Macroeconomic News in Asset Pricing and Reality

Gregory R. Duffee

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Abstract

Permanent innovations to output and transitory innovations to expected output growth are inferred from revisions of Greenbook forecasts of real GDP growth. Stock returns respond strongly to news of transitory expected future growth, yet are largely insensitive to permanent innovations. By contrast, real and nominal bond yields rise with news of higher current output, yet are largely unaffected by news of future economic growth. Benchmark representative-agent, endowment-economy models have considerable difficulty matching these patterns.

Affiliation: Johns Hopkins University. Contact information: voice 410-516-8828, email duffee@jhu.edu.

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How and why do prices of stocks and bonds react to macroeconomic news? Research using equilibrium models bundles these questions together by specifying jointly the properties of macroeconomic shocks, their effects on cash flows, and their effects on agents' discount rates. Many equilibrium models in finance adopt representative agent endowment settings with parsimonious specifications of macroeconomic news. Permanent shocks to the level of aggregate output create uncertainty, as in Mehra and Prescott (1985) and Campbell and Cochrane (1999). Transitory shocks to endowment growth rates generate richer dynamics, as in the models of Kandel and Stambaugh (1990, 1991), Cecchetti, Lam, and Mark (1990, 1993), and Bansal and Yaron (2004). The representative agent might have power utility, recursive utility, or marginal utility that adjusts slowly to habit as in Campbell and Cochrane (1999). The combination of these ingredients determines the reactions of asset prices to macro news.

This paper focuses on the "how" question, an approach that turns out to shed considerable light on the plausibility of various "why" explanations. The only formal structure is a continuous-time model of aggregate real output growth with both permanent innovations and transitory innovations to the conditional mean of output growth. The model imposes no restrictions linking these innovations to asset prices. Instead, the parameters of the output growth model and asset sensitivities to the innovations are estimated using revisions of Federal Reserve Board forecasts of quarterly output growth and contemporaneous variation in aggregate stock prices and bond yields. The time series covers 1975 through 2014. The cross section of forecast revisions ranges from the nowcast to four quarters ahead.

A simple combination of permanent innovations to output and transitory innovations to the output growth rate adequately explains (for the most part) covariances among revisions to output forecasts. Not surprisingly, nowcast revisions are driven almost entirely by news of permanent output innovations. Through the time-averaging embedded in measured quarterly output, this news also drives most of the revisions in one-quarter-ahead forecasts. Innovations in longer-horizon forecasts are driven by news of transitory innovations to the growth rate of output. Estimates of the half-life of these innovations are around three calendar quarters, consistent with a business cycle frequency.

Asset sensitivities to this news are stark and puzzling. News about expected future output growth (the drift of instantaneous log output) is related to stocks but not bonds. Stock prices rise strongly when news arrives of higher expected future output growth. Nominal and real yields are insensitive to the news. This pattern is reversed for permanent innovations. News about permanent innovations in output is related to bonds but not stocks; news of higher current output raises both real and nominal yields. The aggregate stock market reaction is positive but small and statistically indistinguishable from zero. All these results hold across two sample periods with substantially different output dynamics, 1975 through 1996 and 1997 through 2014.

The evidence runs counter to intuition developed in the workhorse representative agent models of Bansal and Yaron (2004) and Campbell and Cochrane (1999). Intuitively, news of higher expected future output growth raises real interest rates and the real-rate component of nominal rates. The representative agent expects marginal utility to decline over time. The agent therefore attempts to borrow from the future to consume today. Stock prices rise in anticipation of higher future cash flows, although the price effects are damped by the higher real rates.

In these models, news of permanently higher output is news of permanently higher cash flows to aggregate equity. Stock prices respond strongly. The news affects real rates less than does news about future economic growth because the representative agent's forecast of future marginal utility differs little from current marginal utility. (Campbell and Cochrane engineer their model to produce zero expected change and thus constant real rates, but that is a parameterization rather than economic intuition.) Nominal yields respond to this news only to the extent that it conveys information about expected inflation. In the data sample studied here, news about permanent innovations to output is unrelated to innovations in inflation forecasts.

Absent preference specifications, economic interpretations of this evidence amount to storytelling. One such story follows the intuition of the endowment economy labeled the "extended model" in Albuquerque, Eichenbaum, Luo, and Rebelo (2016). A representative agent with Epstein-Zin preferences has a very high elasticity of intertemporal substitution and long-lived shocks to their time rate of preference. These preference shocks are exogenously correlated with permanent endowment shocks, such that an unexpectedly higher weight on current consumption corresponds to a permanently higher endowment stream. New Keynesian models offer an endogenous interpretation of this positive correlation. Long-lived demand shocks generate a production response while simultaneously changing real and nominal rates in the same direction. The changes in real rates need to be sufficiently large to keep stock prices largely unchanged.

The results here also illuminate aspects of the stock–bond correlation puzzle. Stock market returns and contemporaneous changes in bond yields are negatively correlated during the 1970s through the late 1990s. As first noted in the academic literature by Li (2002) and Fleming, Kirby, and Ostdiek (2003), the correlation between aggregate stock returns and nominal bond yields abruptly switches sign around 1997. The subsequent work of Campbell, Shiller, and Viceira (2009), Campbell, Sunderam, and Viceira (2017), and Liu (2020) shows similar comovement patterns between stock returns and real bond yields.

Applying the continuous-time output model to Federal Reserve output forecasts for 1975 through 1996 reveals that permanent shocks and growth shocks are negatively correlated over this period. Since positive permanent shocks raise yields and positive growth shocks raise stock prices, this negative correlation between types of output shocks produces a negative correlation between stock returns and bond yields. This macroeconomic negative correlation disappears (more precisely, switches sign and is statistically insignificant) in the later sample of 1997 through 2014. Thus in this later period, macroeconomic shocks induce a weak positive correlation between stock returns and bond yields owing to their common response to permanent output innovations.

This paper is close in spirit to Cieslak and Pang (2020), who impose intuitive restrictions on the joint dynamics of stock returns and bond yields to identify "monetary" and "growth" shocks (as well as risk-premia shocks) from high-frequency asset data. They then ask how these shocks covary with lower-frequency innovations to output and inflation survey forecasts. In other words, they identify different types of economic shocks exclusively from stock returns and bond yields, then ask how these shocks relate to macroeconomic forecast innovations. The complementary approach here identifies different types of economic shocks exclusively from macroeconomic forecast innovations, then asks how these shocks relate to stock returns and bond yields.

The next section describes the data and summarizes important joint properties of output forecasts, stock returns, and changes in bond yields. Section 2 describes the continuous-time model of output dynamics. Section 3 develops formulas that connect the model to forecasts of quarterly output, quarterly output growth, and revisions in these forecasts. Section 4 describes the estimation technique and discusses some features of the model. Section 5 discusses in detail the properties of the estimated model, highlighting discrepancies between standard modeling approaches and the empirical results. Section 6 extends the empirical results to expected inflation and the persistence of bond yields. Section 7 concludes.

1 Comovement of Output Forecasts

Federal Reserve Board staff produce economic forecasts prior to every meeting of the Federal Open Market Committee (FOMC). The forecasts, known as either Greenbook (prior to 2010) or Tealbook (since 2010) forecasts, are available with a five-year lag for all FOMC meetings since 1967. I use the term "Greenbook forecast" regardless of the date of the forecast. Real quarterly output growth (initially GNP, then GDP) is one of the five macroeconomic variables included in every Greenbook forecast, along with quarterly inflation (GNP or GDP deflator), unemployment, industrial production growth, and housing starts. The maximum forecast horizon is only one or two quarters ahead for most of the 1960s. By mid-1974 the maximum horizon is routinely at least four quarters ahead. The empirical analysis in this paper uses Greenbook projections beginning with the final forecast of 1974 through 2014.

Commonly used private-sector macroeconomic forecasts are those from the Survey of Professional Forecasters (SPF; quarterly) and from Blue Chip's Economic Indicators (monthly). Output growth forecasts over horizons similar to Greenbook forecast horizons begin in late 1968 for the SPF and late 1979 for Blue Chip. Greenbook's advantages are frequency (relative to SPF) and greater coverage of an important period, the runup to the Fed's monetarist experiment beginning in late 1979. However, asset prices might respond to different information than do Greenbook forecast revisions, if the private sector has less information, and thus makes substantially different forecasts, then the Fed. We cannot evaluate this hypothesis directly because we do not have private sector forecasts made on the same dates as Greenbook forecasts. Instead, we can ask whether a private sector forecast made on day t can be improved, in an econometric sense, by the most recent Greenbook forecast made prior to day t. If not, this concern is unwarranted.

An existing literature studies this question. Romer and Romer (2000) conclude that Greenbook forecasts would not improve private-sector forecasts of real GDP growth at forecast horizons beyond the current quarter (the nowcast).[1] Faust and Wright (2009) compare

[1]This result contrasts with their conclusion that Greenbook forecasts of inflation are more accurate than private sector forecasts at all horizons. Bernanke and Boivin (2003) also study the accuracy of Greenbook

Greenbook output growth forecasts to those from mechanical forecasting tools (e.g., dynamic factor models). They find, consistent with Romer and Romer, that Greenbook nowcasts are more accurate but their forecasting advantage disappears at other horizons. Reifschneider and Tulip (2007, 2019) compare the accuracy of various output growth forecasts, including Greenbook, Blue Chip, and SPF. They find that differences in accuracy are economically small, and cannot reject at the 5% level the hypothesis of equal forecast accuracy.

On balance, this evidence justifies treating Greenbook forecasts of output growth as proxies for those made by professional forecasts in the private sector. The maintained hypothesis is that professional forecasts are also the forecasts of marginal investors in stock and bond markets.

1.1 Forecast and asset innovations

This subsection describes the data. The Federal Reserve Bank of Philadelphia maintains the Greenbook dataset. Index each Greenbook forecast by $i = 0, \dots, T$, and hence index forecast revisions by $1, \dots, T$. The index corresponds to the order of the FOMC meetings rather than calendar time. Forecast $i = 0$ occurs on December 11 1974 and forecast $i = T = 340$ occurs on December 10 2014. Denote the quarter in which a forecast is made as t_i, an index from 0 (1974Q4) to 160 (2014Q4). Revisions from meeting $i-1$ to meeting i in forecasts of j-ahead real quarter-to-quarter GDP growth are

$$\eta_i^{(j)} \equiv 100 \left(\log E_i^G \left(\left(\frac{GDP_{t_i+j}}{GDP_{t_i+j-1}} \right)^4 \right) - \log E_{i-1}^G \left(\left(\frac{GDP_{t_i+j}}{GDP_{t_i+j-1}} \right)^4 \right) \right). \tag{1}$$

The notation E^G denotes Greenbook forecasts, which are forecasts of quarterly growth rates compounded to an annual horizon. Equation (1) converts Greenbook forecasts to continually compounded growth rates, expressed in annualized percent. The indexes on GDP are quarterly indexes, with j measured relative to the quarter of Greenbook forecast i. For example, $j = 0$ is the nowcast for a given set of Greenbook projections. If $t_i = t_{i-1}$ (the quarter of the current forecast is also the quarter of the previous forecast), the innovation (1) for $j = 0$ is the innovation in successive nowcasts. If $t_i = t_{i-1} + 1$ (the current forecast is in the quarter after the previous forecast), the forecast innovation (1) for $j = 0$ is the nowcast for

forecasts, but do not examine forecasts of real GDP growth.

Greenbook i less the one-quarter-ahead forecast for Greenbook $i-1$.

The forecast innovation horizons studied here are $j = 0$ through $j = 4$. There are 17 missing observations of innovations in four-quarter-ahead output growth. The first missing observation is for the forecast of January 14 1976 and the final missing observation is for the forecast of February 3 1988.

Construction of stock returns from forecast $i-1$ to forecast i uses daily aggregate stock returns maintained by the Center for Research in Security Prices (CRSP). The gross stock return over this period is the product of gross one-day simple returns for all trading days beginning with the day of forecast $i-1$ through the day before forecast i (or the latest trading date prior to forecast i). The log excess stock return is calculated by subtracting the log riskfree return from the log stock return, assuming that the three-month Treasury bill yield as of the date of forecast $i-1$ is the riskfree rate for each day between the two forecasts. The daily three-month Treasury bill yield is from the Federal Reserve's H-15 release.

Changes in nominal bond yields from forecast $i-1$ to forecast i use daily observations of one-year and ten-year nominal Treasury yields from the CRSP Fixed Term Index file. Yields for forecast i are the most recent yields observed prior to the date of the forecast. These are yields as of the day before the forecast unless the forecast date is either a Monday or the day after a bond-market holiday. The contemporaneous change in the ex-ante one-year real yield is the changes in one-year nominal yield less the change in expected inflation over the next year. All of these data are expressed with continuously compounded annual rates. The expected inflation measure is constructed using Greenbook forecasts of the GDP deflator. The construction is sufficiently complicated to be relegated to the Appendix.

1.2 A first look at comovement

Table 1 reports standard deviations for the observed variables, including unconditional standard deviations of the output growth forecasts. Results are displayed separately for the 1975 through 1996 and 1997 through 2014 periods. For brevity, refer to the periods as "early" and "later." The samples have 196 and 144 observations of Greenbook forecast innovations respectively.

The most obvious reason for splitting the sample is apparent from the table. The sample distributions of forecasts differ substantially across the two periods. Reflecting the Great

Moderation, the unconditional standard deviations of output growth forecasts during the later period are a little more than half the size of the corresponding values during the early period. Ignoring differences between these samples may obscure instability in estimation results to follow.

The table reveals that the large wedge between the sample periods in unconditional volatilities is accompanied by a smaller wedge in standard deviations of Greenbook-to-Greenbook forecast innovations. A mechanical implication of this fact is that the Greenbook revisions contribute more to the overall variance of forecasts during the later period. For example, the news during the roughly six-week period between two Greenbook forecasts contributes only 16% of the total variance of the nowcast during the early period, and 20% during the later period.

