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ROY RADNER (1927-2022): PREFACE TO A MEMORIAL ISSUE

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> Roy Radner has been elected a Distinguished Fellow of the American Economic Association. I shall give sufficient evidence for the conclusion that in this election the Association honors itself as much as it honors Radner. Readers who are not familiar with Radner's bibliography should understand that they have missed much that is both beautiful and important. Hahn (1992)[1]
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> I have chosen to illustrate the impact of Herbert Simon's work on economics with an example, in which two different threads of Simon's work eventually combined in a surprising, and I think fruitful, way. The choice of this example is perhaps idiosyncratic, but it has a significant personal basis for me. Radner (1979)[2]

In this memorial issue, the editors are pleased to present not one example, but ten exemplary testimonials in alphabetical order to honor, at least in part, his memory and his achievement from the limited vantage point of our time – what the future impact of this body of work will have is for future generations to discover. Before furnishing a guide for the reader to this volume, a guide especially for the rare set totally unfamiliar with his work, we keep as markers Hahn's tribute in the *Journal of Economic Perspectives,* and Radner's own tributes to Herbert Simon and Jacob Marschak.[3]

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[1]This is the second sentence of Hahn's tribute – he repeats the sentiment as his last sentence, "But perhaps I have done enough to substantiate my initial claim that an Association which honors Radner also honors itself." It is unfortunate that Sveriges Riksbank did not honor itself in this way; see [6].

[2]See Radner's 1979 lunch talk published as item [106]; his example is Simon's 1951 paper titled "A Formal Theory of the Employment Relation," a paper he returns to in several subsequent publications; see in particular item [62]. These items refer to the *curriculum vitae* dated 28 May, 2015 and reproduced as Attachment A at the end of this preface; a B before a numbered item in the sequel refers to the first part of the publications that lists books and monographs. The authors have left Attachment A as is available on the internet as a mark of respect and of historical interest. References to Radner's papers not in this bibliography are cited under his name and date as Attachment B; references to the work of other authors is cited in alphabetical order and numbered.

[3]For the influence of Marschak, see item [54] incorporated in his preface to the second edition in item [B3]. The fact that it is not jointly authored is evident in the use of the first person in the third line of the last paragraph on page xxi. The volume has a bibliography of Marschak, and four

Hahn sections his tribute under five headings: (i) the turnpike, (ii) general equilibrium, (iii) rational expectations, (iv) team theory, (v) private information, incentives and markets, along with an appendix on his turnpike theorem.[4] He cites thirteen papers of Roy Radner's, but stresses how and why his tribute is incomplete.

> In the nature of things, this review could not convey the pleasure to be had from Radner's technical virtuosity. Indeed, [this] may lead to the belief that this is all there is. That would be quite wrong. It has paid no attention to Radner's important empirical work on higher education. Recently, he has also looked at organization of firms again, both empirically and theoretically, for example in his 1989 Marshall Lectures. Not only is he capable of empirical work with the best of them, but he belongs to the very small group of theorists who are very conscious of what has been when he theorizes.[5]

The range of Radner's interests is well-known among the *cognoscenti* but even within this group, there may be surprises. While his interests and contributions to the economics of education and to the economics of climate change, increasingly important in the final part of his career, are well known, how many know of his visceral aversion to war and conflict, his keen commitment to the analysis of corruption and to economic development, and given this interest in planning, how knowledgeable he was of the different literatures on his subject.[6] But with the readers of this journal in mind, we stay in the technical register to the extent that we are able, albeit with Hahn's qualification that even here "the reader has been given crumbs from the table."

The trajectory originates in a 1953 Cowles Discussion Paper on "optimal communication rules for certain types of teams," and is followed by a 1955 publication on a "linear team" that culminates in the 1956 Chicago Ph.D. thesis titled "Team Decision Functions." However, it is in 1961 that Radner catapulted himself to the professional mainstream by figuring out, reportedly in two days, the solution to John Hicks' problem of the "mare's nest." The groundwork for the themes that he was to investigate subsequently were all

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articles by Radner himself. The authors aspire to try and attain the high benchmark of the style of his preface.

[4]In this appendix titled "Radner's turnpike theorem," he returns to an earlier 1964 exposition by Matthews and himself, [20].

