Notice that b_j^t's are independent across j's conditional on $Q_A = q_A$ and I, and that out of I_{A^t} only x_j^t matters for the distribution of bids that could be submitted by a transitory seller j. Therefore,

$$f_b(b^t|Q_A = q_A, I) = \prod_{j=1}^{|A^t|} f_b(b_j^t|Q_A = q_A, I) = \prod_{j=1}^{|A^t|} f_b(b_j^t|Q_j = q_j, I). \tag{22}$$

The last equality holds because the transitory seller knows his quality but not the qualities of his transitory competitors.

On the other hand, applying the law of total probability we obtain

$$\begin{aligned} f_b(b^t|I) &= \sum_{q_A \in \mathbb{Q}_A} \left(f_b(b^t|Q_A = q_A, I) \Pr(Q_A = q_A|I) \right) \\ &= \sum_{q_A \in \mathbb{Q}_A} \left(\left(\prod_{j=1}^{|A^t|} f_b(b_j^t|Q_j = q_j, I) \right) \Pr(Q_A = q_A|I) \right). \end{aligned} \tag{23}$$

We will use this expression later, after we deal with $\Pr(Q_A = q_A|I)$.

First, we write $I_A = I_{A^p} \cup I_{A^t}$ and let $\bar{I}_A$ be a realized set of I_A. Then notice that

$$\begin{aligned} \Pr(Q_A = q_A|I_N, I_A = \bar{I}_A) &= \frac{\Pr(I_{A^p} = \bar{I}_{A^p}, I_{A^t} = \bar{I}_{A^t}|Q_A = q_A, I_N) \Pr(Q_A = q_A|I_N)}{\Pr(I_{A^p} = \bar{I}_{A^p}, I_{A^t} = \bar{I}_{A^t}|I_N)} \\ &= \frac{\Pr(I_{A^t} = \bar{I}_{A^t}, Q_A = q_A|I_N)}{\sum_{\tilde{q}_A \in \mathbb{Q}_A} \left(\Pr(I_{A^t} = \bar{I}_{A^t}, Q_A = \tilde{q}_A|I_N) \right)}. \end{aligned} \tag{24}$$

The second equality holds because the events $I_{A^p} = \bar{I}_{A^p}$ and $I_{A^t} = \bar{I}_{A^t}$ are independent conditional on $Q_A = q_A, I_N$, and the event $I_{A^p} = \bar{I}_{A^p}$ is independent of $Q_A = q_A$ conditional on I_N. We write $\Pr(I_{A^t} = \bar{I}_{A^t}, Q_A = q_A|I_N)$ as

$$\begin{aligned} &\sum_{q_{N\setminus A} \in \mathbb{Q}_{N\setminus A}} \left(\Pr(I_{A^t} = \bar{I}_{A^t}, Q_A = q_A, \text{ and } Q_{N\setminus A} = q_{N\setminus A}|I_N) \right) \\ = &\sum_{q_{N\setminus A} \in \mathbb{Q}_{N\setminus A}} \left(\Pr(I_{A^t} = \bar{I}_{A^t}, Q_N = [q_A; q_{N\setminus A}]|I_N) \right) \\ = &\sum_{q_{N\setminus A} \in \mathbb{Q}_{N\setminus A}} \left(\Pr(I_{A^t} = \bar{I}_{A^t}|Q_N = [q_A; q_{N\setminus A}], I_N) \Pr(Q_N = [q_A; q_{N\setminus A}]|I_N) \right). \end{aligned} \tag{25}$$

As for the summands in the last expression, we define

$$\begin{aligned} p_i^A(q_i) &= \Pr(i \in A^t|Q_i = q_i, I_N) \text{ and} \\ p_i^{N\setminus A}(q_i) &= \Pr(i \in N^t \setminus A^t|Q_i = q_i, I_N), \end{aligned}$$

where q_i denotes the i-th entry of q_N. Thus $p_i^A(q_i) > 0$ only when $i \in A^t$ and similarly $p_i^{N\setminus A}(q_i) >$

0 only when $i \in N^t \setminus A^t$. Rewrite

$$\Pr(I_{A^t} = \bar{I}_{A^t} | Q_N = q, I_N) = \left(\prod_{i=1}^{|a^t|} p_i^A(q_i) \right) \left(\prod_{i=|a^t|+1}^{|N^t|} p_i^{N \setminus A}(q_i) \right).$$

This is because participation decisions of transitory sellers do not depend on the qualities of their transitory competitors since the qualities of transitory sellers are only privately known. On the other hand, we write

$$\Pr(Q_N = q | I_N) = \prod_{i=1}^{|N^t|} \Pr(Q_i = q_i | I_N) = \prod_{i=1}^{|N^t|} \Pr(Q_i = q_i | x_i^t),$$

because potential bidders are randomly drawn from the population characterized by a given vector x and from the whole structure I_N only the information about x_i^t is relevant in this probability. Combining this, we write $\Pr(I_{A^t} = \bar{I}_{A^t}, Q_A = q_A | I_N)$ as

$$\begin{aligned} & \prod_{i=1}^{|N^t|} \Pr(Q_i = q_i | x_i^t) \left(\prod_{i=1}^{|a^t|} p_i^A(q_i) \right) \left(\prod_{i=|a^t|+1}^{|N^t|} p_i^{N \setminus A}(q_i) \right) \quad (26) \\ = & \left(\prod_{i=1}^{|a^t|} p_i^A(q_i) \Pr(Q_i = q_i | x_i^t) \right) \left(\prod_{i=|a^t|+1}^{|N^t|} p_i^{N \setminus A}(q_i) \Pr(Q_i = q_i | x_i^t) \right). \end{aligned}$$

Applying this result to the denominator of (24), we find that the numerator and the denominator have a common factor:

$$\prod_{i=|a^t|+1}^{|N^t|} p_i^{N \setminus A}(q_i) \Pr(Q_i = q_i | x_i^t).$$

After canceling out this factor, we conclude that

$$\Pr(Q_A = q_A | I) = \frac{\prod_{i=1}^{|A^t|} p_i^A(q_i) \Pr(Q_i = q_{A,i} | x_i^t)}{\sum_{\tilde{q}_A} \prod_{i=1}^{|A^t|} p_i^A(\tilde{q}_i) \Pr(Q_i = \tilde{q}_i | x_i^t)} = \frac{\left(\prod_{i=1}^{|A^t|} p_i^A(q_i) \right) \omega(q_A; I_{A^t})}{\sum_{\tilde{q}_A} \left(\prod_{i=1}^{|A^t|} p_i^A(\tilde{q}_i) \right) \omega(\tilde{q}_A; I_{A^t})}, \quad (27)$$

recalling the definition of $\omega(q_a; I_{a^t})$. Now, plugging this expression into (23), we can write

$$f_b(b^t | I) = \sum_{q_A \in \mathbb{Q}_A} \frac{\omega(q_A; I_{A^t}) \prod_{j=1}^{|A^t|} f_b(b_j^t | Q_j = q_j, I) p_i^A(q_i)}{\sum_{\tilde{q}_A} \omega(\tilde{q}_A; I_{A^t}) \prod_{j=1}^{|A^t|} p_i^A(\tilde{q}_i)}. \quad (28)$$

Thus we plug (22), (28) and (27) into (21) to obtain the desired result. ■

D2. Basic Moment Restrictions

For the choice of g_j's, we use functions that vary with the active sellers' (x, q)-group memberships and their composition I_A's. For each auction l such that $I_{A_l} = \bar{I}_A$ for a given composition $\bar{I}_A$ and have a winning seller j in (x, q)-group, we take $g_j(B_l, \bar{I}_A)$ as follows. (Here, we let $A_{l,x,q}$ be the set of active sellers in auction l in (x, q)-group and $A_{l,x}$ that in x-group. Then for each (x, q) and (x', q') and for a given composition $\bar{I}_A$, we consider three types of g_j's (except for constant 1)) as follows.

