Attempting to understand stock–bond comovement

Gregory R. Duffee
Johns Hopkins University

First version October 2017
Current version October 29, 2017

Preliminary and incomplete; probably something wrong somewhere . . .

Abstract

Why does the correlation between stock returns and nominal bond returns vary over time, occasionally switching sign? This paper argues that a successful explanation must be grounded in the time-varying behavior of real rates. Standard models with long-run risk dynamics have substantial difficulty generating the necessary properties of real rates. Time-varying dynamics of real output may well account for the observed patterns, but embedding those dynamics in standard asset-pricing frameworks appears to pose a major challenge.

Voice 410-516-8828, email duffee@jhu.edu. Address correspondence to 552 Wyman Park Building, 3100 Wyman Park Drive, Baltimore, MD 21218. Thanks to Anna Cieslak for extensive discussions on this subject.

1 Introduction

The correlation between stock and nominal bond returns varies widely over time, occasionally switching sign. Figure 1 illustrates patterns first identified by Li (2002) and Fleming, Kirby, and Ostdiek (2003). Daily stock returns and contemporaneous daily changes in long-term nominal Treasury yields move together in the early 1960s. They are strongly negatively correlated from the 1970s through the late 1990s. After an abrupt sign change around 1997, the correlation remains positive through much of the 21st century. (Recall that correlations between stock and bond returns have the opposite sign of those in Figure 1.) This time-variation in daily comovement also holds for monthly, quarterly, and annual horizons.

Three related branches of research explore time-varying correlations.[1] One follows Li (2002) by searching for plausible conditioning information. What macroeconomic and financial variables observed at t predict the correlation at $t+1$? Baele, Bekaert, and Inghelbrecht (2010) illustrates another branch that attempts to explain empirically the conditional second moments in Figure 1 with conditional second moments of plausible fundamentals such as output, inflation, and liquidity. The third branch, and the focus of this paper, interprets Figure 1 using standard asset-pricing frameworks.

This paper evaluates possible mechanisms in asset-pricing models that drive time-varying correlations. Much research claims to successfully embed time-varying correlations in asset-pricing models. For example, Burkhardt and Hasseltoft (2012), David and Veronesi (2013), Song (2017), and Campbell, Sunderam, and Viceira (2017) all construct models in which the conditional covariance between stock returns and nominal yields changes sign over time. Are any or all of these different stories plausible, qualitatively and quantitatively?

Not surprisingly, a key mechanism in much of this research is time-variation in the conditional correlation between expected inflation and expected aggregate cash flows to equity. Macroeconomic dynamics swing from periods of countercyclical expected inflation—stagflation—to periods of procyclical expected inflation. During stagflation, good (bad) news about future cash flows tends to be accompanied by news of lower (higher) expected inflation. Thus when stock prices rise (fall), nominal yields tend to fall (rise). Such a pattern is consistent with the 1970s through late 1990s in Figure 1. Procyclical shocks to expected

[1] A fourth, independent branch follows Fleming et al. (2003) by examining the implications for portfolio management.

inflation generate the opposite correlation, such as in the Great Recession.

This paper makes three points. First, any reliance on the central role of a regime-shifting inflation mechanism is misplaced. There are two key pieces of evidence, both of which can be gleaned from the existing literature. The first empirical observation is that conditional covariances between U.S. stock returns and *nominal* Treasury bond yields move closely with conditional covariances between U.S. stock returns and *real* bond yields. Real yields are those for inflation-indexed debt issued by the U.S. and U.K. governments. For example, both nominal and real conditional covariances are negative in the late 1980s and early 1990s, and both are positive in the 21st century.

The second empirical observation is that conditional volatilities of long-horizon inflation expectations are small. Duffee (2017) uses this empirical result to show that innovations in nominal yields are attributable mostly to news about either expected future real rates or term premia. The same observation applies to conditional covariances between stock returns and nominal yields: Since there is not much news about expected inflation, the sign of its conditional covariance with cash flows cannot matter much.

The second point is that existing models in the spirit of Bansal and Yaron (2004) have substantial difficulty generating sufficiently large shocks to real rates to drive observed variations over time in stock–bond comovement. Models in the literature that generate the necessary comovement tend to rely on unusual specifications, such as money illusion.

The third point is that a time-varying mix of short-term and long-term macroeconomic shocks may help account for the time-variation in stock–bond comovement. This claim is supported by evidence from forecasters' revisions of anticipated real GDP growth across various horizons. It is also supported by evidence from the term structure. Stated roughly, the sign of news at the long end of the term structure is associated with the magnitude of news at the short end of the term structure. When short-rate volatility exceeds long-rate volatility, stock returns and long-maturity yields move inversely. When long-rate volatility exceeds short-rate volatility, the comovement of stock returns and bond yields is reversed.

Naturally, this work builds on much earlier research. Connolly, Stivers, and Sun (2005) is one of the first attempts to link time variation in the stock-bond correlation to macroeconomic fundamentals. Campbell, Shiller, and Viceira (2009), Campbell, Sunderam, and Viceira (2017), and Liu (2017) investigate empirically the comovement between stock returns and real yields. Attempts to interpret properties of the stochastic discount factor using the comovement of stock returns and yields dates to at least d'Addona and Kind (2006).

2 An inflation-centric approach

Expected inflation affects nominal bond yields. This observation worthy of any Principles course can be formalized using an accounting identity borrowed from Campbell and Ammer (1993).

2.1 Accounting definitions of news

Denote the continuously compounded yield on an n-maturity real (inflation-indexed) bond as $y_t^{(n)}$. Shorthand notation for the one-period real bond yield is r_t. Denote the log return to holding an n-period real bond from t to $t+1$ in excess of r_t as

$$ex_{t+1}^{(n)} \equiv \left(ny_t^{(n)} - (n-1) y_{t+1}^{(n-1)} \right) - r_t. \tag{1}$$

Recursive substitution of (1) produces the accounting identity

$$y_t^{(n)} = \frac{1}{n} \sum_{i=1}^{n} r_{t+i-1} + \frac{1}{n} \sum_{i=1}^{n} ex_{t+i}^{(n-i+1)}. \tag{2}$$

This equation says that holding constant the real yield on the left, higher (lower) real short rates over the life of the bond must correspond to lower (higher) excess returns.

Conditioning (2) on an information set Σ produces

$$E\left(y_t^{(n)} | \Sigma \right) = \frac{1}{n} \sum_{i=1}^{n} E\left(r_{t+i-1} | \Sigma \right) + \frac{1}{n} \sum_{i=1}^{n} E\left(ex_{t+i}^{(n-i+1)} | \Sigma \right). \tag{3}$$

When the information set is everything observable at t, including bond yields, this expectation version says that the period-t real yield is the sum of average expected future short-term real rates and average expected excess returns over the life of the bond.

Notation for the innovation in the bond yield from $t-1$ to t is

$$\begin{aligned}\tilde{y}_t^{(n)} &\equiv y_t^{(n)} - E_{t-1} y_t^{(n)} \\ &= \eta_{r,t}^{(n)} + \eta_{ex,t}^{(n)}.\end{aligned} \tag{4}$$

The two components of this news are

$$\begin{aligned}\eta_{r,t}^{(n)} \equiv & E_t \left(\frac{1}{n} \sum_{i=1}^{n} r_{t+i-1} \right) - E_{t-1} \left(\frac{1}{n} \sum_{i=1}^{n} r_{t+i-1} \right), \\ \eta_{ex,t}^{(n)} \equiv & E_t \left(\frac{1}{n} \sum_{i=1}^{n} ex_{t+i}^{(n-i+1)} \right) - E_{t-1} \left(\frac{1}{n} \sum_{i=1}^{n} ex_{t+i}^{(n-i+1)} \right).\end{aligned} \tag{5}$$

The first type of news is the innovation in expected average real rates over the life of the bond, and the second is the innovation in expected average excess returns, again over the life of the bond.

Now turn to nominal bonds. Denote the yield on an n-maturity nominal bond as $y_t^{\$(n)}$. Denote the log change in the price level from $t-1$ to t as π_t. Finally, denote the log return to holding an n-period nominal bond from t to $t+1$ in excess of the short-term real rate and inflation as

$$ex_{t+1}^{\$(n)} \equiv \left(n y_t^{\$(n)} - (n-1) y_{t+1}^{\$(n-1)} \right) - r_t - \pi_{t+1}. \tag{6}$$

The expectation form of the resulting accounting identity is

$$E\left(y_t^{\$(n)} | \Omega \right) = \frac{1}{n} \sum_{i=1}^{m} E\left(\pi_{t+i} | \Sigma \right) + \frac{1}{n} \sum_{i=1}^{n} \left(r_{t+i-1} | \Omega \right) + \frac{1}{n} \sum_{i=1}^{n} E\left(ex_{t+i}^{\$(n-i+1)} | \Sigma \right). \tag{7}$$

Equation (7) says that the period-t nominal yield is the sum of average expected future short-term real rates, average expected inflation, and average expected excess returns over the life of the bond.

In innovation form, we have

$$\begin{aligned}\tilde{y}_t^{\$(n)} &\equiv y_t^{\$(n)} - E_{t-1} y_t^{\$(n)}, \\ &= \eta_{\pi,t}^{(n)} + \eta_{r,t}^{(n)} + \eta_{ex,t}^{\$(n)}.\end{aligned} \tag{8}$$

The news components other than real-rate news are

$$
\begin{aligned}
\eta_{\pi,t}^{(n)} &\equiv E_t\left(\frac{1}{n}\sum_{i=1}^{n}\pi_{t+i}\right) - E_{t-1}\left(\frac{1}{n}\sum_{i=1}^{n}\pi_{t+i}\right), \\
\eta_{ex,t}^{\$(n)} &\equiv E_t\left(\frac{1}{n}\sum_{i=1}^{n}ex_{t+i}^{\$(n-i+1)}\right) - E_{t-1}\left(\frac{1}{n}\sum_{i=1}^{n}ex_{t+i}^{\$(n-i+1)}\right).
\end{aligned} \tag{9}
$$

Note that the news about expected excess returns in (5) differs from the news in (9). The former is for real bonds and the latter is for nominal bonds. For example, risk premia on nominal bonds can vary without corresponding variation in risk premia on real bonds.

2.2 Standard stock-bond comovement intuition

The conditional covariance between the aggregate stock return and the innovation in the n-maturity nominal bond yield is the sum of covariances between stock returns and the three components of nominal news,

$$
\mathrm{Cov}_t\left(ret_{M,t}, \tilde{y}_t^{\$(n)}\right) = \mathrm{Cov}_t\left(ret_{M,t}, \eta_{\pi,t}^{(n)}\right) + \mathrm{Cov}_t\left(ret_{M,t}, \eta_{r,t}^{(n)}\right) + \mathrm{Cov}_t\left(ret_{M,t}, \eta_{ex,t}^{\$(n)}\right). \tag{10}
$$

Changes in the covariance over time must therefore be driven by changes in one or more of the three component covariances. The literature focuses on the second term on the right, the conditional covariance between stock returns and news about expected inflation.

Interpreting U.S. aggregate data from 1952 through 2005 with a dynamic factor model, Piazzesi and Schneider (2007) argue that expected inflation is countercyclical. More precisely, unexpected changes in aggregate consumption growth tend to lead future decreases in inflation. Piazzesi and Schneider combine this evidence with recursive utility to explain why the nominal yield curve slopes up on average: investor fear stagflation. Bansal and Shaliastovich (2013) present similar evidence in building a term structure model that exhibits countercyclical expected inflation.

Aggregate stock prices and news about aggregate consumption growth tend to move together. Therefore the evidence of Piazzesi/Schneider and Bansal/Shaliastovich implies that the conditional covariance between stock returns and news about future inflation is negative.