[5]Writing thirty years later than Hahn, we shall have occasion to emphasize both themes in the sequel: his respect for antecedent work, and his curiosity and the range of his interests. Also see Footnote 6 below.

[6]For the economics of education, see references 8 and 9 in Hahn's tribute (items [B5, B6, B7], as well as items [6, 22, 37, 41, 42]. For the economics of climate change, see his papers dated 2005, 2014, 2015, 2020 and their references, items [105, 107, 108, 112, 113, 114]. For arms-control, see items [65], [B9] and its reviews in *The Journal of Interdisciplinary History* and in *Political Science Quarterly*. In the context of his interests in planning (of which more below), the reader can see item [21], his 1968 review of Nemchinov's volume, and items [B1, 15, 36] that go to the very beginning of his career.

laid out in this 8-year period: linear programming, Bayesian procedures, decision criteria, reliability theory and incomplete information,[7] even though a good half of his publications date to the three decades after Hahn wrote. Nevertheless, in turning to the placement of the ten papers in this memorial issue in the context of our perception of Radner's *oeuvre,* we begin with the subject headings that Hahn worked with.

Hahn's seven-paragraphed description of Radner's contribution to Samuelson's turnpike conjecture can hardly be bettered. The master stroke was to assume a unique von Neumann ray, to free the argument from the vexatious weeds of the higher-dimensional McKenzie-Morishima facets, and to apply the fundamental insight that efficiency (constrained vector maximization) implies the existence of an accounting price system (shadow prices, Lagrange multipliers, dual variables), costs evaluated at which are minimized, a result that is thereby parlayed as the (second) fundamental theorem of welfare economics. The rest is arithmetic. There is by now a river of work on the turnpike theorem, but this is largely in the context of discrete time.[8] In his authoritative exposition, Zaslavski moves along the Radner rails but in the context of an approximate turnpike for agreeable programs in the case of a specific multisectoral model in continuous time.[9] What is perhaps of greater consequence is that the result he offers is set in the context of the consumption turnpike, Samuelson's reformulation of the conjecture from von Neumann's setting to that of Ramsey.[10] This brings the discount factor squarely to the fold, and thereby forces a connection to the work of David Gale.[11]

It is never clear where economic theory ends and mathematical economics begins, and Radner was not always comfortable with the latter designation. If one had to limit his work to one register of pure mathematics, it would surely be functional analysis and his 1967 generalization of the Arrow-Barankin-Blackwell theorem. Even here however it was in the service of applied work

[7]Hoffman's review MR0075516 of item [2] with which we conclude this preface, already highlights some of these themes that were to emerge in the way they did. Radner's incomplete 2015 CV lists seven papers prior to item [8], his 1961 *Restud* paper, 10 books and monographs, 114 publications and 5 papers in process. The point is that out of 120 or so published papers, a good 55 were written after Hahn wrote.

[8]See [27], [17] and their references. Also see the references furnished in Zaslavski's contribution.

[9]See [21–24] and Theorem 6.2 in Zaslavski's paper. Also see [56, 57] for a comprehensive treatment of the RSS (Robinson-Solow-Srinivasan) model.

[10]It is perhaps worth reminding the reader at this point that Radner never returned to turnpike theory with its own twists and turns: see for example, the *straight-down-the turnpike, twisted* and *periodic* turnpikes of Winter, Keller and Samuelson: the reference to Radner's theorem in [55] is particularly noteworthy. As brought out in [16] and their references, the work of consolidation and synthesis remains to be done, Zaslavski's manifold contributions notwithstanding. However, the theory, as it pertains to the RSS model, can now be seen as fully worked out, see [56, 57].

[11]Also see Footnote 13 below. This emphasis on the discount factor is a recurring theme in Radner's later work on repeated games; see items [52, 56, 57, 58], and it is perhaps not altogether fanciful to connect it to McKenzie's "neighborhood turnpike theorem"; also see Footnote 10 above and Footnote 17 below.

and the synthetic, interdisciplinary thrust is fully evident. He motivates his paper with the following statement.[12]

> The problem of characterizing maximal points of convex sets often arises in the study of admissible statistical decision procedures, of efficient allocation of economic resources, and of mathematical programming.