(1) Moments involving a single winning bid:

$$g_j(B_l, I_{A_l}) = B_{j,l} 1\{I_{A_l} = \bar{I}_A\} 1\{j \in A_{l,x,q}\}.$$

(2) Moments involving a winning bid and another bid:

$$g_j(B_l, I_{A_l}) = \sum_{i \in A_{l,x',q'}} \bigg((B_{j,l} - B_{i,l}) 1\{I_{A_l} = \bar{I}_A\} 1\{j \in A_{l,x,q}\}.$$

(3) Moments involving two permanent sellers' bids and one transitory seller's bid:

$$g_j(B_l, I_{A_l}) = \sum_{i \in A_{l,x,q}} \sum_{h \in A_{l,x}} \bigg((B_{j,l} - B_{i,l}) B_{h,l} 1\{I_{A_l} = \bar{I}_A\}.$$

We also use moments in (i) with $B_{j,l}$ replaced by $B_{j,l}^2$, and in (2) with $(B_{j,1} - B_{i,l})$ replaced by $(B_{j,1} - B_{i,l})^2$ and by $(B_{j,1} - B_{i,l}) B_{j,l}$, and in (3) with $(B_{j,l} - B_{i,l}) B_{h,l}$ replaced by $B_j B_{h,l}$, $B_j^2 B_{h,l}$, $B_j x_h$ and $B_j^2 x_h$. In the actual implementation, we re-weight all the moments by the frequency with which relevant observations appear in the data.

D3. Additional Restrictions

We also use additional restrictions that stem from restrictions on transitory sellers' bid distributions, transitory sellers' participation probabilities, and restrictions that come from expected profit conditions.

(a) The restriction associated with transitory sellers' bid distribution:

$$f(b_{a^t} | I) = \frac{1}{J} \sum_{q_a' \in \mathbb{Q}_a} \omega(q_a; I_{a^t}) \prod_{i=1}^{|a^t|} f(b_i | Q_i = q_i, I) \Pr(i \in A^t | Q_i = q_i, I), \tag{29}$$

where

$$J = \sum_{q_a' \in \mathbb{Q}_a} \bigg(\omega(q_a; I_{a^t}) \prod_{i=1}^{|a^t|} \Pr(i \in A^t | Q_i = q_i, I).$$

Moment conditions associated with this restriction relate the empirical moments of $f(b_{a^t}|I)$ to the theoretical moments based on the restriction (29).

(b) The restriction associated with the transitory sellers' participation probability:

$$\begin{aligned} &\Pr(I_{A^t} = \bar{I}_{A^t}|I_N) \\ &= \sum_{q_A \in \mathbb{Q}_A} \left(\prod_{i=1}^{|a^t|} p_i^A(q_i) \Pr(Q_i = q_i|x_i^t) \right) \left(\prod_{i=|a^t|+1}^{|N^t|} p_i^{N \setminus A}(q_i) \Pr(Q_i = q_i|x_i^t) \right). \end{aligned} \tag{30}$$

This restriction is based on (26) in the proof of Proposition 2. Moment conditions associated with this restriction relate the transitory sellers' empirical probability of participation and expected x-characteristics of entrants conditional on I to their theoretical counterpart using (30).

(c) The restriction related to the expected profit condition. This restriction summarizes optimal participation behavior. It is summarized by the threshold strategy where potential bidders with entry cost draws below the ex-ante expected profit participate in the auctions and those with higher draws stay out. This implies that in equilibrium: for each $q^t \in \mathbb{Q}_x$ and $\theta_j = (t, x_j, q^t)$,

$$\Pr(j \in A^t|\theta = \theta_j, I_N) = F_E(\mathbf{E}[\Pi(\theta_j, I_N)]),$$

where $F_E(.)$ is the distribution function of entry costs E.

D4. Estimation of the Sellers' Cost Distribution

We estimate the distributions of the seller's costs conditional on the seller's attributes by combining the bid distributions of permanent sellers with the corresponding inverse bid functions for a given composition of the set of potential bidders $\bar{I}_N$:

$$\hat{F}_C(c|x, q) = \hat{G}_{p,x,q}(\hat{\xi}^{-1}_{p,x,q}(c; \bar{I}_N)|I_{N_l} = \bar{I}_N),$$

where $\hat{\xi}^{-1}_{p,x,q}(c; \bar{I}_N)$ denotes the inverse of the estimated inverse bid function of a permanent seller in group (x, q).[4] Recall that the distribution of costs depends only on sellers' attributes (both observable and unobservable), (x, q), not on whether the seller is permanent or transitory. On the other hand, the sellers' bidding strategy and thus the distribution of bids depends on his full type $\theta = (\rho, x, q)$, $\rho \in \{\text{p, t}\}$ which is reflected in the expression above. Notice that we are using the distributions of bids and inverse bid functions associated with permanent sellers. The estimated inverse bid function, $\hat{\xi}_{p,x,q}(b; \bar{I}_N)$, is derived from the first order condition of the

[4] We implement kernel smoothing over $\bar{I}_N$ when estimating $\hat{G}_{p,x,q}(.|I_{N_l} = \bar{I}_N)$ or $\hat{P}(j \text{ wins } | B_{j,l} = b, I_{N_l} = \bar{I}_N)$, $\frac{\partial}{\partial b}\hat{P}(j \text{ wins } |B_{j,l} = b, I_{N_l} = \bar{I}_N)$ and $\Pr(i \text{ is active}|\theta, I_{N_l} = \bar{I}_N)$.

corresponding permanent seller's optimization problem: for a permanent seller i in group (x,q),

$$\hat{\xi}_{p,x,q}(b,\bar{I}_N) = b - \frac{\hat{P}(j\,\text{wins} \mid B_{j,l} = b,\, I_{N_l} = \bar{I}_N)}{\frac{\partial}{\partial b}\hat{P}(j\,\text{ wins } |B_{j,l} = b,\, I_{N_l} = \bar{I}_N)},$$

where $\theta_j = (p,x,q)$.

We assess the magnitude of the costs of entering the auction using the conditions derived from the optimality of the permanent sellers' participation behavior. In the model, the participation probability satisfies the following equation

$$F_E(\Pi(\theta,\bar{I}_N)|x,q) = \Pr(j\,\text{is active}|\theta, I_{N_l} = \bar{I}_N), \quad (31)$$

where $F_E(.|x,q)$ is the distribution function of the entry costs and $\Pi(\theta,\bar{I}_N)$ the ex-ante expected profit of a seller with type θ.

We assume that the distribution of entry costs is given by the truncated normal distribution (truncated at zero) with seller-type-specific mean and standard deviation. We estimate the parameters of these distributions using a minimum distance estimation procedure based on the restrictions in equation (31) for various (x,q)-groups of permanent sellers and for time periods characterized by the high/low presence of potential bidders from different (x,q)-groups. The probability $\Pr(i\,\text{is active}|\theta, I_{N_l} = \bar{I}_N)$ can be directly identified from the data. As for the expected profits conditional on entry, we compute them from the estimated distributions of bids and costs and the sellers' beliefs about their competitors' participation strategies approximated by the participation behavior observed in the data.

E. Details of the Numerical Algorithm

Here we summarize the numerical algorithm used to solve for a type-specific equilibrium bidding and participation strategies $\{\sigma_k^*,\, p_k^*\}_{k=1,\dots,K}$.[5] It combines insights from projection methods[6]and the numerical approach developed in Marshall, Meurer, Richard, and Stromquist (1994). To simplify the presentation we summarize the method for the case when (a) all sellers are permanent; (b) α and U_0 are independent. Further, note that sellers' strategies depend on vector I_N summarizing the number of potential sellers by group. We suppress this dependence below in the interest of clarity of exposition.

