Expertise and Reputations in Markets for Credence Services

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Abstract

The waste of resources spent on unnecessary services-such as sham car repairs and dental and medical treatments-is prevalent and well documented. This paper studies the role of providers' expertise and reputations in credence service markets and their effect on the incidence of the fraudulent prescription of unnecessary services and treatments.

Keywords: Credence service markets, reputation, experts' competence, dynamic stochastic games of incomplete information

1 Introduction

Credence services-such as car repairs and maintenance, medical and dental treatments, plumbing repairs, and taxi services, to mention just a few examples-are char-

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acterized by information asymmetry between providers (who have the expertise required to assess the need for service) and customers (who lack the ability to assess the necessity of the prescribed service *both before and after* the service is rendered). When, in addition, cost considerations lead to the bundling of the diagnosis and the provision of service, and providers have incentives to prescribe unnecessary services, this asymmetry leads to fraudulent prescriptions and the provision of unnecessary services.

In the wake of Darby and Karni (1973), who introduced the concept of credence goods, numerous studies have documented and analyzed fraudulent prescriptions of unnecessary services and treatments in the markets for credence goods and services.[1] Most of the literature dealing with credence goods and services presumes that the experts (service providers) *can always correctly diagnose the problem and identify the necessary treatment.* This presumption, while a convenient simplification, disregards the fact that can correctly identify the problem and necessary treatment. An expert's limitation results in to two types of errors: overestimation and underestimation of the severity of a problem and the prescription of unnecessary or insufficient service, respectively.

Expertise is the providers' private information, but records of their recommendations and the subsequent outcomes are public and determine providers' reputations.[2] Reputations affect the price of service and the provider's income. Consequently, rep-

[1]Dulleck and Kerschbamer (2006) and Balafoutas and Kerschbamer (2020) survey of these studies and provides references.

[2]The assumption that the record is public information is invoked to simplify the exposition. It is intended to capture information spread by channels, such as word of mouth or review platforms, that affect providers' reputations.

utation concerns affect providers' prescription decisions and the prevalence of fraud.

This paper studies the effects of providers' levels of expertise and their reputations on the prices of service and the perpetration of fraud in credence service markets.

In credence service markets in which providers have limited, or bounded, levels of expertise, providers do not have complete control over their reputations. Consequently, the pattern of fraud perpetrated in these markets and the prices charged are quite different from those that would be exist if the providers' expertise allowed them to always diagnose problems correctly.

To grasp this point, consider an expert who can always diagnose a problem and identify the necessary (and sufficient) service required to address it. Such a provider can enhance his reputation by always prescribing the necessary service. The cumulative evidence will prove him right every time, and his reputation for being correct will grow.

By contrast, if the provider's expertise is limited, even if he always prescribes service that, to his best judgment, is what is needed, he may err by prescribing services that prove either insufficient or unnecessary. Consequently, the cumulative evidence is a stochastic process and the corresponding provider's reputation a random variable that the provider may influence but cannot control.

Providers' reputations shape customers' demand for their services, which in turn, affect prices, profits, and inclination to commit fraud. Analysis of these aspects of the market is the main thrust of this paper.

Consider a market for a service populated by customers and two providers. When a customer detects symptoms, (e.g., equipment malfunction, physical malaise) she must choose between seeking service or ignoring the problem (e.g., to service or to retire malfunctioning equipment). If she chooses to address the problem, she selects a provider to diagnose it and prescribe treatment. that, if accepted, provides the treatment. The customer may reject the provider's prescription and seek a second opinion, in which case, the second provider diagnoses the problem and prescribes treatment. The customer may then accept the second provider's recommendation or forgo addrssing the problem.

The presumption is that underlying the symptoms is a true state, which is the cause of the problem. The providers' signals indicate the likelihoods of the underlying states. The quality of a provider's signal depends on his expertise. In particular, the greater the provider's expertise, the more informative is his signal. Fraud is said to be committed when the provider prescribes treatment that is unlikely to be necessary given his signal.

Providers' reputations reflect customers' perceptions of the adequacy of their prescriptions. More specifically, providers' reputations are customers' probabilistic beliefs that they prescribe the necessary and sufficient treatments. As such, the providers' reputations amalgamate their expertise and the honesty of their prescription strategies. The customers' beliefs and the providers' reputations evolve as evidence about the correctness of their prescription accumulates.

Markets for credence goods and services encompass a variety of institutions with specific essential features. Consequently, the analysis of these markets requires market-specific models that capture these idiosyncratic characteristics.[3]

[3]See Chiu and Karni (2021) and Karni (2024) for examples.

To highlight the roles of expertise and reputations, I model the market of credence services as a dynamic stochastic game of incomplete information consisting of a sequence of stage games. The stage games are linked by the evolution of the providers' records of prescription-outcome pairs. These records contain information about the experts' expertise, which, in turn, affects the providers' reputations in the stage games. Each stage game consists of two interlinked parts. The first part models the providers' interactive pricing decisions. The second part is a prescription subgame depicting the interaction among customers and providers. The analysis invokes the concept of perfect Bayesian equilibrium.

The next section describes the credence service market. Section 3 presents formal definitions and discusses of expertise and reputations. Section 4 describes the stage game and establishes the existence of Perfect Bayesian Equilibrium (PBE). Section 5 discusses the economic implications of expertise, reputations, and liability in shaping the equilibrium in the credence service market and the ensuing fraudulent behavior. Section 6 provides concluding remarks and discusses of the related literature. The appendix contains the proofs.

2 The Credence Service Market

2.1 Overview

Consider a market for credence services populated by two providers, A and B, and numerous potential customers. The providers are characterized by their levels of expertise, which are their private information. The levels of expertise are fixed at the outset and are maintained throughout the sequences of the stage games. When presented with symptoms indicating a problem a provider's expertise determines his likelihoods of correctly identifying the underlying cause and the treatment necessary to resolve it.[4]

At the outset of each stage game, providers' records are inherited from the preceding stage game. These records reflect their past performance (i.e., their recommendations and subsequent outcomes).

Customers are characterized by their types, which include the values, $v \in V = [0, \bar{v}]$, they attach to fixing the problem; their search costs, $\theta \in \Theta = [0, \bar{\theta}]$; and the costs, $b \in \mathbb{B} = [0, \bar{b}]$, of the inconvenience of having to fix the problem if the prescribed service proved to be insufficient. The customers' characteristics are assumed to be independent random variables whose cumulative distribution functions, denoted, F_v, F_θ, F_b, respectively, are assumed to be common knowledge. The customers' characteristics are their private information.

At the outset of each stage game, the providers post their hourly service prices. Problems occur randomly, so the arrival of customers on the market is an exogenous stochastic process. The arrival of customers at a provider's service station, however, is in part endogenous, as it depends on the providers' prices and reputations.

To simplify the exposition, I assume that there are two states: a minor problem, denoted by ω_L, and a major problem, denoted by ω_H, one of which is true (i.e., is the cause of the problem). I assume that fixing a major problem requires more

[4] I assume that providers are not constrained by capacity and are able to accommodate all customers in need of service without delay. Chiu and Karni (2012) and Karni (2024) address the possible effect of waiting time.

service time than fixing a minor one. With slight abuse of notation, let ω_L and ω_H denote the number of service hours required to fix the minor and major problems, respectively (i.e., $\omega_L < \omega_H$). Let $\Omega := \{\omega_L, \omega_H\}$ denote the state space.

Customers lack the expertise required to diagnose the problem they experience and determine, either in advance or after the fact, whether the prescribed service is necessary. If a provider prescribes the high level of service and the customer accepts it, then the problem is solved, and the customer concludes that the prescribed service was sufficient. She cannot, however, be sure that the service was necessary (i.e., whether the service provided was the minimal required to solve the problem).

