Supplemental On-line Appendix for
"The Dynamic Auction Environment with Subcontracting"

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Supplemental on-line Appendix consists of three parts. In the first part we present the analysis of equilibrium outcomes' sensitivity to the restrictiveness of reserve price imposed by the government. The second part studies the sensitivity of contractors' strategies and of equilibrium outcomes to the availability of subcontracting opportunities as summarized by the flatness of the subcontracting supply schedule. The third part of an on-line Appendix contains the results of the analysis exploring the effects of the policies which regulate access to subcontracting: (a) imposing upper limit on the fraction of the work that can be subcontracted; (b) requiring that a certain amount of work should be subcontracted to so-called disadvantaged businesses.

Robustness Analysis: Reserve Price

We study sensitivity of equilibrium outcomes to the restrictiveness of reserve price by simulating the equilibrium of the games with and without subcontracting under the distributions of reserve price with the means that are 5%, 10% and 15% higher (and lower) than the mean of the distribution of reserve price calibrated from the data. The results are reported in tables 1 and 2 below.

Table 1: Summary of Equilibrium Variables

Reserve Price ($\Delta\mu_R$)	Subcont.	Allocated Projects	Number of Bidders	Firm's Profit ($ mln)	Procur. Cost ($ mln)	Work Share	Backlog ($ mln)	Difference in Backlog ($ mln)
-15%	Yes	0.43	1.1	1.36	4.41	0.82	2.11	2.17
	No	0.37	1.03	1.56	4.72	1	2.5	2.47
	$\Delta_{NS,S}$	-14.0%	-6.4%	14.7%	7.0%	22.0%	18.5%	13.8%
-10%	Yes	0.45	1.13	1.38	4.52	0.8	2.16	2.18
	No	0.38	1.04	1.63	4.93	1	2.68	2.53
	$\Delta_{NS,S}$	-15.6%	-8.0%	18.1%	9.1%	25.0%	24.1%	16.1%
-5%	Yes	0.48	1.16	1.42	4.65	0.77	2.21	2.2
	No	0.39	1.05	1.68	5.14	1	2.86	2.56
	$\Delta_{NS,S}$	-18.8%	-9.5%	18.3%	10.5%	29.9%	29.4%	16.4%
+0%	Yes	0.51	1.19	1.44	4.76	0.73	2.21	2.16
	No	0.41	1.07	1.76	5.35	1	3.06	2.64
	$\Delta_{NS,S}$	-19.6%	-10.1%	22.2%	12.4%	37.0%	38.5%	22.2%

Note: See the comment under table 2.

Table 2: Summary of Equilibrium Variables (continued)

Reserve Price ($\Delta\mu_R$)	Subcont.	Allocated Projects	Number of Bidders	Firm's Profit ($ mln)	Procur. Cost ($ mln)	Work Share	Backlog ($ mln)	Difference in Backlog ($ mln)
+0%	Yes	0.51	1.19	1.44	4.76	0.73	2.21	2.16
	No	0.41	1.07	1.76	5.35	1	3.06	2.64
	$\Delta_{NS,S}$	-19.6%	-10.1%	22.2%	12.4%	37.0%	38.5%	22.2%
+5%	Yes	0.53	1.21	1.47	4.87	0.71	2.18	2.16
	No	0.42	1.11	1.8	5.54	1	3.22	2.7
	$\Delta_{NS,S}$	-20.8%	-8.3%	22.5%	13.7%	40.9%	47.7%	25.0%
+10%	Yes	0.56	1.24	1.5	4.97	0.67	2.17	2.13
	No	0.43	1.13	1.86	5.75	1	3.45	2.78
	$\Delta_{NS,S}$	-23.2%	-8.9%	24.0%	15.7%	49.3%	59.0%	30.5%
+15%	Yes	0.59	1.27	1.51	5.07	0.65	2.16	2.1
	No	0.45	1.14	1.89	5.94	1	3.63	2.8
	$\Delta_{NS,S}$	-23.7%	-10.2%	25.2%	17.2%	53.9%	68.1%	33.3%

Note: This table reports the equilibrium outcomes under alternative distributions of the reserve price. The third column reports the fraction of projects allocated in equilibrium; the fourth column reports the expected number of bidders which realizes in individual auction; the fifth, sixth and seventh columns report for allocated projects the firm's profits, procurement costs and the expected fraction of work performed in house respectively.