However, Burkhardt and Hasseltoft (2012), hereafter B/H, find that the relation between consumption growth and inflation changes over time. From 1930 through 1970, the correlation between annual consumption growth and annual inflation ranges from positive to modestly negative. Subsequently the correlation turns sharply negative, in the neighborhood of -0.6 during 1970 through 2000. The correlation then switches sign again, to about 0.6 from 2010 through 2010. B/H use this and related evidence to motivate a regime-shifting model that interprets the sign change in the stock-bond covariance in the 1990s as a consequence of a shift from a countercylical inflation regime to a procyclical inflation regime.

B/H report correlations with inflation levels rather than news about expected future inflation. Song (2017) expands on the evidence of Piazzesi and Schneider by constructing an explicit regime-shifting dynamic factor model of consumption growth and inflation. He confirms that expected inflation is largely countercyclical prior to 2000 and procyclical after.

2.3 A benchmark long-run risk model

Much research into stock-bond comovement adopts the long-run risk (LRR) setting pioneered by Bansal and Yaron (2004). One approach pairs the LRR dynamics of real quantities with an exogeneous inflation process. The joint dynamics determine prices of nominal bonds, real bonds, and equity, as in Bansal and Shaliastovich (2013).

An example of the assumed macroeconomic specification is taken from B/H. There are two regimes. One has countercyclical inflation expectations and the other has procyclical inflation expectations. The regime in period t is s_t. Regime dynamics are determined by a Markov transition matrix.

Denote log aggregate consumption by c_t, log aggregate cash flows to equityholders as d_t, and the log change in the price level as π_t. Each variable has a time-varying conditional expectation. Inflation is conditionally heteroskedastic. Their dynamics are

$$\Delta c_{t+1} = \mu_c + x_{c,t} + \sigma_c \eta_{c,t+1}, \tag{11}$$

$$\Delta d_{t+1} = \mu_d + \phi x_{c,t} + \psi \sigma_c \eta_{d,t+1}, \tag{12}$$

$$\pi_{t+1} = \mu_\pi + x_{\pi,t} + V_t^{1/2} \eta_{\pi,t+1}. \tag{13}$$

Consumption and cash-flow growth share the conditional mean component $x_{c,t}$. The parameter $\phi > 1$ captures the idea that equity is leveraged. The innovations $\eta_{c,t}$, $\eta_{d,t}$, and $\eta_{\pi,t}$ are iid standard normal shocks.

The state variables are the regime, the conditional expectations of consumption growth and inflation, and a state variable that drives time-varying volatilities. The dynamics of these state variables are

$$V_{t+1} = \overline{V} + \nu_{\pi}\left(V_t - \overline{V}\right) + \sigma_{\nu}\omega_{t+1}, \tag{14}$$

$$\begin{pmatrix} x_{c,t+1} \\ x_{\pi,t+1} \end{pmatrix} = \begin{pmatrix} \beta_{11}(s_{t+1}) & \beta_{12}(s_{t+1}) \\ 0 & \beta_{22}(s_{t+1}) \end{pmatrix} \begin{pmatrix} x_{c,t} \\ x_{\pi,t} \end{pmatrix} + \begin{pmatrix} \Omega_{11}(s_{t+1}) & \Omega_{12}(s_{t+1}) \\ \Omega_{21}(s_{t+1}) & \Omega_{22} \end{pmatrix} \begin{pmatrix} \sigma_c \epsilon_{c,t+1} \\ V_t^{1/2}\epsilon_{\pi,t+1} \end{pmatrix}. \tag{15}$$

The innovations ω_t, $\epsilon_{c,t}$ and $\epsilon_{\pi,t}$ are iid standard normal shocks.

The matrices β and Ω differ across the two regimes.[2] The parameters β_{11}, β_{22}, and Ω_{11}, are all positive in both regimes, although they differ in magnitude across the regimes. Cyclicality is determined by the remaining parameters:

$$s_t = \begin{cases} \text{countercyclical,} & \equiv \beta_2 < 0, \Omega_{12} < 0, \Omega_{21} < 0; \\ \text{procyclical,} & \equiv \beta_2 > 0, \Omega_{12} > 0, \Omega_{21} > 0. \end{cases}$$

The countercyclical regime is characterized by negatively correlated shocks to the conditional expectations of consumption growth and inflation. In addition, high (low) expected inflation forecasts decreasing (increasing) conditional consumption growth. These patterns are reversed in the procyclical regime.

The representative agent has recursive preferences,

$$U_t = \left[(1-\delta)\, C_t^{\frac{1-\gamma}{\theta}} + \delta\left(E_t\left[U_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \tag{16}$$

where γ is the coefficient of relative risk aversion (CRRA), ψ is the elasticity of intertemporal substitution (EIS), and $\theta = (1-\gamma)/(1-(1/\psi))$. Following Bansal and Yaron (2004), both the CRRA and the EIS are assumed to be greater than one.

Denote the log price/dividend ratio for aggregate equity by pd_t. Denote the log prices of

[2] The parameters μ_c and μ_π in (11) and (13) also differ across regimes in the parameterized model of B/H.

real and nominal bonds by $p_t^{(n)}$ and $p_t^{\$(n)}$ respectively. Their functional forms are

$$pd_t = A_0(s_t) + A_1(s_t)x_{c,t} + A_2 x_{\pi,t} + A_3 V_t, \tag{17}$$

$$p_t^{(n)} = D_{0,n}(s_t) + D_{1,n}(s_t)x_{c,t} + D_{2,n} x_{\pi,t} + D_{3,n} V_t, \tag{18}$$

$$p_t^{\$(n)} = D_{0,n}^{\$}(s_t) + D_{1,n}^{\$}(s_t)x_{c,t} + D_{2,n}^{\$} x_{\pi,t} + D_{3,n}^{\$} V_t. \tag{19}$$

Properties of asset prices in this setting are intuitive. Higher expected consumption growth corresponds to higher real short-term interest rates, as investors attempt to borrow from the future to spend today. Therefore $D_{1,n} < 0$. Nominal bonds inherit this property from real bonds, thus $D_{1,n}^{\$} < 0$. Higher expected consumption growth raises expected cash flows to equity more than it increases discount rates, therefore $A_1 > 0$. With both an EIS and a CRRA greater than one, higher volatility simultaneously lowers real short rates through the precautionary savings channel and and lowers stock prices (and total wealth) through higher risk premia. Therefore $D_{3,n} > 0$ and $A_3 < 0$.

Higher expected inflation lowers nominal bond prices, thus $D_{2,n}^{\$} < 0$. When inflation expectations are countercyclical, higher expected inflation lowers real rates (investors attempt to save for the coming bad times) and lowers equity valuations, $D_{2,n} > 0$ and $A_2 < 0$. Nominal bonds are speculative assets, thus an increase in volatility raises risk premia, lowering nominal bond prices; $D_{3,n}^{\$} < 0$. When inflation expectations are procyclical, these sensitivities are reversed. Higher expected inflation raises real rates and raises equity values. Nominal bonds are a hedge so higher volatility raises their prices.

These signs are summarized in Table 1.

Table 1. Price sensitivity of assets to the factors of a regime-shifting long run risk model

	Expected Cons Growth	Expected Inflation		Volatility	
	Both Regimes	Counter-C	Pro-C	Counter-C	Pro-C
Equity	+	−	+	−	−
Real bonds	−	+	−	+	+
Nominal bonds	−	−	−	−	+

This model has no difficulty generating the time-varying correlations displayed in Figure 1. The countercyclical regime generates a negative correlation between stock returns and changes in nominal yields because higher expected inflation is bad news for the economy (stagflation). The procyclical regime generates a positive correlation because higher expected inflation is good news.

Other researchers follow the lead of B/H. Song (2017) and Campbell, Pflueger, and Viceira (2015) partially endogenize inflation with regime shifts in monetary policy. (The latter paper uses habit formation rather than LRR, as discussed in Section 5.3.) David and Veronesi (2013) have unobserved regimes that differ in their conditional covariance between inflation expectations and equity cash flows, creating a filtering problem for agents. Campbell, Sunderam, and Viceira (2017) use a continuous state variable to capture time-varying covariances with expected inflation rather than one that jumps from regime to regime.

Although this is a popular framework, the empirical evidence presented in the next section casts considerable doubt on this simple countercyclical/procyclical interpretation of the variation over time in covariances between stock returns and changes in nominal yields.

3 Evidence of comovement with real and nominal yields

This section characterizes the empirical time-series relation between two covariances: The covariance between stock returns and changes in nominal yields, and the covariance between stock returns and changes in real yields. The preceding language is cumbersome, so I often use the shorthand terms 'nominal covariance' and 'real covariance.'

Two reasons underlie the focus on covariances rather than correlations. First, covariances arise naturally in asset-pricing models such as the B/H model discussed in the previous section. Second, as documented below, an important empirical regularity is missed if we look only at correlations.

3.1 Data description

The daily value-weighted return to the Center for Research in Security Prices (CRSP) index proxies for the aggregate U.S. stock return. The daily return to the Financial Times Stock Exchange (FTSE) index proxies for the aggregate U.K. stock return.

Daily observations of a ten-year nominal Treasury coupon bond yield are from the CRSP Fixed Term file. That file also reports the duration of the bond. The first nominal yield observation is in June 1961. Daily observations of ten-year real Treasury yields are from the Treasury Department's TIPS website. These ten-year constant maturity yields are interpolated from secondary market quotes. The data begin in January 2003. D'Amico, Kim, and Wei (2008) show that TIPS yields prior to approximately mid-2003 exhibit erratic behavior behavior that they plausibly attribute to market inefficiencies. Durations of these bonds are determined using the formula for par ten-year bonds.

Daily observations of ten-year Bank of England real and nominal yields are from the Bank of England web site. These are zero-coupon yields interpolated from coupon bonds. Bank of England yields are available beginning in January 1985. These yields, like all other data used here, are available through the end of 2016. Yields are expressed on an annual basis.

3.2 Realized covariances

Figure 2 displays realized covariances between daily stock returns and daily changes in bond yields for 44-day rolling samples. Stock returns are measured in percent and changes in yields are measured in basis points. Panel A is the covariance counterpart to Figure 1. Realized covariances between changes in nominal Treasury yields and U.S. stock returns are modestly negative from the beginning of the sample in 1961 through the late 1970s. They are strongly negative during the Fed monetarist experiment period, then slowly drift higher until the late 1990s. A striking sign change in 1997 is followed by consistently positive covariances through the end of 2016.

Panel B focuses on the 2003 through 2016 sample and includes the covariance between stock returns and changes in real bond yields. Similar information is in Figure 4 of Campbell, Sunderam, and Viceira (2017). The nominal and real realized covariances closely track each other and are almost all positive. The introduction of the TIPS market coincides with a period of stable and low forecasts of long-run inflation in the U.S. Thus it is not too surprising that their realized covariances with equities are similar; during the sample, nominal bonds are economically close to real bonds. Absent heroic assumptions, these data do not allow us to infer the link between the nominal and real covariances at times when inflation concerns are greater.

I extend the sample with real yields back to 1985 using inflation-indexed debt issued by the Bank of England. This choice introduces some potential noise in the results. As discussed in Campbell, Shiller, and Viceira (2009), yields on inflation-indexed bonds can diverge substantially across countries. The underlying consumption baskets differ. Perhaps more importantly, illiquidity and other sources of segmentation prevent complete market integration. However, Campbell et al. (2009) show that U.S. and U.K. real yields have closely tracked each other since roughly 2003 (when U.S. real yields no longer exhibit erratic behavior).

Rolling realized nominal and real covariances are plotted in Panel C. Contemporaneous realized real covariances use the U.S. stock return from day $t-1$ to day t the change in the Bank of England real yield from $t-1$ to $t+1$. This two-day yield change picks up news that arrives when the U.S. market is open on day t and the U.K. market is already closed.

Panel C reveals three important patterns. First, during the 2003–2016 period, realized covariances between U.S. stock returns and UK real bond yields behave similarly to those displayed in Panel B for TIPS yields. This pattern supports the use of U.K. real yields as proxies for U.S. real yields. Second, realized equity covariances with real yields, like those with nominal yields, switch sign in the late 1990s. Third, the covariances with real yields are muted versions of covariances with nominal yields. Although they move together, the covariances with nominal yields are more volatile than those with real yields.