In the second contribution to this issue that we consider, Becker takes as his point of departure the Koopmans-Kantorovitch results on sustaining technologically efficient production plans as profit-maximizing plans chosen at suitably-determined (shadow) prices in an infinite-commodity context of Debreu, Hurwicz and Malinvaud (DHM). Becker's state-of-the-art exposition is sketched in the setting of a partially ordered topological vector space, and it also connects to "applications in infinite-dimensional general equilibrium theory initiated in the mid-1980's," a theory built on the work of Bewley, Stigum and Peleg-Yaari that advanced the 1953-1954 DHM achievement.

It is of interest that Radner's 1967 theorems on the price-decentralization of inter-temporal efficiency are in the same issue as Gale's pioneering work on development planning and his independent sighting of Kuhn-Tucker-Karush theory of constrained optimization as the relevant benchmark.[13] It is through this channel that Radner's assumption of a unique von-Neumann ray is transmuted into Brock's assumption of a unique golden-rule stock, and his existence theorem for weakly maximal programs in a multi-sectoral undiscounted Ramseyian setting.[14] In their contribution to this issue, Akao-Ishii-Kamihigashi-Nishimura (AIKN) look at existence issues in such non-concave optimality problems in a discounted setting. With non-concavity precluding explicit duality considerations, they offer a primal argument based on the necessity conditions furnished by Pontryagin's maximum principle. Sighting Romer's 1986 analysis of "cake-eating, chattering, and jumps," they are very much in the 1967 Gale-Radner world.

The subtle issues of non-existence brought out by AIKN are only a step away from the issues of economic survival first raised by Koopmans. Radner was very much concerned with circumstances where the state variables are run down to zero and his 1998 Nancy Schwartz lecture (item [93]) was titled

[12]It is worth noting that he had already completed this paper when he turned to his great 1967 paper on efficiency pricing; see [33] for the influence of these two papers in subsequent work – Radner is cited 37 times in this paper.

[13]It is of interest that the two papers make no reference to each other, almost like two grand liners passing each other: both analyses originate in efficiency pricing, though the former is phrased through the theory of constrained optimization, and the latter through the geometric form of the Hahn-Banach theorem. Also see Footnote 11 above.

[14]To fully appreciate the fecundity of this approach, one powered by Radner's value-loss lemma, see [36] in the context of the economics of forestry, and the recent resuscitation of work in development planning by Khan-Mitra through the RSS and RSL models – in addition to the references in Zaslavski's contribution, see [16]. We shall also have occasion to refer to [12] in the context of stochastic growth.

"Economic Survival."[15] In this connection, the Bhattacharya-Majumdar (BM) contribution to this volume "explores the implications of harvesting in the context of extinction or endangerment of a renewable resource." The authors take their cue from the discrete time model presented in Majumdar-Radner (see item [72]) who "obtained similar bounds on initial stocks and harvesting levels for a class of concave regeneration functions satisfying appropriate endpoint properties." The use of a model as a synthetic instrument to connect a variety of economic phenomena is again evident. Majumdar-Radner write:

> While it is easy to indicate the obvious limitations of a dynamic model that fails to come to grips with survival, it is difficult to make a move towards any "general" theory: the economic and ethical implications or "extinction" or "failure" appear to be quite specific to the role or the agent in the economy. It is hard to contemplate a framework that can adequately deal simultaneously with, for example, consumers facing extinction during a catastrophic famine, firms facing extinction in a Schumpeterian world, and banks facing "runs" in a period of liquidity or confidence crisis.

Radner was especially interested in the contrast between the "implications of maximizing the chance of survival with those of expected utility maximization," and it was natural that he would connect survival issues to those of bankruptcy and turn to bank-runs. He did so in item [94], and in a diffusion model, gave conditions for profit maximization to lead to bankruptcy almost surely. We would surely be amiss not to highlight the crisp and clean nature of the solution as a bang-bang policy simply on the grounds that none of the contributions in this issue tackle bank-runs.