Robustness Analysis: Subcontracting Schedule

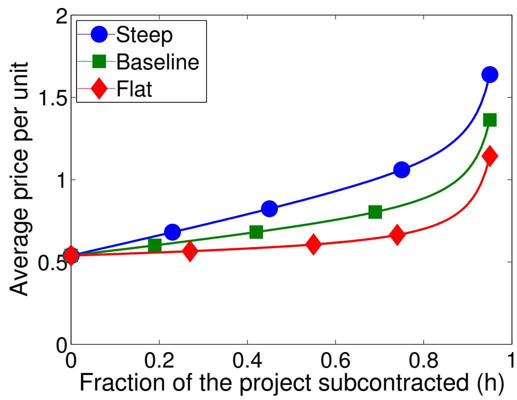
We consider two schedules in addition to the one obtained in calibration exercise (baseline schedule): a schedule which is steeper and a schedule which is flatter than the baseline schedule. The parameters for these alternative supply functions are given in table 3 and they are shown jointly with the baseline subcontracting schedule in the Figure 1. We find that all the properties we documented in the paper are preserved under these permutations. The magnitudes of the effects are generally increasing in the flatness of the subcontracting supply curves.

Table 3: Parameters of Subcontracting Schedules

	Subcontracting schedule		
	Steep	Baseline	Flat
Intercept (γ_1)	0.65	0.65	0.65
Linear part (γ_2)	0.08	0.04	0.01
Hyperbolic part (γ_3)	0.04	0.04	0.04

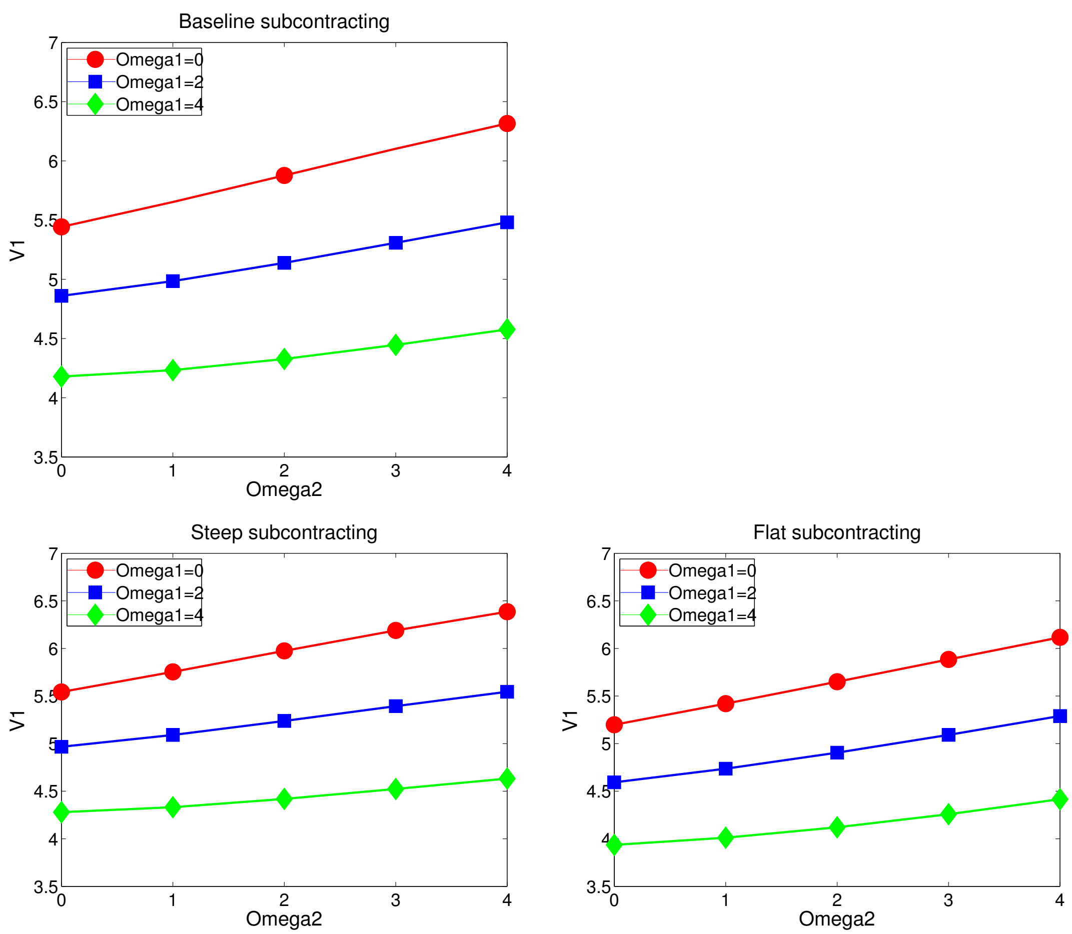
The simulations are based on the subcontracting supply schedules given by equation $P(hx) = \gamma_1 + \gamma_2 hx + \gamma_3 \frac{hx}{x-hx}$. The various parameter combinations used in the paper are summarized in the table above.