In choosing her first provider, the customer is guided by the posted service prices and the providers' reputations. If, after obtaining the first provider's prescription, the customer chooses to seek a second opinion, the second provider diagnoses the problem and prescribes treatment, which the customer either accepts or forgoes the service altogether.[5] Seeking a second opinion involves search, or switching, cost, θ, the presence of which makes the bundling of diagnosis and provision of service cost-effective.

If the anticipated cost of the prescribed service exceeds the value the customer assigns to service, she forgoes it. If the customer accepts the recommended service and the problem persists (i.e., the recommended service fails to fix the problem), the provider is supposed to rectify his mistake at no additional charge.This aspect of the transaction is dubbed *liability*.[6] In such a case the customer still bears the cost, b,

[5] To simplify the exposition, I assume that returning to the first provider for service is not an option.

[6] In some markets (e.g., the market for medical and dental services), misdiagnosis may result in malpractice payouts.

representing the inconvenience to her (e.g., loss of time and money) of having to deal with the problem.

The information asymmetry in this market is two-sided. Customers' private information is depicted by the parameters (θ, v, b). Providers' private information is their level of expertise, denoted e_j, $j \in \{A, B\}$, reflecting their inherent talents and/or acquired skills. When a customer shows up reporting symptoms, the provider receives a noisy signal in the set $S = \{s_L, s_H\}$ that informs him about the true state. The quality of the signal, which depends on the provider's expertise, determines the provider's posterior distribution on the states in the information set, Ω. By definition, the greater the provider's expertise, the higher the quality of the signal he receives. Formally, $e_j := \Pr_j (s_k \mid \omega_k)$, $k \in \{L, H\}$. Expertise is the ability to identify the true state more often than not. Hence, e_A and e_B take values in [1/2,1].[7] I further assume that, from the customers' viewpoint, the providers' levels of expertise are independent and identically distributed random variables. Let η denote the prior cumulative distribution function of e_j, $j \in \{A, B\}$ and assume that it has full support, is differentiable, and is common knowledge.

Ex post, after the inspection, there is additional information asymmetry; the provider is better informed about the likelihoods of the two states.

Let $p_j \in V$, $j \in \{A, B\}$, denote the price per hour of service of provider j, and let c denote the (constant) marginal hourly operating cost of the service station, which I assume to be the same for the two providers.[8] Then provider j's hourly profit is

[7]There is no essential loss of generality by including $1/2$ in the range of the expertise. The analysis that follows will not change if the range is $[q, 1]$ for some $q > 1/2$.

[8]This simplifying assumption is introduced in order to highlight the roles of expertise and reputations by not including additional sources of asymmetry between the providers.

$p_j - c \geq 0$ for services sold and $-c$ for services the provider is required to fix free of charge for problems he misdiagnosed. Consequently, to increase their profits and avoid the risk of loss as a result of prescribing insufficient treatment, providers have incentives to prescribe ω_H more often than necessary.

2.2 The Credence Service Market Game

Interactions between the customers and service providers in the credence service market are modeled as sequence of stage games of incomplete information. Before the $(n+1) - th$ stage game begins, the record of past prescription-outcome pairs , $x^n = (x_1, ..., x_n)$, is public knowledge. Depending on the sequence of moves by providers and customers, each data point, x_i, consists of a single prescription or two prescriptions and the corresponding outcomes. More specifically, each data point corresponds to one of the following prescription-outcome scenarios.

Scenario 1. The first provider, j_1, prescribes a service that is *revealed to be unnecessary* (i.e., the first provider prescribes ω_H; the customer seeks a second opinion; and the second provider prescribes ω_L, which happens to be the true state). This Scenario is denoted ϱ_1.

Scenario 2. The first provider, j_1, prescribes ω_H, which is *revealed to be necessary and sufficient* (i.e., the first provider prescribes ω_H; the customer seeks a second opinion; and the second provider prescribes ω_L, which turns out to be insufficient). This scenario is denoted ϱ_2.

Scenario 3. The first provider, j_1, prescribes ω_L, which is *revealed to be the true state* (i.e., the customer accepts the first provider's prescription, which turns out to be sufficient). This scenario is denoted ϱ_3.

Scenario 4. The first provider prescribes ω_L, which is *revealed to be insufficient* (i.e., the customer accepts the first provider's prescription and the problem persists). This scenario is denoted ϱ_4.

If the first provider prescribes ω_H and the customer either accepts it or rejects it and accepts the prescription, ω_H, of the second provider, the problem is solved regardless of whether the true state is ω_L or ω_H. If the customer rejects the second provider's prescription, the true state is not revealed; no information is revealed about the necessity of the prescribed service.[9]

Let $X = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4\}$ denote the set of these scenarios. Then $x^n \in X^n$ is a record of scenarios after the n-th stage game.

Providers' strategies in the stage game consist of two parts. In the first part, providers post their hourly service prices. I assume that, if the two providers post the same price, provider whose record is better is the leader and, as the name suggests, posts his price first. The provider with worse record, dubbed the follower, observes the leader's price before posting his own. Without loss of generality, suppose that A happens to be the the leader in the stage game under consideration. In the second part of the stage game, referred to as the *prescription subgame*, providers play their respective prescription strategies.

Given the record $x^n \in X^n$, the leader's strategy in the stage game consists of two

[9] These are the only scenarios on the equilibrium paths of the stage games. Specifically, if a customer plans on rejecting the prescription ω_L of provider j, she will not choose j for her first provider.

maps representing the pricing and prescription strategies, respectively. Formally,

$$\varphi_A : [1/2, 1] \times X^n \to V$$

and

$$\alpha_A : [1/2, 1] \times X^n \times V^2 \to \Delta(\Omega)^S,$$

where $\alpha_A(s \mid e_A, x^n, p_A, p_B) \in \Delta(\Omega)$ denotes the probability distribution on Ω according to which A prescribes a treatment upon receiving the signal $s \in S$.

The follower observes the price of the leader and chooses his own strategy. Formally, B's strategy consists of two maps depicting the pricing and prescription strategies , respectively,

$$\varphi_B : [1/2, 1] \times X^n \times V \to V,$$

and

$$\alpha_B : [1/2, 1] \times X^n \times V^2 \to \Delta(\Omega)^S.$$

3 Expertise and Reputation

The analysis of the effects of providers' expertise and reputations on the service prices and the level of fraud perpetuated in credence service markets requires a clear concept of what is meant by expertise and reputation.

3.1 Expertise

Given $e_j \in [1/2, 1]$, the joint probability distribution on $\Omega \times S$ is given by:

Prior	1-μ	μ
S$\diagdown\Omega$	ω_L	ω_H
s$_L$	e$_j(1-\mu)$	(1-e$_j$)μ
s$_H$	(1-e$_j$)$(1-\mu)$	e$_j\mu$

By Bayes' rule, provider $j's$ posterior beliefs about the likelihoods of the two states are

$$\mu_j\left(\omega_H \mid s_H\right)=\frac{e_j\mu}{e_j\mu+\left(1-e_j\right)\left(1-\mu\right)},\ \mu_j\left(\omega_L \mid s_H\right)=\frac{\left(1-e_j\right)\left(1-\mu\right)}{e_j\mu+\left(1-e_j\right)\left(1-\mu\right)} \quad (1)$$

and

$$\mu_j\left(\omega_H \mid s_L\right)=\frac{\left(1-e_j\right)\mu}{\left(1-e_j\right)\mu+e_j\left(1-\mu\right)},\ \mu_j\left(\omega_L \mid s_L\right)=\frac{e_j\left(1-\mu\right)}{\left(1-e_j\right)\mu+e_j\left(1-\mu\right)}. \quad (2)$$

The higher the provider's expertise, the greater the probability that the signal indicates the true state. Formally, $\mu_j\left(\omega_k \mid s_k\right)>\mu_{j'}\left(\omega_k \mid s_k\right)$, $k\in\{L,H\}$ if and only if $e_j>e_{j'}$.