Table 2 reports contemporaneous correlations among the forecast innovations and asset changes. Like Table 1, this table displays substantial differences between the early and later periods. The sign of the anticipated persistence of output innovations changes. In the early period, nowcast innovations are negatively correlated with innovations in expectations of economic growth beyond one quarter ahead. In the later period, nowcast innovations are positively correlated with innovations in expectations at all horizons. Stock prices react accordingly, incorporating the news about current and expected future economic activity. Good news about current output corresponds to a weak drop in stock prices during 1975–1996 (correlation of -0.04) and to an increase in stock prices during 1997–2014 (correlation of 0.31).

As noted in the introduction, correlations between stocks and bonds also differ in sign across the two periods. The correlation between stock returns and changes in the ten-year nominal yield (which move inversely with its return) is -0.35 in the early sample and 0.24 in the later sample.

Principal components analysis offers a complementary perspective on comovement. Panel A of Table 3 reports, for both sample periods, the fraction of total variance explained by the principal components of the covariance matrix of innovations in expected output growth. The first two principal components explain more than 80% of the total variance. Figure 1 displays loadings of these two principal components. For now, ignore the dashed lines in the figure. In the early period, the first component loads on shorter-horizon and longer-horizon forecast innovations with opposite signs. In the later period, the first component loads with the same sign on all of the forecast innovations.

Panel B of Table 3 reports covariances of the first two principal components with stock returns and changes in bond yields. For now (again), ignore the "Estimated Model" sections of the panel. The first component drives, in a statistical sense, a positive covariance between the nowcast and changes in both real and nominal bond yields. This pattern holds for both periods, although the magnitudes are much smaller in the later period. The second component drives a positive covariance between expected future output growth and stock returns, while leaving bond yields largely unaffected.[2] The discussion in Section 5.1 compares properties of the empirical principal components to those implied by the estimated dynamic model of output.

The evidence in Tables 1 and 2, along with Figure 1, tells us that macroeconomic dynamics in the early period are markedly different from those in the later period. (This paper is hardly the first to make that point.) A key question is whether these differences carry over to the sensitivity of asset prices to macroeconomic innovations. The empirical analysis in Section 4 helps answer the question by reporting separate estimation results for the two sample periods.

2 Output Dynamics and Asset Price Responses

This section describes a parsimonious model of instantaneous aggregate output growth. The dynamics mimic those of standard endowment economy models. Here, however, nothing links the reduced-form output dynamics to aggregate consumption, or a stochastic discount factor. Therefore the connections among aggregate stock returns, bond yields, and news about output are also specified in reduced form.

2.1 Continuous time output dynamics

Time is continuous and measured in quarter-years. Quarter-end dates are integers. Instantaneous log output is denoted y_t. Innovations to this process are created by increments to

[2]Recall from Figure 1 that in the early period, the second principal component is positively associated with innovations in expected future output growth. The sign is reversed in the later period.

two independent Brownians,

$$dB_{m,t} = \begin{pmatrix} dB_{p,t} & dB_{x,t} \end{pmatrix}'. \tag{2}$$

The "m" subscript distinguishes macroeconomic innovations from non-macroeconomic innovations introduced in Section 2.3. One Brownian has a "p" subscript to indicate that it only creates permanent shifts in output. Another has an "x" subscript to indicate that it also affects state variables that drive the conditional mean of output growth.

Instantaneous output growth follows a diffusion process

$$dy_t = \mu_t dt + \begin{pmatrix} \sigma_{yp} & \sigma_{yx} \end{pmatrix} dB_{m,t}, \tag{3}$$

where μ_t is the instantaneous drift. Both Brownian increments create permanent shifts in the level of output. This setup allows innovations to the conditional mean of output growth to be correlated with innovations to permanent changes.

The instantaneous drift follows a scalar Ornstein-Uhlenbeck process under the baseline specification,

$$\mu_t = x_{1,t}, \qquad dx_{1,t} = -\gamma_1 x_{1,t} dt + \sigma_1 dB_{x,t}. \tag{4}$$

Unconditional means play no role in the empirical work, which focuses on forecast revisions. Therefore the unconditional mean is set to zero, as implied by (4).

Conditional means in macro-finance models are often modeled as scalar O-U processes. That said, there is no economic reason to assume innovations to the conditional mean die out at a constant rate over time. Moreover, the combination of (3) and (4) answers by construction the question "Why are nowcast revisions negatively correlated with longer-horizon forecast revisions in the early period?" Mechanically, this pattern requires $\sigma_{yx}\sigma_1 < 0$. In words, permanent innovations and drift innovations must be negatively correlated.

A more general setup allows the conditional mean process to have interesting dynamics determined by two state variables. Sum two state variables to determine the conditional mean,

$$\mu_t = x_{1,t} + x_{2,t}. \tag{5}$$

Each state variable follows its own scalar O-U process. The state variables are exposed to the same single Brownian innovation,

$$dx_{i,t} = -\gamma_i x_{i,t} dt + \sigma_i dB_{x,t}, \qquad i = 1, 2. \tag{6}$$

Since there is a common single Brownian process driving the conditional mean, instantaneous innovations in the drift of (3) are perfectly correlated (or perfectly negatively correlated) with instantaneous innovations in forecasts of future drifts. Formally, given time-t values of the x variables, the expectation of the drift as of some future date $t + h$ is

$$\begin{aligned} f_{t,h} \equiv & E(\mu_{t+h} | x_{1,t}, x_{2,t}) \\ = & x_{1,t} e^{-\gamma_1 h} + x_{2,t} e^{-\gamma_2 h}, \qquad h > 0. \end{aligned} \tag{7}$$

This conditional expectation follows a diffusion process with dynamics

$$df_{t,h} = -\left(\gamma_1 x_{1,t} e^{-\gamma_1 h} + \gamma_2 x_{2,t} e^{-\gamma_2 h}\right) dt + \left(\sigma_1 e^{-\gamma_1 h} + \sigma_2 e^{-\gamma_2 h}\right) dB_{x,t}. \tag{8}$$

Equation (8) shows that the common Brownian creates innovations in expectations of future drifts at all horizons.

A few parametric examples illustrate the flexibility of the conditional mean process for output. Consider how a Brownian increment affects the current drift and expected future drifts. Figure 2 plots the diffusion term of (8) as a function of the horizon h, for various sets of parameters. Case 1 is the baseline for a single Ornstein-Uhlenbeck state variable; $x_{2,t}$ is identically zero. Expected future drifts decay with the horizon at a constant exponential rate. With Case 2, the decay in expected future drifts combines two different exponential rates, decaying quickly at first, then more slowly. Case 3 exhibits a sign change. A Brownian increment that raises the current drift also lowers expected future drifts for horizons a little more than a quarter ahead. Finally, with Case 4 the Brownian increment affects only expected future drifts, not the current drift. The expected drift rises for a little more than a quarter, then slowly dies out.

Section 4 discusses results for both the one-factor and two-factor versions of conditional mean dynamics. The discussion concludes that the additional flexibility of the two-factor version is unhelpful in fitting Greenbook output forecast revisions.

2.2 Information lags

Agents' output forecasts made at t might not incorporate the history of Brownian increments to output through t. A parameter in the model allows forecasters to recognize the state of the economy with a lag. They know where the economy was in the recent past rather than where it is at the date of the forecast. In line with the reduced-form nature of the model, there is no explicit mechanism that ties down the lag length.

Before getting into more details, it is worth discussing how the setup here differs from an approach to imperfect information that is more common in the literature. The common setting is one in which agents immediately observe output (or consumption, or any other component of output), yet never observe directly key properties of output dynamics. For example, in Veronesi (2000) the true expected growth rate of the economy is stochastic and unobserved. In Johannes, Lochstoer, and Mou (2016), agents must learn about both parameter values and true stochastic growth rates. Agents are continually engaging in Bayesian updating to predict what they cannot observe, resulting in slow learning over time. Pástor and Veronesi (2009) concisely review asset-pricing implications of this type of learning. By contrast, the setup here ignores long-run swings in beliefs while focusing on routine short-run dynamics.

The possibility that forecasters observe the macroeconomic state with a lag is consistent with other evidence. Recall from Romer and Romer (2000) and Faust and Wright (2009) that Greenbook nowcasts are more accurate than private-sector nowcasts or atheoretic forecasts. Nowcasts would be identical if agents could observe the current state of the economy. Sims (2002) discusses how Fed staff have (or claim to have) an advantage in understanding how to combine large amounts of disaggregated data to produce an accurate nowcast.

In an asset-pricing context, a high-frequency literature beginning with Schwert (1981) documents that prices respond to macroeconomic news about the past. Fleming and Remolona (1997) and Balduzzi, Elton, and Green (2001) list a variety of announcements that move Treasury bond prices, including nonfarm payrolls and durable goods orders. Such announcements also move aggregate stock prices, with sensitivities that depend on economic conditions. McQueen and Roley (1993) and Boyd, Hu, and Jagannathan (2005) show that stock prices respond to industrial production and unemployment announcements.

Econometric identification of the lag length from the covariance matrix of forecast revisions follows from the math of Working (1960). The intuition is easier to explain by assuming that output is a continuous time random walk, hence for this discussion assume the drift of output is constant. Figure 3 displays the standard picture of triangular weights on Brownian increments that sum to first-differenced time-averaged output. Consider forecasts made at successive dates τ_1 and τ_2 in the figure, which are near the end of calendar quarter two. At both dates, the nowcast is a prediction of output growth from quarter one to quarter two, while the one-quarter-ahead forecast is a prediction of output growth from quarter two to quarter three.

If forecasters observe Brownian increments without a lag, the news from τ_1 to τ_2 is primarily about the one-quarter-ahead forecast. The nowcast weights (in blue) are close to zero, while the one-quarter-ahead weights (in red) are much larger. Therefore the conditional variance of the nowcast revision is much smaller than the conditional variance of the one-quarter-ahead forecast revision.

If, however, forecasters observe output with a lag L, the relative variances of forecast revisions can be reversed. Forecasters observe the Brownians from $\tau_1 - L$ to $\tau_2 - L$. In the figure, L is chosen so that the news revealed from τ_1 to τ_2 is primarily about the nowcast, leading to a larger conditional variance of the nowcast revision than the conditional variance of the one-quarter-ahead forecast revision.

The broad point is that conditional variances (and covariances) of nowcast and one-quarter-ahead forecasts depend on the dates of the forecasts and the information lag, if any. Adding a stochastic conditional drift of instantaneous output complicates the math considerably, but does not alter the basic intuition.

2.3 Adding stock returns and bond yields

I do not attempt to couple these output dynamics with a pricing kernel. Instead, I ask in a purely reduced-form setting how sensitive stock prices and bond yields are to output news. Aggregate stock returns and bond yields respond to the same Brownian increments that drive output. These variables also have sources of variation independent of the Brownian processes that drive output.

The instantaneous dynamics of the log of the value of aggregate stock market, the one-year ex-ante real yield, the one-year nominal yield, and the ten-year nominal yield are

$$d\begin{pmatrix} S_t \\ r_{1yr,t} \\ n_{1yr,t} \\ n_{10yr,t} \end{pmatrix} = \underbrace{\Omega_a}_{4\times 2} dB_{m,t-L} + \underbrace{\Sigma_a}_{4\times 4} dB_{nm,t}. \tag{9}$$

The subscript "a" denotes "assets." The time subscript on the macroeconomic Brownian vector indicates that stocks and bonds react to the Brownian increments with a lag length L, consistent with the lag in forecasters' observation of these same innovations. Non-macro variation is created by $B_{nm,s}$, a length-four vector of Brownians. The vector is nothing more than a residual picking up all variation in the stock market that is orthogonal to macroeconomic innovations. Without loss of generality, Σ_a is the lower triangular Cholesky decomposition of the instantaneous covariance matrix of the non-macro innovations in stock returns and bond yields.

Equation (9) ignores conditional means. The empirical analysis applies (9) to returns and changes in bond yields between FOMC meetings. Over a six-week period, the contributions of conditional means to total variation are small.

2.4 A useful vector representation

Output in a quarter is the integral of the instantaneous output during the quarter. Grossman, Melino, and Shiller (1987) and Breeden, Gibbons, and Litzenberger (1989) first explore using time-averaged data in a finance context, building on the statistical properties described by Working (1960). Derivations here use the approximation that the log of time-averaged output is the integral of the log of instantaneous output.

Define the cumulated instantaneous log output from an arbitrary date zero to t as

$$Y_t \equiv \int_0^t dy_s ds \tag{10}$$

Recall that time is measured in quarters. Therefore log output during the quarter beginning at $t-1$ and ending at t is

$$Y_t^Q = Y_t - Y_{t-1}. \tag{11}$$

Solving for forecasts of log quarterly output (and first-differenced log quarterly output) requires the joint dynamics of instantaneous output, the state variables that drive the conditional mean of instantaneous output, and cumulative output. Stack these in the vector[3]

$$X_{m,t} \equiv \begin{pmatrix} y_t & x_{1,t} & x_{2,t} & Y_t \end{pmatrix}'. \tag{12}$$

The state vector's continuous dynamics are

$$dX_{m,t} = K_m X_{m,t} dt + \Omega_m dB_{m,t}, \tag{13}$$

$$K_m = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & -\gamma_1 & 0 & 0 \\ 0 & 0 & -\gamma_2 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \qquad \Omega_m = \begin{pmatrix} \sigma_{yp} & \sigma_{yx} \\ 0 & \sigma_1 \\ 0 & \sigma_2 \\ 0 & 0 \end{pmatrix}. \tag{14}$$

The discrete time conditional expectation and covariance of the macroeconomic state vector are the usual Ornstein-Uhlenbeck forms, which use matrix exponentials. The formulas are

$$E\left(X_{m,t+s} | X_{m,t}\right) = \exp(K_m s) X_{m,t}, \tag{15}$$

$$V_m(s) \equiv \text{Cov}\left(X_{m,t+s} | X_{m,t}\right) = \int_0^s \exp(K_m u)\Omega_m \Omega_m' \exp(K_m u)' du. \tag{16}$$

The Appendix explains how to solve (16) analytically. The notation for the covariance indicates that the covariance depends only on s, not separately on t and $t+s$.