> The optimal withdrawal policy is an 'overflow policy': the withdrawal rate is zero if the asset level is below a 'barrier', and equal to the maximum rate if the asset level is at least equal to the barrier.[16]

In their contribution, Benhabib-Brunet-Hager (BBH) look at the condition for the survival of firms in the context of technological progress through innovation. They study several models of growth in which a continuum of firms, driven by innovation through imitation, and with a focus on the interaction between the two, show that:

> Stochastic imitation and innovation can make the distance of the productivity frontier to the lowest productivity level fluctuate, and this distance can occasionally become large. Alternatively, if we fix the length of the support of the productivity distribution because firms too far from the frontier cannot survive, the number of firms can fluctuate randomly."

It is a result that Radner would surely appreciate.

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[15]Peter Hammond reminds us that this was also delivered as the Arrow Lecture at Stanford, with Radner connecting it to Friedman's questionable claim on firm-longevity in *Capitalism and Freedom.* For ongoing work on extinction and survival, see [35], [11] and their references.

[16]Through an explicit differential equation, he derived the "optimal policy for the control of the drift (yield) and volatility (risk) of the earnings and furnished for the corresponding value function."

These last two contributions of BM and BBH revolve around the introduction of randomness and stochasticity, and it is not often appreciated to the extent that it perhaps ought, that along with Mirrlees, Phelps, Stigum, Ross, Mirman and Brock, Radner was one of the pioneers of Ramseyian optimal stochastic growth; see items [24, 28, 29, 52].[17] It is this consideration that allow us to frame the contribution of Torre-Marsiglio-Mendivil-Privileggi (TMMP) on iterated functional systems: it connects both to issues of stochastic growth and thence to epidemiology, but also more generally to environmental economics. TMMP exploit the condensation term of the fractal operator to highlight the trade-offs between short-term and long-term policies. In a sentence, they note:

> Far-sighted policies lead to asymptotic invariant probability distributions concentrating more mass on high levels of aggregate consumption together with small numbers of infectives, while the invariant distribution reached through short-sighted policies, besides concentrating more mass on low levels of aggregate consumption together with large numbers of infectives, exhibits an additional layer of (uniform) uncertainty generated by the condensation term.

Their contribution is also very much in keeping with Roy Radner's concerns and especially his writing during the latter part of his illustrious career.[18]

The discussion of the turnpike theorem led us willy-nilly to Radner's subsequent work in economic dynamics, and we now turn to static equilibrium theory. Under his second category of general equilibrium theory, Hahn sights Radner's "achievement to be of the highest order." In six lucid paragraphs, he reads the 1968 paper, item [20], as being inspired by statistical decision theory, and shows how a rather severe critique of the theory can be mounted by the introduction of spot prices and through them, the introduction of externalities and "network information" in the Arrow-Debreu-McKenzie conception. The limitation of a single budget constraint leaves "unaccounted [economic phenomena] such as a market in shares, money and liquidity" and even the maximand of an individual firm is far from clear. From a technical point of view, Hahn connects to the technicalities resulting from a lack of "free disposal" and the boundedness from below of the consumption sets.[19]

[17]Given his interests in economic planning, and his command of turnpike theory, it could not be otherwise. In item [28], in his bibliographic note, Radner cites forthcoming work by Brock-Mirman, and distinguishes between undiscounted and discounted cases. The following reference to Jeanjean's demonstration is also noteworthy: the observation is made that Jeanjean "demonstrated the existence of "Lagrangian multipliers" associated with the constraint that programs be stationary, but these multipliers do not seem to lend themselves to an economic interpretation as prices in the usual sense." Also see [12], whose authors write, "In the theory of existence and asymptotic properties of programs that are optimal in various senses, adaptations of the 'value loss' arguments [dating back to Radner (1961)] figure prominently."

[18]From the viewpoint of the theory of optimum stochastic growth, the reader is referred to [38] and the additional references given in the TMMP contribution. For a recent re-direction of the theory towards examples and exemplifications, a theme always emphasized by Radner, see [32] in the context of the stochastic version of the RSS model.

[19]It is interesting that the theory that takes these considerations into account has still not attained a form that can be sighted as adequate, much less complete; see [2] for an ongoing attempt.

> Indeed, this paper illustrates a recurring situation in which the purely formal project of providing an existence proof confronts economists in the clearest possible terms with their lack of real understanding of certain crucial elements. It was inevitable that Radner should continue to pursue these attractive hares.