Li (2002) observes that variation over time in the U.S. stock/nominal-bond relation is shared across many countries including the U.K. Panel D plots nominal and real realized covariances using entirely U.K. data. As in Panel C, both nominal and real covariances switch sign in the late 1990s, and real covariances are less volatile over time than nominal covariances.

Figures similar to Panel C, using U.K. data, appear in earlier literature. Campbell et al. (2009) use data beginning in the early 1990s to plot annual real and nominal realized correlations. Liu (2017) also plots annual realized correlations, extending the sample back to 1985. These figures illustrate the comovement of nominal and real covariances, although the muted time series behavior of real covariances is difficult to detect in correlations. Daily changes in real yields are less volatile than daily changes in nominal yields, largely offsetting the smaller time-series variation of real covariances.

3.3 Conditional covariances

Conditional covariances rather than realized covariances are central to asset-pricing models. Realized covariances equal conditional covariances plus orthogonal noise. The persistence of the realized covariances in Figure 2 indicates that lags of realized covariances contain substantial forecasting information. Denote successive, nonoverlapping periods of 44 days by t, $t+1$, and so on. Denote realized variances and covariances during period t by $\Omega_{t,i,j}$, where i and j are "s," "n," or "r," indicating stocks, nominal yields, and real yields respectively. Conditional covariances are constructed by ordinary least-squares (OLS) projections

$$\Omega_{t,s,i} = b_{0,i} + \sum_{k=1}^{L} b'_{k,i} X_{t-k} + \epsilon_{t,s,i}, \qquad i \in \{n, r\}, \tag{20}$$

where the set of instruments is either parsimonious or expanded,

$$X_t = \left(\Omega_{t,s,n} \quad \Omega_{t,s,r}\right)' \quad \text{or} \quad X_t = \left(\Omega_{t,s,n} \quad \Omega_{t,s,r} \quad \sqrt{\Omega_{t,s,s}} \quad \sqrt{\Omega_{t,n,n}} \quad \sqrt{\Omega_{t,r,r}}\right)'. \tag{21}$$

Fitted values are estimated conditional covariances.

The fitted values allow us to investigate how tightly linked nominal conditional covariances are to real conditional covariances. Regress the former on the latter,

$$\widehat{\Omega}_{t,s,n} = a_0 + a_1 \widehat{\Omega}_{t,s,r} + e_t. \tag{22}$$

Since the explanatory variable is a generated regressor from (20), asymptotic standard errors are constructed using Generalized Methods of Moments (GMM). The moments are the OLS moments of the two regressions in (20) and the OLS moments of the single regression (22). Estimation is performed separately for the three sets of data underlying Panels B, C, and D of Figure 2: U.S. only, U.S. combined with U.K. real yields, and U.K. only.

Table 1 reports estimation results for (22), as well as R^2s of the regressions (20) that produce the fitted conditional covariances. The main message of Table 1 does not depend on the sample period, the instrument choice, or the number of lags of instruments used to construct conditional covariances. Variations in conditional real covariances explain, in an R^2 sense, the bulk of the variation in conditional nominal covariances. All of the R^2s for (22) are well above a half. Nominal covariances respond much more than one-to-one;

point estimates of (22) range from about 1.7 to 2.1. Asymptotic t-statistics testing the null hypothesis of a one-to-one relation range from 1.25 to 2.4.[3]

Figure 3 plots fitted conditional covariances for the 1985–2016 period. Panel A displays covariances with U.S. stock returns and Panel C displays covariances with U.K. stock returns. Naturally, these are less noisy than the realized covariances plotted in Figure 2.

Another intuitive way to express the comovement of stocks and bonds is with market betas. Daily changes in yields are multiplied by negative duration, converting the changes to daily bond returns. Then fitted conditional covariances between stock and bond returns are divided by fitted conditional variances of stock returns. These fitted variances are produced following (20), using lagged realized variances as instruments.

Panels B and D of Figure 3 display the fitted conditional market betas. Here I briefly discuss betas with respect to the U.S. stock market (Panel B); results for the U.K. stock market are similar.[4] Conditional nominal betas between 1985 and 1996 ranged from roughly zero to 0.8, averaging 0.17. Conditional real betas during the same period ranged from -0.05 to about 0.4, averaging only 0.05. After 1996, conditional nominal betas ranged from -0.6 to 0.35, averaging -0.11. Conditional real betas ranged from -0.6 to 0.1, averaging -0.09.

The most important message to take from this evidence is that nominal and real covariances move together over time, including switching signs together. This conclusion tightly constraints potential explanations for time-varying covariances. Whatever economic forces drive variation in nominal covariances must simultaneously drive smaller variation in real covariances.

In the context of the accounting framework of Section 2.1, innovations to nominal bond yields can be written as the sum of the innovation in a real bond yield and a component specific to nominal bonds:

$$\tilde{y}_t^{\$(n)} = \left[\eta_{r,t}^{(n)} + \eta_{ex,t}^{(n)}\right] + \left[\eta_{\pi,t}^{(n)} + \left(\eta_{ex,t}^{\$(n)} - \eta_{ex,t}^{(n)}\right)\right].$$

[3] The reported test statistics assume serially uncorrelated errors for (20) and use the Newey-West adjustment for seven lags of moving-average residuals for (22). Varying the number of Newey-West lags has almost no effect on the test statistics.

[4] Campbell et al. (2009) display realized betas using U.K. data for a shorter sample.

Then the conditional covariance of nominal yields with stock returns is

$$\begin{aligned}\mathrm{Cov}_t\left(ret_{M,t},\tilde{y}_t^{\$(n)}\right)=&\left[\mathrm{Cov}_t\left(ret_{M,t},\eta_{r,t}^{(n)}\right)+\mathrm{Cov}_t\left(ret_{M,t},\eta_{ex,t}^{(n)}\right)\right]+\\&\left[\mathrm{Cov}_t\left(ret_{M,t},\eta_{\pi,t}^{(n)}\right)+\mathrm{Cov}_t\left(ret_{M,t},\eta_{ex,t}^{\$(n)}-\eta_{ex,t}^{(n)}\right)\right].\end{aligned}\tag{23}$$

The first square bracket on the right side contains covariances with innovations in real yields. The second square bracket contains covariances with nominal-specific components. The evidence presented here tells us this first term must move together with the second term.

3.4 Revisiting the benchmark LRR model

A glance at Table 1 makes clear the benchmark LRR model is inconsistent with the evidence presented above. The sign of the covariance between stock returns and real yields is positive regardless of the regime. In the LRR model, news of higher expected consumption growth raises both stock prices and expected future short-term real rates. Thus real bonds are always hedging assets. News of higher volatility lowers both stock prices, through higher risk premia, and real yields, through precautionary savings and lower risk premia.

Another generic feature of the LRR framework in Section 2.3 is the limited scope for variations in real rates. Two key features of the long-run risk approach are (a) variations in expected consumption growth are small but persistent, and (b) the EIS is high. In combination, these features imply that short-term real rates do not vary much over time. From quarter to quarter, news about expected future ex-ante real rates is small. Thus innovations in nominal bond yields must be driven primarily by news about expected inflation or expected excess turns. A more detailed discussion of the drivers of nominal yields follows.

4 How big is news about expected inflation?

Equation (8) decomposes innovations in nominal yields into news about expected inflation, news about expected real rates, and news about expected excess returns–all over the life of the bond. There is a corresponding variance decomposition,

$$\begin{aligned}\mathrm{Var}_t\left(\tilde{y}_t^{\$(n)}\right) =& \mathrm{Var}_t\left(\eta_{\pi,t}^{(n)}\right) + \mathrm{Var}_t\left(\eta_{r,t}^{(n)}\right) + \mathrm{Var}_t\left(\eta_{ex,t}^{\$(n)}\right) \\ &+ 2\mathrm{Cov}_t\left(\eta_{\pi,t}^{(n)}, \eta_{r,t}^{(n)}\right) + 2\mathrm{Cov}_t\left(\eta_{\pi,t}^{(n)}, \eta_{ex,t}^{\$(n)}\right) + 2\mathrm{Cov}_t\left(\eta_{r,t}^{(n)}, \eta_{ex,t}^{\$(n)}\right). \end{aligned} \tag{24}$$

The first subsection below discusses empirical evidence concerning this decomposition. We then turn to properties of (24) implied by macro-finance models that attempt to explain time-varying comovement between stock returns and nominal yields.

4.1 Empirical evidence

This subsection draws on some results in Duffee (2017). In that paper I construct quarterly estimates of long-horizon expected inflation for the period 1968Q4 through 2013Q4. The forecasting tool is a trend-cycle model. Inflation in quarter t is the sum of three components. One is a martingale, another follows a persistent stationary process, and the third is a serially uncorrelated shock. The model's parameters are estimated using consensus inflation forecasts over short horizons–up to six quarters ahead–from surveys. Therefore the expectations should be interpreted as those of survey respondents.

The top panel of Figure 4 illustrates how well the model-implied forecasts line up with consensus forecasts of long-horizon inflation from the same survey respondents. (The model model extrapolates short-horizon survey forecasts because they are available more often than long-horizon survey forecasts and because surveys do not ask about all of the horizons of interest.) The black line is model-implied forecasts at of expected CPI inflation from five years ahead to ten years ahead, based on short-horizon Blue Chip consensus forecasts. The blue circles are Blue Chip consensus forecasts of inflation over the same horizon. They are almost indistinguishable from the model forecasts. The only noticeable deviations are in the early 1980s, when the Blue Chip long-horizon forecast is for GDP inflation rather than CPI inflation.

The bottom panel of Figure 4 illustrates one of the conclusions of Duffee (2017). Nominal bond yields fluctuate much more than do measures of expected inflation over the life of the bond. The solid line is expected average inflation over the next ten years and the dashed line is the ten-year nominal yield. A more precise statement of the visual evidence in Figure 4 is that innovations in bond yields, defined in an accounting sense by (8), are largely driven by the two components *other* than news about expected inflation over the life of the bond. I use the estimated trend-cycle model to calculate quarterly news about expected future inflation, denoted $\eta_{\pi,t}^{\$(n)}$ in (9). This is the innovation, from quarter $t-1$ to quarter t, in average expected inflation from quarter t to quarter $t+k$. I estimate the innovations for five-year ($k=20$) and ten-year ($k=40$) horizons. These innovations are calculated for every quarter from 1968Q4 through 2013Q4.

Table 3 reports standard deviations of the expected-inflation news. It also reports standard deviations of quarterly changes in five-year and ten-year nominal yields. These yields are nearly indistinguishable from martingales, thus the standard deviations are very close to the standard deviations of the left side of (8). The table reveals that for the full sample 1968Q4 through 2013Q4, as well as for every interesting subsample, standard deviations of yield innovations substantially exceeded standard deviations of news about expected inflation. The former are typically more than twice as large as the latter. Cram (2016) and Bauer and Rudebusch (2017) confirm and extend this evidence.

4.2 The LRR benchmark

Two key features of the long-run risk approach of Bansal and Yaron (2004) are (a) shocks to expected consumption growth are small, and (b) the EIS is high. In combination, these features imply that short-term real rates do not vary much over time. From quarter to quarter, news about expected future real rates is small. Duffee (2017) explains why news about expected excess bond returns is also small in plausible specifications of Bansal and Yaron's setup. Shocks to conditional variances are small, and the effects of these shocks on bond risk premia are proportional to average bond risk premia—which are also small.