A straightforward extension adds assets to this macroeconomic state vector. Define an augmented state vector as

$$X_t = \begin{pmatrix} X_{m,t}' & S_{t+L} & r_{1yr,t+L} & n_{1yr,t+L} & n_{10yr,t+L} \end{pmatrix}'. \tag{17}$$

Agents observe all elements of X_t at time $t+L$. The dynamics of the augmented state vector are

$$dX_t = K X_t dt + \Omega dB_{m,t} + \Sigma dB_{nm,t+L}, \tag{18}$$

[3]Thanks to Scott Joslin for proposing this framework, which considerably simplifies the math relative to an earlier version of the paper.

$$K \equiv \begin{pmatrix} K_m & 0_{4\times 4} \\ 0_{4\times 4} & 0_{4\times 4} \end{pmatrix}, \qquad \Omega = \begin{pmatrix} \Omega_m \\ \Omega_a \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} 0_{4\times 4} \\ \Sigma_a \end{pmatrix}. \tag{19}$$

Equation (15), absent the "m" subscript, is the conditional mean of the augmented state vector. The conditional variance is (16) plus an additional term for the non-macro innovations,

$$V(s) \equiv \mathrm{Cov}\left(X_{t+s}|X_t\right) = \int_0^s \exp(Ku)\Omega\Omega' \exp(Ku)' du + s\Sigma\Sigma'. \tag{20}$$

The Appendix explains how to solve (20) analytically.

3 Discrete Time Forecasting Innovations

This section derives formulas for time-t forecasts of first-differenced log consumption, and related formulas for revisions in forecasts from time t_1 to time t_2. The formulas are unavoidably complicated because multiple cases are possible. A nonzero information lag adds considerably to the complications. The math is necessary but has no economic content. Readers content to assume the existence of such formulas without knowing how they are derived can safely skip this section.

3.1 Forecast formulas

The change in log output from quarter $T+j-1$ to quarter $T+j$, where T is an integer, is given by first-differencing (11):

$$\Delta Y^Q_{T+j} = Y_{T+j} - 2Y_{T+j-1} + Y_{T-j-2}. \tag{21}$$

Agents at a non-integer date t in quarter T forecast (21). The time until the next quarter-end is, using the floor function,

$$d(t) = 1 + \lfloor t \rfloor - t. \tag{22}$$

For example, at $t = 3.2$, the quarter ends at $t + d(t) = 4$ and the time until the next quarter-end is $d(t) = 0.8$. Using this definition and the identity $T \equiv t + d(t)$, the forecast is

$$E_t\left(\Delta Y^Q_{t+d(t)+j}\right) = E_t\left(Y_{t+d(t)+j}\right) - 2E_t\left(Y_{t+d(t)+j-1}\right) + E_t\left(Y_{t+d(t)-j-2}\right). \tag{23}$$

When agents observe the macroeconomic state X_m with a lag L, this conditional expectation is

$$\begin{aligned} E_t\left(\Delta Y^Q_{t+d(t)+j}\right) =& e_4' E\left(X_{m,t+d(t)+j}|X_{t-L}\right) - 2e_4' E\left(X_{m,t+d(t)+j-1}|X_{m,t-L}\right) \\ &+ e_4' E\left(X_{m,t+d(t)-j-2}|X_{m,t-L}\right) \end{aligned} \tag{24}$$

where e_n is a conformable vector with one in element n and zeros elsewhere. Although the right side of (24) has three expectations of random variables, one or two may be known at t. For example, with no information lag ($L = 0$), both previous quarter-end values are known for the nowcast ($j = 0$) at t, and one quarter-end value is known for the one-quarter-ahead forecast ($j = 1$). A function for the known number of values, given $L \geq 0$, is

$$\mathcal{N}(j,t) = \begin{cases} 2, & L \leq 1 - j - d(t); \\ 1, & 1 - d(t) - j < L \leq 2 - j - d(t); \qquad L \geq 0. \\ 0, & 2 - j - d(t) < L; \end{cases} \tag{25}$$

The expectations in (24) can be expressed using (25), the formula for conditional expectations of the state variable (15), and the following function,

$$\Theta(j,t) \equiv \begin{cases} \exp(K_m(j + d(t) + L)), & \mathcal{N}(j,t) = 2; \\ \exp(K_m(j + d(t) + L)) - 2\exp(K_m(j - 1 + d(t) + L)), & \mathcal{N}(j,t) = 1; \\ \exp(K_m(j + d(t) + L)) - 2\exp(K_m(j - 1 + d(t) + L)) & \\ \quad + \exp(K_m(j - 2 + d(t) + L)), & \mathcal{N}(j,t) = 0. \end{cases} \tag{26}$$

Then (24) is

$$E_t\left(\Delta Y^Q_{t+d(t)+j}\right) = \begin{cases} e_4'\Big[\Theta(j,t)X_{m,t-L} - 2X_{m,j-1+d(t)} + X_{m,j-2+d(t)}\Big], & \mathcal{N}(j,t)=2; \\ e_4'\Big[\Theta(j,t)X_{m,t-L} + X_{m,j-2+d(t)}\Big], & \mathcal{N}(j,t)=1; \\ e_4'\Theta(j,t)X_{m,t-L}, & \mathcal{N}(j,t)=0. \end{cases} \tag{27}$$

The next subsection derives formulas for discrete-time innovations in this expectation.

3.2 Forecast revisions

Consider two successive forecasts made at t_1 and t_2. The forecasts are made at most a quarter apart, a fact that reduces slightly the complexity of the formulas. Formally, the forecast dates satisfy

$$\text{Maintained Assumption:} \qquad t_1 < t_2 \leq t_1 + 1. \tag{28}$$

The forecast dates are otherwise arbitrary. From (28), the forecasts are either made in the same calendar quarter or in successive quarters. An indicator variable equals one if the dates are in the same calendar quarter, and zero otherwise.

$$\mathcal{I}_{same}(t_1,t_2) = \begin{cases} 1, & \lfloor t_1 \rfloor = \lfloor t_2 \rfloor; \\ 0, & \lfloor t_1 \rfloor < \lfloor t_2 \rfloor. \end{cases} \tag{29}$$

The forecast innovation for some fixed quarter-end date is the forecast at t_2 less the forecast at t_1. Since the forecasts might be made in different quarters, we need unambiguous notation to refer to the horizon of the forecast. For the remainder of this paper the index j refers to the forecast horizon as of the later date t_2. The forecast horizon at t_1 is either j or $j+1$, depending on whether the forecast dates are in the same quarter. The forecast revision is

$$\epsilon_{j,t_2} \equiv \begin{cases} E_{t_2}\left(\Delta Y^Q_{t_2+j+d(t_2)}\right) - E_{t_1}\left(\Delta Y^Q_{t_1+j+d(t_1)}\right), & \mathcal{I}_{same}(t_1,t_2)=1; \\ E_{t_2}\left(\Delta Y^Q_{t_2+j+d(t_2)}\right) - E_{t_1}\left(\Delta Y^Q_{t_1+j+1+d(t_1)}\right), & \mathcal{I}_{same}(t_1,t_2)=0. \end{cases} \tag{30}$$

The left side notation of (30) specifies only the latter date because the earlier date is the date of the immediately preceding forecast. It is therefore unambiguous.

Forecast revisions are created by innovations to the state vector X_m. Denote discrete-time innovations in the state vector as

$$\tilde{X}_{m,t,t+s} \equiv X_{m,t+s} - E_t X_{m,t+s}. \tag{31}$$

Formulas for the forecast revision (30) are complicated by the possible dependence of the revision on two discrete-time innovations in the state vector. One is the innovation in expected future states owing to the change in the state from its value at $t_1 - L$, observed at t_1, to its value at $t_2 - L$, observed at t_2. The other is the possible realization of a quarter-end state that is unknown at t_1 and known at t_2. For example, with $L = 0$, agents at t_2 know one more quarter-end value of the state than agents at t_1 if t_2 and t_1 are not in the same calendar quarter.

Define an indicator variable that equals one if a quarter-end value is observed between two dates and zero otherwise. At most one quarter-end value will be revealed because the forecast dates are not more than one quarter apart. The indicator variable is the difference between the number of known quarter-end values at the second date less the number of known quarter-end values at the first date. Using the function (25), the variable is

$$QEND(t_1, t_2) = \mathcal{N}(0, t_2) - \mathcal{N}(1 - \mathcal{I}_{same}(t_1, t_2), t_1). \tag{32}$$

The Appendix defines the functions $\Theta^*(j, t_1, t_2)$ and $p(t)$. Using these functions, the forecast innovation is

$$\epsilon_{j,t_2} = \begin{cases} e_4' \Theta(j, t_2) \tilde{X}_{m,t_1-L,t_2-L}, & QEND(t_1, t_2) = 0; \\ e_4' \Theta^*(j, t_1, t_2) \tilde{X}_{m,t_1-L,t_1+d(t_1)-p(t_1)} & \\ \qquad + e_4' \Theta(j, t_2) \tilde{X}_{m,t_1+d(t_1)-p(t_1),t_2-L}, & QEND(t_1, t_2) = 1. \end{cases} \tag{33}$$

Covariances among forecasts at different horizon lead to testable restrictions of this model.

3.3 Covariances among forecast revisions

Consider the forecast innovations from t_1 to t_2 for N quarter horizons j_{min} through j_{max}. Stack the innovations in the length-N vector

$$\epsilon_{m,t_2} \equiv \begin{pmatrix} \epsilon_{j_{min},t_2} & \ldots & \epsilon_{j_{max},t_2} \end{pmatrix}'. \tag{34}$$

The "m" subscript refers to macroeconomic revisions. The innovation vector has expectation zero. Denote the covariance matrix by

$$C_m(t_1, t_2, j_{min}, j_{max}) \equiv \mathrm{Cov}_{t_1}\left(\epsilon_{m,t_2}\right). \tag{35}$$

As the notation indicates, the covariance depends on the dates of the forecasts and the forecast horizons. The dates matter, not just the difference between the dates, because covariances depend on whether quarter-end values are realized.

Define two matrices that stack sensitivities of the forecast revisions to innovations in the macroeconomic state vector. The matrices are denoted P_m and P_m^*. The rows of the matrix P_m are

$$P_m(t, j_{min}, j_{max})_{[j-j_{min}+1,:]} = e_4'\Theta(j,t), \quad j = j_{min}, \ldots, j_{max}. \tag{36}$$

The rows of P_m^* are

$$P_m^*(t_1, t_2, j_{min}, j_{max})_{[j-j_{min}+1,:]} = e_4'\Theta^*(j, t_1, t_2), \quad j = j_{min}, \ldots, j_{max} \tag{37}$$

Using this notation, the form of the covariance matrix depends on whether a quarter-end value is realized. The cases are

$$\begin{aligned} &C_m\left(t_1, t_2, j_{min}, j_{max} \middle| QEND(t_1, t_2) = 0\right) = \\ &\qquad P_m(t_2, j_{min}, j_{max})V_m(t_2 - t_1)P_m(t_2, j_{min}, j_{max})'; \end{aligned} \tag{38}$$

$$C_m\left(t_1,t_2,j_{min},j_{max}\middle|QEND(t_1,t_2)=1\right)=$$
$$P^*_m(t_1,t_2,j_{min},j_{max})V_m(d(t_1)+L-p(t_1))P_m(t_1,t_2,j_{min},j_{max})^{*\prime}$$
$$+P_m(t_2,j_{min},j_{max})V_m(t_2-t_1-(d(t_1)+L-p(t_1)))P_m(t_2,j_{min},j_{max})'. \tag{39}$$

The function $V_m(s)$ is defined in (16).

3.4 Covariance formulas for forecast and asset innovations

The discrete-time innovations in stock returns and bond yields between two FOMC forecasts on t_1 and t_2 are

$$\epsilon_{s,t_2}=S_{t_2}-S_{t_1},\quad \epsilon_{r,t_2}=r_{1yr,t_2}-r_{1,t_1},\quad \epsilon_{nk,t_2}=n_{(k)yr,t_2}-n_{(k)yr,t_1},\ k=1,10. \tag{40}$$

Stack the N discrete-time forecast innovations and four discrete-time asset innovations in the length-$(N+4)$vector

$$\epsilon_{t_2}\equiv\begin{pmatrix}\epsilon'_{m,t_2} & \epsilon_{s,t_2} & \epsilon_{r,t_2} & \epsilon_{n1,t_2} & \epsilon_{n10,t_2}\end{pmatrix}'. \tag{41}$$

Following the approach of Section 3.3, define the matrices

$$P(t_2,j_{min},j_{maxx})\equiv\begin{pmatrix}P_m(t_2,j_{min},j_{max}) & 0_{N\times4}\\ 0_{4\times4} & I_{4\times4}\end{pmatrix},$$
$$P^*(t_1,t_2,j_{min},j_{max})\equiv\begin{pmatrix}P^*_m(t_1,t_2,j_{min},j_{max}) & 0_{N\times4}\\ 0_{4\times4} & I_{4\times4}\end{pmatrix}. \tag{42}$$

Again following Section 3.3, the covariance matrix of the innovation vector (41) is

$$C_{m+a}(t_1,t_2,j_{min},j_{max})\equiv\mathrm{Cov}_{t_1}\left(\epsilon_{t_2}\right), \tag{43}$$

$$C_{m+a}\left(t_1,t_2,j_{min},j_{max}\middle|QEND(t_1,t_2)=0\right)=$$
$$P(t_2,j_{min},j_{max})V(t_2-t_1)P(t_2,j_{min},j_{max})'+\Sigma\Sigma'(t_2-t_1); \tag{44}$$

$$
\begin{aligned}
&C_{m+a}\left(t_1, t_2, j_{min}, j_{max} \middle| QEND(t_1, t_2) = 1\right) = \\
&\quad P^*(t_1, t_2, j_{min}, j_{max}) V(d(t_1) + L - p(t_1)) P(t_1, t_2, j_{min}, j_{max})^{*\prime} \\
&\quad + P(t_2, j_{min}, j_{max}) V(t_2 - t_1 - (d(t_1) + L - p(t_1))) P(t_2, j_{min}, j_{max})'. \qquad (45)
\end{aligned}
$$

The function $V(s)$ is defined in (20).