This pursuit continued in his pioneering 1972 paper on sequential economics, and more importantly, in his subsequent 1979 path-breaking paper on rational expectations equilibria; items [25, 39] respectively.

It is in the context of the 1972 paper that the Chatterji-Kajii (CK) contribution, and its connection to the Hicks-Grandmont temporary equilibrium notion, can be most productively read. Grandmont [18] himself documents the move from static to dynamic theory, a move that synthesizes Walrasian general equilibrium with its Cournotian game-theoretic counterpart.

> Roy Radner ' s pioneering work on the existence and properties of dynamic equilibria, including expectations, changed the tools with which economists were able to analyze uncertainty, dynamic interactions in general equilibrium models, financial markets, as well as in dynamic strategic games under conflict, partnership or teams.

Framing their contribution in the light also of their own earlier work, CK focus on a set of price forecasts and interpret their solution concept as a "generalization of an equilibrium with perfect foresight to an efficient equilibrium with ambiguity that possesses a self-fulfilling property which is weaker than perfect foresight." But before we move to Hahn's third subsection and his third category of "rational expectations," the last word ought to be his even for this second category Hahn titles "general equilibrium."

> It should be emphasised that Radner's paper was the first rigorous study of general equilibrium under rational expectations. (In an Arrow-Debreu economy no assumptions on price expectations are needed.) This assumption was later appropriated by macroeconomists (who were inspired by Muth, 1961, not Radner), not always, it must be said, to its advantage.[20]

But it is really the third category of "rational expectations" that Hahn documents Radner's "beautiful results on the role of prices as carriers of information: the price function is revealing is generically correct for a finite state space. It is not so for an infinite state space (if the signal and price space are of equal dimensions)." Cotter [9, 10] isolates the *information map* from the space of random variables to the space of information formalized by the sub-σ-field it reveals, and identifies the basic difficulty that this map is not injective, and thereby "no discussion of its inverse is possible, [and that only] by perturbing price functions with a smooth convolution, demand can

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It may be worth stating in this context that even though Radner never worked on economies (or games) with a continuum of agents, he surely worked on measure-theoretic economies and games. Also see Footnote 29 below and the text it footnotes.

[20]In this connection the reader may want to see the entry [26] that connects the concept, and the Jordan-Radner JET symposium, to Chapter 12 of Keynes' *General Theory,* an entry whose paragraphs had the benefit of Radner's oversight and his criticism.

be made continuous." There is no contribution in this issue that pursues this line of research, but it is precisely here that Radner escapes the stranglehold of conventional Walrasian (Arrow-Debreu-McKenzie) theory, and fully moves to non-cooperative game theory and to economic interdependence.[21] Furthermore, leaving aside its substantive contribution, this paper, in its composition around a two-person example, is unexcelled as a piece of exposition and as a pedagogical instrument.[22]

There is also no contribution to the fourth category in Hahn's taxonomy – team theory. Radner began with team theory and kept returning to it, by himself or with a succession of distinguished authors.[23] Rosenblatt's review of item [7] is a full recognition of the importance he attached to communication in an interactive environment, and specifically in the context of Marschak's 1955 theory of teams; the earlier item [9] is a further specification to the context of quadratic payoff functions. An extended quotation from Rosenblatt's prescient preview is worthwhile given the current interest in strategic communication for contexts saturated with cheap-talk and fake news.

> The paper derives values of information structures and the corresponding optimal decision functions for the following situations: (a) complete communication, complete information, and routine; (b) no communication, and a case of complete informational decentralization; (c) partitional communication; (d) dissemination of independent information; (e) error in instruction; (f) complete communication of erroneous observations; (g) management by exception, reporting exceptions and "emergency conference". The terminology is given some organizational motivation and some of the results are interpreted.[24]

From "team theory," the move to a "game" was only natural. Hahn's fifth and final classification is a residual grab-bag labelled "private information, incentives and markets,"[25] and he limits himself to a "brief account of his game-theoretic study of an organizational problem," conceiving of an organization as a game and contrasting it with a team whose members have the same objective.

We have already seen how Radner's earliest contribution to Walrasian general equilibrium theory emphasized private information and measurability of

[21]In addition to the original pieces, items [39, 48, 75], see the important advance in [3, 4], especially given current concern with Berk-Nash equilibria that take model misspecification explicitly into account in the optimizing calculus.