Figure 1: Subcontracting Price Schedules



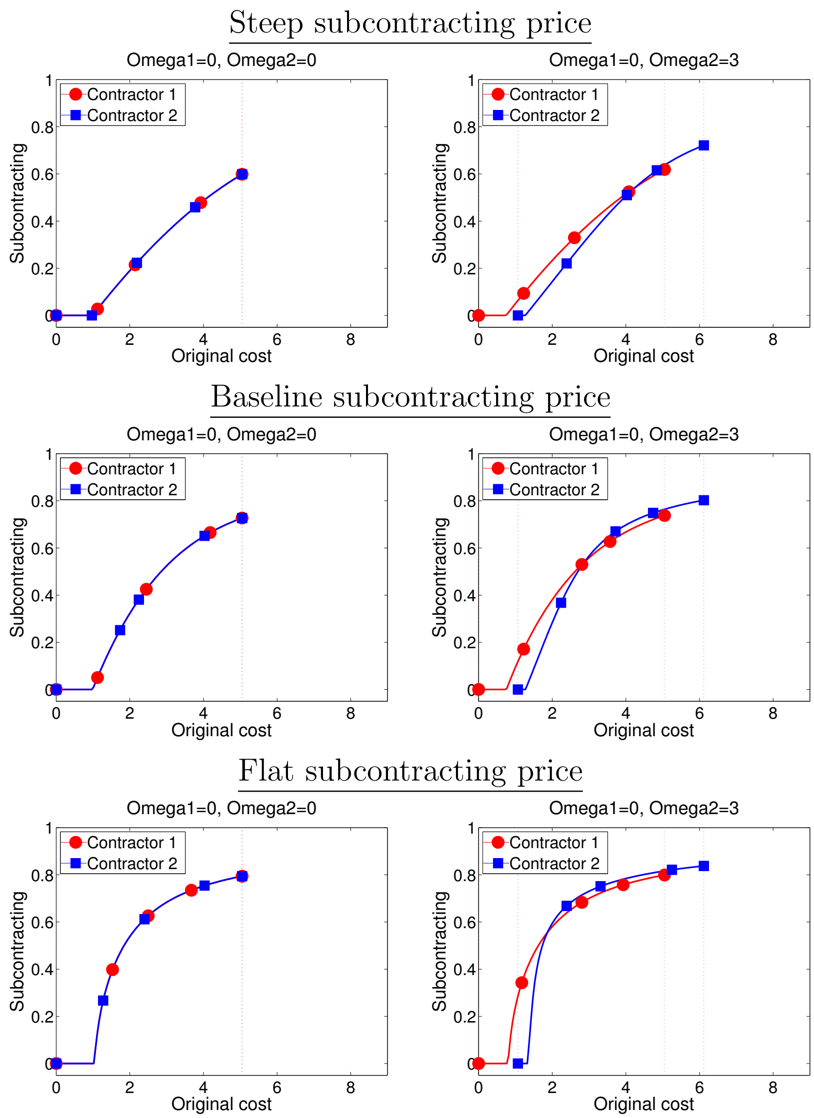
This figure shows various subcontracting supply functions we use in simulations. It plots per unit price of subcontracting services (vertical axis) versus the subcontracted amount (horizontal axis).