4 Model Estimation

The evidence in Section 1.2 motivates estimating the model separately for the 1975 through 1996 sample and the 1997 through 2014 sample. The early sample has 196 Greenbook forecast dates and the latter has 144 such dates.

The Ornstein-Uhlenbeck dynamics imply that discrete-horizon forecast errors and asset innovations are jointly normally distributed with covariance matrix (43). Since the distribution of the forecast errors is known (conditional on parameters), maximum likelihood estimation is asymptotically efficient. Statistical inference is performed using the outer product estimate of the information matrix.

4.1 Estimation mechanics

The data are described in Section 1.1. Greenbook forecast i is at time τ_i. Innovations from forecast $i-1$ in quarterly GDP growth forecasts are observed, as well as contemporaneous aggregate excess stock returns, changes in the one-year ex-ante real rate, the one-year nominal yield, and the ten-year nominal yield. Section 1.1 notes that 17 of the 340 observations have forecast innovations for $j = 0$ through $j = 3$. The others have innovations through $j = 4$. Denote the maximum horizon for observation i by i_{max}.

The demeaned aggregate excess stock return is a proxy for the stock return innovation. I use yield changes as proxies for yield innovations. Research since Duffee (2002) notes that random-walk forecasts of long-term nominal yields are difficult to beat. By contrast, the shape of the term structure contains information that helps predict future changes in short-term yields. However, Duffee (2018), in an exercise similar to those performed here, concludes that properties of short-horizon changes in one-year yields are very close to properties of innovations constructed using conditioning information in the term structure.

The notation distinguishes observed innovations from the model's version of these innovations by using superscripts, as in (1). Stack the observed innovations from forecast $i-1$ to forecast i in the vector

$$\epsilon_i^{observed} = \left(\begin{array}{cccccccc} \epsilon_i^{(0)} & \epsilon_i^{(1)} & \ldots & \epsilon_i^{(i_{max})} & \epsilon_i^{(stock)} & \epsilon_i^{(r1)} & \epsilon_i^{(n1)} & \epsilon_i^{(n10)} \end{array} \right)'. \quad (46)$$

This vector is the empirical equivalent of the model's vector of innovations (41), with two caveats. First, observed innovations of expected output are assumed to be contaminated by iid normally distributed measurement error. Measurement error breaks the tight link in the model between innovations in longer-horizon forecasts. In the model, forecasts for horizons greater than a quarter ahead are affected only by the innovation to the drift of output growth, and thus are almost perfectly correlated. This measurement error is appropriately interpreted as a catch-all picking up missing components in the model, such as occasional news about more-distant output growth that is not captured by the parsimonious model of Section 2.

Second, the model's covariances are scaled to match those of the data. Recall from (1) that the observed innovations of expected output growth are expressed at annualized rates. The model of Section 2 measures time in quarters. The relevant elements of the model's covariance matrices are scaled to match the use of annualized rates.

Stack the model parameters in a vector ρ. This vector contains parameters that describe output dynamics and parameters that describe asset innovations. They are, for the general case of two state variables driving the instantaneous drift of log output,

$$\rho = \left(\begin{array}{cccccccccc} \sigma_{yp} & \sigma_{yx} & \sigma_1 & \sigma_2 & \gamma_1 & \gamma_2 & L & \sigma_{err} & \mathrm{vec}(\Omega_a)' & \mathrm{vech}(\Sigma_a)' \end{array} \right)', \quad (47)$$

where σ_{err} is the standard deviation of iid measurement error in observed innovations of expected output growth. The 4x2 matrix Ω contains the sensitivities of the aggregate stock return and bond yields to the Brownians. The lower triangular 4x4 matrix Σ_a contains the 10 parameters that determine covariances among stock returns and bond yields owing to innovations orthogonal to innovations in forecasted output.

A few additional estimation details are in the Appendix.

4.2 Preferred models

The dynamic model describes the drift of instantaneous output with a single Brownian and either one or two state variables. The discussion in this subsection compares estimated versions of these models and explains why the next section considers only one-factor versions.

Consider a positive innovation to the Brownian that drives the drift state variables in the two-state drift specification (6). The innovation alters expected future drifts as expressed in (7). The response of the h-ahead expected drift to this innovation is the diffusion term in (8). Figure 4 plots these responses for the estimated models. The units on the vertical axis are in percentage points per quarter. For example, a value of 0.25 at $h = 0$ means that a Brownian increment $dB_{x,t}$ raises the $h = 0$ drift of instantaneous log output by $(0.0025)dB_{x,t}$.

Panels A and B in the figure reveal that for the 1975 through 1996 sample, adding a second factor to the conditional mean specification contributes almost nothing to the estimated drift dynamics. The shapes of the drift functions are economically and statistically indistinguishable. The two-factor version, with two additional parameters, has a log-likelihood only 0.7 greater than the one-factor version. The only clear difference between the two panels is that the standard errors are tighter with the more parsimonious specification.[4]

Panels C and D tell a different story for the later period that is, at first glance, perplexing. The drift function of the estimated one-factor model for 1997 through 2014 is almost identical to that of the same model for 1975 through 1996. The two-factor version basically combines the one-factor version with something like a Dirac delta function. The value at $h = 0$ is about -2, well outside the bounds of the graph. The estimate of the speed of mean reversion for the second factor is on the (arbitrary) boundary of the allowed range of 5.0 imposed in estimation. It is worth recalling that the estimates are produced with low-frequency data. No high-frequency data on short-horizon forecasts are available to contradict the estimated shape in Panel D.

There is less to the two-factor dynamics than meets the eye. The shape of the two-factor drift function helps fit forecast revisions at two FOMC meetings with particularly large revisions: September 2001 and March 2009. These revisions are unusual, both in their magnitudes and patterns. For both, the forecast revision in two-quarter-ahead output growth is below -2.25%. These revisions exceeded (in absolute value) revisions at any other

[4]Standard errors are computed using the delta method.

horizon for these dates. By contrast, as Table 1 shows, on average volatilities of revisions decline with forecast horizon.

Both of these FOMC meetings are near the end of a calendar quarter. For reasons insufficiently interesting to discuss in detail, the estimated two-factor model for 1997 through 2014 produces relatively high volatilities of two-quarter-ahead forecast revisions for forecast dates near the end of a calendar quarter. The log-likelihood of the estimated two-factor model exceeds that of the estimated one-factor model by about 11.5, a difference that is almost entirely accounted for by these two observations.

A better visual comparison of the one-factor and two-factor models is displayed in Figure 5. The figure displays the model's implications for expected future cumulative log output. A forecaster at t knows the state of the macroeconomy through $t - L$. Consider forecasting the average cumulative change in log output from $t - L$ through some future date $t + h$. Formally, the forecast is

$$g_{t,h} \equiv E_t \left(\frac{Y_{t+h} - Y_{t-L}}{h + L} \right). \tag{48}$$

Writing the forecast in terms of the model,

$$g_{t,h} \equiv \frac{1}{h + L} e_4' \left(\exp \left(K_m (h + L) \right) - I \right) X_{m,t-L}. \tag{49}$$

This function follows a diffusion process

$$dg_{t,h} = \{\text{drift}\} dt + \frac{1}{h + L} e_4' \left(\exp \left(K_m (h + L) \right) - I \right) dB_{m,t-L}. \tag{50}$$

A permanent innovation to instantaneous log output observed at t produces an innovation in (50) that is constant across h. A drift innovation observed at t produces an innovation in (50) that depends model parameters and h.

Figure 5 displays the estimated diffusion components of (50) for the two estimated models of 1997 through 2014.[5] The effects of the "level" Brownian are almost identical across the two models. The effects of the "drift" Brownian are almost identical for horizons greater than a quarter ahead. The two-factor model has a somewhat larger effect for horizons less than a quarter, a feature that drives up the correlation between nowcast innovations and

[5] Standard errors are computed using the delta method. The units on the vertical axis are in percentage points.

one-quarter-ahead innovations.

The modest difference between Panels A and B in Figure 5 does not justify using a two-factor model of instantaneous drift dynamics. There are no insights gleaned from the two-factor model that are not in the one-factor version. Therefore the models discussed in detail in the next section are the one-factor versions.

Parameter estimates for the one-factor models are reported in Table 4 (1975 through 1996) and Table 5 (1997 through 2017). Volatility parameters in the dynamic model ($\sigma_{yp}, \sigma_{yx}, \sigma_1, \sigma_{err}$) are all expressed in percentage terms. Their reported values in the tables are 100 times their natural-unit values. Parameter estimates are reported primarily for completeness. Properties of the estimated models are explored in in the next section.

5 Interpreting the Estimated Models

This section first discusses how well the estimated models fit the data. It then outlines similarities and differences in output dynamics between the two sample periods. After this analysis, the section turns to asset responses to news—the most important and surprising results. The section concludes by highlighting how much of the output and asset news is unexplained by the model.

5.1 Goodness of fit

We first check whether the estimated model accurately describes the covariances among forecast innovations, stock returns, and changes in bond yields. Naturally, the model's simple structure is incapable of reproducing all of the rich properties of sample covariances. On balance, it parsimoniously captures the bulk of observed covariation.

Figure 1 provides visual evidence to evaluate the model's accuracy in fitting covariances among output forecast innovations. Recall that Section 1.2 uses this figure to discuss the principal components of the sample covariance matrix. The figure also displays principal components of the mean model-implied covariance matrix of forecast revisions.

Table 6 provides detailed information comparing the sample mean outer product of the forecast revisions and asset innovations with model-implied equivalents. The model-implied covariance matrix of innovations for Greenbook date τ_2 depends on τ_2 and the date of the previous Greenbook date τ_1. The table reports the sample mean of each date's model-implied covariance matrix.

The figure and table document that for the most part, the estimated models capture sample covariances among forecast revisions, especially in the early period. Panel A of Figure 1 shows that the model's implied loadings of the first two principal components closely match those of the data, aside from the loading of the first PC on one-quarter-ahead revisions. This one-quarter-ahead mismatch is also apparent in the table. In the early sample, the sample correlation between revisions in the nowcast and the one-quarter-ahead forecast is 0.38, while the average model-implied covariances produce a correlation of only 0.16.

The performance of the estimated model for the later period (1997 through 2014) is a little weaker. As in the early period, the estimated model undershoots the correlation between nowcast and one-quarter-ahead revisions. The sample correlation of 0.57 modestly exceeds the model-implied correlation of 0.48. Since the model's correlation is too low, the parameter estimates trade off fitting the covariance between these forecasts and fitting the variance of the nowcast. Panel B of Figure 1 shows the model-implied loading of the nowcast on the first principal component is a little high and the loadings of the longer-horizon forecasts are a little low. The loading of the one-quarter-ahead forecast on the second principal component is lower (more negative) than the sample loading.

Following the spirit of Figure 1, Panel B of Table 3 reports covariances between the first two principal components of forecast innovations and asset innovations. The signs are all correct, aside from one covariance that is approximately zero in the data and in the model. The match between sample and model-implied covariances is fairly close in the later period. The most significant mismatch between the data and the model is in the early period. The estimated model underestimates somewhat the magnitudes of all of the covariances. For example, sample covariances between the first principal component of forecast innovations and changes in bond yields are about 1.1 to 1.2 times the corresponding model-implied covariances.

In sum, the simple model of output dynamics captures qualitatively the joint behavior of forecast and asset innovations. The model's quantitative limitations result in some model-implied covariances that are too small relative to the data.

5.2 Estimated macroeconomic dynamics

Section 1 uses raw forecast revisions to document the well-known fact that output news in 1975 through 1996 is more volatile than such news in 1997 through 2014. It also documents the less well-known fact that nowcast revisions and longer-horizon revisions are negatively correlated in the early sample and positively correlated in the later sample.

This discussion explains where these empirical patterns show up in the estimated dynamic model of output. Briefly, the higher volatility in the early period is concentrated entirely in the volatility of permanent innovations to output. Permanent and drift innovations are strongly negatively correlated in the early period, and uncorrelated in the later period.

Recall that Tables 4 and 5 display parameter estimates for the 1975 through 1996 and 1997 through 2014 samples respectively. As discussed in Section 4.2, the univariate properties of the conditional mean processes in the early and later samples are almost identical. Point estimates of the volatility of drift innovations σ_1 and the persistence of the innovations γ_1 are very close. The standard deviation of innovations is slightly lower in the later period, but the persistence of these innovations is slightly greater. The half-life of innovations is 2.8 quarters in the early period and 3.0 quarters in the later period. Panels A and C illustrate the similarity of the two drift functions.

The tables reveal another similarity between the periods. They share a common estimated lag between realizations of macro innovations and the observation of the realizations. The point estimate is a little more than half a month.

Since conditional mean dynamics are unchanged, permanent innovations must be more volatile in the early period. The instantaneous variance of permanent innovations to log output is (informally)

$$\mathrm{Var}(dy) = \left(\sigma_{yp}^2 + \sigma_{yx}^2\right) dt.$$

The early-period instantaneous variance is 2.6 times the later-period instantaneous variance.

Although the univariate conditional mean process is approximately the same in both periods, joint properties involving the conditional mean are decidedly not. The instantaneous correlation between permanent innovations and drift innovations is

$$\mathrm{Cor}(dy, d\mu) = \frac{\sigma_{yx}}{(\sigma_{yp}^2 + \sigma_{yx}^2)^{1/2}}.$$

This correlation is -0.47 in the early period and almost exactly zero in the later period. The negative correlation between innovations in the nowcast and longer-horizon forecasts is a consequence of the negative correlation between permanent and drift innovations.

The dynamic model of Section 2 is entirely real; inflation is unmentioned. Empirical evidence in Section 6 reveals another difference between the early and later periods. In the early period, drift innovations are negatively correlated with innovations in expected inflation. In the later period, drift innovations are positively correlated with innovations in one-quarter through four-quarter ahead expected inflation.

Output dynamics in the early period differ substantially from output dynamics in the later period. The next subsection shows that nonetheless, asset responses to permanent and drift innovations are similar across the two periods.