3.2 Reputation

Providers' reputations are the customers' perceived probabilities that they prescribe the correct service to address their problem. Customers' perceptions of the likelihoods that the prescriptions are necessary and sufficient for the problem at hand are based on the providers records. The probability that provider j prescribes *truthfully* (i.e., prescribes ω_k when he receives the signal s_k is $\alpha_j\left(s_k \mid e_j,x^n,p\right)\left(\omega_k\right)=1$,

$k \in \{L, H\}$) and $p = (p_A, p_B)$.

If ω_k is the true state, the probability that provider j prescribes *correctly* (i.e., that he prescribes ω_k) is

$$\rho_j(e_j, x^n, p)(\omega_k(j)) := e_j \alpha_j \left(s_k \mid e_j, x^n, p\right)(\omega_k) + (1 - e_j)\, \alpha_j \left(s_{k'} \mid e_j, x^n, p\right)(\omega_k),$$

$k, k' \in \{L, H\}$, where $k \neq k'$.

Because his expertise is the provider's private information, the customer's belief that the provider j prescribes correctly (i.e., the provider's *reputation*) is

$$r_j\left(x^n, p\right) := \int_{1/2}^{1} \rho_j(e, x^n, p) d\eta_j\left(e \mid x^n\right),$$

where

$$\eta_j\left(e_j \mid x^n\right) = \frac{Pr(x_n \mid e_j)\eta_j(e_j \mid x^{n-1})}{\int_{1/2}^{1} Pr(x_n \mid e') d\eta_j(e' \mid x^{n-1})},$$

for every $x_n \in X$ and $x^n = (x^{n-1}, x_n)$. Then $\eta_j\left(e_j \mid x^n\right)$ represents the customers' posterior beliefs regarding provider j's expertise at the outset of the n-th stage subgame, and $Pr(x_n \mid e_j)$ is the probability of the scenario x_n. The latter probability depends on the expertise and prescription strategies of providers, as well as the strategies of customers. As these strategies depend on the prices, by amalgamating the providers' expertise and prescription strategies, the providers' reputations depend on both their records and prices.

4 The Stage Game

The first move in the stage game is by Nature, who selects the customer's type (θ, v, b) and the state $\omega \in \Omega$ according to commonly known joint densities $f_b(b) f_\theta(\theta) f_v(v)$ and probabilities $\mu, (1 - \mu)$, respectively.

To analyze the stage game, it is convenient to start with the prescription subgame. This approach facilitates the derivation of the demands for the providers' services and the corresponding equilibrium prices.

4.1 The prescriptions subgame

4.1.1 Preliminaries

The reader may find it helpful think about each provider as represented by a pair of agents, one in charge of the pricing decision and the other in charge of the prescription decision. Invoking this formulation, one can think of the stage game as a game played by the customer and the two pairs of agents. Seen in this way, players in the prescription subgame are the customer and the two agents representing the providers in the subgame.

At the start of a subscription subgame, Γ_E, Nature assigns a customer a state in Ω. The customer observes the prices of the two providers and, based on their records and assess their reputations. If the value she attached to the fixing the problem exceeds the minimal expected cost of service, she seeks treatment and chooses the least costly provider to be her first provider, $j_1 \in \{A, B\}$.

Upon obtaining the first provider's prescription, the customer updates her beliefs about the likelihoods of the prescriptions of the second provider and recalculates the expected cost of seeking his opinion. The customer must then choose between accepting the first provider's prescription, thereby terminating the game, and seeking a second opinion.

After receiving the second prescription, the customer must either accept or reject it. In the latter case she forgoes the service altogether. In either case the customer's decision terminates the stage game.[10]

4.1.2 Customers' strategies, beliefs, and payoffs

I assume that customers are myopic, in the sense that, upon noticing a problem, they consider solely the next interaction with providers.[11] Given the providers' records and prices, the customer's first decision is between forgoing and seeking service. Based on the prices posted by the two providers, and their reputations, a customer in need of service anticipates the expected cost of alternative courses of action and pursues the least costly one.

Given $r(x^n, p) := (r_A(\mathrm{x}^n, p), r_B(x^n, p))$, let $\Psi_j\left(p_j \mid b, r(x^n, p)\right)$ denotes the anticipated expected cost of service of a customer whose inconvenience cost is b if she chooses $j \in \{A, B\}$ and anticipates accepting the first provider's prescription.[12] By

[10]Note that customers who choose the less reputable provider, B, for their first provider would never seek a second opinion. Thus, all customers who show up at A's service facility are customers who choose A as their first provider. Consequently, provider A knows that he is the customer's first provider. By contrast, if the prices are such that the less reputable provider attracts some customer types, then provider B does not know whether he is the customer's first or second provider.

[11]For a model of a credence good market that includes client loyalty and its effect on the perpetration of fraud, see Karni (2024).

[12]This type of customer's search cost makes it too costly to seek a second opinion even if the first provider prescribes ω_H.

definition

$$\Psi_j\left(p_j \mid b, r(x^n,p)\right) = r_j\left(x^n,p\right)\left[\mu\omega_H + (1-\mu)\,\omega_L\right]p_j + \left(1 - r_j(x^n,p)\right)(1-\mu)\,b.$$

The customer always accepts the first provider's prescription if he prescribes ω_L, or she wouldn't have chosen him in the first place. If the customer anticipates seeking a second opinion if prescribed ω_H she anticipates updateing her beliefs based on the prescription of the first provider. Formally, upon receiving the first provider's prescription, $\omega_k(j_1)$, $j_1 \in \{A, B\}$, the customer updates her beliefs about the states using Bayes' rule, as follows:

$$\mu\left(\omega_H \mid \omega_k(j_1), r(x^n,p)\right) = \frac{r_{j_1}(x^n,p)\mu}{r_{j_1}(x^n,p)\mu + (1 - r_{j_1}(x^n,p))(1-\mu)} \tag{3}$$

and

$$\mu\left(\omega_L \mid \omega_k(j_1), r(x^n,p)\right) = 1 - \mu\left(\omega_H \mid \omega_k(j_1), r(x^n,p)\right). \tag{4}$$

Then,

$$\Psi_{j_2}\left(p_{j_2} \mid b, r(x^n, p, \omega_H(j_1)\right) =$$

$$r_{j_2}\left(x^n,p\right)\left[\mu\left(\omega_H \mid \omega_k(j_1), r(x^n,p)\right)\omega_H + \mu\left(\omega_L \mid \omega_k(j_1), r(x^n,p)\right)\omega_L\right]p_j +$$

$$(1 - r_j(x^n,p))\mu\left(\omega_L \mid \omega_k(j_1), r(x^n,p)\right)b.$$

Thus, the expected cost of a customer who anticipates seeking a second opinion is

$$\Psi\left(b, r(x^n, p)\right) = r_{j_1}\left(x^n, p\right)(1-\mu)\,\omega_L p_{j_1} + (1-\mu)\,\Psi_{j_2}\left(p_{j_2} \mid b, r(x^n, p, \omega_H(j_1)\right) - \theta.$$

For such customer, $\Psi\left(b, r(x^n, p)\right) \leq \Psi_j\left(p_j \mid b, r(x^n, p)\right).$

A customer chooses to forgo seeking service if and only if

$$v \leq \min\left\{\Psi_{j_1}\left(p_{j_1} \mid \ b, r(x^n, p)\right), \Psi_{j_2}\left(p_{j_2} \mid \ b, r(x^n, p)\right)\right\}, \Psi\left(b, r(x^n, p)\right).$$

in which case no subgame is initiated. Henceforth, I consider only the case in which the customer decides to seek service. In her first move, the customer chooses A as her first provider if $\Psi_A\left(p_A \mid b, r(x^n, p)\right) \leq \Psi_B\left(p_B \mid b, r(x^n, p)\right)$. Otherwise she visits provider B first.