Figure 2: Value Function



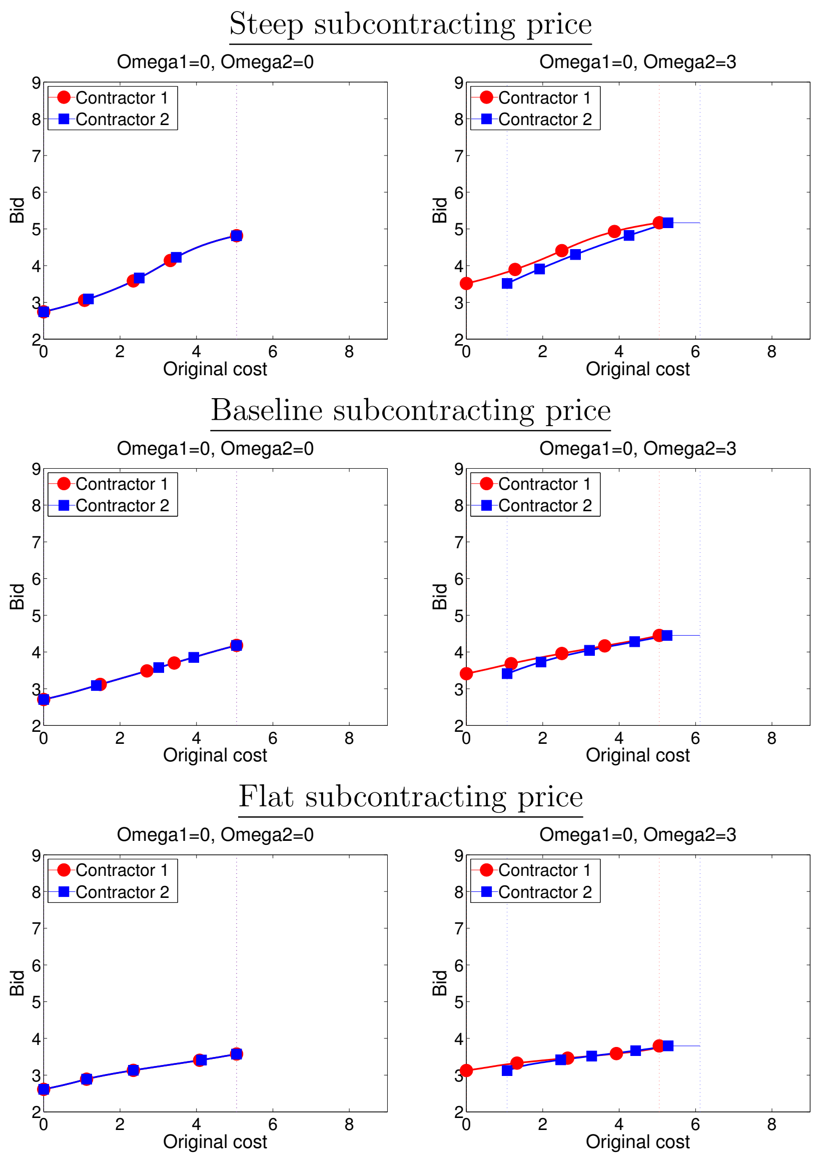
This figure shows sections of value function that correspond to different levels of own backlog. It graphs value function against the level of competitor's backlog.