[22]We shall have further occasion to refer in the sequel to the use of examples and exemplification in the corpus; our second epigraph already alludes to this.

[23]Whereas one can make the argument that "bounded rationality" and "team theory" inform much of Radner's writings, the editors count nine pieces as directly on the theory of teams: items[B4, 2, 5, 7, 9, 26, 38, 54, 83]. Footnote 10 in item [83] notes how the "analyses of the cost and value of information in an organizational setting intersects with studies of team-theoretic models of a hierarchical form in which the acquisition of information by managers is time-consuming."

[24]See *Mathematical Reviews,* review number MR0151314. Also item [38] with its 5 references.

[25]An obvious play on the title of his 1987 edited volume in honor of Leo Hurwicz, items [B8, 62], a scholar whose influence on Roy Radner was profound; see the two-part item [32] for example.

individual actions: this 1968 paper, item [20], motivated his contributions to *communicative interaction* and *rational equilibria.*[26] His fifth subheading notwithstanding, Hahn's subsection is sharply focused on items [56, 57] on repeated games. In five succinct paragraphs, he explicates the intuition that the "possibility of punishment for deviations by one player by the others in the repeated setting will bring about an efficiency improvement."

> Radner demonstrates that there exists a "trigger strategy" or "review strategy" which will bring about "approximate" efficiency. [If] there is a myopic "arrangement" which is efficient and dominates a short-period Nash outcome, then this arrangement can be sustained in an infinitely repeated game with [these] strategies.

Hahn provides a detailed discussion of the result, drawing special attention to the remarkable application of the law of iterated logarithm.[27]

Whereas we have nothing to present on repeated games in this issue – and Radner's contribution to this subject surely ranks as foundational, and has perhaps again not received the prominence it merits[28] – it is worth recognizing that just as his work in intertemporal general equilibrium theory is, in hindsight, squarely based on his 1968 contribution to its atemporal counterpart, so perhaps his work in repeated games can be seen as taking root from his pioneering work with Rosenthal on the existence of pure-strategy Nash equilibria in simultaneous-play, one-shot games of private information. It is important to see this contribution as changing the subject from the earlier 1973-1974 attempts of Harsanyi and Aumann at purification of Nash's mixed-strategy equilibria. The first relied on perturbations of games with a finite number of agents each with a finite number of pure-strategies to *purify* the mixed-strategy equiibria of the original game; and the second utilized independent random devices to provide an equivalence theorem. As in 1968, Radner-Rosenthal took the reliance on private information and measurability of strategies with respect to it, and formulate an explicit game of private information for which they show the existence of pure-strategy equilibria by assuming information to be diffused and dispersed (*independent* and *atomless*) by connecting it to [51].[29] In

[26]His 1982 survey in item [49] is titled "The role of private information in markets and other organizations." An year earlier, he had surveyed the subject of "equilibrium under uncertainty" in item [42]. This is also one of his entries in *The New Palgrave*; see item [66].

[27]Even though Hahn does not do so, one may also cite here the remarkable application of the classical law of large numbers to communication patterns in item [38].

[28]Hahn does not include any paper on repeated games among the 13 that he chooses, but Mailath-Samuelson [34] note both the 1980 paper as well as the Radner-Maskin-Myerson example; they include all of the relevant references. A text or a survey that does not discuss this work reflects more on itself than on the work. The editors count five papers related directly to repeated games: the pioneering 1980 item [44] followed up by the 1985-1987 items [56, 57, 58, 59].

[29]See items [47, 103]; also see the Working Paper listed below as Radner 1980 which has an explicit existence proof that was eliminated from the published version. This version limits itself to the acknowledgement that "We are indebted to R. B. Wilson, who introduced us to the issues dealt with in this note and who first suspected that the results of Schmeidler (1973) were relevant." This is not to say that Radner gave up on correlated information structures: in the light of the examples

his contribution, Askoura provides a self-contained and comprehensive treatment of large non-anonymous games, and also connects to co-operative game theory through the core. It is well known that Radner relied on the core for a variety of problems in industrial organization; see items [77, 97, 100.]