5.3 Asset responses to macroeconomic innovations

How do innovations to the permanent and drift components of log output affect assets? For concreteness, consider stock returns. The discrete-time stock return innovation from τ_1 to τ_2 is, from the instantaneous log dynamics (2.3),

$$\epsilon_{s,\tau_2} = \Omega_{s,1}\Delta B_{p,\tau_2} + \Omega_{s,2}\Delta B_{x,\tau_2-L} + \sigma_s'\Delta B_{nm,\tau_2-L}, \tag{51}$$

where the difference operators denote the change in the Brownians from their observed values at τ_1 to their observed values at τ_2. The first Brownian on the left of (51) affects only the level of instantaneous log output, while the second affects both the level and the drift.

To disentangle the level and the drift, rewrite (51) as

$$\begin{aligned}\epsilon_{s,\tau_2} = {} & \underbrace{\frac{\Omega_{s,1}}{\sigma_{yp}}}_{\gamma_{s,P}} \underbrace{(\sigma_{yp}\Delta B_{p,\tau_2-L} + \sigma_{yx}\Delta B_{x,\tau_2-L})}_{\text{permanent innovation}} \\ & + \underbrace{\left(\frac{\Omega_{s,2}}{\sigma_1} - \frac{\Omega_{s,1}}{\sigma_{yp}}\frac{\sigma_{yx}}{\sigma_1}\right)}_{\gamma_{s,D}} \underbrace{\sigma_1\Delta B_{x,\tau_2-L}}_{\text{drift innovation}} + \sigma_s'\Delta B_{nm,\tau_2}.\end{aligned} \tag{52}$$

The term labeled "permanent innovation" is the cumulative news about permanent shocks to instantaneous log output. The sensitivity of the aggregate stock return to this news, denoted $\gamma_{s,P}$, is identified from the loading of the stock return on the first Brownian. The term labeled "drift innovation" is the cumulative news about innovations to the drift of instantaneous log output. Since this news is correlated with the permanent innovation, the sensitivity of the aggregate stock return to pure drift news ($\gamma_{s,D}$) is the total sensitivity to the second Brownian less the implied response to the permanent component of this news.

Table 7 reports estimates of these sensitivities to permanent and drift innovations. The evidence is striking. The stock market responds to drift innovations but not permanent innovations. Bond yields exhibit the opposite pattern.

The point estimates imply that an unexpected temporary increase of 1% in the annual growth rate of output (drift per quarter of instantaneous log output increases by 0.25%) raises stock prices by 2.3% in the early sample and 6.7% in the later sample. The standard errors are tight. An unexpected 1% jump in output (permanent shock to log output of 0.01) raises annualized yields around 45 to 85 basis points, again with tight standard errors. The other point estimates are economically small and statistically indistinguishable from zero.

Almost any representative-agent model is consistent with the stock price response to drift innovations. Forward-looking agents expect firms to have higher cash flows when they expect the economy to grow faster. However, the other evidence in Table 7 is difficult to square with benchmark models. Neither the one-year real yield nor one-year and ten-year nominal yields respond to drift innovations. Yet models with news about expected future economic growth, such as Kandel and Stambaugh (1990, 1991), Cecchetti et al. (1990, 1993), and Bansal and Yaron (2004), all imply that when agents anticipate good times ahead, they attempt to borrow from the future to consume today. This increased desire to borrow raises real rates, and raises nominal rates in the absence of an offsetting change in expected inflation or inflation risk premia.

News about expected inflation does not mask a real-rate response to drift innovations in the nominal yields. The response of expected inflation to drift innovations is economically small with an unstable sign. Section 6 documents that in the early period, drift innovations are associated with modest decreases in expected inflation. In the later period such innovations are associated with slight increases in expected inflation. The results of Table 7 are consistent with these patterns of expected inflation, combined with the absence of a real-rate response to drift innovations. In the early sample, the estimated responses of nominal yields to drift innovations are slightly negative, while in the later sample these responses are slightly positive.

Asset responses to news of permanent changes in output pose a related set of issues. Good news about current and expected future levels of output leads a representative agent with recursive utility to anticipate higher future cash flows to equity and flat marginal utility. In such a setting, stock prices increase and real yields are unchanged, directly conflicting the evidence of Table 7. The sensitivity of nominal yields to permanent innovations is not driven by news about expected inflation. Section 6 documents expected inflation news is unrelated to permanent innovations in both the early and later periods.

Different asset responses are implied by models in which a representative agent has habit formation preferences. With short-horizon habit formation as in, say, Smets and Wouters (2003, 2007), positive permanent shocks immediately raise both the value of equity and surplus consumption. Since surplus is short-lived, agents anticipate surplus will quickly fall. All else equal, this effect lowers short-horizon real bond yields as investors attempt to shift consumption ahead for times of expected low surplus. Habit formation that looks back further in time, as in Campbell and Cochrane (1999), produces the same qualitative effect for longer-horizon maturities.

This negative correlation between innovations in permanent output and real rates can be erased, as in Campbell and Cochrane (1999), or reversed, as in parameterizations of Wachter (2006), by appropriate dynamics of conditional precautionary saving. Conditional volatilities of marginal utility need to decline by the right amount when news of higher permanent output arrives, putting upward pressure on interest rates. The increase in interest rates needs to be sufficient to keep stock prices unchanged as news of higher future cash flows arrives. But then why, when news of higher output drift arrives, does the same effect not raise yields? The combination of results in Table 7 pose

Persistent shocks to a representative agent's time rate of preference can explain, at least qualitatively, the results in Table 7 for permanent innovations without running afoul of the results for drift innovations. Albuquerque, Eichenbaum, Luo, and Rebelo (2016) and Schorfheide, Song, and Yaron (2018) rely on these types of shocks to explain variations in real rates. In these endowment economies, nothing links innovations to preferences with innovations in output dynamics, although links can be imposed exogenously as in the extended model of Albuquerque et al. (2016). New Keynesian models offer an endogenous link. Positive demand shocks (agents suddenly prefer to save less and spend more) raise output at the same time they raise real rates. The two effects must offset to keep stock prices unchanged.

5.4 Missing pieces

Section 2's parsimonious model fails to explain much of the observed variation in stock returns, bond yields, and longer-horizon forecast innovations. Table 8 uses variance decompositions to document the magnitude of the missing pieces.

The components of the decomposition are the permanent and drift innovations defined in (52), and an "other" piece. The permanent and drift innovations are correlated, thus the variance decomposition includes a covariance term. The "other" category combines the non-macro variation of stock returns and the noise in output forecast innovations. In the model, this component is orthogonal to the others. The model's point estimates imply the variance decompositions reported in Panel A of Table 8.

In both the early and later periods, permanent innovations in output drive almost all of the variation in nowcast revisions of output growth. Through time-averaging, they also drive a substantial part of one-quarter-ahead revisions in output growth. By construction, they cannot explain any part of innovations in longer-horizon forecasts. Innovations in drift dynamics explain between a quarter to a half of the variance of longer-horizon forecasts for the early period. In other words, most of the variance at longer horizons is unexplained. In the later sample the model does a little better, explaining between half and 70% of these longer-horizon innovations.

The most embarrassing empirical property of the model is the weak connection of output dynamics to aggregate stock returns and changes in bond yields. In general, the term structure literature struggles to find macroeconomic variables that explain, at least statistically, changes in bond yields; Duffee (2013) offers a handbook discussion of the (lack of) evidence. Depending on the time period and the instrument, Table 8 attributes between 60% to 85% of yield variation to the unexplained component.

The explanatory power for excess stock returns is puzzling. In the later period, over half of the variance of stock returns is explained by innovations to the drift of output growth. But in the early period, less than 10% of the variance is explained.

Panel B of Table 8 provides indirect evidence that the estimated model misses some of the output-induced variation in assets. The table reports model-implied correlations among stock returns and changes in bond yields. In the early period, "output news" induces negative correlations between stock returns and changes in bond yields. "Non-output news" also induces negative correlations. In the later period, all of these signs switch. This is either a remarkable coincidence or evidence that output news is buried within the "non-output news" label.

6 Other Properties of the Macroeconomic Dynamics

Section 2's model is narrowly focused on output dynamics and immediate sensitivities of stock prices and bond yields to output shocks. This section extends the use of the model to draw some empirical conclusions about inflation dynamics and the persistence of bond yields.

6.1 Inferring shocks

In the model, the covariance matrix of the macroeconomic state vector X_m, combined with the loadings of stock returns and yield changes on innovations to the vector, determine the joint covariance matrix of observed variables. We can roughly interpret the estimation technique as inferring properties of the unobserved macroeconomic state vector from the sample covariance matrix of observed variables.

This section extends the estimation logic by applying the estimated model to regressions of the unobserved Brownians on the observed variables. The regressions are

$$\Delta B_{y,\tau_2-L} = \chi_y' \epsilon_{\tau_2} + e_{y,\tau_2}, \tag{53}$$

$$\Delta B_{x,\tau_2-L} = \chi_x' \epsilon_{\tau_2} + e_{x,\tau_2}. \tag{54}$$

Analytic expressions for the regression coefficients are functions of the model's parameters. Since the parameters and right-hand side variables are observed, fitted values of the unobserved changes in the Brownians are readily calculated.

Given these fitted values, fitted "permanent innovations" and "drift innovations" are constructed following (52). These are, for Greenbook observation i,

$$\widehat{PERM}_i \equiv \hat{\sigma}_{yp}\widehat{\Delta B}_{p,i} + \hat{\sigma}_{yx}\widehat{\Delta B}_{x,i}, \tag{55}$$

$$\widehat{DRIFT}_i \equiv \hat{\sigma}_1\widehat{\Delta B}_{x,i}. \tag{56}$$

The remainder of this section uses these fitted time series to answer two questions. First, what is the relation between revisions in inflation forecasts and these Brownian changes? Second, are the contemporaneous responses of yields to the Brownians reversed over the next few months?

6.2 Inflation expectations

I construct innovations in Greenbook forecasts of expected inflation (GDP deflator) in the same way that I construct innovations of forecasts of output growth with (1). The Appendix contains details. As with output growth, at each Greenbook forecast there are revisions of inflation expectations for horizons ranging from the nowcast through four quarters ahead.

Table 9 reports estimates of regressions of these inflation forecast revisions on the fitted sum of Brownian increments to the level of output and the drift of output growth as defined by (55). The regressions are, for $k = 0$ through $k = 4$ and Greenbook observation i,

$$\eta_{\pi,i}^{(k)} = b_{k,0} + b_{k,1}\widehat{PERM}_i + b_{k,2}\widehat{DRIFT}_i + e_{\pi,i}^{(k)}. \tag{57}$$

Standard errors are adjusted only for generalized heteroskedasticity, not for the generated regressor problem.

The results show that for both the early and later samples, permanent innovations to output are largely unrelated to innovations in inflation expectations. Only one of the 10 estimated coefficients (two samples, five forecast horizons) is statistically different from zero. For all ten the economic significance is small. A one percent permanent innovation corresponds to a change in expected inflation of no more than 12 basis points. For comparison, Table 7 reports that a one standard deviation innovation changes bond yields by 45 to 85 basis points.

By contrast, drift innovations and innovations in expected inflation are connected. In the early period 1975–1996, innovations in output drift and expected inflation are negatively correlated. An unexpected temporary increase of 1% in the annual growth rate of output (drift per quarter increases by 0.25%) corresponds to inflation expectation innovations of around -25 b.p. In the later period, a positive drift shock lowers the inflation nowcast while simultaneously raising expected inflation over the four quarters. The sum of innovations in expected inflation during quarters one through four almost precisely offsets the nowcast innovation.

6.3 Persistence of yields

Real and nominal yields react to permanent innovations to output. How persistent are these reactions? Are the responses of yields reversed quickly? I investigate this question by regressing changes in yields on current and lagged permanent and drift innovations. Using the notation of (46) for changes in yields, the regressions have the form (here, for the one-year real rate)

$$\eta_i^{(r1)} = b_{r1,0} + b_{r1,1}\widehat{PERM}_i + b_{r1,2}\sum_{j=1}^{6}\widehat{PERM}_{i-j} + e_i^{(r1)}. \tag{58}$$

If changes in yields associated with contemporaneous changes in the level of output are partially reversed through the next six Greenbook meetings (about three quarters), the coefficient on the sum of lagged permanent innovations will be negative. The coefficient will be zero if, on average, changes are not reversed within three months. As in Section 6.2, standard errors are adjusted only for generalized heteroskedasticity, not for the generated regressor problem.

Results are in Table 10, and are easy to summarize. There is no evidence that the yield changes are reversed during the next few months. The coefficients on the lags are economically small, at most 1/10 the sizes of the coefficients on the contemporaneous innovation. They are also statistically insignificant.

7 Conclusion

The macroeconomy is subject to a variety of shocks. The nature of these shocks determine how investors alter their valuations of stocks and bonds. Much of the asset-pricing literature describes macroeconomic shocks differently than does the macroeconomic literature. The broad point of this paper is that the asset-pricing descriptions need to be confronted with data.

The results here are not encouraging from an asset-pricing perspective. However, this research focuses on particular types of shocks: those inferred from Greenbook revisions of short-run (a year or so) forecasts of economic growth. These are not the only types of macroeconomic shocks, and much of the variation in stock returns and bond yields is not explained by Greenbook revisions. The challenge for future work is identifying other macroeconomic shocks and characterizing how they affect asset prices.

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Table 1. Volatilities of Expected Output Growth, Stock Returns, and Changes in Yields

Observations of expected real GDP growth for the current quarter and future quarters are from Greenbooks. Changes in forecasts, excess aggregate stock returns, changes in nominal Treasury bond yields, and the change in the one-year ex ante real yield are measured between the dates of the two Greenbook forecasts. The "Uncon" columns report unconditional standard deviations for output growth forecasts. The "Greenbook to Greenbook" columns report the square root of mean squared forecast innovations and changes in bond yields. They also reports the standard deviation of excess stock returns. Output growth and yields are expressed in annualized percentage points. Excess stock returns are expressed in percent.