Customers' strategies. Denote by a, r, and f, respectively, the customer's decisions to accept the first provider's prescription, reject it and seek a second opinion, and forgo service altogether. Customers' strategies map from their types and the providers' reputations and prices to contingent plans specifying their responses to the providers prescriptions.

Formally, the customer strategy consists of two mappings: The first mapping, $\sigma_1 : \Theta \times V \times \mathbb{B} \times [0,1]^2 \times [0,\bar{v}]^2 \rightarrow \{\Delta(a,r,f)\}^{\Omega}$, depicts the customer's plan of action following her visit to the first provider. Specifically, $\sigma_1\left(\omega_k \mid \theta, v, b, r(x^n, p)\right)(y)$ denotes the probability of choosing her move randomly according to the probabilities distribution $y \in \{a, r, f\}$ if the first provider's prescription is ω_k, $k \in \{L, H\}$.[13]

[13]Thus, $\sigma_1\left(\omega_k \mid \theta, v, b, p, r(x^n)\right)(y) \geq 0, \forall y \in \{a,r,f\}$, $\sum_{y\in\{a,r,f\}} \sigma_1\left(\omega_k \mid \theta, v, b, p, r(x^n)\right)(y) = 1.$

If after receiving the first provider's prescription the customer decides to seek a second opinion, her strategy is the mapping $\sigma_2 : \Theta \times V \times \mathbb{B} \times \Omega \times [0, \bar{v}]^2 \times [0,1]^2 \rightarrow \{\Delta(a, f)\}^{\Omega}$, which depicts the plan of action contingent on the second provider's prescription.[14] Specifically, if the first provider prescribes $\omega_{k'}$ then $\sigma_2(\omega_k \mid \theta, v, b, r(x^n, p), \omega_{k'})(y)$ denotes the probability of choosing $y \in \{a, r\}$ if the second prescription is ω_k, $k \in \{L, H\}$. Let $\Sigma := \{\sigma = (\sigma_1, \sigma_2)\}$ denote the set of the customer's strategies in the prescription subgame.

The customers' system of beliefs. Customers' beliefs include the likelihoods of the states and the levels of the providers' expertise. In particular, a customer's prior beliefs regarding the likelihoods of the states ω_H and ω_L are μ and $(1-\mu)$, respectively.

Let $\zeta(\omega_k(j_1), r(x^n, p), x^n) = (\mu, \mu(\omega_H \mid \omega_k(j_1), r(x^n, p)), \eta(\bullet \mid x^n))$ denote the customer's system of beliefs following her visit to the first provider.[15] Clearly, the customer's beliefs depend on her choice of the first provider and the providers' reputations.

The customer's payoffs. Given the prices p_A and p_B and the first provider's prescription, the customer's expected payoffs are as follows:

If the first provider prescribes ω_L, the customer accepts the prescription.[16] In this case, her payoff is $v - \omega_L p_{j_1} - \mu(\omega_H \mid \omega_L(j_1), r(x^n, p))b$. If the first provider prescribes ω_H and the customer accepts the prescription, her payoff is $v - \omega_H p_{j_1}$.

[14]Recall that, should the customer decide to seek a second opinion, she must accept the second provider's prescription or reject it and, thereby, forgoing fixing the problem.

[15]$\mu(\omega_H \mid \omega_k(j_1)$ is given in equation (3).

[16]If the customer rejects the first provider's prescription ω_L, it must hold that either the prior expected cost of the second provider is lower than that of the first, in which case she would have chosen to visit the second provider first, a contradiction.

The customer rejects the first provider's prescription ω_H if

$$v-\omega_H p_{j_1} < v-\theta-\Psi_{j_2}\left(p_{j_2} \mid b, r(x^n,p), \omega_H\right),$$

where $\Psi_{j_2}\left(p_{j_2} \mid\ b, r(x^n,p), \omega_H\right)$ is the expected cost of service provided by the second provider.

If the customer rejects the first provider's prescription and seeks a second opinion, her payoffs are as follows: If the second provider prescribes ω_L, the customer accepts it, and her payoff is $v-\omega_L p_{j_2}-\mu\left(\omega_H \mid r(x^n,p), \omega_H\left(j_1\right), \omega_L(j_2)\right)b>0$. [17] If the second provider prescribes ω_H, the customer accepts the prescription if $v-\omega_H p_{j_2}>0$ and her payoff is $v-\theta-\omega_H p_{j_2}>0$. Otherwise, she forgoes fixing the problem, and her payoff is 0.

4.1.3 The providers' strategies, beliefs and payoffs

Providers' strategies were discussed in section 3.2. The providers' prior beliefs about the likelihoods of the states ω_H and ω_L are μ and $1-\mu$, respectively. Their posterior beliefs are given by equations (1) and (2).

Demand for services depends on providers reputations. Consequently, when planning their moves in the prescription subgame, providers include the effect of their current decisions on the subsequent demand for their services.

Let $W_j\left(r_A\left(x^n,p\right), r_B\left(x^n,p\right)\right)$ denote the expected present value of provider j's the future profits generated by the equilibrium strategies given the providers' repu-

[17] $\mu\left(\omega_H \mid r(x^n,p)\omega_H\left(j_1\right), \omega_L(j_2)\right)$ denotes the customer's posterior belief that the true state is ω_H conditional on the two prescriptions.

tations. I denote by C the cost associated with having to compensate a customer for prescribing undertreatment.[18]

Consider the payoff of provider $j \in \{A, B\}$. If he prescribes ω_L, it is accepted.[19] If it subsequently proves to be sufficient (i.e., ω_L happens to be the true state), A's payoff is

$$P_j\left(\omega_L\left(j\right) \mid \omega_L\right) = \left(p_j - c\right)\omega_L + W_j\left(r(x^{n+1}, p)\right) - W_j\left(r\left(x^n, p\right)\right). \tag{5}$$

If j is the customer's first provider, then $x_{n+1} = \varrho^3$ is depicted by Scenario 3; if he is the customer's second provider, then $x_{n+1} = \varrho^1$. In either case, $r_j\left(x^{n+1}, p\right) > r_j\left(x^n, p\right)$ and $r_{j'}\left(x^{n+1}, p\right) = r_{j'}\left(x^n, p\right)$. Thus, $\left(p_j - c\right)\omega_L + W_j\left(r(x^{n+1}, p)\right) - W_j\left(r\left(x^n, p\right)\right) > 0$.

If, however, ω_L proves to be insufficient, then j's payoff is

$$P_j\left(\omega_L\left(j\right) \mid \omega_H\right) = \left(p_j - c\right)\omega_L - C + W_j\left(r\left(x^{n+1}, p\right)\right) - W_j\left(r\left(x^n, p\right)\right), \tag{6}$$

where $x_{n+1} = \varrho^4$ is depicted by Scenario 4. In this case $r_j\left(x^{n+1}, p\right) < r_j\left(x^n, p\right)$ and $r_{j'}\left(x^{n+1}, p\right) = r_{j'}\left(x^n, p\right)$. Thus, $W_j\left(r\left(x^{n+1}, p\right)\right) - W_j\left(r\left(x^n, p\right)\right) < 0$. Hence, given $s \in S$, provider j's expected payoff is

$$P_j\left(\omega_L\left(j\right) \mid s\right) := P_j\left(\omega_L\left(j\right) \mid \omega_L\right)\mu_j\left(\omega_L \mid s\right) + P_j\left(\omega_L\left(j\right) \mid \omega_H\right)\mu_j\left(\omega_H \mid s\right). \tag{7}$$

[18]This cost consists of having to fix the problem at the provider's expense (i.e., $(\omega_H - \omega_L)c$; it may also include malpractice payouts.

[19]Rejection of ω_L is off the equilibrium path, as mentioned earlier.