Figure 3: Subcontracting Strategy



This figure shows subcontracting strategy under various backlog configurations and for different subcontracting supply schedules.

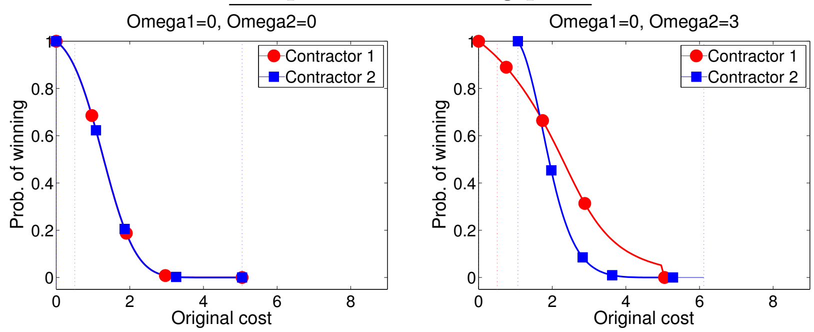
Figure 4: Bidding Strategies



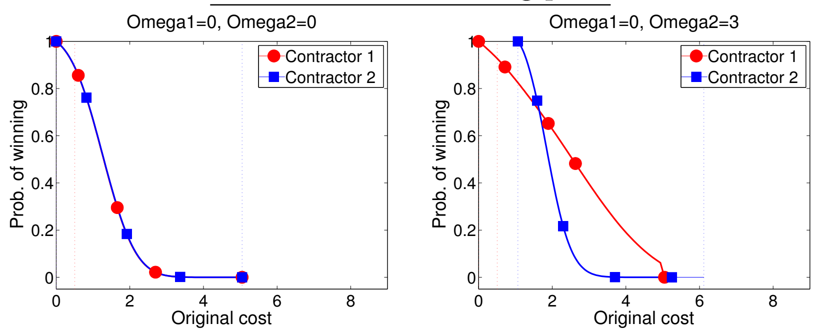
This figure shows bid functions under various backlog configurations and for different subcontracting supply schedules.