Therefore LRR models that attempt to match the observed volatilities of innovations to nominal yields must have substantially more volatility of expected-inflation news than we observe in the data. Table 3 reports quarterly standard deviations of news about expected inflation over five-year and ten-year horizons for various models. Standard deviations implied by the model of Burkhardt and Hasseltoft (2012) are two to three times larger than those implied by the SPF consensus forecasts. Standard deviations specific to each regime in Song

(2017) are similar.[5] Table 3 reports that Song's model generates standard deviations of quarterly nominal yield innovations similar to those observed in the data. However, it does so almost entirely through news about expected inflation. The ratio of the first term on the right of (24) to the total variance on the left is close to one for all regimes.[6]

Table 3 also reports standard deviations of expected-inflation news and nominal yield innovations for the model of David and Veronesi (2013). The standard deviations for news about expected inflation are in line with the standard deviations from SPF estimates. Hence not surprisingly, the standard deviations of nominal yield innovations are smaller than we observe in the data. Yet the ratio of (expected-inflation news volatilities) to (yield innovation volatilities) are well below one. David and Veronesi's model generates much more news about expected future real rates than does a benchmark LRR model–even though David and Veronesi use power utility! How is this possible?

4.3 Non-standard sources of variation in real rates

4.3.1 Money illusion

David and Veronesi's (hereafter D/V) model has a representative agent who consume aggregate consumption C_t. The agent's per-period utility is

$$U_t = e^{-\rho t} \frac{C_t^{1-\gamma}}{1-\gamma}.$$

Denote the price level by Q_t. Then the usual real and nominal stochastic discount factors (SDFs), or ratio of marginal utility at the future date τ to the marginal utility at the current date t, are

$$\frac{M_\tau}{M_t} = e^{-\tau(\tau-t)} \left(\frac{C_\tau}{C_t}\right)^{-\gamma}, \tag{25}$$

$$\frac{M_\tau^\$}{M_t^\$} = e^{-\tau(\tau-t)} \left(\frac{C_\tau}{C_t}\right)^{-\gamma} \left(\frac{Q_t}{Q_\tau}\right). \tag{26}$$

[5] Thanks to Dongho for sharing the standard deviations underlying his Table E-7. The table does not report numbers for the ten-year horizon.

[6] Calculating standard deviations for B/H is on the to-do list.

With the real SDF (25), real bond prices are

$$P_t^{(n)} = e^{-\rho n} E_t \left(\frac{C_{t+n}}{C_t} \right)^{-\gamma}. \tag{27}$$

In D/V, log consumption is normally distributed with constant drift and diffusion. Therefore the price of an n-maturity bond calculated using (27) is constant over time. The real yield curve is fixed.

However, D/V do not use these SDFs to price assets. Instead, they follow Basak and Yan (2010) by assuming investors have a form of money illusion. The extreme version of money illusion in Basal and Yan is that investors discount real cash flows using the nominal SDF (26). More generally, the real and nominal SDFs have a degree of money illusion indexed by δ,

$$\frac{M_\tau}{M_t} = e^{-\tau(\tau-t)} \left(\frac{C_\tau}{C_t} \right)^{-\gamma} \left(\frac{Q_t}{Q_\tau} \right)^{\delta}, \tag{28}$$

$$\frac{M_\tau^{\$}}{M_t^{\$}} = e^{-\tau(\tau-t)} \left(\frac{C_\tau}{C_t} \right)^{-\gamma} \left(\frac{Q_t}{Q_\tau} \right)^{1+\delta}. \tag{29}$$

D/V use an estimate $\hat{\delta} = 0.8$. Therefore all else constant, increase in expected inflation of one percentage point raises real interest rates by 80 basis points. Nominal interest rates increase by 180 basis points. Nominal bonds are priced as leveraged bets on inflation.

This form of money illusion has a variety of useful empirical implications. Real and nominal yields move together, with nominal yields more volatile than real yields. (In D/V, they are perfectly correlated.) A sign change in the covariance between firms' expected cash flows and expected inflation produces a sign change in covariances of both real and nominal yields with stock returns. Yet the relevance of money illusion to pricing real bonds is not obvious. The best source of data is the U.K. Barr and Campbell (1997) estimate that monthly innovations in one-year real rates and one-year inflation expectations are negatively correlated, a result inconsistent with money illusion. (Monthly correlations of innovations for the ten-year horizon are close to zero.) Some visual evidence is in Figure 5, which displays three-year nominal and real yields for the Bank of England. It also displays annual inflation for the year beginning with month t. Realized annual inflation appears to be more closely tied to the nominal yield than the real yield, especially during the late 1980s/early 1990s and the 1995–2000 period.

4.3.2 Preference shocks

Albuquerque, Eichenbaum, Luo, and Rebelo (2016) argue that shocks to investors' time rate of preference account for substantial variation in riskfree rates. Denoting the time rate of preference by λ_t, the LRR version of their specification can be written as

$$U_t = \left[(1-\delta)\,\lambda_t C_t^{\frac{1-\gamma}{\theta}} + \delta \left(E_t \left[V_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}. \tag{30}$$

This specification generalizes Equation (16). Asset prices are affected by the relative change in the time rate of preference. The log change is the preference state variable, denoted

$$\log\left(\lambda_{t+1}/\lambda_t\right) = \Lambda_{t+1}.$$

This change is known a period in advance. The dynamics are

$$\Lambda_{t+1} = \rho_\Lambda \Lambda_t + \sigma_\Lambda \epsilon_{\Lambda,t}. \tag{31}$$

When Λ_t is high, investors place a relatively high value on consumption at $t+1$. This produces a high desire to save, lowering the riskless rate. The persistence of real rates owing to preference shocks is determined by ρ_Λ.

Schorfheide, Song, and Yaron (2017) perform a detailed econometric analysis of the LRR model. One of their conclusions is that preference shocks as described here are necessary to explain much of the observed variation in risk-free rates. Albuquerque et al. (2016) estimate the persistence parameter in (31) to be almost indistinguishable from one, producing a near unit-root in the riskfree rate. Preference shocks help explain the observed high volatility of bond-yield innovations relative to the volatility of expected-inflation innovations, as discussed by Cram (2016).

In addition, their dynamics can be reverse-engineered to explain the observed time-variation in covariances between stock returns and both nominal and real rates. The necessary modeling assumption is that the countercyclical regime described in Section 2.3 is characterized by both a negative covariance between expected consumption growth and expected inflation *and* a positive covariance between expected consumption growth and the preference shock in (31). The latter covariance must be sufficiently large to overcome the standard positive effect of expected consumption growth on the riskfree rate. Then in the countercyclical regime, higher expected consumption growth is accompanied by lower real yields, lower expected inflation, and thus lower nominal yields. Nominal yields incorporate both real yields and inflation, thus they covary more (in absolute value) with stock returns than do real yields.

A qualitatively similar reduced-form model is described by Campbell, Sunderam, and Viceira (2017). Rather than describe preferences or cash flows to equity, Campbell et al. (2017) proceed directly to the real SDF. They model the log SDF from period t to $t+1$ as homoskedastic. A stripped down version of their model writes the return to equity as perfectly negatively correlated with the SDF, implying that equity has the maximum Sharpe ratio in the economy. The equations are

$$m_{t+1} = -r_t - \frac{\sigma_m^2}{2} - \epsilon_{m,t+1}, \tag{32}$$

$$r_{equity,t+1} = E_t(r_{equity,t+1}) + \sigma_e \epsilon_{m,t+1}, \quad \sigma_e > 0, \tag{33}$$

where r_t is the one-period riskless rate and the innovation is homoskedastic.

The riskfree rate dynamics are (again, in this stripped-down version) are

$$r_{t+1} = \overline{r}(1-\phi_r) + \phi_r r_t + \psi_t \epsilon_{r,t+1}. \tag{34}$$

Innovations to the riskfree rate are heteroskedastic because of the presence of the state variable ψ_t. The innovation $\epsilon_{r,t}$ is homoskedastic with fixed covariance to the SDF innovation,

$$\mathrm{Cov}_t(\epsilon_{m,t}, \epsilon_{r,t}) = \sigma_{mr} > 0. \tag{35}$$

The state variable ψ_t has AR(1) dynamics

$$\psi_{t+1} = \overline{\psi}(1-\phi_\phi) + \phi_\psi \psi_t + \epsilon_{\phi,t+1}. \tag{36}$$

Since ψ_t can change sign, the conditional covariance between the shock to equity and the shock to the riskfree rate can change sign. Finally, specify the process for inflation as

$$\pi_{t+1} = \overline{\pi} + \xi_t + \epsilon_{\pi,t+1}, \tag{37}$$

where the time-varying component of the conditional mean of inflation is shares the heteroskedasticity of the riskfree rate,

$$\xi_{t+1} = \psi_\xi \xi_t + \psi_t \epsilon_{\xi,t+1}. \tag{38}$$

The covariance between the homoskedastic innovations in (38) and (34) is fixed and positive,

$$\mathrm{Cov}_t(\epsilon_{r,t}, \epsilon_{\xi,t}) = \sigma_{r\xi} > 0. \tag{39}$$

When the state variable $\psi_t < 0$, expected inflation is countercyclical. An unexpected drop in the SDF (an unexpected increase in equity valuation) corresponds to a decline in expected inflation. It also corresponds to a decline in the riskfree rate. Real and nominal bonds are therefore both speculative assets, rising in value when the economy does well. When $\psi_t > 0$, expected inflation is procyclical, as is the riskfree rate. Real and nominal bonds bonds are both hedging assets.

In one sense, this model successfully explains the variation over time in conditional covariances between stock returns and both real and nominal yields. It has all the required moving parts. But treating the riskfree rate (or preference shocks) as an exogeneous state variable with properties tailored to produce the desired outcome is something less than satisfactory.

5 Consumption-based models of real-rate variation

Denoting the marginal utility of wealth by MU_t, the Euler equation for a one-period real bond satisfies

$$r_t \equiv -\log P_t^{(1)} = -\log \left[E_t \left(\frac{MU_{t+1}}{MU_t} \right) \right]. \tag{40}$$

Here, marginal utility is viewed from the perspective of some fixed past date t_0. Equation (27) is the power utility version of (40). Consumption-based models have various channels through which real rates can vary.

5.1 Trend and mean-reverting shocks to consumption

With recursive utility and conditional log-normality of both consumption growth and the return to total wealth, standard calculations produce

$$r_t = (\text{time rate of preference}) + \frac{1}{EIS} E_t \left(\Delta c_{t+1}\right) - \text{Jensen's inequality terms.} \tag{41}$$

The sign of the covariance between shocks to the short-term real rate and shocks to consumption is determined by consumption dynamics. Assume that consumption has both difference-stationary and trend-stationary components shocks. Log consumption follows

$$c_{t+1} - c_t = \overline{g} + x_t + \epsilon_{c,t+1} - (1-\phi) \sum_{i=0}^{\infty} \phi^i \epsilon_{c,t-i}, \quad \epsilon_{c,t} \sim N(0, \sigma_{c,t}^2), \tag{42}$$

where $x_{c,t}$ is the standard LRR part of conditional expected consumption growth,

$$x_{t+1} = \theta x_t + \epsilon_{x,t+1}, \quad \epsilon_{x,t} \sim N(0, \sigma_{x,t}^2), \tag{43}$$

and the shocks $\epsilon_{c,t}$ produce trend-stationary innovations to consumption, in the sense that the effect of the period-t shock on long-distant log consumption $c_{t+\tau}$ goes to zero.

This consumption process allows for both positive and negative correlations between aggregate stock returns and real yields. Positive realizations of the shocks $\epsilon_{c,t}$ and $\epsilon_{x,t}$ are good news for both agents and for equity valuations, given plausible utility parameters. With the former shock, current and expected future consumption both rise. With the latter, expected future consumption rises (as well as expected future growth rates of consumption). Yet the former shock lowers the riskfree rate at t, as investors foresee future decreases in consumption growth rates. The latter shock increases the riskfree rate at t, while the latter increases the riskfree rate at t. The idea that the relative importance of these two types of shocks might underlie time-varying covariances between stock returns and real yields dates to at least Campbell et al. (2009).