Recall that the entry threshold is set so that it is equal to the expected profit from entry which realizes if competitors enter the auction with probabilities $\{p_k^*\}$ and everybody follows

[5]Note that participation strategies are summarized by entry thresholds t_k^*, $k = 1,..,K$ so that realized probabilities of entry are given by $p_k^* = G_E(t_k^*)$, $k = 1,..,K$. We describe the algorithm in terms of the probabilities of entry rather than thresholds. However, the entry thresholds can always be recovered from probabilities of entry.

[6]See, for example, Bajari (2001).

equilibrium bidding functions to construct bids. Hence

$$p_k^* = G(\bar{\pi}_k), \text{ and } \bar{\pi}_k = \mathbf{E}\left[\Pi_k\left(\sigma_k(C), C; \bar{p}^*, \sigma^*\right)\right].$$

The numerical algorithm consists of two loops. The outside loop implements an iterative procedure which solves for $\bar{p}^*$ such that equation (32) is satisfied. The inside loop computes for a given vector $\bar{p}$ the optimal bidding functions which are then used in conjunction with the vector $\bar{p}$ to compute expected profit from entry. The iterative procedure in the outside loop is fairly standard whereas the procedure for computing bidding strategies is specifically developed for this setting. That is why from here on we focus on the inside loop which computes bidding strategies. We suppress the dependence of $\Pr(A)$ on $\bar{p}$ from notation for simplicity.

1. Seller chooses bidding and strategy to maximize the following objective function:

$$(b - c_i)\sum_A \left(\Pr(U_0 \leq \alpha q_{\kappa(i)} - b + \epsilon_i \text{ and } \alpha q_{\kappa(j)} - B_j + \epsilon_j \leq \alpha q_{\kappa(i)} - b + \epsilon_i \,\forall\, j \in A\right)\Pr(A)$$

which can be re-written as

$$(b - c_i)\sum_A \left(\int_\alpha \int_{\epsilon_i} F_{U_0}(q_{\kappa(i)} - \alpha b + \epsilon_i)\right.$$
$$\times \prod_{j \neq i} \left(\int_{c_j}\left(F_\epsilon(\Delta_{i,j} q - \alpha(b - \sigma_{\kappa(j)}(c_j)) + \epsilon_i) dF_{\kappa(j)}(c_j) dF_\epsilon(\epsilon_i) dF_\alpha(\alpha)\Pr(A).\right.\right.$$

Here $\kappa(i)$ denotes the quality group of seller i. Summation is over all possible sets of entrants, A, and $\Pr(A)$ reflects the probability that a given set of entrants is realized.

2. For the derivation simplicity we re-write sellers' strategies in terms of 'offers', $\omega_i = \mu_\alpha q_{\kappa(i)} - b_i$ that are functions of seller's surplus, $s_i = \mu_\alpha q_{\kappa(i)} - c_i$. That is, seller's strategy is now given by $\gamma_k : S_k \to R$. This is just a simple re-parametrization of the seller's problem which now becomes

$$(s_i - \omega_i)\sum_A \left[\left(\left(\int_\alpha \int_{\epsilon_i} F_{U_0}(\omega_i + \alpha_0 q_{\kappa(i)} + \epsilon_i)\right.\right.\right.$$
$$\left.\times \prod_j \left(\int_{s_j} F_\epsilon(\omega_i - \gamma_{\kappa(j)}(s_j) + \alpha_0(q_{\kappa(i)} - q_{\kappa(j)}) + \epsilon_i) dF_{\kappa(j)}(s_j) dF_{\epsilon_i}(\epsilon_i) dF_\alpha(\alpha_0)\right.\right]\Pr(A),$$

where $\alpha_0 = \alpha - \mu_\alpha$.

3. Then, the first order conditions for this problem are given by

$$\sum_A \left(J_{1,A} + J_{2,A}\right)\Pr(A) = 0,$$

where

$$
\begin{aligned}
J_{1,A} &= -\int_{\alpha_0}\int_{\epsilon}\Bigg(F_{U_0}(\omega_i + \alpha_0 q_{\kappa(i)} + \epsilon_i) \\
&\quad \times \prod_j \Bigg(\int_{s_j} F_\epsilon(\omega_i - \gamma_{\kappa(j)}(s_j) + \alpha_0 \Delta_{i,j} q + \epsilon_i) dF_{\kappa(j)}(s_j) dF_{\epsilon_i}(\epsilon_i) dF_{\alpha_0}(\alpha_0) \\
J_{2,A} &= (s_i - \omega_i)\int_{\alpha_0}\int_{\epsilon}\Bigg(\Bigg\{ f_{U_0}(\omega_i + \alpha_0 q_{\kappa(i)} + \epsilon_i), \\
&\times \prod_j \Bigg(\int_{s_j} F_\epsilon(\omega_i - \gamma_{\kappa(j)}(s_j) + \alpha_0 \Delta_{ij} q + \epsilon_i) dF_{\kappa(j)}(s_j) \\
&+ F_{U_0}(\omega_i + \alpha_0 q_{\kappa(i)} + \epsilon_i) \sum_{j_1=1}^{L}\Bigg(\int_{s_{j_1}} f_\epsilon(\omega_i - \gamma_{\kappa(j_1)}(s_{j_1}) + \alpha_0 \Delta_{ij} q + \epsilon_i) dF_{\kappa(j_1)}(s_{j_1}) \\
&\times \prod_{j \neq j_1} \Bigg(\int_{s_{j_1}} F_\epsilon(\omega_i - \gamma_{\kappa(j)}(s_j) + \alpha_0 \Delta_{ij} q + \epsilon_i) dF_{\kappa(j)}(s_j)\Bigg)\Bigg\}\Bigg(dF_{\epsilon_i}(\epsilon_i) dF_\alpha(\alpha_0).
\end{aligned}
$$

4. Following Marshall, Meurer, Richard, and Stromquist (1994) we divide the support of S_k into small intervals. Further, we approximate each of the $\gamma_k(.)$ functions by a polynomial of $(s_k - \tilde{s}_k^l)$ where $\tilde{s}_k^l$ is a centroid of l's interval on the support of S_k. Specifically, we assume that $\gamma_k(s) = \sum_{p=0}^{\infty} a_{k,p}^{(l)}(s - \tilde{s}_k^l)^p$ on interval l. We also use their technique for representation of the nonlinear function of a bidding strategy in the form of a polynomial of $(s - \tilde{s}_k^l)$ (see non-uniform case). We use spline approximation of the estimated densities to obtain coefficients in the polynomial expansion of the outside functions. The exact expression of the polynomial expansion of the first order conditions is 12 pages long and is available from the authors upon request.

5. The polynomial expansion of first order conditions discussed in point (4) is summarized by a set of coefficients in front of the polynomial terms. To obtain $n-$th order approximation of the offer function we set the first n coefficients of the first order conditions expansion to zero and solve for a set of $\{a_{k,p}^{(l)}\}$ coefficients that satisfy this restriction. This part of our algorithm is borrowed from the projection methods. We deviate from the algorithm in Marshall, Meurer, Richard, and Stromquist (1994) at this point because the expressions for the coefficients in a first order conditions representation are non-linear functions of $\{a_{k,p}^{(l)}\}$ and thus we are unable to obtain an iterative expression similar to that in Marshall, Meurer, Richard, and Stromquist (1994).

6. The set of coefficients is obtained for a given set of starting points (boundary conditions). Once the set of coefficients is obtained we compute approximation error associated with a solution obtained under such starting point. Unlike a standard auction model the multi-attribute auction model does not have a singularity on either end because of ϵ and integra-

tion over ϵ. On the other hand, we do not solve for the coefficients so that the first order condition holds exactly (or arbitrary close). For this reason we do not target an objective function which reflects the fit of numerical solution at the boundary. Instead, we compute an error for each subinterval and then average it over the intervals. We incorporate this error into an iterative mechanism which searches for an optimal boundary condition. The search stops once the targeted precision is reached. In this our approach resembles the projection method.