Figure 5: Probability of winning

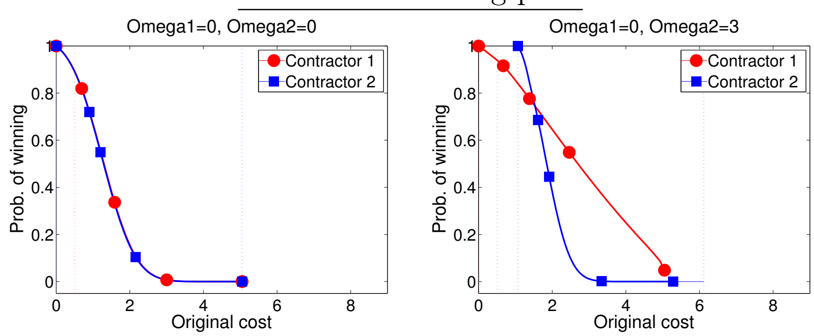
Steep subcontracting price



Baseline subcontracting price



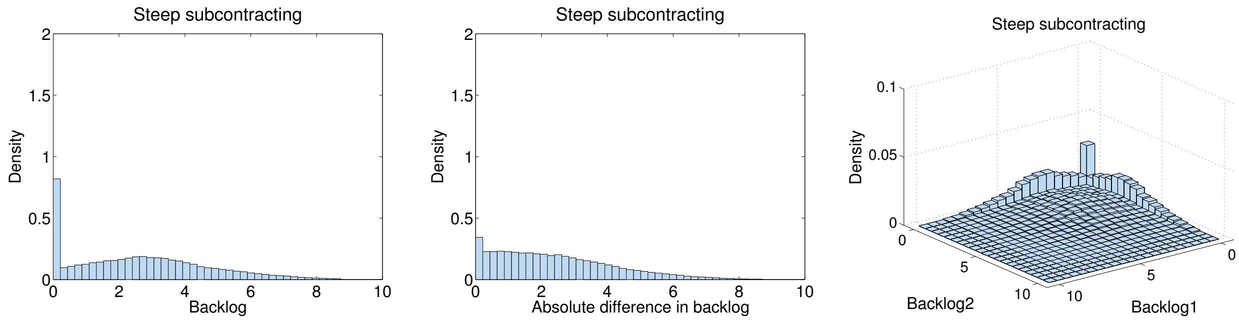
Flat subcontracting price



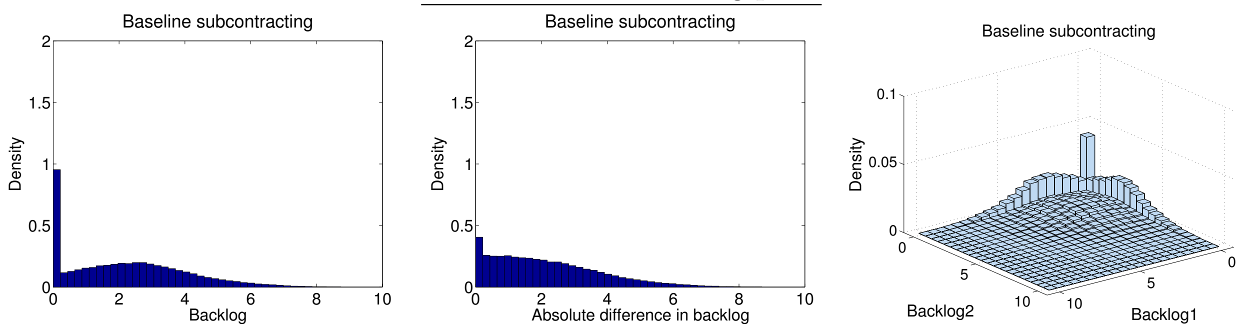
This figure shows probability of winning under various backlog configurations and for different subcontracting supply schedules.

Figure 6: Stationary Distribution of Backlogs

Steep subcontracting price



Baseline subcontracting price



Flat subcontracting price

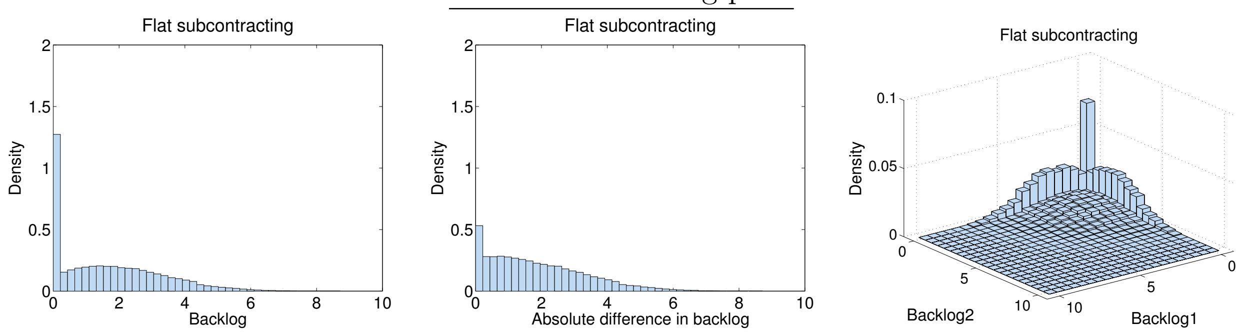


Table 4: Expected Bids

Subcontracting price schedule	$\omega_1 = 0$ $\omega_2 = 0$	$\omega_1 = 0.5$ $\omega_2 = 0$	$\omega_1 = 1$ $\omega_2 = 0$	$\omega_1 = 0$ $\omega_2 = 0.5$	$\omega_1 = 0$ $\omega_2 = 1$
Steep	0.34	0.37	0.39	0.36	0.37
Baseline	0.33	0.35	0.37	0.35	0.36
Flat	0.32	0.34	0.35	0.33	0.34

This table reports expected bid levels conditional on the state and for different subcontracting supply functions.