A straightforward way to augment this real model with inflation is through a Taylor rule. Since (42) and (43) do not depend on a monetary policy rule, inflation is neutral here. The simplest Taylor rule in this setting is

$$i_t = \bar{i} + \alpha(\pi_t - \pi^*), \tag{44}$$

where i_t is the one-period nominal rate and the coefficient α satisfies the Taylor stabilization principle of $\alpha > 1$. Combine this with the Fisher equation

$$i_t = r_t + E_t(\pi_{t+1}) \tag{45}$$

to produce

$$\alpha(\pi_t - \pi^*) = E_t\left(\pi_{t_1} - \pi^*\right) + (r_t - \bar{r}). \tag{46}$$

The model is closed with the dynamics of the riskfree rate. A simple special case is homoskedasticity of both types of consumption shocks, as well as a shared decay rate $\theta = \phi < 1$. In this case the riskfree rate follows an AR(1) process,

$$r_{t+1} = \bar{r} + \phi(r_t - \bar{r}) + v_{t+1}, \quad v_{t+1} = (1/EIS)(\epsilon_{x,t+1} - (1-\phi)\epsilon_{c,t+1}). \tag{47}$$

Following, say, Davig and Leeper (2007), who solve a regime-shifting version of this model, the solution–in other words, a specification of inflation in terms of the riskfree rate–is

$$\pi_t - \pi^* = \frac{1}{\alpha - \rho}(r_t - \bar{r}). \tag{48}$$

Since $\alpha > 1$ and $\rho < 1$, inflation moves in the same direction as the riskfree rate. Therefore variations in nominal yields are magnified versions of varitions in real yields.

Viewed through the lens of this model, variations over time in the covariance between stock returns and bond yields is driven by variations in the relative volatilities of difference-stationary and trend-stationary shocks. The 1970s and 1980s are characterized by largely trend-stationary shocks, while the late 1990s and 2000s are characterized by largely difference-stationary shocks. In both periods, the covariation of nominal yields exceeds (in absolute value) the covariation with real yields because of an aggressive monetary policy rule.

The endowment process of consumption (42) and (43) can be motivated in a production economy with capital, in which there are both difference-stationary and trend-stationary shocks. Liu (2017) constructs and estimates this type of model. Other motivations are easy to imagine. For example, the difference-stationary component could be productivity shocks in endogenous growth model, while the trend-stationary component could be something as simple as government spending shocks.

Difference-stationary shocks have the flavor of long-run influences on the economy, while trend-stationary shocks are shorter-term shocks. If so, the former type of shock should affect current and expected future real rates into the distant future, while the latter type of shock should have a much greater effect on current real rates than it has on expected future real rates. This intuition suggests that when the former type of shock dominates, long-rate volatility should be close to short-rate volatility. When the latter type dominates, short-rate volatility should be high relative to long-rate volatility.

This intuition is consistent with the evidence in Table 4 and Figure 6. I construct rolling estimates of conditional standard deviations of daily changes in three-month and ten-year nominal Treasury yields. The sample period is 1961 through 2016. The conditioning procedure is similar to that used for conditional covariances described in Section 3.3. The two time series of standar deviations are plotted in the top panel of Figure 6. The bottom panel displays the difference—short less long—along with conditional covariances between daily U.S. aggregate stock returns and daily changes in the ten-year bond yield.

The pattern in the bottom panel is striking. Periods of relatively high (low) short-maturity standard deviations correspond to periods of relatively low (high) covariances between stock returns and the ten-year yield. The relation is visually strong but clearly not linear. For example, the spread between the three-month and ten-year volatilities is very high in late 1973 and 1974 (the oil crisis), while the covariance is only slightly less than zreo. Fifteen years later the spread in volatilities was similarly high, and the covariance is much lower.

Nonetheless, Table 4 reports results of estimating a linear relation; regressing the conditional covariance on the short–long difference. The estimated coefficient is negative for the full sample as well as in early and late subsamples. The results are statistically strong only for the full sample. Standard errors are from GMM estimation with seven Newey-West lags, as in Section 3.3.

5.2 Some puzzling evidence

The core idea in the previous subsection is that for part of the sample–say, in the 1970s–the news investors receive at time t about macroeconomic growth differs in its dynamic implications from the implications of macroeconomic news in another part of the sample–say, the 2000s.

This section discusses related evidence from the Survey of Professional Forecasters (SPF) predictions of GDP. Each quarter, beginning in 1968Q4, respondents forecast the quarter-to-quarter percentage change in GDP for the current quarter (the nowcast) and the next four quarters ahead. Therefore successive forecasts allow the construction of model-free innovations in forecasts. The quarter-t nowcast less the quarter-$(t-1)$ one-quarter-ahead forecast is the nearest-horizon forecast, while the quarter-t three-quarter-ahead forecast less the quarter-$(t-1)$ four-quarter-ahead forecast is the longest available horizon forecast.

I use consensus forecasts to produce a panel of real GDP growth forecast innovations, from 1969Q1 through 2016Q2.[7] I construct the covariance matrix of innovations for three nonoverlapping time periods, then calculate principal components for each period. The ending point of the first sample is 1996Q4, which is the date at which the sign change in the stock-bond correlation occurs. The ending point of the second sample is the beginning of the financial crisis, and the ending point of the third sample is the end of the available data. The loadings of the first two principal components are all plotted in Figure 7.

The plots are almost identical across the three samples. The first principal component is a positive shock to GDP at each forecast horizon. The magnitude of the shock drops montonically and swiftly from the nowcast to the three-quarter-ahead forecast, which is almost unaffected by the shock. The second principal component is a negative shock to the nowcast, accompanied by positive shocks at horizons one through three quarters ahead.

The first PC can be labeled a short-lived shock to expected growth rates, or a difference-stationary shock. The second PC is harder to characterize. A negative shock to the nowcast GDP growth is close to completely offset by a positive shock to expected one-quarter-ahead growth. If there were no loadings of the PC on longer-horizon forecast innovations, this first PC would be a trend-stationary shock. But the longer-horizon forecasts of GDP growth move in the same direction as the one-quarter-ahead forecast innovation.

[7] A few individual forecasts are missing in the early part of the sample.

Table 5 contains puzzling evidence about these principal components. It reports the magnitude of the PCs and their correlations with stock returns and changes in bond yields. A positive realization of the first PC—positive innovations in forecasted future real GDP at all horizons—corresponds to higher stock prices, higher nominal yields, and (for the most part) higher real yields. These correlations hold across all three samples, and are precisely the patterns implied by the previous subsection's model. Higher expected growth pushes up stock prices, real rates, and, through the Taylor rule, nominal rates.

Correlations involving the second PC are much more difficult to interpret. A positive realization of the second PC—a negative innovation to the real GDP nowcast, and positive innovations to expected future GDP growth—corresponds to higher stock prices, lower nominal yields, and lower real yields. The stock-price reaction is plausible; cash flow news from higher expected future GDP growth more than offsets the news of the lower nowcast. But the sign of the change in the real yield contradicts standard intuition. The second PC unambiguously raises future prospects relative to the present. This type of shock should raise real rates, as investors attempt to borrow from the future. Nominal yields change in the same way. Given the behavior of the real rate, this change is consistent with the toy Taylor rule model of the previous section.

Understanding the properties of the second PC may be the key to understanding why the comovement between stock returns and bond yields changes over time. The first PC generates a positive comovement between stock returns and yields, while the second generates a negative comovement. Table 5 reports that the magnitude of the second PC is much greater during the pre-1997 sample than during either post-1996 samples. This evidence suggests that the sign change in the comovement is a consequence of the decline over time in the magnitude of second-PC type shocks.

5.3 Habit formation

Readers might be encouraged by the evidence in the previous subsection. It suggests that time-varying stock-bond comovement are related to fundamentals such as shocks to economic growth rates. But we are far removed from a coherent explanation grounded in a consumption-based preference framework. As discussed in Section 4.2, long-run risk models with high EIS produce only small shocks to real yields. This model feature, combined with the evidence that little of the volatility of shocks to nominal yields is attributable to inflation, leads us to consider alternative consumption-based frameworks.

Habit formation preferences are a natural choice because of their flexibility in specifying the determinants of real rates. Campbell and Cochrane (1999) introduce a state variable called surplus consumption that affects marginal utility. Surplus has the AR(1) dynamics

$$s_{t+1} = (1 - \theta_0)\overline{s} + \theta_0 s_t + \lambda(s_t)\epsilon_{c,t+1}, \tag{49}$$

where the shock $\epsilon_{c,t+1}$ is the contemporaneous shock to consumption growth. The volatility function $\lambda(s_t)$ is an inverse function of surplus. A brief review of the riskfree rate mechanics is helpful. A bad shock to consumption corresponds to a bad shock to surplus consumption. Marginal utility is high both because of the unexpected decline in consumption and the unexpected decline in surplus.

The mean reverting process of (49) implies that the effect of the shock is expected to die out over time. This effect drives up real rates, since investors anticipate better times (higher surplus) ahead. However, low surplus also raises the volatility of surplus $\lambda(s_t)$. This raises the desire for precautionary savings, driving down real rates. The extension of Campbell/Cochrane preferences by Wachter (2006) allows for parameterizations that make the net effect either positive or negative.

Campbell, Pflueger, and Viceira (2015), attempting to explain time-varying comovement between stocks and bonds, generalize these preferences even further by specifying a more general functional form for surplus. They use

$$s_{t+1} = (1 - \theta_0)\overline{s} + \theta_0 s_t + \theta_1 x_t + \theta_2 x_{t-1} + \lambda(s_t)\epsilon_{c,t+1}, \tag{50}$$

where x_t is the demeaned output gap at period t. This specification allows expected future real rates at different horizons to react non-monotonically to shocks. For example, a bad shock at $t+1$ lowers surplus at $t+1$ through the final term in (50). It also lowers expected surplus at $t+2$ through the AR(1) component (the second term on the right) and the first lag of the output gap (the third term). The parameters determine whether surplus is expected to rise or fall from $t+1$ to $t+2$. The same logic applies to expected surplus at $t+3$, working through the second lag of the output gap. For example, the parameters can be chosen such that the bad shock at $t+1$ lowers the one-period real yield, as investors anticipate even lower surplus ahead, and raises the two-period real yield, as investors anticipate that surplus will rise rapidly after the next period.

The tradeoff created by specifications such as (50) is standard. Greater flexibility can create an opportunity to explain the time-variation in stock–bond comovement, but the inherent reverse engineering forces researchers to look beyond the stock–bond comovement in order to truly test the model.

6 Concluding comments

To be written ...

References

Albuquerque, Rui, Martin Eichenbaum, Victor Xi Luo, and Sergio Rebelo, 2016, Valuation risk and asset pricing, *Journal of Finance* 71, 2861-2903.

Baele, Lieven, Geert Bekaert, and Koen Inghelbrecht, 2010, The determinants of stock and bond return comovements, *Review of Financial Studies* 23, 2374–2428.

Bansal, Ravi, and Ivan Shaliastovich, 2013, A long-run risks explanation of predictability puzzles in bond and currency markets, *Review of Financial Studies* 26, 1-33.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: a potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481-1509.

Barr, David F., and John Y. Campbell, 1997, Inflation, real interest rates, and the bond market: a study of UK nominal and inflation-linked government bond prices, *Journal of Monetary Economics* 39, 361-383.

Basak, Suleyman, and Hongjun Yan, 2010, Equilibrium asset prices and investor behaviour in the presence of money illusion, *Review of Economic Studies* 77, 914–936.

Bauer, Michael D., and Glenn D. Rudebusch, 2017, Interest rates under falling stars, Working paper, Federal Reserve Bank of San Francisco.

Burkhardt, Dominic, and Henrik Hasseltoft, 2012, Understanding asset correlations, Swiss Finance Institute Research Paper 12-38.

Campbell, John Y., and John Ammer, 1993, What moves the stock and bond markets? A variance decomposition for long-term asset returns, *Journal of Finance* 48, 3-37.

Campbell, John Y., and John H. Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205-251.

Campbell, John Y., Carolin Pflueger, and Luis M. Viceira, 2015, Monetary policy drivers of bond and equity risks, Working Paper.