	1975–1996 196 obs.		1997–2014 144 obs.	
	Uncon	Greenbook to Greenbook	Uncon	Greenbook to Greenbook
Expected Output Growth				
Nowcast	3.15	1.26	1.84	0.83
One Q Ahead	2.40	0.79	1.47	0.69
Two Q Ahead	1.86	0.62	1.17	0.54
Three Q Ahead	1.64	0.53	1.05	0.45
Four Q Ahead	1.38	0.41	0.92	0.35
Assets				
Excess Stock Return		5.31		5.91
Δ One Yr Real Yield		0.80		0.28
Δ One Yr Nom Yield		0.79		0.29
Δ Ten Yr Nom Yield		0.47		0.32

Table 2. Correlations Among Innovations in Expected Output Growth, Stock Returns, and Changes in Yields

Innovations in expected output growth are Greenbook forecasts of j quarter ahead growth in real GDP less the previous Greenbook forecast for the same period. Excess aggregate stock returns, changes in nominal Treasury bond yields, and the change in the one-year ex ante real yield are measured between the dates of the two Greenbook forecasts.

	Expected Output Growth						One Yr	One Yr
						Stock	Real	Nominal
	Nowcast	$j=1$	$j=2$	$j=3$	$j=4$	Return	Yield	Yield
A. 1975–1996 (196 obs., mean of 41 days between forecasts)								
Nowcast	1.00							
$k=1$	0.38	1.00						
$k=2$	−0.18	0.37	1.00					
$k=3$	−0.33	0.09	0.69	1.00				
$k=4$	−0.31	−0.05	0.36	0.70	1.00			
Stock Return	−0.04	0.13	0.24	0.24	0.10	1.00		
1 Yr Real Yield	0.39	0.16	−0.09	−0.17	−0.26	−0.23	1.00	
1 Yr Nom Yield	0.42	0.16	−0.22	−0.28	−0.34	−0.29	0.94	1.00
10 Yr Nom Yield	0.40	0.13	−0.20	−0.25	−0.29	−0.35	0.70	0.76
B. 1997–2014 (144 obs., mean of 45.6 days between forecasts)								
Nowcast	1.00							
$k=1$	0.57	1.00						
$k=2$	0.29	0.63	1.00					
$k=3$	0.22	0.44	0.77	1.00				
$k=4$	0.16	0.24	0.55	0.68	1.00			
Stock Return	0.31	0.53	0.62	0.56	0.54	1.00		
1 Yr Real Yield	0.37	0.35	−0.02	−0.08	−0.11	0.12	1.00	
1 Yr Nom Yield	0.47	0.51	0.19	0.04	−0.05	0.29	0.87	1.00
10 Yr Nom Yield	0.32	0.20	0.06	−0.03	0.00	0.24	0.52	0.57

Table 3. Principal Components of Innovations in Expected Output Growth

Notes to Table 1 describe the data. Principal components of innovations in expected real output growth are constructed separately for two sample periods. Panel B reports covariances among the first two principal components, aggregate excess stock returns, and changes in bond yields. All values are expressed in annualized percentage points. The panel also reports corresponding properties of the dynamic model of output growth described in Section 2.

A. Fraction of Variance Explained by Each Component

Sample	Obs	1st	2nd	3rd	4th	5th
1975–1996	196	0.583	0.255	0.103	0.043	0.015
1997–2014	144	0.613	0.230	0.102	0.033	0.021

B. Covariances With Stock Returns and Changes in Yields

	Variance	Stock Return	One Year Real Yield	One Year Nominal Yield	Ten Year Nominal Yield
1975–1996 (196 observations)					
Data					
First PC	1.86	−0.61	0.47	0.52	0.29
Second PC	0.79	1.27	−0.05	−0.11	−0.06
Estimated Model					
First PC	1.74	−0.37	0.39	0.47	0.24
Second PC	0.80	1.08	0.01	−0.05	−0.02
1997–2014 (144 observations)					
Data					
First PC	1.06	3.47	0.09	0.14	0.08
Second PC	0.41	−1.38	0.05	0.04	0.04
Estimated Model					
First PC	1.19	3.37	0.13	0.18	0.11
Second PC	0.52	−1.84	0.05	0.03	0.05

Table 4. Model Parameter Estimates, 1975 through 1996

The model and estimation method are described in Sections 2 and 4 respectively. Asymptotic standard errors are in parentheses. In Panel B, asterisks represent asymptotic two-sided p-values versus zero of 10%, 5% and 1%.

A. Output Growth Dynamics

σ_{yp}	σ_{yx}	σ_1	γ_1	L	σ_{err}
0.59	−0.32	0.26	0.25	0.18	0.11
(0.04)	(0.08)	(0.03)	(0.03)	(0.03)	(0.003)

B. Loading of Stock Returns and Bond Yields on Macro Innovations

Innovation	Stock Return	One Year Real Yield	One Year Nominal Yield	Ten Year Nominal Yield
Level	0.33	0.46***	0.50***	0.26***
	(0.71)	(0.11)	(0.10)	(0.05)
Drift	2.27***	−0.16	−0.33***	−0.16*
	(0.76)	(0.12)	(0.13)	(0.09)

C. Cholesky Factorization of Non-Output Components of Stock Returns and Bond Yields

	Stock Return	One Year Real Yield	One Year Nominal Yield	Ten Year Nominal Yield
Stock Return	7.61			
	(0.41)			
One Year Real Yield	−0.26	1.10		
	(0.12)	(0.05)		
One Year Nominal Yield	−0.29	0.96	0.35	
	(0.13)	(0.04)	(0.01)	
Ten Year Nominal Yield	−0.19	0.38	0.12	0.45
	(0.05)	(0.04)	(0.04)	(0.03)

Table 5. Model Parameter Estimates, 1997 through 2014

The model and estimation method are described in Sections 2 and 4 respectively. Asymptotic standard errors are in parentheses. In Panel B, asterisks represent asymptotic two-sided p-values versus zero of 10%, 5% and 1%.

A. Output Growth Dynamics

σ_{yp}	σ_{yx}	σ_1	γ_1	L	σ_{err}
0.41	0.02	0.23	0.23	0.18	0.07
(0.02)	(0.07)	(0.03)	(0.05)	(0.01)	(0.002)

B. Loading of Stock Returns and Bond Yields on Macro Innovations

Innovation	Stock Return	One Year Real Yield	One Year Nominal Yield	Ten Year Nominal Yield
Level	0.66	0.21***	0.24***	0.19***
	(0.81)	(0.04)	(0.03)	(0.06)
Drift	6.11***	0.02	0.10	0.02
	(0.64)	(0.06)	(0.06)	(0.05)

C. Cholesky Factorization of Non-Output Components of Stock Returns and Bond Yields

	Stock Return	One Year Real Yield	One Year Nominal Yield	Ten Year Nominal Yield
Stock Return	5.78			
	(0.44)			
One Year Real Yield	0.04	0.34		
	(0.03)	(0.03)		
One Year Nominal Yield	0.05	0.28	0.17	
	(0.03)	(0.02)	(0.01)	
Ten Year Nominal Yield	0.11	0.17	0.08	0.35
	(0.05)	(0.04)	(0.04)	(0.02)

Table 6. Empirical and Model-Implied Mean Outer Products of Innovations

The data are innovations in expected output growth from Greenbook forecasts for quarters $k = 0$ (nowcast) through $j = 4$ ahead, Greenbook-to-Greenbook excess stock returns, and contemporaneous changes in one-year ex ante real rates, one-year nominal Treasury yields, and ten-year nominal Treasury yields. The table reports the sample mean outer product matrix for these data and the corresponding implied mean outer product matrix using parameter estimates of Section 2's model with a single state variable determining the drift of instantaneous output. All values are expressed in annualized percentage points.

A. 1975–1996 (196 obs.)

	Expected Real GDP Growth						One Yr	One Yr	Ten Yr
	Nowcast					Stock	Real	Nominal	Nominal
	$j = 0$	$j = 1$	$j = 2$	$j = 3$	$j = 4$	Return	Yield	Yield	Yield
Empirical									
Nowcast	1.58								
$j = 1$	0.38	0.62							
$j = 2$	−0.14	0.18	0.38						
$j = 3$	−0.22	0.04	0.23	0.28					
$j = 4$	−0.16	−0.02	0.09	0.15	0.17				
Stock Return	−0.29	0.55	0.80	0.67	0.21	28.16			
1 Yr Real Yield	0.39	0.09	−0.04	−0.07	−0.09	−0.96	0.64		
1 Yr Nom Yield	0.42	0.10	−0.11	−0.12	−0.11	−1.20	0.59	0.62	
10 Yr Nom Yield	0.24	0.05	−0.06	−0.06	−0.06	−0.83	0.26	0.28	0.22
Model									
Nowcast	1.68								
$j = 1$	0.16	0.60							
$j = 2$	−0.16	0.15	0.39						
$j = 3$	−0.12	0.12	0.16	0.31					
$j = 4$	−0.09	0.09	0.12	0.09	0.26				
Stock Return	−0.29	0.61	0.68	0.53	0.41	28.34			
1 Yr Real Yield	0.38	0.06	−0.05	−0.04	−0.03	−0.99	0.67		
1 Yr Nom Yield	0.45	0.03	−0.10	−0.08	−0.06	−1.24	0.63	0.67	
10 Yr Nom Yield	0.23	0.02	−0.05	−0.04	−0.03	−0.79	0.27	0.29	0.22

B. 1997–2014 (144 obs.)

	Expected Real GDP Growth Nowcast $j=0$	$j=1$	$j=2$	$j=3$	$j=4$	Stock Return	One Yr Real Yield	One Yr Nominal Yield	Ten Yr Nominal Yield
Empirical									
Nowcast	0.69								
$j=1$	0.33	0.48							
$j=2$	0.14	0.24	0.29						
$j=3$	0.09	0.14	0.19	0.20					
$j=4$	0.05	0.06	0.10	0.11	0.12				
Stock Return	1.48	2.12	1.93	1.47	1.10	34.94			
1 Yr Real Yield	0.09	0.07	0.00	−0.01	−0.01	0.20	0.08		
1 Yr Nom Yield	0.12	0.10	0.03	0.01	0.00	0.50	0.07	0.08	
10 Yr Nom Yield	0.09	0.05	0.01	0.00	0.00	0.45	0.05	0.05	0.10
Model									
Nowcast	0.93								
$j=1$	0.30	0.47							
$j=2$	0.11	0.22	0.26						
$j=3$	0.09	0.17	0.15	0.20					
$j=4$	0.07	0.14	0.12	0.09	0.15				
Stock Return	1.57	2.34	1.84	1.47	1.17	35.56			
1 Yr Real Yield	0.13	0.04	0.01	0.00	0.00	0.23	0.08		
1 Yr Nom Yield	0.16	0.08	0.03	0.02	0.02	0.53	0.07	0.09	
10 Yr Nom Yield	0.12	0.04	0.01	0.00	0.00	0.44	0.05	0.06	0.10

Table 7. Asset Reactions to Output News

The model and estimation method are described in Sections 2 and 4 respectively. Stock returns and bond yields react to news about permanent innovations in instantaneous log output and news about the drift of instantaneous log output. The table reports estimates of the sensitivity of stock returns and bond yields to the news, as defined in equation (52). The units are percentage point responses to percentage point changes in either output or the growth rate of output. Standard errors are in parentheses. Asterisks represent asymptotic two-sided p-values versus zero of 10%, 5% and 1%.

	1975–1996		1997–2014	
	Permanent	Drift	Permanent	Drift
Excess Stock Return	0.56	9.30***	1.60	26.68***
	(1.19)	(3.16)	(1.98)	(3.86)
Δ One Yr Real Yield	0.78***	0.31	0.52***	0.03
	(0.16)	(0.47)	(0.10)	(0.19)
Δ One Yr Nom Yield	0.84***	−0.26	0.57***	0.39*
	(0.15)	(0.50)	(0.09)	(0.24)
Δ Ten Yr Nom Yield	0.43***	−0.10	0.46***	0.05
	(0.08)	(0.31)	(0.16)	(0.21)

Table 8. Model-Implied Variance Decompositions and Asset Correlations

The model and estimation method are described in Sections 2 and 4 respectively. Three components explain the variation in innovations to expected output growth, excess stock returns, changes in one-year ex ante real rates, and changes in one-year and ten-year nominal Treasury yields. The components are permanent innovations to log output, innovations to the drift of log output, and catch-all shocks unrelated to output dynamics. Panel A reports fractions of total variance explained by the components, as well as the covariance between permanent and drift innovations. Totals may not sum to one owing to rounding. Panel B reports model-implied correlations among excess stock returns and bond yields.

A. Variance Decompositions

	Estimates for 1975–1996				Estimates for 1997–2014			
	Permanent	Drift	Covar	Other	Permanent	Drift	Covar	Other
Expected Output Growth								
Nowcast	1.03	0.05	−0.20	0.11	0.81	0.08	0.02	0.09
One Q Ahead	0.53	0.48	−0.32	0.32	0.27	0.54	0.02	0.17
Two Q Ahead	0.00	0.51	0.00	0.49	0.00	0.69	0.00	0.31
Three Q Ahead	0.00	0.39	0.00	0.61	0.00	0.59	0.00	0.41
Four Q Ahead	0.00	0.27	0.00	0.73	0.00	0.48	0.00	0.52
Assets								
Excess Stock Return	0.00	0.09	−0.01	0.92	0.01	0.52	0.01	0.47
Δ One Yr Real Yield	0.18	0.00	−0.03	0.84	0.27	0.00	0.00	0.73
Δ One Yr Nom Yield	0.21	0.00	0.02	0.76	0.32	0.05	0.01	0.63
Δ Ten Yr Nom Yield	0.17	0.00	0.02	0.81	0.18	0.00	0.00	0.82

B. Correlations Among Stock Returns and Changes in Bond Yields

	Due to Output News			Due to Non-Output News		
	Stock Return	Δ One Yr Real Yield	Δ One Yr Nom Yield	Stock Return	Δ One Yr Real Yield	Δ One Yr Nom Yield
1975–1996						
Δ One Yr Real Yield	−0.20	1		−0.23	1	
Δ One Yr Nom Yield	−0.44	0.97	1	−0.27	0.94	
Δ Ten Yr Nom Yield	−0.42	0.97	1.00	−0.31	0.66	0.69
1997–2014						
Δ One Yr Real Yield	0.19	1		0.11	1	
Δ One Yr Nom Yield	0.49	0.95	1	0.16	0.85	1
Δ Ten Yr Nom Yield	0.21	1.00	0.96	0.27	0.44	0.49

Table 9. Relation between Innovations in Inflation Forecasts and Output Forecasts

Revisions in Greenbook forecasts of inflation from zero (nowcast) to four quarters ahead are regressed on contemporaneous fitted shocks to the level of log real GDP and the instantaneous drift of log real GDP. The shocks are inferred from continuous-time dynamic model of output that is estimated using revisions in Greenbook forecasts of real output growth, stock returns, and changes in real and nominal yields. The model is estimated separately over the samples 1975 through 1996 and 1997 through 2014. Asymptotic standard errors are adjusted for generalized heteroskedasticity.