Table 5: Expected Subcontracting Levels

Subcontracting price schedule	$\omega_1 = 0$ $\omega_2 = 0$	$\omega_1 = 0.5$ $\omega_2 = 0$	$\omega_1 = 1$ $\omega_2 = 0$	$\omega_1 = 0$ $\omega_2 = 0.5$	$\omega_1 = 0$ $\omega_2 = 1$
Steep	0.06	0.11	0.16	0.06	0.07
Baseline	0.09	0.17	0.25	0.10	0.10
Flat	0.15	0.26	0.36	0.15	0.16

This table reports expected subcontracting levels conditional on the state and for different subcontracting supply functions.

Table 6: Expected Probability of Winning

Subcontracting price schedule	$\omega_1 = 0$ $\omega_2 = 0$	$\omega_1 = 0.5$ $\omega_2 = 0$	$\omega_1 = 1$ $\omega_2 = 0$	$\omega_1 = 0$ $\omega_2 = 0.5$	$\omega_1 = 0$ $\omega_2 = 1$
Steep	0.50	0.43	0.36	0.57	0.64
Baseline	0.50	0.42	0.36	0.58	0.64
Flat	0.50	0.42	0.35	0.58	0.65

This table reports expected probability of winning the contract conditional on the state and for different subcontracting supply functions.

Table 7: Moments of the Distributions of Private Costs

Steeper subcontracting price								
ω_1	$E(c\bar{x})$	std$(c\bar{x})$	$E(\phi(c))$	std$(\phi(c))$	$E(\phi(c)_S)$	std$(\phi(c)_S)$	$E(\phi(c)_D)$	std$(\phi(c)_D)$
0.00	1.30	0.64	2.39	0.59	1.38	0.70	1.016	0.125
0.50	1.60	0.79	2.79	0.67	1.68	0.84	1.103	0.169
1.00	1.90	0.91	3.13	0.72	1.99	0.92	1.137	0.203
Baseline subcontracting price								
ω_1	$E(c\bar{x})$	std$(c\bar{x})$	$E(\phi(c))$	std$(\phi(c))$	$E(\phi(c)_S)$	std$(\phi(c)_S)$	$E(\phi(c)_D)$	std$(\phi(c)_D)$
0.00	1.31	0.64	2.29	0.54	1.44	0.73	0.856	0.200
0.50	1.60	0.78	2.65	0.59	1.74	0.84	0.910	0.256
1.00	1.89	0.89	2.94	0.61	2.02	0.87	0.919	0.296
Flat subcontracting price								
ω_1	$E(c\bar{x})$	std$(c\bar{x})$	$E(\phi(c))$	std$(\phi(c))$	$E(\phi(c)_S)$	std$(\phi(c)_S)$	$E(\phi(c)_D)$	std$(\phi(c)_D)$
0.00	1.32	0.64	2.20	0.47	1.51	0.75	0.692	0.299
0.50	1.58	0.78	2.48	0.47	1.78	0.80	0.705	0.342
1.00	1.88	0.90	2.72	0.46	2.04	0.80	0.683	0.359

The table summarizes variation in private costs of contractor 1 for three configurations of backlogs, keeping the backlog of the opponent at 0. The second and third columns show the mean and the standard deviation of the original private costs (i.e. costs before subcontracting is taken into account) whereas the fourth and the fifth columns present the mean and the standard deviation of the static part of the effective private costs. Finally, the sixth and the seventh columns contain the mean and the standard deviation of the full effective private costs (i.e. costs after subcontracting is taken into account that also include the continuation value of winning).