We have already had occasion to refer to the equilibrium composition of an industry in the context of the BBH contribution, and this emphasis on industrial composition and organization takes flight in Radner's pioneering work in demand theory; see items [102, 104] on what he referred to as *viscous* demand, and built on the work of Rosenthal and others on "slowly changing consumer loyalties," the terminology issuing from viscosity phenomena in stochastic optimal control. As he notes in his 2003 paper, his analysis "casts a new light on various economic phenomena including the importance of market share in investment, kinked demand curves, and competitive pricing."[30] In his contribution to the issue, Hildenbrand focusses on Gorman's 1981 question as to the properties of Engel curves deriving from specific functional form of the demand function. Hildenbrand notes:

> Gorman motivates his analysis by the claim or hope that his result is useful for estimating demand systems from survey data. It is well-known that for a general population that is heterogeneous in income and behavior, the Engel function curve of the population [as defined in his paper] is neither homogenous in prices and income, nor is the Slutsky substitution matrix symmetric, even if all individual demand functions have these properties. This fact is a serious obstacle to the application of Gorman ' s results.

Hildenbrand's paper is a first serious attempt to deal with this obstacle. Instead of Lie group arguments used by Russell, Jerison and others, he relies on a 1949 characterization by Schwartz of closed and shift-invariant linear subspaces of the linear space of complex-valued continuous functions endowed with the topology of uniform convergence on compact sets.[31]

The final contribution we consider in this preface, that of Mitra-Ok, can be introduced through Radner's manifold commitments to growth and development, to the alleviation of poverty, to the avoidance of social conflict, and to the importance he attached to data and to empirical work.[32] Within economic theory proper, as opposed to operations research or to applied mathematics, it

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in item [47], he turns to approximation theorems in [51]. This change of subject in articulated in some detail in [30, 31], and as documented there, this notion of diffused and dispersed private information has given rise to a mature theory based on general actions sets at the hands of Yeneng Sun and his followers; see [28] for an early synthetic treatment.

[30]Radner's meticulousness, and his respect for antecedent scholarship, on continuous-time stochastic game theory in this instance, is especially evident in his acknowledgement to his 2003 paper; also see Anderson [1] and Stinchcombe [53, 54] and his reference to earlier work of Cotter's.

[31]For a statement and framing of Schwartz's result see Cameron's reviews MR17471 and MR23948 in *Mathematical Reviews.* For the exchange between Jerison and Russell, see [42] and its references; also [43] and the references to texts on exterior differential systems quoted therein. For the importance of mean demand functions, also see [45].

[32]He listed the "statistical theory of data mining" as a current research interest in 2015.

can also be tied to his interest in planning, and through development planning, to the register of development economics. This remained a constant throughout his long career, even though his paper, item [84], in honor of the Indian economist, Sukhomoy Chakravarty, was a singularly isolated contribution to mainstream economics in its involvement with the literature on cost-benefit analysis, on tax reform, on welfare measures, and implicitly to the subject of welfare economics.[33]

Mitra-Ok present sufficient conditions for a "non-negative rearrangeable real map f to have a larger L_p-norm than another such function g for every $p \geq 1$." This is a problem with a venerable history, and the authors give a comprehensive treatment by taking as their point of departure their own extension of the classic Tomic-Weyl submajorization theorem to the category of all non-negative rearrangeable maps. They emphasize that the work goes beyond its own mathematical interest and has direct implications for inequality measures: the *compromise inequality* measure due to Ebert, the *absolute income movement index* of the authors themselves, and the P_p-index of poverty due to Foster, Greer, and Thorbecke.[34] The point is that these rankings depend on the value of p that is chosen, and the authors' show in the specialized context of (finite-dimensional) Euclidean spaces, that there exist sufficient conditions for a unanimous ranking to take place.

At the end of this framing of the ten contributions to this issue in the light of a rich and varied multi-dimensional corpus, the editors feel constrained to say that only time will tell how many developments in economics, and especially in economic theory and in mathematical economics, will find their origins already marked out in Radner's *oeuvre.* In his tribute, Hahn was limited by necessity to the first cut of thirty-seven years of the trajectory, the years 1955 to 1992, but given the luxury of an additional thirty years, a completion to 1992, we can sight *decision, information* and *organization* as signature words. As regards the first, there is a little irony in that Savage's student never contributed to axiomatic decision theory, but his command of the subject is evident in the volume he edited with C. B. McGuire, item [B3], in honor of Jacob Marschak.