I assume that the cost C and the loss of future profits from the loss of reputation, are sufficiently large so that $P_j\left(\omega_L\left(j\right) \mid s_L\right) > 0$ and $P_j\left(\omega_L\left(j\right) \mid s_H\right) < 0$. Thus, by equation (7), j would never prescribe ω_L if the signal received is s_H.

If j prescribes ω_H and it is accepted, then regardless of the true state, his payoff is $\left(p_j - c\right)\omega_H > 0$. If j's prescription is rejected and the customer forgoes service, or if she seeks a second opinion and the second provider prescribes ω_H, j's payoff is 0.

If the customer decides to seek a second opinion and j' prescribes ω_L, which turns out to be true (i.e., Scenario 1) then j's payoff is $W_j(r(^{n+1}, p)) - W_j(r(^{n}, p)) < 0$. Hence,

$$P_j\left(\omega_H\left(j\right) \mid s_L\right) = \sigma_1(a)\left(p_j - c\right)\omega_H + \sigma_1(r) r_{j'}(x^n, p)(W_j(r^{n+1}, p) - W_j(r^n, p)).$$

If the customer decides to seek a second opinion and j' prescribes ω_L, which turns out to be insufficient (i.e., Scenario 2) then j's payoff is $W_j(r(^{n+1}, p)) - W_j(r(^{n}, p)) < 0$. Hence,

$$P_j\left(\omega_H\left(j\right) \mid s_L\right) = \sigma_1(a)\left(p_j - c\right)\omega_H + \sigma_1(r) r_{j'}(x^n, p)(W_j(r^{n+1}, p) - W_j(r^n, p)).$$

4.2 The providers' pricing game

The more reputable provider, A, plays the role of a leader and, as such, posts his price first. Provider B, the follower, observes the leader's price before posting his own.

4.2.1 The demands for credence services

The follower's demand. By definition, A is perceived to be more likely than B to prescribe correctly. By accepting A's prescription ω_H, the customer is less likely to purchase unnecessary services; by accepting A's prescription ω_L, the customer is less likely to be provided with insufficient services. Consequently, the only customers who choose B for their first provider are those whose value v exceeds the expected cost and the cost, b, of being inconvenienced is sufficiently low that they prefer being prescribed insufficient service and receiving the additional remedial service at no charge.

Given $r(x^n, p)$, let b^* be defined by the equation

$$\Psi_A \left(p_A \mid b^*, r(x^n, p)\right) = \Psi_B \left(p_B \mid b^*, r(x^n, p)\right).$$

Then, the probability that a customer chooses B for her first provider is

$$F_b(b^*)(1 - F_v(\Psi_B \left(p_B \mid b^*, r(x^n, p)\right)))+$$

$$(1 - F_b(b^*, r(x^n, p)))[F_v(\Psi_B \left(p_B \mid b^*, r(x^n, p)\right)) - F_v(\Psi_A \left(p_A \mid b^*, r(x^n, p)\right))].$$

If $p_A \leq p_B$, then the only customers who chose B for their first provider come from the lower tail of the distribution F_b. If the lower tail is thin, the share of customers who choose B for their first provider is small. To attract additional customers, B must lower his price. The lower price increases the value of b^* which attracts more customers from the lower tail of the distribution F_b as well as all customers whose valuations, v, are such that $v \in [\Psi_B(p_B \mid b^*, r(x^n,p)), \Psi_A(p_A \mid b^*, r(x^n,p))))$. I refer to the probability of the set of these customers as B's *extensive-margin demand*, and denote it by $J_B(r(x^n,p),p)$.

A customer prescribed ω_H by A is in possession of new information that allows her to update her beliefs about the likelihoods of the states. The posterior distribution on the states allows her customer to revise her assessment of the likelihoods of the signals received by B and, given B's prescription strategy, the corresponding expected cost of seeking a second opinion from B.

Let $\Psi_B(p_B \mid b, r(x^n,p), \omega_H(A))$ denote the updated anticipated cost of seeking a second opinion from provider B. If the customer anticipates accepting B's prescription, her expected cost conditional on $\omega_H(A)$ is $\Psi_B(p_B \mid b, r(x^n,p), \omega_H(A))$.[20] Customers for whom $p_A\omega_H - \Psi_B(p_B \mid b, r(x^n,p), \omega_H(A)) \leq \theta$ accept provider $A's$ prescription. All other customers seek a second opinion.

Let $z_j(\omega_k; r(x^n,p))$ denote the probability that provider j prescribes ω_k given record x^n and the price vector p. Formally,

$$z_j(\omega_k; r(x^n,p)) = r_j(x^n,p)\Pr(s_k \mid e_j) + (1 - r_j(x^n,p))Pr(s_{k'} \mid e_j).$$

For customers who anticipate seeking a second opinion, the expected cost of choosing

[20]The expected cost is

$$r_B(x^n,p)[\mu(\omega_H \mid \omega_H(A))\omega_H + \mu(\omega_L \mid \omega_H(A)\,\omega_L]p_B + (1 - r_B(x^n,p))b\mu(\omega_H \mid \omega_H(A)))\,.$$

A for their first provider is

$$\Psi_B(p_B \mid \theta, b, r(x^n,p), \omega_H(A), p_A) = z_A(\omega_H, r(x^n,p))(\Psi_B(p_B \mid b, r(x^n,p), \omega_H(A)) + \theta) +$$

$$z_A(\omega_L, r(x^n,p))[\omega_L p_A + (1 - r_A(x^n,p))\mu(\omega_H \mid \omega_H(A))\, b].$$

Thus, the probability that a customer who chooses A as her first provider seeks a second opinion from B is

$$D_B(p \mid r(x^n,p), p) := z_A(\omega_H; r(x^n,p), p) \int_{\mathbb{B}} F_\theta(p_A\omega_H - \Psi_B(p_B \mid \theta, b, x^n, \omega_H(A), p_A)\, dF_b(b)\,. \tag{8}$$

I refer to $\mathrm{D}_B(p \mid r(x^n,p))$ as B's *intensive-margin demand.*

The probability that a new customer arriving on the market will seek a prescription from B is

$$K_B(p \mid r(x^n,p)) := D_B(p \mid r(x^n,p), p) + J_B(r(x^n,p), p). \tag{9}$$

The expected hours of service demanded of provider B is

$$Q_B(p_A, p_B \mid r(x^n,p)) =$$

$$K_B(p \mid r(x^n,p))(z_B(\omega_L; r(x^n,p), p))\,\omega_L + z_B(\omega_H; r(x^n,p), p)(1 - F_v(\omega_H p_B))\,\omega_H,$$

where $1 - F_v(\omega_H p_B)$ is the probability that the customer accepts provider B's prescription ω_H.

4.2.2 The leader's demand

A new customer arriving on the market chooses the more reputable provider to visit first if her inconvenience cost $b > b^*$ and the value she assigns to the service exceeds the expected cost, $\Psi_A \left(p_A \mid b^*, r(x^n, p)\right))$, of consulting A. The probability that a new customer arriving on the market will choose A as her first provider thus

$$Pr(A = j_1) = (1 - F_v \left(\Psi_A \left(p_A \mid b^*, r(x^n, p)\right)\right))(1 - F_b(b^*, r(x^n, p))).$$

I refer to these customers as A's *extensive-margin demand*.

All of these customers accept prescription ω_L. Some customers who are prescribed ω_H will seek a second opinion. The probability of the set of these customers is $F_\theta(\theta^*(p, b, r(x^n, p), \omega_H (A))$, where $\theta^*(p, b, r(x^n, p), \omega_H (A)) = \omega_H p_A - \Psi_B \left(p_B \mid b, r(x^n, p), \omega_H (A), p_A\right).$

Non of the customers who choose B as their first provider seeks a second opinion if prescribed ω_L. However, a customer who choses B for her first provider and is prescribed ω_H seeks a second opinion if $\Psi_A \left(p_A \mid b, r(x^n, p), \omega_H(B)\right) < v < \omega_H p_B$. These customers accept prescription ω_L and forgo service if A prescribes ω_H.