Campbell, John Y., Robert J. Shiller, and Luis M. Viceira, 2009, Understanding inflation-indexed bond markets, *Brookings Papers on Economic Activity*, Spring, 79-120.

Campbell, John Y., Adi Sunderam, and Luis M. Viceira, 2017, Inflation bets or deflation hedges? The changing risks of nominal bonds, *Critical Finance Review* 6, 263–301.

Connolly, Robert, Chris Stivers, and Licheng Sun, 2005, Stock market uncertainty and the stock-bond return relation, *Journal of Financial and Quantitative Analysis* 40, 161–194.

Cram, Roberto Gómez, 2016, How important are inflation expectations for the nominal yield curve, Working paper, Wharton.

David, Alexander, and Pietro Veronesi, 2013, What ties return volatilities to price valuations and fundamentals?, *Journal of Political Economy* 121, 682–746.

d'Addona, Stefano, and Axel H. Kind, International stock-bond correlations in a simple affine pricing model, *Journal of Banking and Finance* 30, 2747–2765.

D'Amico, Stefania, Don H. Kim, and Min Wei, 2017, Tips from TIPS: The informational content of Treasury Inflation-Protected Security prices, *Journal of Financial and Quantitative Analysis*, forthcoming.

Davig, Troy, and Eric M. Leeper, 2007, Generalizing the Taylor principle, *American Economic Review* 97, 607–635.

Duffee, Gregory R., 2017, Expected inflation and other determinants of Treasury yields, *Journal of Finance*, forthcoming.

Fleming, Jeff, Chris Kirby, and Barbara Ostdiek, 2003, The economic value of volatility timing using "realized" volatility, *Journal of Financial Economics* 67, 473–509.

Li, Lingfeng, 2002, Macroeconomic factors and the correlation of stock and bond returns, Yale ICF Working Paper 02-46.

Liu, Yunting, 2017, The real and nominal determinants of stock and bond returns comovement, Working paper, Johns Hopkins.

Piazzesi, M., Schneider, M. (2007). "Equilibrium yield curves". NBER Macroeconomics Annual 2006. MIT Press, Cambridge, MA.

Schorfheide, Frank, Dongho Song, and Amir Yaron, 2017, Identifying long-run risks: a Bayesian mixed-frequency approach, *Econometrica*, forthcoming.

Song, Dongho, 2017, Bond market exposures to macroeconomic and monetary policy risks, *Review of Financial Studies* 30, 2761–2817.

Wachter, Jessica A., 2006, A consumption-based model of the term structure of interest rates, *Journal of Financial Economics* 79, 365-399.

Table 2. Conditional covariances between stock returns and changes in government bond yields

Second moments of daily aggregate stock returns and daily changes in nominal and real bond yields are calculated for 44-day nonoverlapping rolling samples. Denote these by $\Omega t, i, j$, where i and j can be "s" for stocks, "n" for nominal bond yields, and "r" for real bond yields. Conditional covariances between stock returns and changes in bond yields are projections of sample covariances on lagged second moments. The table reports the instruments (which second moments and the number of lags) and the R^2 of the projections. The table also reports the results of regressing conditional covariances with nominal yields on conditional covariances with real yields. Generalized method of moments estimation produces asymptotic standard errors on the regression coefficient, using a Newey-West adjustment with seven lags.

			R^2s		Regression of nominal covar on real covar	
			Nominal	Real		
Obs	Instruments	Lags	Covar	Covar	Coef	R^2
A. 2003–2016, US stock market, Treasury nominal and real bonds						
75	$\Omega_{t,s,n}, \Omega_{t,s,r}$	6	0.59	0.49	2.13	0.77
					(0.51)	
B. 1985–2016, US stock market, Treasury nominal, Bank of England real bonds						
179	$\Omega_{t,s,n}, \Omega_{t,s,r}$	6	0.52	0.48	1.91	0.69
					(0.65)	
181	$\Omega_{t,s,n}, \Omega_{t,s,r}, \sqrt{\Omega_{t,s,s}},$	4	0.52	0.51	1.74	0.61
	$\sqrt{\Omega_{t,n,n}}, \sqrt{\Omega_{t,r,r}}$				(0.60)	
C. 1985–2016, UK stock market, Bank of England nominal and real bonds						
179	$\Omega_{t,s,n}, \Omega_{t,s,r}$	6	0.46	0.40	1.73	0.87
					(0.30)	
181	$\Omega_{t,s,n}, \Omega_{t,s,r}, \sqrt{\Omega_{t,s,s}},$	4	0.51	0.45	1.66	0.82
	$\sqrt{\Omega_{t,n,n}}, \sqrt{\Omega_{t,r,r}}$				(0.31)	

Table 3. Standard deviations of news about expected inflation and yield innovations

Quarterly shocks to average expected inflation over five-year and ten-year horizons are estimated by Duffee (2017) from a model that assumes inflation is the sum of a martingale and an AR(1) component. The table reports standard deviations of the shocks for various sample periods, along with standard deviations of quarterly changes in five-year and ten-year nominal Treasury yields. Also reported are corresponding standard deviations implied by three macro-finance models. Each has regime shifts that drive inflation dynamics and stock-bond correlations. Unconditional standard deviations of shocks (excluding Song) and regime-specific standard deviations are reported.

		Infl Horizon		Yields	
Source	Period	5 Years	10 Years	5-year	10-year
SPF estimates	1968Q4–2013Q4	23	21	61	56
	1968Q4–1979Q2	27	25	50	42
	1979Q3–1982Q4	33	25	128	124
	1983Q1–2008Q2	16	16	55	49
	2008Q3–2013Q4	8	7	42	44
Burkhardt and Hasseltoft (2012)	Unconditional, 1965–2011	74	41		
	Countercyclical infl regime	82	45		
	Procyclical infl regime	40	22		
Song (2017)	1963–2014				
	Countercyclical/Active Fed	79		71	
	Countercyclical/Passive Fed	104		99	
	Procyclical/Active Fed	44		54	
David and Veronesi (2013)	Unconditional, 1958–2010	26	19	47	33
	Regime 1*	12	10	21	17
	Regime 2	30	20	53	35
	Regime 3	17	12	30	20
	Regime 4	48	29	84	49
	Regime 5	9	8	16	14
	Regime 6	21	15	39	28

* States are unobserved by investors. Reported are conditional standard deviations for quarter t, conditioned on subjective probability that the true state at t is Regime i.

Table 4. Conditional second moments of daily changes in nominal bond yields and stock returns

Standard deviations of daily changes in three-month and ten-year nominal bond yields are calculated for 44-day nonoverlapping rolling samples. Sample covariances between daily aggregate stock returns and daily changes in the ten-year nominal yield are calculated for the same samples. Conditional standard deviations and the conditional covariance are projections of sample values on lagged standard deviations and the lagged covariance. Three lags are used, for a total of nine instruments. The table reports the R^2 of the projections. The table also reports the results of regressing the conditional covariance on the difference between the three-month conditional standard deviation and the ten-year conditional standard deviation. Generalized method of moments estimation produces asymptotic standard errors on the regression coefficient, using a Newey-West adjustment with seven lags.

		R^2s			Regression of covar on the SD spread	
Sample	Obs	3-month	10-year	Covar	Coef	R^2
1961–2016	320	0.70	0.63	0.52	−0.31	0.26
					(0.10)	
1961–1989	162	0.73	0.68	0.37	−0.13	0.20
					(0.08)	
1990–2016	154	0.32	0.44	0.48	-0.42	0.06
					(0.57)	

Table 5. Principal components of innovations in GDP forecasts and their relation to stocks and bonds

Quarterly innovations to forecasts of future growth of real GDP are constructed using consensus forecasts from the Survey of Professional Forecasters. Innovations are available for one through four quarters ahead. Principal components of these four time series are calculated for three sample periods. The table reports total variance of forecast innovations explained by the first two principal components. It also reports contemporaneous correlations of these PCs with three time series: the quarterly excess return to the CRSP value-weighted index, the change in ten-year nominal Treasury yield, and the change in the ten-year inflation-indexed bond issued by the Bank of England. Surveys are conducted in the middle of the quarter, therefore the stock and bond series are also mid-quarter to mid-quarter.

	Fraction of total Variance Explained		Contemporaneous Correlations					
			First PC			Second PC		
Sample	1st	2nd	Stock	Nominal	Real	Stock	Nominal	Real
1969Q1–1996Q4 (107 obs; 46 for the real bond)	0.58	0.29	0.23	0.30	0.15	0.19	−0.35	−0.19
1997Q1–2007Q4 (44 obs)	0.88	0.07	0.36	0.37	0	0.14	−0.11	−0.09
2008Q1–2016Q2 (34 obs)	0.94	0.04	0.73	0.35	0.14	0.18	−0.19	−0.35

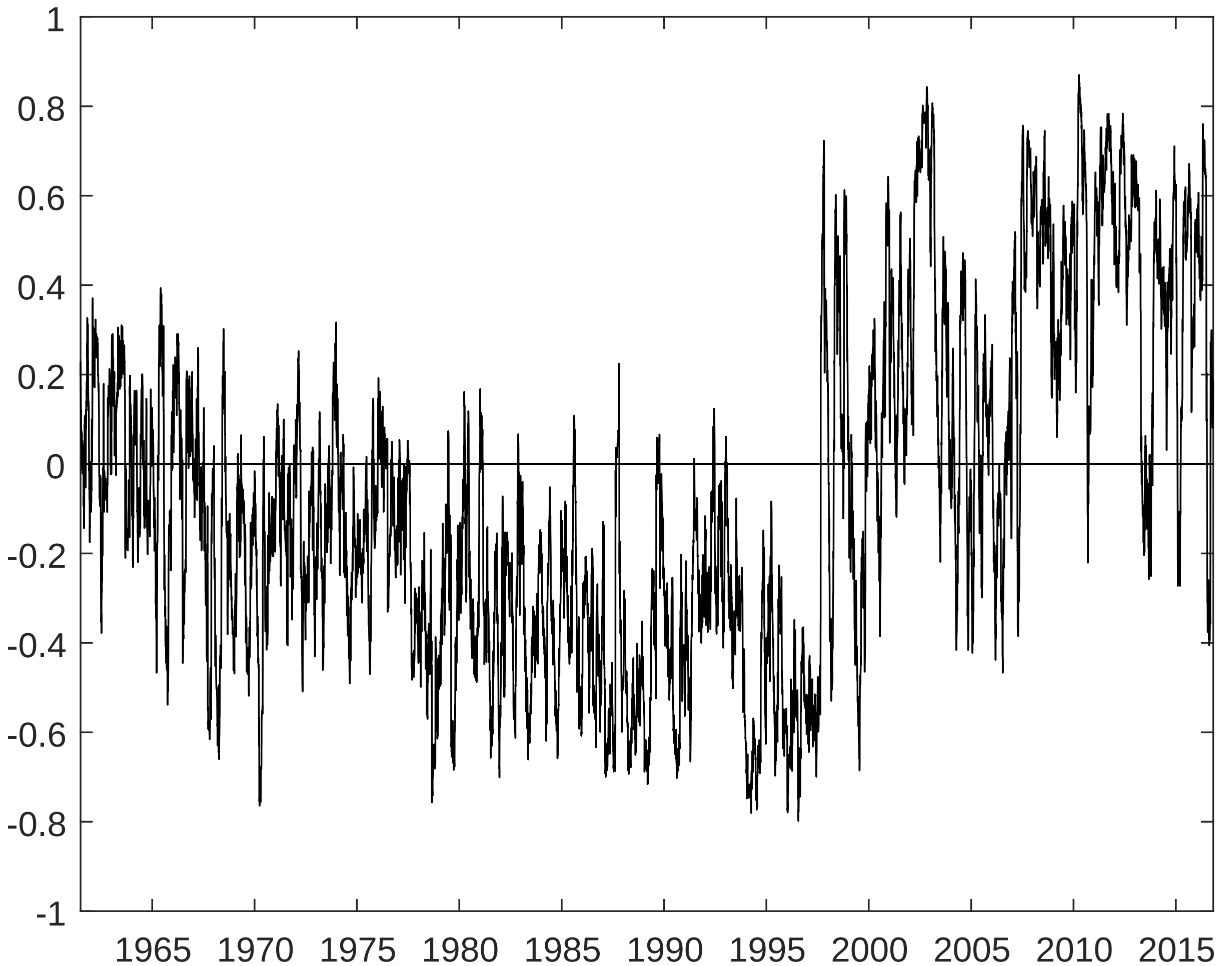

Figure 1. Rolling sample correlations between daily stock returns and changes in ten-year nominal Treasury yields

Contemporaneous correlations between the daily return to the U.S. aggregate stock market and the daily change in the yield on a ten-year Treasury coupon bond are constructed with overlapping 44-day (two-month) samples. The sample range is July 1961 through December 2016.