7. We have verified that this algorithm converges to a vector of equilibrium bidding functions in the case when all the relevant distributions are uniform.

F. Additional Empirical Results

F1. Estimated Quality Structure

In Table 2, we report the estimated quality structure for each given number of the groups.

Table 2: Estimated Quality Structure for a Given Number of Groups

	Number of Groups	$K = 1$	$K = 2$	$K = 3$	$K = 4$	$K = 5$	$K = 6$
	1	52	45	33	2	2	2
	2	0	7	12	31	26	24
	3	0	0	7	12	5	2
	4	0	0	0	7	12	5
	5	0	0	0	0	7	12
	6	0	0	0	0	0	7
$\bar{V}$		9.21	2.61	1.77	0.85	0.73	0.31
$Q(K)$		10.03	4.22	4.11	4.24	4.81	5.22

This table shows the estimated quality group structures for the various numbers of quality groups for Eastern European suppliers with the medium levels of average reputation score. Rows 1-6 record the number of suppliers estimated to belong to a respective group. Rows 7 and 8 record the value of the $p - value$ component of the criterion function and the value of the criterion function. The results are based on the penalty function $Kg(L) = K \log \log L$. Results indicate that the number of groups most supported by the data is equal to three.

Table 3: Number of Observations by Group

Country Group	Reputation Score	Number of Observations (across pairs of sellers) smallest	average
North America	Low	122	511.7
North America	Medium	149	393.7
North America	High	131	240.6
Eastern Europe	Low	175	403.5
Eastern Europe	Medium	137	344.7
Eastern Europe	High	133	252.7
South and Eastern Asia	Low	110	647.7
South and Eastern Asia	Medium	167	547.8
South and Eastern Asia	High	155	258.2

This table reports the number of observations available for the use in the classification algorithm. Specifically, a row corresponds to a cell defined by values of observable seller characteristics. The first column reports the smallest number of observations across pairs of sellers in a given cell that can be used to construct the value of the index for one of sellers in a pair; the second column reports the average number of such observations for a given cell.

F2. Robustness Analysis of Classification Results

In this section we verify robustness of our results to some assumptions of the model.

Unobserved Auction Heterogeneity. In our model, the distribution of buyers' utility weights and the distribution of competitors' costs are invariant (from sellers point of view) across projects conditional on projects' observable characteristics. A researcher may be concerned that in the data for a given project sellers may have access to additional information about either of these objects, i.e. the empirical environment may be characterized by unobserved auction heterogeneity. Here, we first discuss how such unobserved heterogeneity may matter for our classification procedure. Then, we explain some of the robustness analyses we performed in order to reassure the reader about these concerns.

Let us consider the case when sellers have access to additional information (unavailable to the researcher) about buyers' utility weights for a specific project while the distribution of sellers' costs remains free from unobserved project heterogeneity. Notice that the index in Proposition 1 which establishes quality rankings between sellers i and j by comparing their performance in two sets of auctions: for seller i we use auctions where seller i participates but seller j does not and for seller j we use auction where seller j participates but i does not. If unobserved heterogeneity is present then sellers may select into participation on the basis of the project unobserved heterogeneity. This would impact quality ranking in in two ways: (a) the distribution of buyer utility weights in the set of auctions where i participates may be different from the distribution of utility weights in the set of auctions where j participates; (b) due to selection into participation the distribution over sets of competitors in auctions where i participates may be different from the distribution over the sets of competitors in the auctions where j participates. In contrast, if sellers have additional information (unobserved by the researcher) about the competitors' costs but the distribution of buyers' utility weights does not vary across auctions in unobserved (to the researcher) way then the issue in (a) no longer arises. However, it is still possible that sellers' participation decisions may vary across realizations of unobserved heterogeneity in which case the distributions over the sets of competitors would differ.

To verify robustness of our results to 'no unobserved heterogeneity' assumption we re-estimate a quality group structure while comparing permanent sellers under a varying set of circumstances. First, (1) we compare sellers i and j using data for the projects where they both belong to a subset of potential sellers who contacted the buyer but may not have submitted a bid; further (2) we re-estimate classifications using variously defined subsets of the original dataset, i.e. imposing higher degree of homogeneity in terms of size and duration. In the first exercise we would expect to observe a lower degree of selectivity into participation relative to the benchmark case. In the second case, we use different subsets of projects so that the underlying distribution of unobserved heterogeneity if it is present would differ across these subsets. We would expect the estimation results to differ across specifications and to be different from our benchmark specification if unobserved project heterogeneity affects this environment to an important degree.

We find that quality group structures estimated under these specifications are very similar to the estimates we obtain under our benchmark specification. The difference between the estimated group structures across the specifications is always under 3% of the total number of permanent sellers. The confidence intervals are somewhat larger under alternative specifications. This is not surprising since the classification procedure in these cases is based on a much lower number of auctions. These results indicate that unobserved project-level heterogeneity, even if present, is unlikely to be large. This is perhaps not surprising given that we observe many important parameters of the project and we have access to the professional assessment of the project size.

Fixed Seller Quality. In the interest of tractability we also assume that the seller's quality is constant for all projects that share the same observable characteristics. Next, we verify robustness of our results to this assumption. Recall that in order to improve the power of our test we aggregate the test statistics over the interval of prices that belong to the intersection of the supports of the distributions of bids submitted by sellers i and j. To implement the robustness check we split the interval of bids we used in the original classification procedure into two sub-intervals and then re-do the classification for each sub-interval. We find that the estimated group structures are basically the same, with the difference among the three estimates being under 5% of the number of permanent sellers. These findings alleviate our concerns about the potential variation in sellers' unobserved quality across projects. Indeed, if such project-specific variation in seller's quality were important, we would expect that the seller would be more likely to be classified as high quality in the auctions where he submits high bids. However, we do not find any substantial evidence of such regularity.

F3. Auxiliary Parameters in Baseline Model

Tables 4, 5 and 6 report parameters of the estimated distributions of bids and probabilities of participation for permanent and transitory sellers in the case of our benchmark model. We estimate that the number of reputation scores and an average reputation score matter for transitory bidders in a statistically significant way. The results show how these variables impact transitory sellers' prices (bids). For example, having no reputation scores bears a negative premium of close to 8% relative to the price charged by a seller with more than six scores. On the other hand, having a positive but small number of scores erodes this negative premium to 4% or 3%. The average reputation score does not appear to be important when the number of scores is really small. However, the difference between 9 points and 10 is rewarded with a 5% premium if the number of scores is moderate. This is comparable to the 7% premium documented above for the case of a long-run average reputation score that corresponds to the large number of scores.