The expected hours of service demanded of provider A are

$$Q_A \left(p_A, p_B \mid r(x^n, p)\right) =$$

$$Pr(A = j_1)[z_A \left(\omega_L; r(x^n, p)\right)\omega_L + z_A \left(\omega_H; r(x^n, p)\right)(1 - F_\theta(\theta^*))\omega_H]$$

$$z_B \left(\omega_H; r(x^n, p)\right)) \left[F_v(\omega_H p_A) - F_v \left(\Psi_A \left(p_A \mid b^*, r(x^n, p), \omega_H(B)\right)\right]\omega_L.$$

From the properties of the distribution F_v, F_θ, andF_b and the reputations effect of changing prices, it follows that demand for both providers is continuous function and monotonic decreasing in the provider's own price and monotonic increasing in the rival's price (i.e., $\partial Q_j\left(p_j, p_{j'} \mid r(x^n, p)\right)/\partial p_j < 0$ and $\partial Q_j\left(p_j, p_{j'} \mid r(x^n, p)\right)/\partial p_{j'} > 0$, $j, j' \in \{A, B\}$).

4.3 Stage-game equilibrium

The equilibrium concept by which the stage game is analyzed is Perfect Bayesian Equilibrium (PBE). By definition, such equilibrium consists of a strategy profile $((\varphi_A^*, \alpha_A^*), (\varphi_B^*, \alpha_B^*), \sigma^*)$ and a system of beliefs ζ^* that is derived from the strategy profile using Bayes rule whenever possible. To prove the existence of equilibrium of the stage game, it is convenient to establish the existence of equilibrium of the prescription subgame given the prices and then establish the equilibrium of the pricing game.

4.3.1 Prescription subgame equilibrium

Given the prices p_A, p_B, a PBE of the prescription subgame Γ_E consists of a profile of prescription strategies $\vartheta^*(p_A, p_B) = (\alpha_A^*, \alpha_B^*, \sigma^* \mid p_A, p_B)$ and a system of beliefs $\zeta^*(p_A, p_B)$. The system of beliefs specifies a probability $\zeta^*\left(y \mid p_A, p_B\right) \in [0, 1]$ for each decision node ι in Γ_E such that $\Sigma_{\iota \in I} \zeta^*\left(y \mid p_A, p_B\right) = 1$, for all information sets I; the strategy profile $\vartheta^*(p_A, p_B)$ is sequentially rational given the system of beliefs $\zeta^*(p_A, p_B)$ (i.e., given the system of beliefs $\zeta^*(p_A, p_B)$, the player who moves at any information set maximizes his expected utility given the strategies of the other players); the system of beliefs $\zeta^*(p_A, p_B)$ is derived from the strategy profile

$\vartheta^*(p_A, p_B)$ using Bayes' rule whenever possible.

Proposition 1: *For all given prices $p = (p_A, p_B)$ and providers' reputations, $\mathsf{r}_A(x^n, p)$, $\mathsf{r}_B(x^n, p)$, there exists a PBE of the prescription sugame Γ_E.*

The proof is by application of Kakutani's (1941) fixed point theorem.

4.3.2 Equilibrium of the providers' pricing game

Given the (conditional on the prices) equilibrium of the subgame, the equilibrium of the pricing game is a pair of prices that are the providers' best responses against each other in the pricing game. Thus, the the pricing strategies, the prescription strategies and the customers' are best responses against each other. Moreover, by construction, the system of beliefs of both the customers and the providers are obtained by application of Bayes' rule.

Let $Q_A^*(p_A, p_B \mid r(x^n, p))$ and $Q_B^*(p_A, p_B \mid r(x^n, p)$ denote the demand for service facing the two providers when they implement the equilibrium prescription strategies.

Definition 2: An *equilibrium of the providers' pricing game* is a pair of prices (p_A^*, p_B^*) such that $p_j^* \in \arg\max_{p_j \in [0,\bar{v}]}(p_j - c)Q_j^*(p_j, p_{j'}^* \mid r(x^n, p))$, $j, j' \in \{A, B\}$.

The next proposition asserts the existence of equilibrium of the providers' pricing game.

Proposition 2. *There exists an equilibrium of the providers' pricing game.*

Taken together, Propositions 1 and 2 establish the existence of a PBE of the stage game.

Theorem: *For every given record, $x^n \in X^n$, there exista a PBE of the stage game.*

The proofs of the propositions and the theorem are in the Appendix.

5 Equilibrium Analysis

5.1 The prevalence of fraud

Fraud is said to be committed when a provider prescribes ω_H upon receiving the signal s_L. Put differently, a provider commits fraud when he prescribes a service that he knows to be more likely than not unnecessary.

A stage game equilibrium is said to be *fraud-free* if $\alpha_j^*(e_j, x^n, p_A^*, p_B^*)(s_k) = \delta_{\omega_k}$. The market is said to be fraud-free if the stage games equilibria are fraud-free for all $x^n \in X^n, n = 1, 2, ...$ Fraud is said to be perpetrated in the market if it is not fraud-free (i.e., if the sequence includes stage games in which some providers implement prescription strategies that call for prescribing ω_H when the signal they receive is s_L).

To understand the effect of providers' expertise and reputations on the prevalence of fraud, it is useful to start with the benchmark case in which it is common knowledge that providers can always identify the problem correctly (i.e., $e_j = 1, j \in \{A, B\}$). In this case, there exists no fraud-free equilibrium of the stage game. To grasp this assertion, suppose, by way of negation, that there is an equilibrium in which providers prescribe truthfully. Since they prescribe the necessary and sufficient service, customers' best-response is to accept their prescriptions.

Consider a stage game in which a provider diagnoses the problem as ω_L. Knowing that the customer accepts his prescription, if instead of prescribing truthfully, as required by the fraud-free strategy, the provider deviates and prescribes ω_H he increases his payoff by $(\omega_H - \omega_L)p_j$. Moreover, because no information on whether the prescription was necessary is revealed, the provider's reputation remains intact. Truthful prescriptions are, thus, not equilibrium strategies.

In general, the prevalence of fraud in the markets for credence services depends on the distributions of the customer types and providers' expertise. Because they are determined, in part, by the prescription-outcome record, providers' reputations are random variables that may drift up or down as the record enhances or diminishes their reputations. The main point here is that the providers do not fully control the evolution of their reputations.

The question is what conditions induce a provider to commit fraud. To answer this question, consider provider $j \in \{A, B\}$ and suppose that he receives the signal s_L. If j prescribes ω_H and it is accepted, his payoff is $P_j\left(\omega_H\left(j\right) \mid s_L, a\right) = (p_j^* - c)\omega_H > 0$. If it is rejected, j's expected payoff depends on the second provider's prescription and the actual state. In particular, if j' prescribes ω_H, the first provider's reputations is intact, and his payoff is 0. By contrast, if j' prescribes ω_L and it turns out to be sufficient (i.e., Scenario 1 holds), then j suffers reputation loss; and if ω_L turns out to be insufficient (i.e., Scenario 2 holds), then j's reputation is enhanced.

As j received the signal s_L, his posterior beliefs are $\mu_j(\omega_H \mid s_L), \mu_j(\omega_L \mid s_L)$. Moreover, his beliefs that j' will obtain the signals s_L and s_H, respectively, are

$$\pi_{j'}(s_L) = \bar{e}_{j'}(x^n)\mu_j(\omega_L \mid s_L) + (1 - \bar{e}_{j'}(x^n))\mu_j(\omega_H \mid s_L)$$

and

$$\pi_{j'}(s_H) = 1 - \pi_{j'}(s_L),$$

where $\bar{e}_{j'} = \int_{[1/2,1]} ed\eta(e \mid x^n)$.