	1975–1996			1997–2014		
Horizon of Inflation Forecast	Level Shocks	Growth Shocks	R^2 [Obs]	Level Shocks	Growth Shocks	R^2 [Obs]
Nowcast (zero ahead)	−0.237	−1.353***	0.108	0.058	−1.541***	0.146
	(0.166)	(0.310)	[196]	(0.383)	(0.352)	[144]
One quarter ahead	0.006	−0.841**	0.096	0.013	0.703**	0.075
	(0.097)	(0.320)	[196]	(0.141)	(0.322)	[144]
Two quarters ahead	0.009	−0.955***	0.154	0.067	0.391***	0.079
	(0.085)	(0.296)	[196]	(0.072)	(0.131)	[144]
Three quarters ahead	0.038	−0.869***	0.223	−0.021	0.281**	0.047
	(0.060)	(0.180)	[196]	(0.041)	(0.128)	[144]
Four quarters ahead	0.118**	−0.295*	0.106	0.079	0.344***	0.179
	(0.054)	(0.151)	[179]	(0.050)	(0.117)	[144]

Table 10. Changes in Yields and Innovations in Output Forecasts

Changes in bond yields from the date of one Greenbook forecast to the next Greenbook forecast are regressed on contemporaneous and lagged permanent innovations to the level of real GDP. The innovations are inferred from continuous-time dynamic model of real output that is estimated using revisions in Greenbook forecasts of real output growth, stock returns, and changes in real and nominal yields. The model is estimated separately over the samples 1975 through 1996 and 1997 through 2014. Asymptotic standard errors are adjusted for generalized heteroskedasticity.

Explanatory Variable	Δ One Year Real Yield	Δ One Year Nominal Yield	Δ Ten Year Nominal Yield
A. 1975–1996			
Contemporaneous Innovation	0.897***	1.035***	0.577***
	(0.225)	(0.236)	(0.107)
Sum of Lags 1–6 of	−0.016	−0.017	0.026
Innovation	(0.088)	(0.081)	(0.037)
R^2	0.179	0.240	0.221
Obs	186	190	190
B. 1997–2014			
Contemporaneous Innovation	0.607***	0.708***	0.556***
	(0.078)	(0.101)	(0.081)
Sum of Lags 1–6 of	0.065	0.053	−0.044
Innovation	(0.043)	(0.042)	(0.059)
R^2	0.273	0.345	0.191
Obs	138	138	138

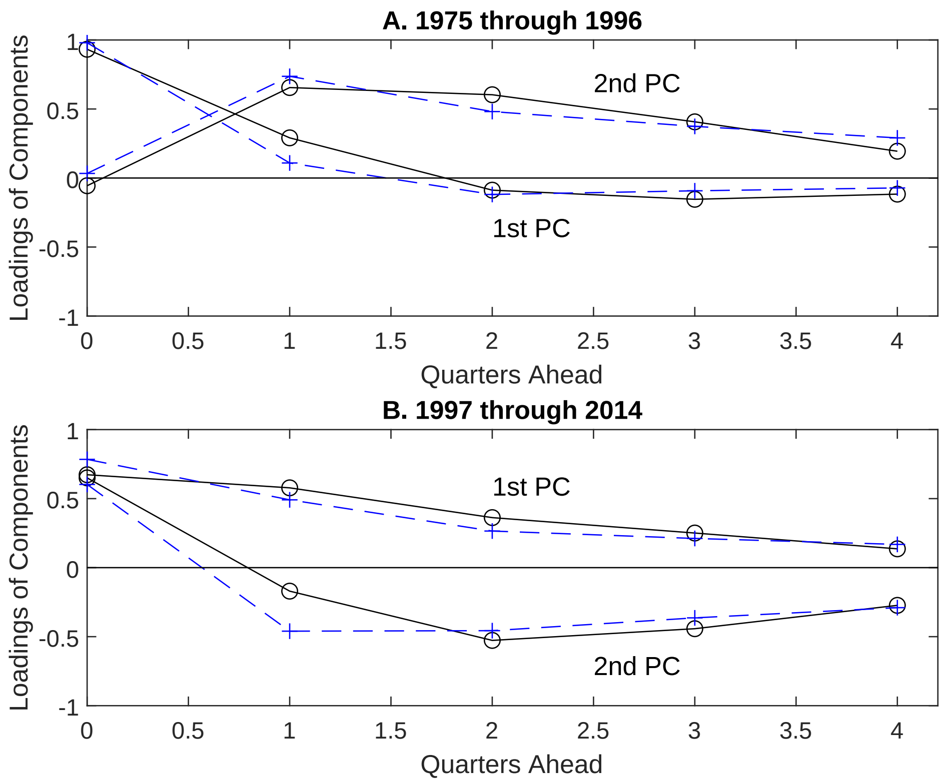

Figure 1. Principal components of innovations in Greenbook forecasts of GDP growth, 1975 through 2014. The data are forecast-to-forecast innovations in Greenbook predictions of real GDP growth zero quarters ahead (the nowcast) through four quarters ahead. The figure displays with solid lines the loadings of the first two principal components for an early and a later sample. The dashed lines are population properties of the dynamic model of output growth in Section 2, estimated separately for the two sample periods. The estimated versions use one factor to specify the drift of instantaneous output.

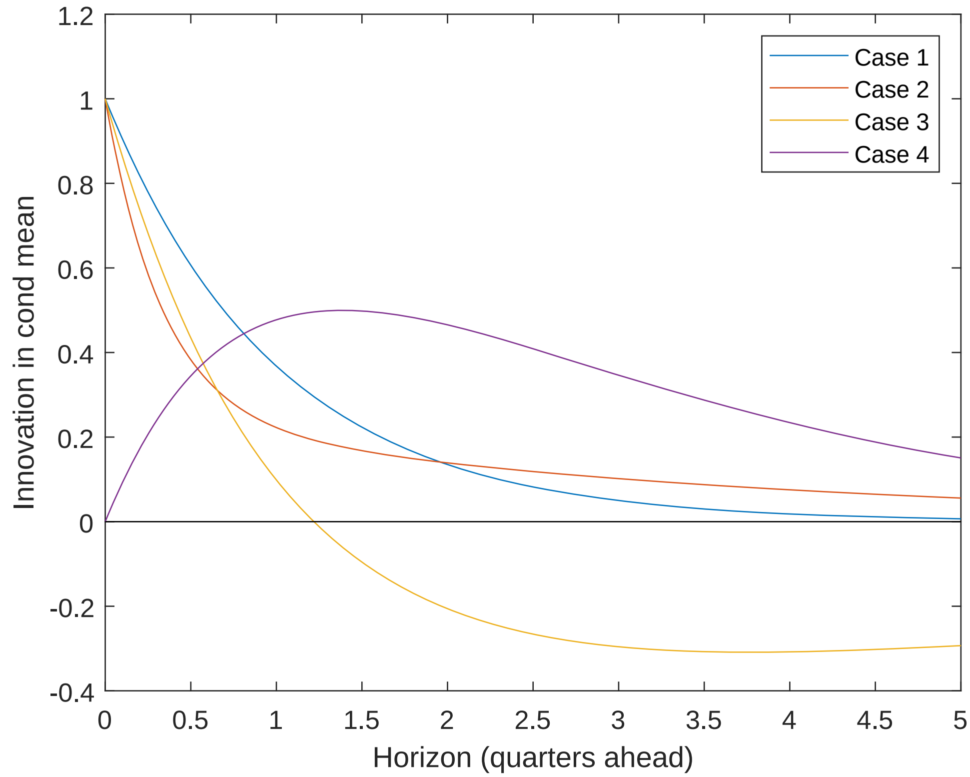

Figure 2. Examples of a model's innovations to conditional means of output growth. A Brownian increment at $t = 0$ alters conditional means of instantaneous real output growth at all horizons $h > t$. The figure displays shapes of the h-ahead innovations for various parameters of the model in Section 2. Case 1 sets $\sigma_1 = 1, \gamma_1 = 1$, $\sigma_2 = 0$. Case 2 sets $\sigma_1 = 0.75, \gamma_1 = 3, \sigma_2 = 0.25$, $\gamma_2 = 0.3$. Case 3 sets $\sigma_1 = 1.5$, $\gamma_1 = 1$, $\sigma_2 = -0.5$, $\gamma_2 = 0.1$. Case 4 sets $\sigma_1 = 2$, $\sigma_2 = -1$, $\gamma_1 = 1$, $\gamma_2 = 0.5$.

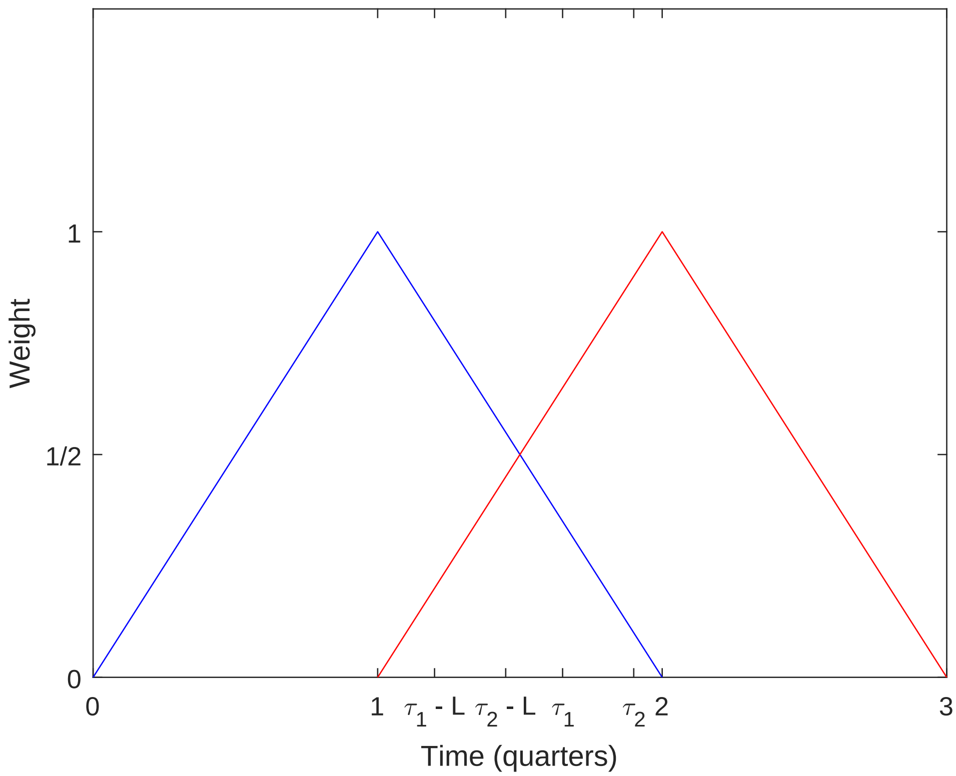

Figure 3. Weights on Brownian increments that comprise quarterly changes in output. Instantaneous output follows a continuous time random walk. Quarterly output is the average of instantaneous output during the quarter. Quarter-end dates are integers. Quarter-two output less quarter-one output is an integral of the Brownian increments to instantaneous output from time zero to time two with weights given by the blue line. The red line represents the weights for quarter-three output less quarter-two output. Forecast dates are τ_1 and τ_2, with instantaneous output observed through $\tau_1 - L$ and $\tau_2 - L$ respectively.

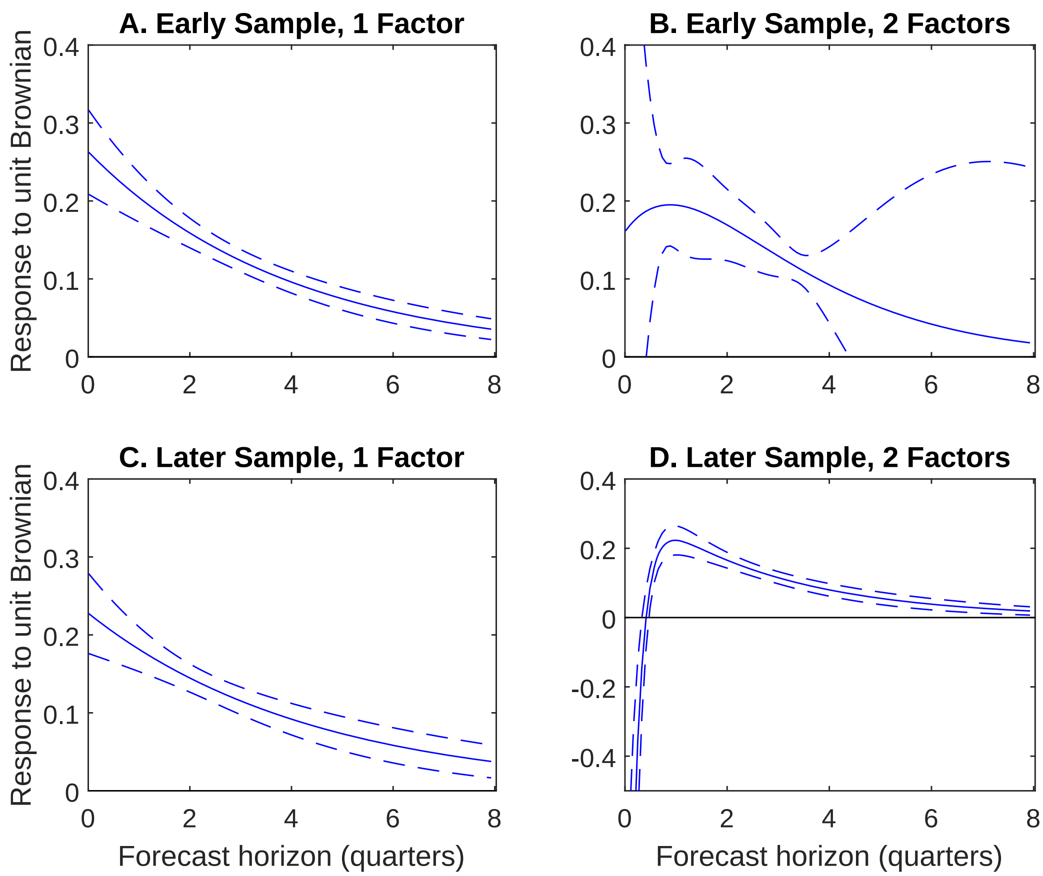

Figure 4. Estimated shapes of drift functions of instantaneous log output. In the model of Section 2, a Brownian increment alters the expected drift of instantaneous log output at all future horizons h. The figure displays the estimated response of these expected h-ahead drifts to a Brownian increment. The four estimated models differ in their sample periods (1975 through 1996 and 1997 through 2014) and in the number of factors included in the drift function. Also displayed are plus/minus two standard error bounds on the responses.