> This paper, in its role as an introduction to the volume, provides a brief survey of the theory of rational choice. Eleven aspects of the theory are discussed, with all but the last (stochastic choice) to be treated in greater detail in subsequent chapters of this volume. The topics introduced include choice theory under uncertainty, dynamics in choice theory, independence

[33]Even though published in 1993, this paper was already cited in the first edition of Hal Varian's *Microeconomic Analysis* dated 1978. One the authors also recalls a conversation in which Radner singled out to him the importance of this piece for applied theory. This importance was fully underscored by Schlee's connection of Radner's conception to Debreu's coefficient of resource utilization, and thereby to index number theory and to other welfare measures as in [48] and its references; also see [46, 47] and ongoing extensions in [49, 50].

[34]For a comprehensive treatment of measures of upward mobility, see [41] and their references.

and separability, various aspects of information in choice theory and sequential decision making.

We have already had occasion to underscore how the importance of information runs through all of Radner's contributions to the lines of research that can be traced to Walras and Cournot. He brought to bear private and differential information on the analysis of solution concepts in both economies and games, and in terms of the desiderata of Walrasian equilibrium theory, focussed on prices as aggregators of information. However, we note here a glaring omission in that we have not considered his work on the commodification and the value of information itself, and how this value may exhibit increasing returns to scale.[35] Within a decade of the Radner-Stiglitz result, he had turned his attention to information processing proper in a variety of organizational frameworks; see items [78, 83, 89] in particular.

In concluding this preface to a memorial issue, we go beyond our strict introductory mandate to make three additional points concerning the corpus that we have used to give the ten contribution an overall rubric. First, an emphasis on approximation runs through its entirety: the turnpike theorem, the existence of approximate pure-strategy equilibria of correlated interaction in finite games, approximate decentralization of technologically efficient programs, approximate optimality of Nash equilibria in the setting of repeated games, approximately constant-returns-to-scale industries. Second, given Radner's emphasis on examples, and the care and he comprehensiveness with which he laid out the relationship to the antecedent literature, the corpus is an exemplary testament that the line between research and teaching is never very clear-cut. His bibliographic notes at the end of the paper vividly illustrate how he wrote in a subject after he had developed a command of it rather than learning the subject by writing in it. Third, Radner's eminence in Walrasian general equilibrium and in economic dynamics and planning obscures the fact that an entire career could be built solely on his contributions to the discipline of operations research and telecommunications: there is, for example, another glaring omission in our (this) account: this is the importance and salience of regulation and de-regulation, and it was this that gave him his command of principal-agent theory. In any case, even a rough listing of Radner's contributions to OR brings out the depth of his contribution to this register, one in which he retained an interest right till the end.[36]

[35]We refer of course to the highly influential 1983 Radner-Stiglitz non-concavity result, item [50]; for a definitive extension, see [8]. Also see an alternative non-Bayesian conception in ongoing work reported as [5]. It is also worth noting, and surely amusing, that there is no reference to Radner in the index or the list of references in [40].

[36]This list includes studies of series-parallel systems, spare-parts demand, opportunistic replacement of parts, failing equipment, failure-restoration processes, behavioral models of cost reduction and stochastic control, deregulation of long-distance telecommunication, sequential approaches to regulation, price caps, subscriber-line charges, sealed-bid mechanism, optimal scheduling.

Finally, we close by noting that Paul Samuelson in his tribute to Abba Lerner, dates the birth of a scholar to his first publication. The editors of this memorial issue can hardly do better than to end with Hoffman's evaluation of Radner's first solo publication:

> If each of the activity levels of x_j of a linear-programming problem is controlled by a separate decision-maker, knowing only the partial information y_j about the precise coefficients of the matrix, bill of goods and objective function (although the probability distribution of these coefficients is known to all), then the group of decision makers are a "linear team". The author shows that the problem of finding the best decision rules is a linear programming problem in the space of decision functions and that the dual leads to a system of probability distributions of implicit prices.

It is fascinating how these themes were to animate Roy Radner's concerns for the entirety of his illustrious career.

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IV. Current Research

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