Table 4: Participation Decision and Bid Distribution (Baseline Model)

	I(T)	II(T)	I(P)	II(P)
Mean				
Constant	0.552** (0.009)	-2.105** (0.019)	0.585** (0.052)	-2.173** (0.009)
No Ratings	-0.083** (0.025)	-0.33* (0.018)		
$0 < \text{Ratings} \leq 3$	-0.043** (0.006)	0.005 (0.021)		
$3 < \text{Ratings} \leq 6$	-0.021* (0.011)	0.005 (0.009)		
Number of Ratings			0.0071 (0.063)	0.003 (0.007)
Average Score 1	-0.005 (0.005)	-0.004 (0.011)		
Average Score 2	0.050** (0.002)	0.011** (0.003)		

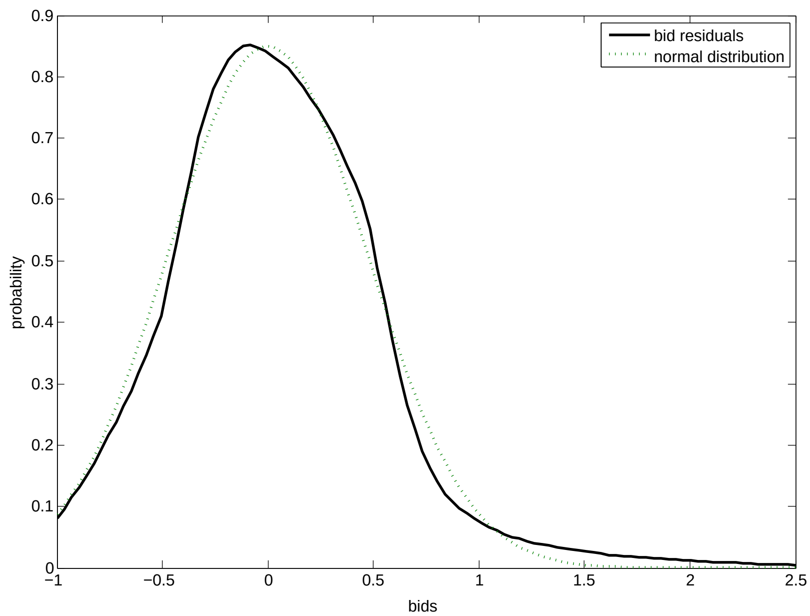
This table reports the effects of the covariates and the group premiums on sellers' bid distribution and participation decisions for the specification 2 presented in the paper. Columns I(T), II(T), and I(P), II(P) report estimated coefficients for the bid distribution and probability of participation of the transitory and permanent sellers respectively. "Average Score 1" and "Average Score 2" denote interactions of the current average score variable with the indicators for $0 \leq Ratings \leq 3$ and $3 \leq Ratings$. The stars, $**$, indicate that a coefficient is significant at the 95% significance level.

Table 5: Participation Decision and Bid Distribution (Baseline Model)

	Score	Q	I(T)	II(T)	I(P)	II(P)
Mean, Quality Groups Fixed Effect						
North America	Low	1	-0.218** (0.023)	0.231** (0.021)	-0.307** (0.046)	0.031 (0.027)
North America	Low	2	-0.173** (0.018)	-0.042 (0.022)	-0.271** (0.044)	-0.053** (0.025)
North America	Medium	2	-0.038** (0.022)	-0.139** (0.023)	-0.062 (0.046)	-0.105** (0.026)
North America	High	1	-0.173** (0.012)	0.134** (0.023)	-0.214** (0.041)	0.165** (0.017)
North America	High	2	-0.108** (0.021)	-0.265** (0.031)	-0.166** (0.039)	-0.193** (0.031)
Eastern Europe	Low	1	-0.026** (0.017)	0.232** (0.021)	-0.083* (0.043)	0.205** (0.017)
Eastern Europe	Low	2	-0.062* (0.022)	-0.194** (0.013)	-0.176** (0.038)	-0.108** (0.021)
Eastern Europe	Medium	1	-0.199** (0.035)	0.034** (0.015)	-0.246** (0.044)	0.013 (0.012)
Eastern Europe	Medium	2	-0.192** (0.024)	-0.245** (0.021)	-0.226** (0.048)	-0.198** (0.022)
Eastern Europe	Medium	3	-0.128** (0.032))	-0.257** (0.017))	-0.167** (0.051)	-0.232** (0.029)
Eastern Europe	High	1	-0.191** (0.024)	-0.131** (0.023)	-0.248** (0.038)	-0.257 (0.034)
Eastern Europe	High	2	-0.178** (0.043)	-0.091** (0.031)	-0.249** (0.051)	-0.099** (0.012)
Eastern Europe	High	3	-0.132** (0.022)	-0.221** (0.011)	-0.172** (0.038)	-0.206** (0.029)
South-East Asia	Low	1	-0.004 (0.034)	0.251** (0.012)	0.086** (0.043)	0.288** (0.023)
South-East Asia	Low	2	-0.246** (0.034)	-0.231** (0.012)	-0.226** (0.041)	-0.204** (0.021)
South-East Asia	Low	3	-0.359** (0.044)	-0.075** (0.011)	-0.434** (0.043)	-0.051 (0.033)
South-East Asia	Medium	1	-0.108* (0.063)	-0.075** (0.021)	-0.196** (0.044)	-0.117** (0.017)
South-East Asia	Medium	2	-0.183** (0.028)	-0.071** (0.031)	-0.231** (0.038)	0.033** (0.010))
South-East Asia	Medium	3	-0.253** (0.035)	-0.074** (0.023)	-0.374** (0.045)	-0.059** (0.027)
South-East Asia	High	1	-0.112** (0.037)	-0.195** (0.011)	-0.178** (0.038)	-0.105** (0.012)
South-East Asia	High	2	-0.095** (0.024)	-0.299** (0.014)	-0.065** (0.053)	-0.281** (0.034)
Std Error			0.207** (0.009)		0.238** (0.011)	

This table reports the effects of the covariates and the group premiums on sellers' bid distribution and participation decisions for the specification 2 presented in the paper. Columns I(T), II(T), and I(P), II(P) report estimated coefficients for the bid distribution and probability of participation of the transitory and permanent sellers respectively. The stars, ∗∗, indicate that a coefficient is significant at the 95% significance level.

Figure 1: Assumption on the Distribution of Bids



This figure verifies suitability of the parametric assumption about the distribution of bids used in estimation. In order to construct this plot we purge the bids of permanent sellers of the observable variation (sellers' country groups, reputation score group, quality group, the number of potential sellers by group). Then we construct a nonparametric density of the residuals from this regression. We plot the density of bid residuals jointly with the distribution of normal density with appropriate standard deviation and zero mean.

Table 6: Participation Decision and Bid Distributions: Competitive Effects (Baseline Model)

	Score	Q	I(T)	II(T)	I(P)	II(P)
Mean						
North America	low	1	0.021 (0.017)	0.021 (0.013)	0.003 (0.002)	0.011** (0.005)
North America	low	2	0.011 (0.012)	0.015 (0.012)	-0.005 (0.003)	0.011** (0.004)
North America	medium	1	-0.023** (0.011)	-0.089** (0.011)	-0.002 (0.002)	-0.004 (0.004)
North America	medium	2	-0.015* (0.008)	-0.031** (0.015)	-0.004** (0.002)	-0.007* (0.003)
North America	high	1	0.052 (0.031)	0.001 (0.0078)	-0.007** (0.002)	-0.012** (0.003)
North America	high	2	-0.026** (0.012)	-0.085** (0.021)	-0.008** (0.003)	-0.016** (0.004)
Eastern Europe	low	1	-0.005 (0.011)	0.001 (0.007)	-0.001 (0.002)	0.002 (0.005)
Eastern Europe	low	2	-0.007** (0.002)	-0.025** (0.012)	-0.005* (0.0026)	-0.009** (0.0026)
Eastern Europe	medium	1	-0.005 (0.003)	-0.007 (0.004)	-0.003* (0.001)	-0.007** (0.002)
Eastern Europe	medium	2	0.034 (0.031)	0.016 (0.012)	0.002 (0.004)	0.001 (0.003)
Eastern Europe	medium	3	-0.021** (0.011)	-0.016 (0.015)	0.001 (0.002)	-0.004 (0.004)
Eastern Europe	high	1	-0.011 (0.012)	-0.012 (0.017)	-0.002 (0.003)	-0.003 (0.005)
Eastern Europe	high	2	0.008 (0.005)	0.007 (0.004)	0.001 (0.005)	0.003 (0.002)
Eastern Europe	high	3	-0.007 (0.004)	-0.011 (0.019)	0.002 (0.003)	-0.0001 (0.003)
South-East Asia	low	1	0.001 (0.011)	-0.017* (0.003)	-0.002* (0.001)	-0.001 (0.001)
South-East Asia	low	2	-0.003 (0.004)	-0.004 (0.008)	-0.008** (0.003)	-0.019** (0.002)
South-East Asia	low	2	-0.023** (0.011)	-0.026** (0.011)	-0.001 (0.002)	0.001 (0.003)
South-East Asia	medium	1	-0.011 (0.012)	-0.013 (0.011)	-0.004 (0.003)	-0.001 (0.005)
South-East Asia	medium	2	-0.003 (0.011)	-0.002 (0.011)	-0.002 (0.002)	-0.004** (0.001)
South-East Asia	medium	3	0.023 (0.033)	0.015 (0.014)	-0.001 (0.002)	0.007 (0.005)
South-East Asia	high	1	-0.004 (0.003)	-0.007 (0.004)	-0.002* (0.001)	0.001 (0.001)
South-East Asia	high	2	-0.024** (0.012)	-0.039** (0.018)	-0.004* (0.002)	-0.007** (0.003)