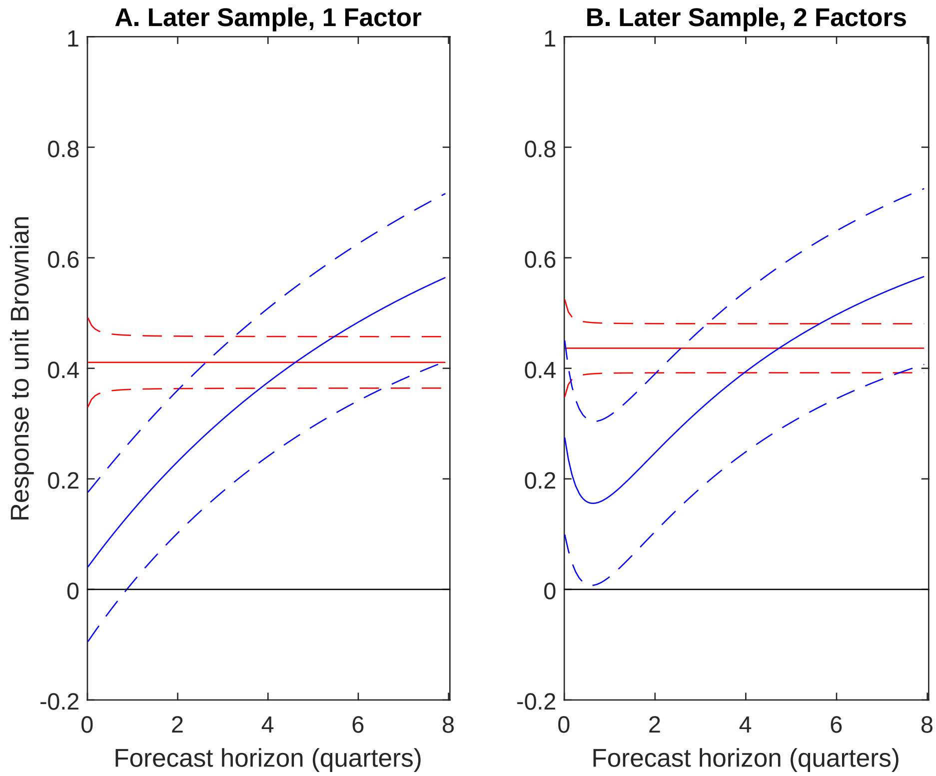

Figure 5. Estimated shapes of innovations to expected average cumulative log output. In the model of Section 2, forecasters observe Brownian increments at t that change the level and drift of instantaneous log output. They alter their expectations of expected average future cumulative log output between the date of the shock and the horizon h. A "level" Brownian produces a level change in average cumulative output, while a "drift" Brownian has both a level component and a drift component. The figure displays estimated innovations to these expectations for the sample period 1997 through 2014 and for two versions of the model. Also displayed are plus/minus two standard error bounds on the responses.

8 Appendix

This appendix provides more details of derivations in the text.

8.1 Calculation of the ex-ante one-year real yield

Section 1.1 in the text describes the data, including the one-year ex-ante real yield. This is nominal one-year yield less a measure of expected inflation. I use Greenbook forecasts of inflation for the GDP deflator. For the purposes of constructing a real yield, the correct measure of expected inflation is from the date of the Greenbook forecast to the same date in the next year. However, Greenbook forecasts of expected inflation are forecasts of the percentage change in average price level in quarter $k-1$ to the average price level in quarter k. Assume that Greenbook forecasts are for price levels from one quarter's midpoint to the next quarter's midpoint. Also assume that expected inflation over each midpoint-to-midpoint is constant. In other words, for Greenbook forecast i, expected inflation for any arbitrary day in the future is a step function in time, where steps occur at quarter midpoints.

Denote the log of forecasted inflation by (following equation (1) in the text for forecasted output growth)

$$\pi_i^{(k)} \equiv 100 \log E_i^G \left(\left(\frac{DEFLATOR_{t_i+k}}{DEFLATOR_{t_i+k-1}} \right)^4 \right). \tag{A-1}$$

For Greenbook forecast i, define the fraction of the quarter remaining as f_i. For example, a Greenbook forecast made in the first week of a quarter has a fraction f_i close to one, while a Greenbook forecast made in the last week of a quarter has a fraction f_i close to zero. Then expected one-year inflation at i is measured by

$$EXP\_INFL_i = \begin{cases} \frac{1}{4}((f_i - 0.5)\pi_i^{(0)} + \sum_{k=1}^{3} \pi_i^{(k)} + (1.5 - f_i)\pi_i^{(4)}), & f_i > 0.5; \\ \frac{1}{4}((f_i + 0.5)\pi_i^{(1)} + \sum_{k=2}^{4} \pi_i^{(k)} + (0.5 - f_i)\pi_i^{(5)}), & f_i \leq 0.5. \end{cases} \tag{A-2}$$

There are a few observations for which $f_i < 0.5$ and the five-quarter-ahead forecast is missing. In these cases the five-quarter-ahead expectation is proxied by the four-quarter-ahead expectation. The first Greenbook forecasts in 1976 and 1977 are missing four-quarter-ahead inflation expectations. For these two observations, expected one-year inflation is set to missing.

8.2 State vector conditional covariance matrices

Equation (16) expresses the covariance of the state vector at $t+s$ conditional on the state vector at t. This conditional covariance does not depend on either t or the state vector. Write the matrix K_m from (14) in Jordan normal form,

$$K_m = V_m J_m V_m^{-1}. \tag{A-3}$$

The matrices are

$$V_m = \begin{pmatrix} 0 & \gamma_1^{-1} - \gamma_2^{-1} & -\gamma_1^{-1} & \gamma_2^{-1} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ \gamma_1^{-1} - \gamma_2^{-1} & \gamma_2^{-2} - \gamma_1^{-2} & \gamma_1^{-2} & -\gamma_2^{-2} \end{pmatrix}, \qquad J_m = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\gamma_1 & 0 \\ 0 & 0 & 0 & -\gamma_2 \end{pmatrix}. \tag{A-4}$$

Then the conditional covariance of the state vector is

$$\mathrm{Cov}\left(X_{m,t+s} | X_{m,t}\right) = \int_0^s V_m e^{J_m u} V_m^{-1} \Omega_m \Omega_m' V_m^{-1\prime} e^{J_m u \prime} V_m' du. \tag{A-5}$$

Rewrite this as

$$\mathcal{V}_m(s) \equiv \mathrm{Cov}\left(X_{m,t+s} | X_{m,t}\right) = V_m \left(\int_0^s e^{J_m u} \mathcal{M}_m e^{J_m u \prime} du \right) V_m', \tag{A-6}$$

defining

$$\mathcal{M}_m \equiv V_m^{-1} \Omega_m \Omega_m' V_m^{-1\prime} \tag{A-7}$$

and recognize that

$$e^{J_m u} = \begin{pmatrix} 1 & u & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{-\gamma_1 u} & 0 \\ 0 & 0 & 0 & e^{-\gamma_2 u} \end{pmatrix}. \tag{A-8}$$

Given the matrix $\mathcal{M}$ and the exponential form (A-8), the integration of the 4×4 matrix in (A-6) can be done analytically element by element. It helps to recall

$$\int x e^{\alpha x} dx = \alpha^{-1} e^{\alpha x}(x - \alpha^{-1}) + C.$$

Analytic solutions for discrete-horizon covariances of the full state vector X_t defined in (17) are solved using the same technique. Simply drop m subscripts from (A-3) through (A-8). The Jordan decomposition of K in (19) is

$$K = VJV^{-1}, \tag{A-9}$$

$$V = \begin{pmatrix} V_m & 0_{4\times 4} \\ 0_{4\times 4} & I_{4\times 4} \end{pmatrix}, \qquad J = \begin{pmatrix} J_m & 0_{4\times 4} \\ 0_{4\times 4} & 0_{4\times 4} \end{pmatrix}. \tag{A-10}$$

The matrix exponential of Ju is

$$e^{Ju} = \begin{pmatrix} e^{J_m u} & 0_{4\times 4} \\ 0_{4\times 4} & I_{4\times 4} \end{pmatrix}. \tag{A-11}$$

Plug these matrices into a version of (A-5) without the m subscripts to determine the covariance matrices in (44) and (45).

8.3 Forecast Innovations

The expression for forecasts at date t is (27). The forecast depends on the macroeconomic state observed at t (dated $t - L$) and possibly known prior quarter-end values of the state. When no quarter-end states are observed between t_1 and t_2, the updated forecast depends only on the difference between the state observed at t_2 and the forecast of that state at t_1. The logic explains the $QEND(t_1, t_2) = 0$ case of the forecast innovation in (33).

The more complicated case is when agents at t_2 observe during the interval $(t_1\ t_2]$ one of the quarter-end values of output. The forecast innovation then requires tracking the evolution of the state during two separate time intervals: the one from $t_1 - L$ to the newly-observed quarter-end value, and the one from the observed quarter-end value to $t_2 - L$. Therefore we need to know the date of the observed quarter-end value. This depends on the information lag length.

Recall that $t+d(t)$ is the ending time of the current quarter from the perspective of agents at t. As of time t, the next quarter-end value revealed is the quarter-end at $t + d(t) - p(t)$, an integer. The function $p(t)$ is

$$p(t) = \lfloor t \rfloor - \lfloor t - L \rfloor. \tag{A-12}$$

If $p(t) = 0$, then agents at t know all of the quarter-end values for quarter-ends prior to t. If, however, $p(t) = 1$, agents at t don't know the previous quarter-end value; the information lag is too long. In principle, $p(t) = 2$ is possible. The first interval for which we track innovations in the state vector is from $t_1 - L$ to $t_1 + d(t_1) - p(t_1)$. The second interval is from the ending date of the first interval to $t_2 - L$.

The total forecast innovation depends on the expectation of the state vector at $t_2 - L$ given its realization at the end of the first interval. This expectation is

$$E(X_{t_2-L} | X_{t_1+d(t_1)-p(t_1)}) = \Gamma(t_1, t_2) X_{t_1+d(t_1)-p(t_1)}, \tag{A-13}$$

$$\Gamma(t_1, t_2) \equiv \exp(K_m(t_2 - L - (t_1 + d(t_1) - p(t_1)))). \tag{A-14}$$

The forecast innovation has separate components for the state innovations over the two intervals,

$$\epsilon_{j,t_2} = e_4' \Theta^*(j, t_1, t_2) \tilde{X}_{m,t_1-L,t_1+d(t_1)-p(t_1)} + e_4' \Theta(j, t_2) \tilde{X}_{m,t_1+d(t_1)-p(t_1),t_2-L}, \tag{A-15}$$

where

$$\Theta^*(j, t_1, t_2) = \begin{cases} \Theta(j, t_2)\Gamma(t_1, t_2) - 2I, & \mathcal{N}(j, t_2) = 2; \\ \Theta(j, t_2)\Gamma(t_1, t_2) + I, & \mathcal{N}(j, t_2) = 1; \\ \Theta(j, t_2)\Gamma(t_1, t_2), & \mathcal{N}(j, t_2) = 0. \end{cases} \tag{A-16}$$

8.4 Estimation details

The covariance matrix of the observed vector (46) is, expressly indicating the dependence on the parameter vector ρ,

$$V_{total}(\tau_{i-1}, \tau_i, i_{max}; \rho) \equiv \text{Cov}(\epsilon_i^{observed}) = V_{full}(\tau_{i-1}, \tau_i, 0, i_{max}; \rho) + V_{err}(\rho), \tag{A-17}$$

$$V_{err}(\rho) = \begin{pmatrix} \sigma_{err} I_{N\times N} & 0_{N\times 4} \\ 0_{4\times N} & 0_{4\times 4} \end{pmatrix}. \tag{A-18}$$

Dropping constant terms, the log likelihood function is

$$\begin{aligned} l(\epsilon_1^{observed}, \ldots, \epsilon_T^{observed}; \rho) = &-\frac{1}{2}\sum_{i=1}^{T}\left(\left|V_{total}(\tau_{i-1}, \tau_i, i_{max}; \rho)\right|\right. \\ &\left. + \ \epsilon_i^{(observed)\prime} V_{total}^{-1}(\tau_{i-1}, \tau_i, i_{max}; \rho)\epsilon_i^{(observed)}\right). \end{aligned} \tag{A-19}$$

Four observations of the one-year ex ante risk free yield are missing. (The appropriate measures of expected inflation are unavailable in the Greenbook forecasts.) These four Greenbook dates are included in the sum of (A-19) by dropping the relevant row and column from the model-implied covariance matrix. Estimation of the information matrix excludes these four dates entirely because the score vector for these observations contains elements that are identically zero.