The probabilities that j' prescribes ω_L and ω_H, respectively, are

$$\xi_{j'}(\omega_L \mid s_L) := r_{j'}(x^n, p^*)\pi_{j'}(s_L) + (1 - r_{j'}(x^n, p^*))\pi_{j'}(s_H),$$

and

$$\xi_{j'}(\omega_H \mid s_L) := (1 - r_{j'}(x^n, p^*))\pi_{j'}(s_L) + r_{j'}(x^n, p^*))\pi_{j'}(s_H).$$

Thus, j's expected payoff if he prescribes ω_H upon receipt of the signal s_L is

$$\Pi_j(\omega_H \mid s_L) := \bar{\sigma}_1(a \mid \omega_H)(p_j - c)\omega_H + \bar{\sigma}_1(r \mid \omega_H)\xi_{j'}(\omega_L \mid s_L)\times$$

$$[\mu_j(\omega_L \mid s_L))W_j(r((x^n, \varrho^1), p^*))) + \mu_j(\omega_H \mid s_L))W_j(r((x^n, \varrho^2), p^*)) - W_j(r(x^n, p^*)))].$$

where $(\bar{\sigma}_1(a \mid \omega_H)$ and $(\bar{\sigma}_1(r \mid \omega_H)$ are the expectations that the customer accepts or rejects j's prescription ω_H, respectively.[21]

Consider next the expected payoff of j of prescribing ω_L upon receiving the signal s_L. This prescription is accepted; if the true state is ω_L, j's payoff is $p_j^*(w_L - c)$ and his reputation is enhanced (i.e., Scenario 3 holds). If the true state is ω_H, then j's payoff is the loss $c(\omega_H - \omega_L)$, due to liability and the loss of reputation because the prescription proves to be insufficient (i.e., Scenario 4 holds). Thus, j's expected payoff is

[21]This is the expectation over $\Theta \times V \times \mathbb{B} \times \ell$ of $\sigma_1(\omega_H \mid \theta, v, b, r(x^n))\ell(a))$.

$$\Pi_j(w_L \mid s_L) := e_j[p_j^*(w_L - c) + W_j(r((x^n, \varrho^3), p^*)) - W_j(r(x^n, p^*))] -$$

$$(1 - e_j)[c(\omega_H - \omega_L) + W_j(r(x^n, p^*)) - W_j(r((x^n, \varrho^4), p^*)))].$$

Provider j commits fraud if and only if $\Pi_j(w_H \mid s_L) > \Pi_j(w_L \mid s_L)$.

The upshot of this analysis is that, unlike the benchmark case, less than perfect expertise combined with reputation concerns make the existence of a fraud-free equilibrium of stage games possible. Specifically, if $\Pi_j(w_H \mid s_L) < \Pi_j(w_L \mid s_L)$, $j \in \{A, B\}$, then the equilibrium of the stage game is fraud-free.

High expertise levels inhibit fraudulent prescriptions. Intuitively, the greater the provider's expertise, the more confident he is that his diagnosis is correct (i.e., the greater is $\Pi_j(w_L \mid s_L)$). Consequently, the probability of loss of reputation and having to bear the extra cost involved in providing remedial service is smaller. These considerations increase the expected payoff to prescribe truthfully. In addition, the probability that the provider misdiagnoses the problem is smaller, which means that if, having received signal s_L he prescribes ω_H and the customer decides seek a second opinion, the expected loss is larger, which inhibits the provider's inclination to commit fraud.

The effect of the prices on the prevalence of fraud is ambiguous. The higher the price, the greater the marginal profit $(\omega_H - \omega_L)p_j$, incentivizing the prescription of unnecessary service. However, the higher is the price of one provider and the lower the price of his rival, the more likely customers are to seek a second opinion.

Formally, define $\theta^*(b, r(x^n, p), p_{j_1}, p_{j_2}, \omega_H(j_1)) \in \Theta$ by the equation

$$\omega_H p_{j_1} = \Psi_{j_2}\left(p_{j_2} \mid b, r(x^n, p), \omega_H\left(j_1\right)\right) - \theta^*. \tag{10}$$

Then the probability of a customer seeking a second opinion is $F_\theta(\theta^*(b, r(x^n, p), p_{j_1}, p_{j_2}, \omega_H(j_1)))$. Since $\theta^*(b, r(x^n, p), p_{j_1}, p_{j_2}, \omega_H(j_1))$ is a monotonic decreasing function of p_{j_2} and monotonic increasing function of p_{j_1}, the higher the provider's price and the lower is the price of his rival, the less likely he is to retain a customer and, consequently, the less likely he is to commit fraud. Thus, the larger the price difference, the less inclined is the high-price provider and the more inclined is the low-price provider to commit fraud. Moreover, because the providers' reputations and their prices are comonotonic, the reputations tend to reinforce the inhibition the high-price provider and disinhibition of the low-price provider to commit fraud.

5.2 Effect of reputation on price

Providers are concerned with their records because they affect their reputations and the prices they can charge. A better performance record of either provider increases his equilibrium price and lowers the price of his rival. To grasp this claim, recall that the customer perception of the probability of being prescribed the required service is greater the more reputable the provider. Consequently, ceteris paribus, the expected cost to the customer is lower the higher the reputation of a provider. In turn, a better reputation increases the provider's intensive and extensive marginal demand as well as the retention of his customers. To restore equilibrium, the demands must be readjusted by an increase in the price of the provider whose reputation improves, a decrease in the price of the rival provider, or both.

Given the new gap in their reputations, the equilibrium price difference, $p_A^* - p_B^*$, represents the *reputation advantage*. The corresponding difference in their profits is the *reputation rent*. Both reputation advantage and reputation rent increase monotonically when the gap in the their performance records increases.

5.3 Fraudulent behavior: Liability and reputation effects

Fraudulent behavior is motivated by the extra profit, $(\omega_H - \omega_L)(p_j - c)$, it may generate and the avoidance of liability cost.[22]

The effect of reputation on providers' inclinations to commit fraud is ambiguous. The desire to avoid the loss of reputation due to erroneously prescribing insufficient service (Scenario 4) or to enhance his reputation if the true state is ω_H (Scenario 2) increases a provider's inclination to prescribe ω_H when he receives the signal s_L. However, a provider who prescribes ω_H upon receiving the signal s_L, runs the risk that the customer seeks the opinion of a second provider, who prescribes the appropriate service given the true state ω_L (Scenario 1). The potential loss of reputation and the possible enhanced reputation if it turns out that ω_L is the true state (Scenario 3) mitigates the provider's inclination to prescribe unnecessary service.

The balance of these opposing pulls depends on the relative size of the set of customers whose values fall short of the cost $\omega_H p_{j_1}$ of the first provider and exceeds

[22]The provider is liable if he prescribes insufficient service. Hence, the higher the liability cost, the more inclined the provider is to prescribe service that he believes less likely than not to be necessary.

the expected cost of service, $\Psi_{j_2}(p_{j_2} \mid b, r(x^n, p), \omega_H(j_1)$, of the second provider. Specifically, if the probability of the this set of customers is small then the mitigation effect is small and the inclination to commit fraud increases.

6 Concluding Remarks and Related Literature

6.1 Concluding Remarks

This study adds several novel aspect to study of credence service markets. First, it models the market as a stochastic dynamic game of incomplete information consisting of a sequence of stage games linked by the providers' records of prescriptions and outcomes. The model highlights the influence of providers' concern about their performance and its affect on their reputations and inclinations to prescribe unnecessary services.

Second, by allowing for variations in providers' levels of expertise, it opens up the possibility of studying how the interplay between expertise and reputations shapes providers' inclination to commit fraud. In particular, the fact that providers do not have complete control over their reputations introduces a stochastic element to the sequence of stage games and the corresponding stage game equilibrium prices and prescription strategies. The sequence may include stage games whose equilibrium is fraud-free as well as stage games equilibria with varying levels of fraud.