Table 8: Summary of Equilibrium Variables

	Allocated projects	Expected number of bidders	Conditional on allocation			Backlog	Difference in backlog
			Firm's profit	Procurement cost	Work done		
Steep	0.46	1.15	\$1.57M	\$5.04M	0.83	\$2.63M	\$2.42M
	-8.7%	-3.0%	8.6%	5.8%	13.3%	19.2%	12.0%
Baseline	0.51	1.19	\$1.44M	\$4.76M	0.73	\$2.21M	\$2.16M
Flat	0.56	1.24	\$1.29M	\$4.38M	0.62	\$1.67M	\$1.88M
	11.0%	4.1%	-10.5%	-8.1%	-15.6%	-24.5%	-13.0%

This table reports the average values of several variables in the stationary equilibria of the environments with and without subcontracting and for calibrated parameter values.

Policies Restricting Access to Subcontracting

Subcontracting is widely prevalent in the markets for infrastructure construction and maintenance. However, in many cases, as for example in the markets for government procurement, the auctioneer chooses to regulate the primary contractor's decision to subcontract. Often this takes a form of limiting the amount of work that may be subcontracted. In government procurement it is usually not permitted to subcontract more than 40% of the project size. This rule is motivated by concerns about contractors' ability to supervise the work and thus guarantee the quality if a large number of tasks are performed by other firms. Alternatively, many governments require that a fixed fraction of work should be subcontracted to disadvantaged businesses. Such policy intends to support disadvantaged enterprises through their integration into the market place (i.e. it mitigates entry barriers that are allerged to exist for such businesses) and aims to ultimately promote greater competitiveness in the market place. Heuristically the benefits of the described subcontracting restrictions are clear and we do not attempt to measure them empirically in this paper. Rather, we focus on the costs in terms of the procurement expense which arise if contractor is restricted to use a suboptimal subcontracting strategy. We also abstract from the question of whether the disadvantaged businesses are less efficient relative to regular firms. Instead we focus on the effect of imposing lower bound on subcontracting as policies supporting such enterprises effectively do.

Imposing Upper Limit on Subcontracting In this analysis we compute an equilibrium of the dynamic game with subcontracting under the three regimes: (a) no restrictions on subcontracting strategies, (b) restricting the subcontracted fraction of the project to be below 60% of the project (currently prevailing policy), (c) restricting the subcontracted fraction of the project to be below 40% of the project. We find that results for the first policy are similar to those reported in the paper while comparing the equilibria with and without subcontracting. Generally, the cost of procurement for an individual project increases as the policy becomes more restrictive, and the fraction of projects allocated and completed in the equilibrium decreases.

Requiring a Subcontracting Minimum In this analysis we compute an equilibrium of the dynamic game under the following three scenarios: (a) no restriction on subcontracting strategies; (b) requiring 10% minimum, (c) requiring 15% minimum. We find that this policy actually works to decrease the procurement cost for an individual project since subcontracting facilitates symmetrization and thus intensifies competition. The competitive effect dominates the increase in production costs associated with sub-optimal subcontracting levels. This effect, however, is not very large since the policy usually requires only a small increase in the low bound on the fraction of project subcontracted.

Table 9: Policies Restricting Access to Subcontracting

	Allocated projects	Firm's expected profit	Conditional on allocation: Firm's profit	Procurement cost	Work done	Backlog	Difference in backlog
Baseline	0.51	$0.55M	$1.44M	$4.76M	0.73	$2.21M	$2.16M
Aff. action 10%	0.51	$0.54M	$1.41M	$4.74M	0.72	$2.21M	$2.17M
	1.1%	-1.5%	-2.1%	-0.5%	-2.1%	0.2%	0.4%
Aff. action 15%	0.52	$0.52M	$1.38M	$4.69M	0.71	$2.13M	$2.12M
	1.9%	-5.2%	-4.4%	-1.5%	-3.0%	-3.8%	-1.9%
Sub. cap 60%	0.51	$0.55M	$1.45M	$4.81M	0.73	$2.21M	$2.15M
	-0.3%	0.3%	0.4%	1.0%	-0.4%	-0.1%	-0.7%
Sub. cap 40%	0.49	$0.58M	$1.54M	$4.92M	0.76	$2.32M	$2.31M
	-3.7%	4.6%	6.9%	3.4%	4.1%	4.8%	6.9%