This table reports the coefficients summarizing the impact of the various potential competitors on sellers' bid distribution and participation decisions. Columns I(T), II(T), and I(P), II(P) report estimated coefficients for the bid distribution and the probability of participation of transitory and permanent sellers respectively. The results are based on the data set consisting $11,300$ projects. The stars, $**$, indicate that a coefficient is significant at the 95% significance level.

F4. Alternative Specifications

The estimates for the auxiliary objects are available from the authors upon request.

Transitory sellers are an important feature of our setting that potentially introduces methodological challenges. That is why we estimate several specifications that differ in their treatment of transitory sellers. The results of estimation for these specifications are reported below.

Specification one is the most general of out of all the specifications we consider in the paper. It allows the distributions of transitory and permanent sellers' qualities potentially to be different. Under this specification the frequencies of different quality groups in the population of transitory sellers are estimated from the data. The second specification maintains the 'no unobserved heterogeneity' assumption for transitory sellers.

We first compare specifications one to our baseline specification. Tables 7 and 8 report the parameters estimated in the second step of our estimation procedure. Table 7 reports the frequencies for the population of transitory sellers estimated in the second step of the first specification and compares them to the frequency distribution of quality groups in the population of permanent sellers estimated in the first step. The results in Table 7 suggest that the two frequency distributions are very similar, with the transitory sellers' distribution allocating a slightly larger probability mass to the higher quality cells.

Tables 8 and 9 show the estimated parameters of the distribution of buyers' weights. The results for specifications one are reported in column one of these tables. The estimated coefficients are similar to those we obtain under the baseline specification. This specification tends to have somewhat larger standard errors in comparison to baseline specification. It is perhaps not surprising since specification one is substantially more challenging to estimate within the context of our model. Its performance could possibly be strengthened if the model also described the mechanism by which a seller becomes permanent or transitory. We, however, leave investigation of this issue for a separate project.

We perform a further robustness check of our approach with specification two. The estimated coefficients for this specification are reported in column two of Tables 8 and 9. They differ from baseline specification and those for specification one in several important dimensions. First, the estimated variance of ϵ is much higher under this specification. In addition, the estimated quality levels are less dispersed, with high quality levels being substantially lower. In some cases, we estimate quality levels that are not statistically distinct for different quality groups of permanent sellers. These differences reflect an attempt by specification three to rationalize buyers' choices that allocate projects to transitory sellers when permanent sellers with comparable prices are available. Despite this, specification two lags behind baseline specification and specification one in predicting the probability that a project will be allocated to a transitory seller: the predicted probability for specification two is 0.73, whereas baseline specification and specification one get very close to the probability in the data (0.38) with predicted probabilities 0.36 and 0.32 respectively. On the basis of these results we conclude that the assumption of the buyer not being informed about the qualities of transitory sellers does not appear to be consistent with the data.

Table 7: Estimated Quality Distributions of Transitory Sellers

Country Group	Average Score	Permanent Sellers $Q = L$	Permanent Sellers $Q = M$	Permanent Sellers $Q = H$	Transitory Sellers $Q = L$	Transitory Sellers $Q = M$	Transitory Sellers $Q = H$
North America	low	0.33	0.67		0.37**	0.63***	
					(0.19)	(0.23)	
North America	medium	0.31	0.69		0.26	0.74***	
					(0.21)	(0.19)	
North America	high	0.71	0.29		0.65***	0.45***	
					(0.24)	(0.21)	
Eastern Europe	low	0.33	0.67		0.35***	0.65***	
					(0.12)	(0.19)	
Eastern Europe	medium	0.63	0.23	0.13	0.51***	0.28***	0.21***
					(0.04)	(0.11)	(0.05)
Eastern Europe	high	0.07	0.78	0.14	0.12	0.70***	0.17***
					(0.11)	(0.03)	(0.03)
East Asia	low	0.68	0.20	0.12	0.63***	0.24***	0.13
					(0.17)	(0.05)	(0.11)
East Asia	medium	0.09	0.80	0.11	0.12	0.75***	0.13**
					(0.09)	(0.12)	(0.07)
East Asia	high		0.86	0.14		0.78***	0.22***
						(0.21)	(0.04)

This table compares the estimated distribution of transitory sellers' qualities (far right panel) to the distribution of permanent sellers' qualities as implied by the group structure recovered through the classification procedure (see table 4). In this table (***) indicates that the estimated parameter is statistically significant at the 95% significance level.

Table 8: Buyers' Tastes and Quality levels (Alternative Specifications)

Variable	Specification 1	Specification 2
$\log(\sigma_\epsilon)$	-0.732** (0.223)	-0.228* (0.192)
$\log(\sigma_\alpha)$	-1.028** (0.211)	-1.865** (0.033)
μ_{U_0}	-2.213** (0.332)	-1.113** (0.009)
$\log(\sigma_{U_0})$	-0.246* (0.131)	0.136* (0.072)
σ_{α,U_0}	0.149 (0.092)	-0.052 (0.037)

The quality level for South and East Asia, low score, $Q = 1$, is normalized to be equal to zero. The columns in the table show the estimated coefficients and corresponding standard errors for several specifications: specification (1) corresponds to the case when the distribution of transitory sellers' qualities is estimated, whereas specification (2) corresponds to the robustness check where we assume that the buyer is not informed about transitory sellers' qualities and thus treats them as homogeneous conditional on observable characteristics. The stars, $**$, indicate that a coefficient is significant at the 95% significance level.

Table 9: Buyers' Tastes and Quality levels (Alternative Specifications)

Variable	Score Group	Quality Group	Specification 1	Specification 2
Estimated Quality Levels				
North America	low	1	0.062 (0.081)	-0.021** (0.009)
North America	low	2	0.399** (0.065)	0.298** (0.006)
North America	medium	1	0.001 (0.023)	-0.013 (0.008)
North America	medium	2	0.412** (0.057)	0.235** (0.011)
North America	high	1	0.003 (0.032)	-0.019** (0.009)
North America	high	2	0.488** (0.071)	0.261** (0.010)
Eastern Europe	low	1	0.112 (0.102)	0.116** (0.012)
Eastern Europe	low	2	0.703** (0.012)	0.322** (0.002)
Eastern Europe	medium	1	0.111 (0.104)	-0.031** (0.016)
Eastern Europe	medium	2	0.385** (0.031)	0.123** (0.004)
Eastern Europe	medium	3	0.781** (0.035)	0.345** (0.001)
Eastern Europe	high	1	0.001 (0.031)	-0.032** (0.004)
Eastern Europe	high	2	0.289** (0.047)	0.129** (0.005)
Eastern Europe	high	3	0.789** (0.069)	0.351** (0.004)
South and East Asia	low	1	0.000	0.000
South and East Asia	low	2	0.108** (0.043)	0.154** (0.025)
South and East Asia	low	3	0.512** (0.034)	0.178** (0.024)
South and East Asia	medium	1	0.001 (0.101)	-0.001 (0.004)
South and East Asia	medium	2	0.201** (0.059)	0.064** (0.005)
South and East Asia	medium	3	0.535** (0.042)	0.297** (0.003)
South and East Asia	high	1	0.067 (0.041)	0.068** (0.002)
South and East Asia	high	2	0.586** (0.012)	0.301** (0.005)
Pr(transitory seller wins)			0.41	0.23