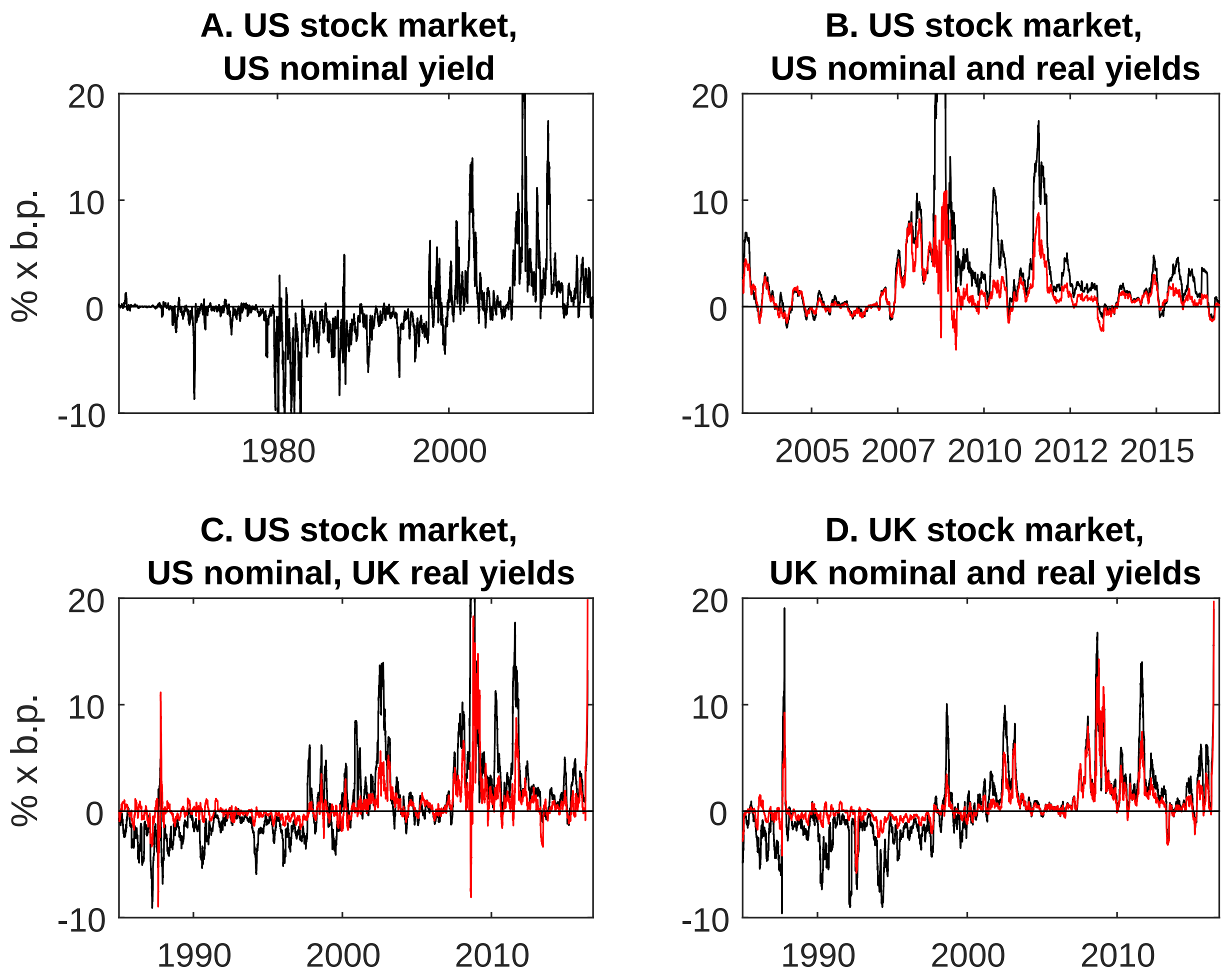

Figure 2. Rolling sample covariances between daily stock returns and changes in ten-year yields

Contemporaneous covariances between daily aggregate stock returns and daily changes in ten-year bond yields are constructed with overlapping 44-day samples. Panel A reports covariances from 1961 through 2016 for the U.S. stock market and a nominal Treasury bond. The period 2003 through 2016 in Panel B adds a Treasury inflation-indexed bond (in red). The period 1985 through 2016 in Panel C uses a nominal Treasury bond (in black) and an inflation-indexed Bank of England bond (in red). The same period in Panel D uses the daily return to the FTSE 100. The bonds are nominal (black) and inflation-indexed (red) Bank of England bonds.

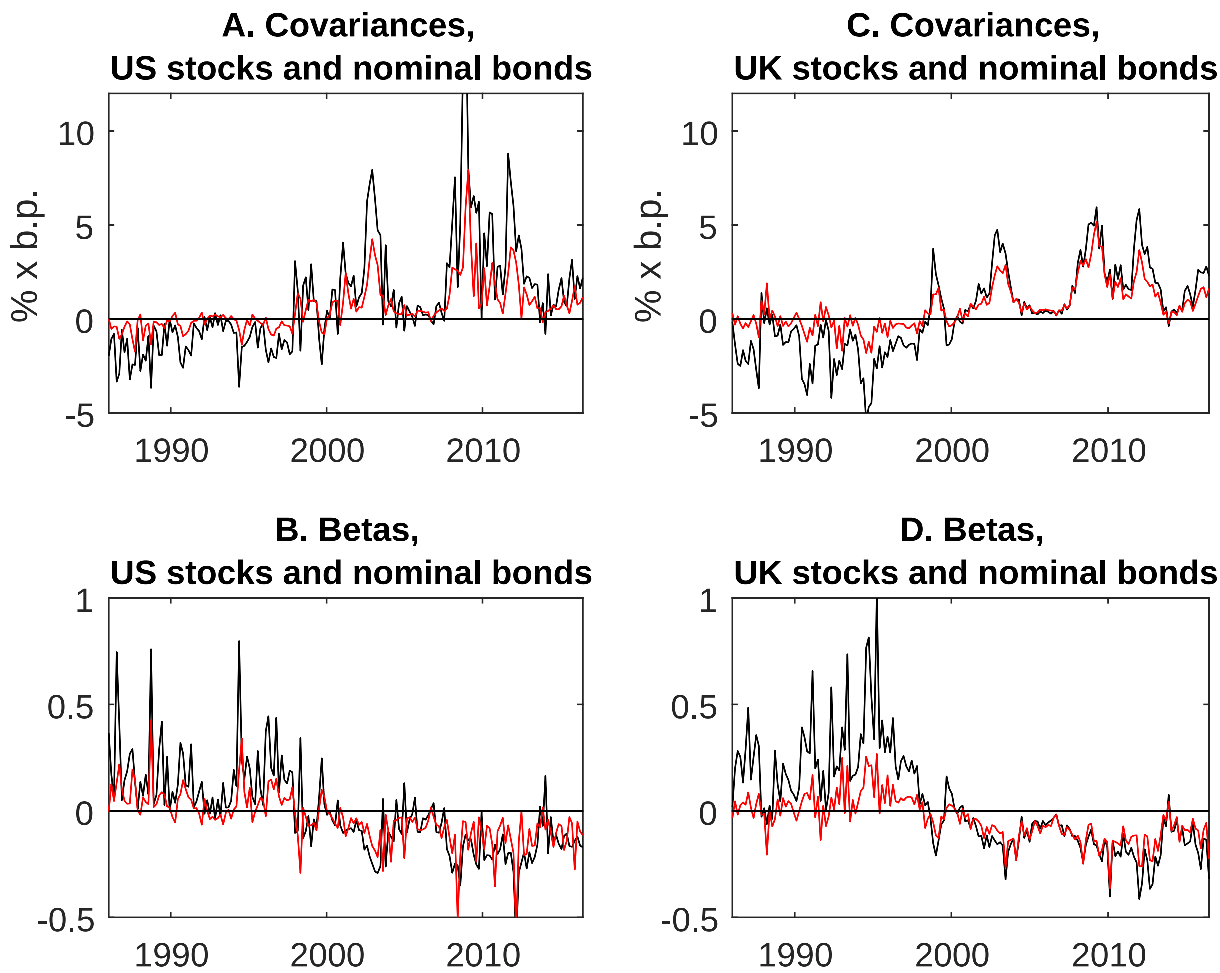

Figure 3. Conditional covariances with stock returns and market betas for nominal and real ten-year bonds

Conditional covariances between daily stock returns and changes in bond yields are projections of sample covariances on conditioning instruments. Panel A is constructed using U.S. stock returns, nominal Treasury yields, and inflation-indexed Bank of England yields. Panel C uses exclusively UK data. For Panels B and D, the yield covariances in Panels A and C are converted to return covariances using duration. Panels B and D report the stock-bond return conditional covariances divided by conditional variances of stock returns. Black lines are for nominal yields and red lines are for inflation-indexed yields. The sample is 1985 through 2016.

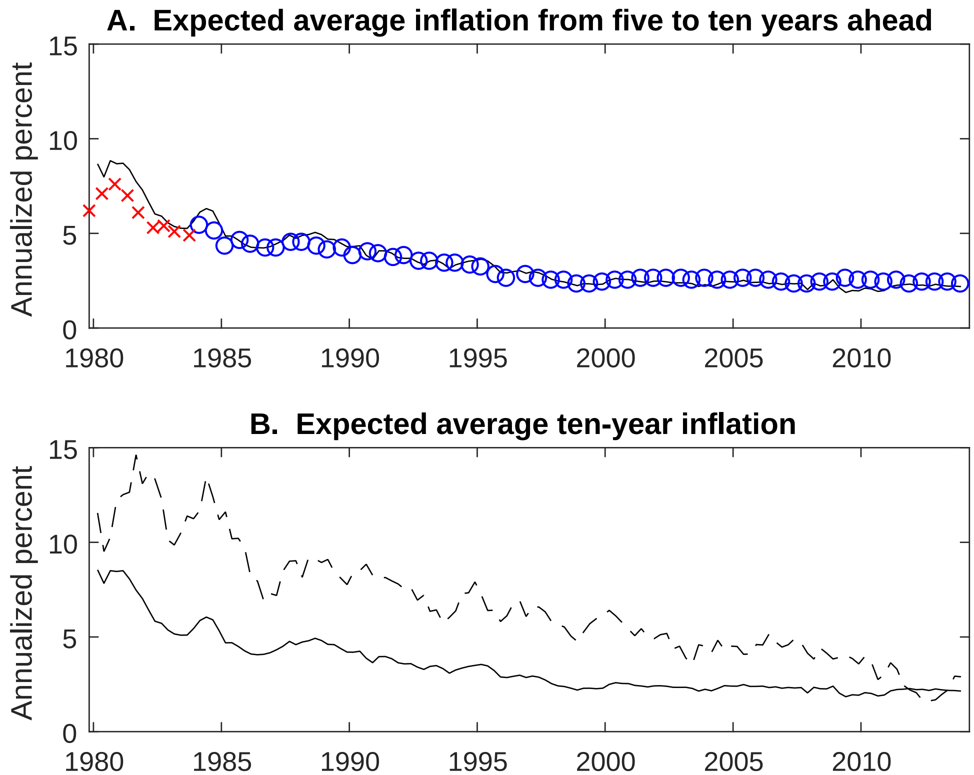

Figure 4. Long-horizon forecasts of inflation

Panel A displays forecasts of average CPI inflation from $t+$ five years to $t+$ ten years. The solid line forecasts are from Duffee (2017), using a model that assumes inflation is the sum of a random walk, an AR(1) process, and white noise. The circles are Blue Chip survey forecasts of CPI inflation over the same horizon. The x's are Blue Chip survey forecasts of GDP inflation over the same horizon. In Panel B, the solid line is an estimate of expected inflation over the next ten years, using the same model. The dashed line is the yield on a ten-year Treasury zero-coupon bond.