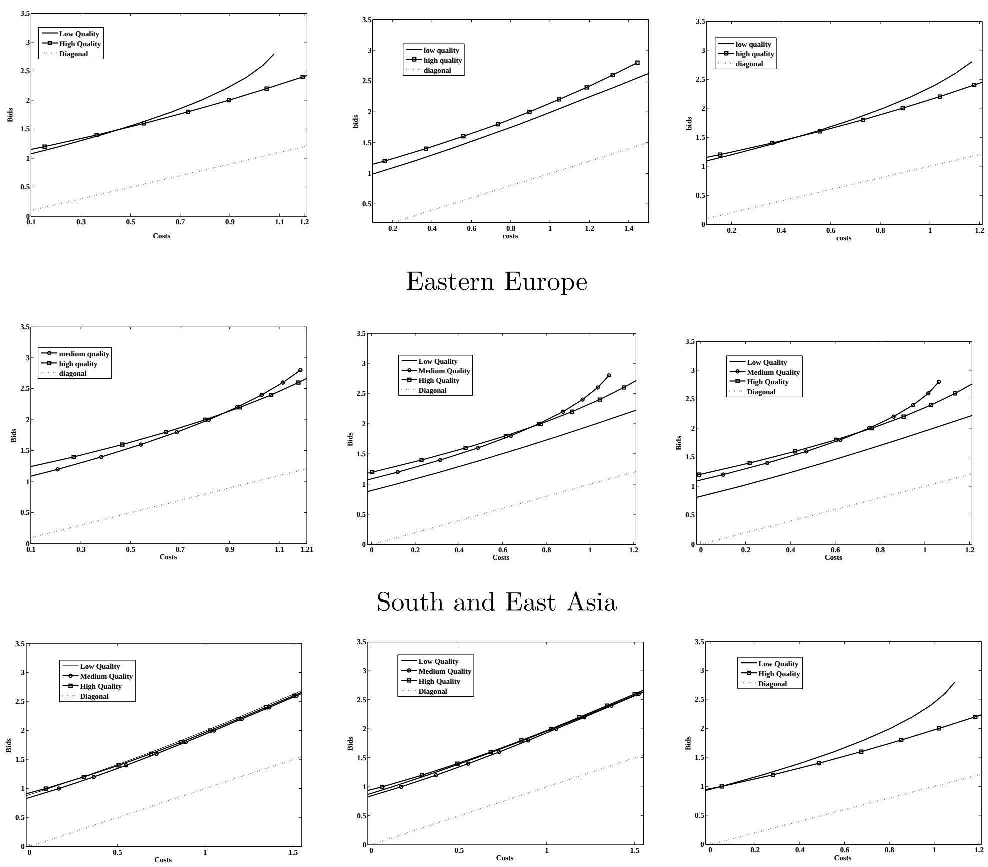
The quality level for South and East Asia, low score, $Q = 1$, is normalized to be equal to zero. Specification (1) corresponds to the case when the distribution of transitory sellers' qualities is estimated, whereas specification (2) corresponds to the robustness check where we assume that the buyer is not informed about transitory sellers' qualities and thus treats them as homogeneous conditional on observable characteristics.

F5. Bidding Functions and the Distribution of Sellers' Costs

Figure 2: Bid Functions

Low Score | Medium Score | High Score

North America / Eastern Europe / South and East Asia



The figure shows the equilibrium bidding strategies of permanent sellers recovered from the first order conditions of bidders' optimization program. The convexity at the upper end of the costs' support arises due to presence of stochastic component in buyers' tastes.

Table 10: Project Costs Distributions (Permanent Sellers)

Country	Score	Quality	Mean		Standard Deviation	
Group	Group	Group	Estimate	Std.Error	Estimate	Std.Error
North America	Low	1	1.239	(0.013)	0.053	(0.006)
North America	Low	2	1.275	(0.012)	0.087	(0.005)
North America	Medium	1	1.627	(0.012)	0.039	(0.005)
North America	Medium	2	1.551	(0.010)	0.073	(0.004)
North America	High	1	1.165	(0.011)	0.053	(0.006)
North America	High	2	1.535	(0.013)	0.099	(0.004)
Eastern Europe	Low	1	1.169	(0.011)	0.037	(0.005)
Eastern Europe	Low	2	1.328	(0.010)	0.079	(0.004)
Eastern Europe	Medium	1	1.576	(0.012)	0.119	(0.005)
Eastern Europe	Medium	2	1.202	(0.009)	0.062	(0.002)
Eastern Europe	Medium	3	1.331	(0.009)	0.101	(0.003)
Eastern Europe	High	1	1.575	(0.011)	0.119	(0.004)
Eastern Europe	High	2	0.981	(0.008)	0.048	(0.002)
Eastern Europe	High	3	1.354	(0.009)	0.089	(0.002)
South and East Asia	Low	1	1.621	(0.010)	0.056	(0.005)
South and East Asia	Low	2	1.119	(0.011)	0.051	(0.002)
South and East Asia	Low	3	1.317	(0.011)	0.169	(0.004)
South and East Asia	Medium	1	1.609	(0.008)	0.107	(0.002)
South and East Asia	Medium	2	1.289	(0.009)	0.121	(0.003)
South and East Asia	Medium	3	1.255	(0.009)	0.124	(0.003)
South and East Asia	High	1	1.074	(0.008)	0.036	(0.002)
South and East Asia	High	2	1.235	(0.008)	0.047	(0.002)

This table summarizes the means and standard errors of the estimated distributions of permanent sellers' project costs.

Table 11: Entry Cost Distribution (Permanent Sellers)

Country Group	Score Group	Quality Group	Mean Estimate	Mean Std.Error	Standard Deviation Estimate	Standard Deviation Std.Error
North America	Low	1	0.081	(0.035)	0.054	(0.022)
North America	Low	2	0.074	(0.041)	0.055	(0.024)
North America	Medium	1	0.072	(0.045)	0.050	(0.021)
North America	Medium	2	0.073	(0.033)	0.057	(0.029)
North America	High	1	0.083	(0.029)	0.058	(0.029)
North America	High	2	0.082	(0.025)	0.059	(0.021)
Eastern Europe	Low	1	0.112	(0.033)	0.072	(0.034)
Eastern Europe	Low	2	0.113	(0.038)	0.067	(0.029)
Eastern Europe	Medium	1	0.103	(0.034)	0.068	(0.031)
Eastern Europe	Medium	2	0.104	(0.021)	0.065	(0.029)
Eastern Europe	Medium	3	0.101	(0.027)	0.064	(0.023)
Eastern Europe	High	1	0.104	(0.021)	0.075	(0.034)
Eastern Europe	High	2	0.103	(0.013)	0.073	(0.032)
Eastern Europe	High	3	0.105	(0.023)	0.076	(0.034)
South and East Asia	Low	1	0.107	(0.043)	0.067	(0.033)
South and East Asia	Low	2	0.118	(0.026)	0.065	(0.036)
South and East Asia	Low	3	0.117	(0.034)	0.063	(0.033)
South and East Asia	Medium	1	0.091	(0.025)	0.059	(0.023)
South and East Asia	Medium	2	0.096	(0.023)	0.061	(0.029)
South and East Asia	Medium	3	0.097	(0.017)	0.060	(0.029)
South and East Asia	High	1	0.104	(0.013)	0.064	(0.033)
South and East Asia	High	2	0.112	(0.021)	0.062	(0.031)

This table summarizes the means and standard errors of the estimated distributions of permanent sellers' entry costs.