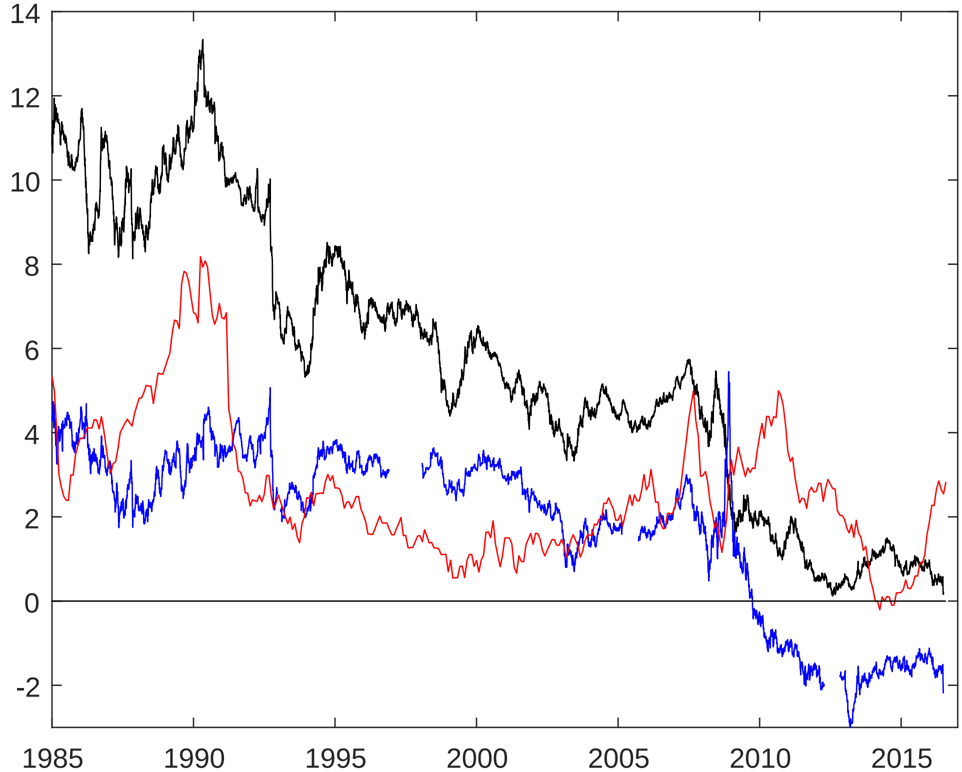

Figure 5. U.K. real and nominal three-year yields and inflation

The red line is CPI inflation in the UK from month t to month $t + 12$. The black line is the three-year yield on a nominal Bank of England bond. The blue line is the three-year yield on an inflation-indexed Bank of England bond. Both yields are zero-coupon yields splined from coupon yields.

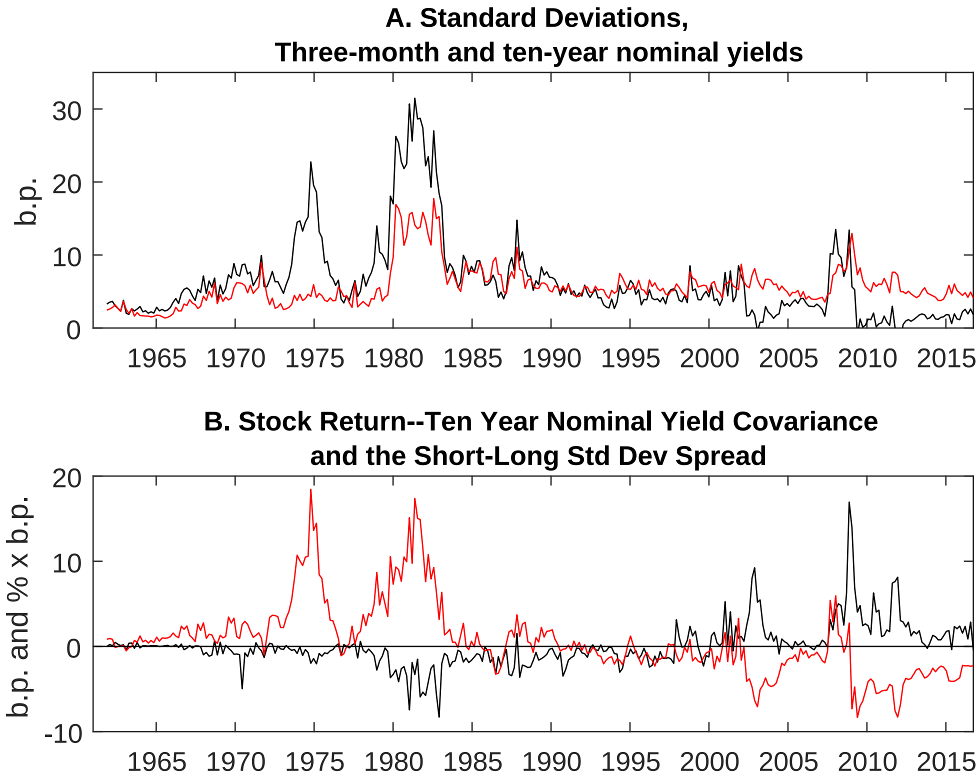

Figure 6. Conditional volatilities of short-term and long-term nominal yields

Conditional standard deviations of daily changes in nominal Treasury yields are projectiosn of sample standard deviations on conditioning instruments. Panel A displays conditional standard deviations of the three-month yield (black) and the ten-year yield (red). Panel B displays their difference (short less long) in red. Panel B also displays the conditional covariance between daily returns to the aggregate U.S. stock market and changes in the ten-year yield. The sample is 1961 through 2016.

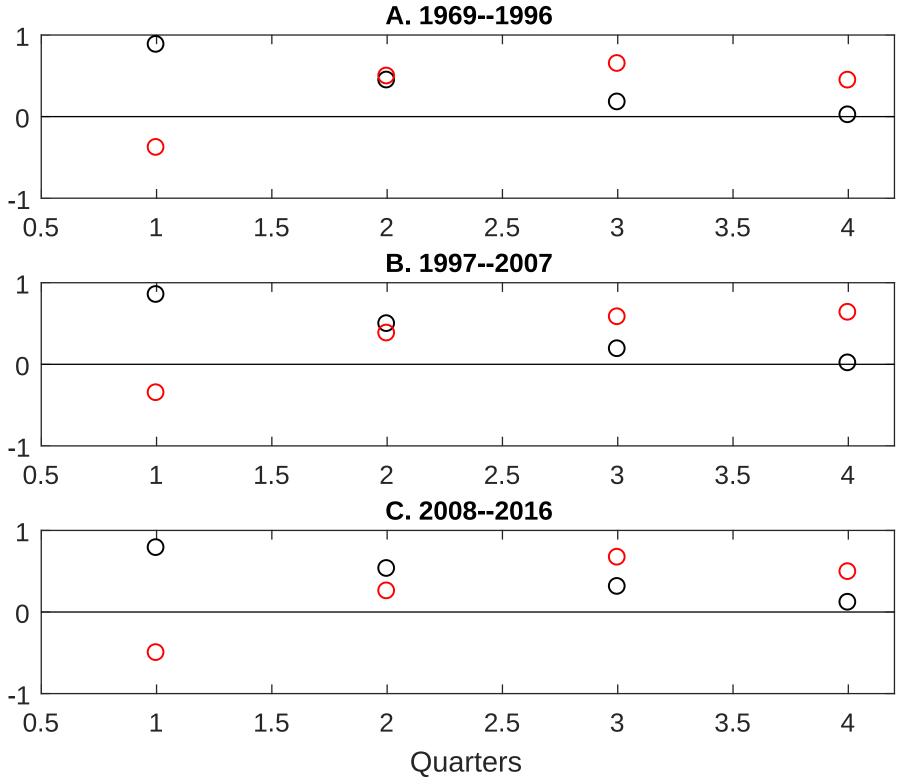

Figure 7. Principal components loadings of innovations in forecasts of GDP growth

The quarter-$(t-1)$ consensus forecasts of GDP growth for one through four quarters ahead, from the Survey of Professional Forecasters, are subtracted from the quarter-t consensus forecasts of GDP growth for zero through three quarters ahead—the same calendar quarters. Covariance matrices these four innovations are constructed for the periods listed in the titles of the three plots. Principal components are then calculated. For each period, loadings of the first (in black) and the second (in red) principal components are plotted. The horizontal axis is the forecast horizon as of quarter $t-1$.

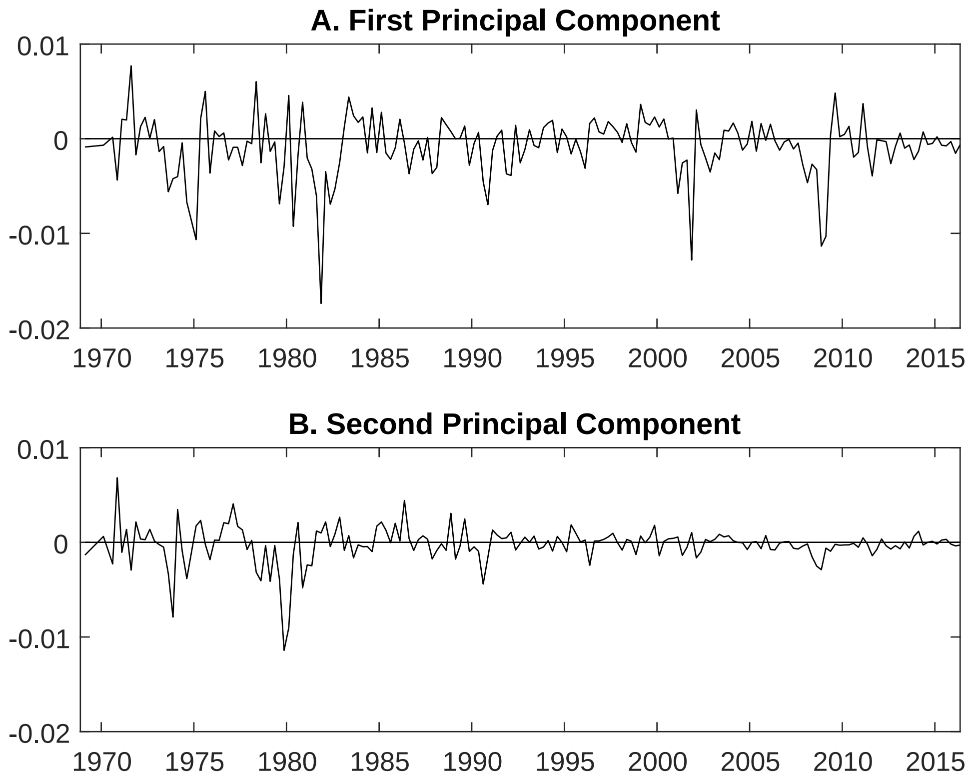

Figure 8. Time series of principal components of innovations in forecasts of GDP growth

The quarter-$(t-1)$ consensus forecasts of GDP growth for one through four quarters ahead, from the Survey of Professional Forecasters, are subtracted from the quarter-t consensus forecasts of GDP growth for zero through three quarters ahead—the same calendar quarters. The covariance matrix of the four innovations is constructed, and principal components of the matrix are constructed. Panel A plots the first principal component and Panel B plots the second.