F6. Participation and Impact of International Trade

In this section we investigate the social cost associated with restrictions on the international trade while emphasizing adjustment in the sellers' participation choices. Specifically, we contrast the market outcomes arising in the auction environment with those that would realize if allocation decisions were implemented by a social planner.[7] We consider two scenarios: (a) the unrestricted match of a buyer to a seller where a social planner has full information about potential bidders' costs and qualities as well as about the realization of the buyer's utility coefficients; (b) an allocation mechanism where a social planner has to pay entry fee in order to learn the sellers' costs but he is fully informed about the sellers' qualities and the realization of the buyer's utility coefficients.

Let us begin by describing the details of these experiments. In experiment (a) a social planner

[7] We are grateful to one of the referees for suggesting this exercise.

chooses the best match for a given buyer among the set of potential bidders associated with his project.[8] In experiment (b) the social planner observes the entry costs of the potential bidders at the time when he decides for which sellers the entry costs should be paid in order to uncover their project costs; after that he observes the project costs of the chosen "entrants".

As in the auction analysis we assume that all potential bidders are permanent and hold the total number of potential bidders fixed at the level observed in the data. Again, the variety of the sellers available to the buyers is restricted by replacing the quality levels and the cost distributions of the foreign sellers with those of the US sellers conditional on the average score group and quality rank. The medium quality sellers are relabeled as either low or high quality so that the original ratio between the sizes of these groups remains constant.

Since the social planner observes the buyer's preferences (α_l and $\epsilon_{j,l}$) and because the composition of the set of potential bidders varies across projects, all seller groups receive a non-zero share of projects. In the experiment (a), we measure social welfare delivered by matching seller j to buyer l as

$$w(q_j, C_{j,l}; \alpha_l, \epsilon_l) = \tilde{u}_{j,l} + (B_{j,l} - C_{j,l}),$$

where $\tilde{u}_{j,l} = -\mu_{U_0} + \alpha_l q_j - B_{j,l} + \epsilon_{j,l}$. Here $\tilde{u}_{j,l}$ reflects buyer l utility from seller j relative to the mean of the outside option. The social planner chooses a seller j_0 from the set of potential bidders associated with project l, N_l, so that social welfare for a given project is maximized: $j_0 = \arg\max_{j \in N_l} w(q_j, C_{j,l}; \alpha_l, \epsilon_l)$. Then, we define

$$W(N_l, \bar{C}_l; \alpha_l, \epsilon_l) = \tilde{u}_{j_0,l} + (B_{j_0,l} - C_{j_0,l}),$$

where $\bar{C}_l$ denoted the vector of project costs of the sellers in N_l. The average per project welfare in this market is given by

$$\bar{W} = \sum_{N_l} \int_{\bar{C}} \int_{\alpha,\epsilon} W(N_l, \bar{C}_l; \alpha_l, \epsilon_l) dF_{\alpha,\epsilon}(\alpha_l, \epsilon_l) dF_C(\bar{C}_l) \Pr(N_l).$$

In the experiment (b), the social welfare for project l is modified as follows

$$W(A_l, \bar{C}_{A_l}, \bar{E}_{N_l}; \alpha_l, \epsilon_l) = \tilde{u}_{j_0,l} + (B_{j_0,l} - C_{j_0,l}) - \sum_{j \in A_l} E_{j,l},$$

where A_l is a subset of potential bidders for whom the social planner chooses to pay the entry fee in order to learn their project cost realizations; $\{E_{j,l}\}_{j \in A_l}$ is the vector of entry costs for this

[8] We do not restrict the set of potential sellers in any way except to allow it to be sufficiently large. Such approach is without a loss of generality. Indeed, in the data the number of sellers who are allocated to the projects in any given week is only a small subset of the sellers who submit messages to the buyers (potential bidders). Therefore, a surplus of potential bidders exists which, theoretically, is available for the social planner to draw on in order to decide on the allocation for a given project. Further, in the analysis with the entry costs only the number of potential bidders selected to draw project costs but not their fraction is important.

Table 12: Further Analysis on the Impact of International Trade

	All Groups	US Sellers Only	Low and High Quality Only
Social planner: Free match			
Total Surplus	1.190	1.179	1.186
Share of outside option	0.003	0.003	0.003
Number of sellers	20	20	20
Social planner: Costly Participation			
Total Surplus before Entry Costs	1.063	1.048	1.036
Social Welfare net of Entry Costs	0.859	0.872	0.858
Share of outside option	0.005	0.006	0.006
Number of sellers	8.88	8	9.2
Composition of sellers	high quality (0.88,5,3)	high quality (8,0,0)	high quality (0,4,4.2)
Auction			
Buyer Surplus	0.746	0.565	0.771
Social Welfare net of Entry Costs	0.777	0.605	0.798
Share of outside option	0.043	0.148	0.040
Number of sellers	3.813	2.151	4.104

This table reports the results of a counterfactual analysis investigating the allocation decisions in this market from social planner's point of view. The analysis emphasizes the importance of participation effects which here are interpreted as the number and composition of the sellers that the social planner chooses to consider. The results for the auction allocations are reported for comparison in the bottom panel of the table. The first column presents the results for the baseline setting where all quality groups are present. The second column presents the outcomes from the setting where foreign potential bidders are replaced by the US potential bidders while preserving the sellers' quality ranks (medium-quality is replaced by high- and low-quality preserving original shares of these quality levels in population). The last column is included for comparison purposes. It reports the results for the intermediate step where medium-quality potential bidders are replaced by high- and medium-quality potential bidders without changing the seller's country of origin. The composition of the sellers shown in the brackets refers to the average number of the sellers by the country of origin listing the US first, then Eastern Europe and South and East Asia last. Buyer and social surpluses are measured relative to the expected value of the outside option.

subset of potential bidders; j_0 is defined as before ($j_0 = \arg\max_{j \in N_l} w(q_j, C_{j,l}; \alpha_l, \epsilon_l)$); and A_l is chosen to maximize the expected welfare for this project with the expectation taken with respect to the distribution of the project costs for the sellers in A_l:

$$A_l = \arg\max_{A \subset N_l} \int_{\bar{C}_A} W(A, \bar{C}_A; \alpha_l, \epsilon_l) dF_C(\bar{C}_A).$$

Notice that in both experiments there is no need to compute prices because they drop out from the welfare expression.

Table 12 summarizes the results of this analysis. As can be easily seen, the unrestricted social planner achieves the highest social surplus before the entry costs. The social planner subject to the entry costs achieves the second highest level of surplus whereas the auction setting delivers the lowest welfare. In the first case the social welfare is high because the planner has an opportunity to choose among a large number of sellers. His decision thus reflects the optimum based on a large number of random draws. In case (b) and in the auction environment, the set of alternatives available to the buyer is substantially reduced since adding an alternative to the choice set is costly. The social planner in case (b) performs better than the auction mechanism because he is able to fully internalize the benefit to the buyer's welfare from a given cost draw. In the auction environment this social benefit is ignored since sellers base their participation decisions only on private profitability. Additionally, prices (and profitability) which endogenously decline in the number of participants further limit entry in the auction environment. Thus, the auction setting is characterized by insufficient entry from the social point of view.

Internalizing the benefits from additional participation helps the social planner to limit the losses associated with the reduction in the variety of seller types. Indeed, under scenario (a), the loss is minimal (1.1%) since the ability to optimize over the cost draws within quality group compensates for the less desirable group-level characteristics of the available seller groups. Under counterfactual scenario (b), the social planner is actually able to achieve higher social surplus after entry costs under reduced variety relative to the baseline case (the surplus before the entry cost is higher in the baseline case). This outcome arises because the US sellers have lower entry costs than the sellers from other countries. As a result under scenario (b) the welfare is improved by 1.3% in contrast to 32% reduction which is realized in the auction market.

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