Third, it analyzes the joint determination of the stage games' equilibrium prices and prescription strategies as best responses against each other.

6.2 Related literature

The literature on the role and economic implications of reputations is multifaceted. Hörner (2002) studies the economic effects of reputations in competitive markets for experience goods. Unlike the approach taken in this paper, according to which providers' reputations are based on their records of past performance, Hörner identifies firms' reputations with their customer bases. The main thrust of this paper is the role of expertise (a hidden characteristic) and the effect it has on providers' reputations and the perpetration of fraud in credence service markets. In contrast, Hörner's main focus is the role of firms' efforts (hidden actions) in delivering high-quality products and the evolutions of their reputations.

Closer to the focus of this paper are the studies by Dulleck, Kerschbamer and Sutter (2011); Mimra, Rasch, and Waibel (2016); and Fong and Ting (2018) on reputations in credence goods markets.

Dulleck et al. (2011) investigate, experimentally, the impact of reputation on experts' commitment of fraud, with and without liability. In their experiment, customers and providers interact over a finite number of periods, during which providers may build reputations with customers based on customers' experiences. They find that reputation has little effect on mitigating welfare loss, because of the inherited asymmetry of information that characterizes credence good markets. Their work does not consider the main concern of this paper, the possibility of imperfect expertise and its interaction with reputations in shaping experts' behavior.

The experimental study by Mimra, Rasch, and Waibel (2016) examines the effects of the pricing and information regimes (competitive versus fixed prices and private versus public information) on providers' performance, reputation, and perpetration of fraud. Both their setup and objectives are different from those of this paper. In particular, because liability is absent form their design, fraud takes the form only of undertreating, and of charging for services not performed. Their findings suggest that price competition undermines the incentive to build reputation by providing correct treatment and, consequently, induces higher levels of fraud than a regime of fixed (regulated) prices. Unlike in this paper, the experts in their study observe the state perfectly.

Invoking a repeated game framework, Fong and Ting (2018) study the interactions between a monopoly expert, who diagnoses the true state correctly, and short-lived customers. The main thrust of their analysis is the impact of liability on the provider's incentive to build, and maintain, a reputation for quality service. Fraudulent behavior involves undertreatment of a serious problem. Under liability, customers are fully compensated for undertreatment. Consequently, customers do not care about the reputation of the expert, which undermines the provider's incentives to build and maintain his reputation. Their work is quite different from this paper, starting with the market structure (monopoly versus duopoly), the lack of concern about the level of expertise, and the definition of fraudulent behavior.

APPENDIX

Proof of Theorem 1

The proof is an implication of Propositions 1 and 2.

Proof of Proposition 1. Fix the prices $p = (p_A, p_B) \in [0, \bar{v}]^2$, the providers' reputations, $r(x^n, p) = (r_A(x^n, p), r_B(x^n, p))$, and expertise $(e_A, e_B) \in [1/2, 1]^2$, and a customer type $(\theta, v, b) \in \Theta \times V \times \mathbb{B}$. Define a best-response correspondence Ξ from $\Sigma \times \Delta(\Omega)^S \times \Delta(\Omega)^S$ to itself

$$\Xi(\sigma(\theta, v, b, r(x^n, p), p), \alpha_A(e_A, r(x^n, p), p), \alpha_B(e_B, r(x^n, p), p)) \rightrightarrows$$

$$(\sigma^*(\theta, v, b, r(x^n, p), p), \alpha_A^*(e_A, r(x^n, p), p), \alpha_B^*(e_B, r(x^n, p), p)), \tag{11}$$

where $\sigma^*(\theta, v, b, r(x^n, p), p), \alpha_A^*(e_A, r(x^n, p), p), \alpha_B^*(e_B, r(x^n, p), p)$ are the customer's and providers' best-responses against $\sigma(\theta, v, b, r(x^n, p), p), \alpha_A(e_A, r(x^n, p), p), \alpha_B(e_B, r(x^n, p), p)$.

To prove that Ξ has a fixed point, we need to show that (a) the set $\Sigma \times \Delta(\Omega)^S \times \Delta(\Omega)^S$ is compact (in the product topology) and convex, (b) given a system beliefs ζ^*, Ξ is upper hemicontinuous and, for every point in its domain, the set Ξ is nonempty and convex, and (c) ζ^* is derived from the strategy profile $\vartheta^* = (\sigma^*(\theta, v, b, r(x^n, p), p), \alpha_A^*(e_A, r(x^n, p), p), \alpha_B^*(e_B, r(x^n, p), p))$, using Bayes' rule whenever possible.

Since Σ and $\Delta(\Omega)^S$ are compact (in the product topology), so is the product $\Sigma \times \Delta(\Omega)^S \times \Delta(\Omega)^S$. Given $\sigma(\theta, v, b, r(x^n), p), \alpha_A(e_A, r(x^n, p), p), \alpha_B(e_B, r(x^n, p), p)$ and $(\sigma'(\theta, v, b), \alpha_A'(e_A, r(x^n, p), p), \alpha_B'(e_B, r(x^n, p), p)$ in $\Sigma \times \Delta(\Omega)^S \times \Delta(\Omega)^S$ and

$\gamma \in [0,1]$, $k \in \{L, H\}$, define

$$(\gamma\sigma_1 + (1-\gamma)\,\sigma_1')\,(\omega_k \mid \theta, v, b, r(x^n, p), p) =$$

$$\gamma\sigma_1\,(\omega_k \mid \theta, v, b, r(x^n, p), p) + (1-\gamma)\,\sigma_1'\,(\omega_k \mid \theta, v, b, r(x^n, p), p)\,,$$

and, for every prescription $\omega_{k'}$ of the first provider,

$$(\gamma\sigma_2 + (1-\gamma)\,\sigma_2')\,(\omega_k \mid \theta, v, b, r(x^n, p), p, \omega_{k'}) =$$

$$\gamma\sigma_2\,(\omega_k \mid \theta, v, b, r(x^n, p), p, \omega_k') + (1-\gamma)\,\sigma_2'\,(\omega_k \mid \theta, v, b, r(x^n, p), p, \omega_k')\,.$$

Hence, $(\gamma\sigma_1 + (1-\gamma)\,\sigma_1')\,(\omega_k \mid \theta, v, b, r(x^n, p), p) \in \Delta\{a, r, f\}$ and

$$(\gamma\sigma_2 + (1-\gamma)\,\sigma_2')\,(\omega_k \mid \theta, v, b, , r(x^n, p), p, \omega_{k'}) \in \Delta\{a, f\}.$$

Thus, Σ is convex.

For every $s \in S$ and $j \in \{A, B\}$ define

$$(\gamma\alpha_j\,(e_j, r(x^n), p))\,(1-\gamma)\,\alpha_j'\,(e_j, r(x^n), p))(s) =$$

$$\gamma\alpha_j\,(e_j, r(x^n, p), p)\,(s) + (1-\gamma)\,\alpha_j'\,(e_j, r(x^n, p), p)\,(s)\,.$$

Then,

$$\gamma\alpha_j\,(e_j, r(x^n, p), p) + (1-\gamma)\,\alpha_j'\,(e_j, r(x^n, p), p)) \in \Delta\,(\Omega)^S\,.$$

Hence, $\Sigma \times \Delta\,(\Omega)^S \times \Delta\,(\Omega)^S$ is convex.

Consider next the customer's best-response correspondence. Regardless of the second provider's strategy, $\sigma_1^*(\omega_L \mid \theta, v, b, r(x^n, p), p)(a) = 1$. To see this, suppose by way of negation that this is false. Then the expected payoff of seeking a second opinion must exceed the payoff of accepting the best possible prescription of the first provider. But in this case the customer could go directly to the second provider and saves herself the search cost. A contradiction.

Consider next, the case in which the customer's first provider prescribes $\omega_H(j_1)$. By definition,

$$\omega_H p_{j_1} < (=, >) \Psi_{j_2}(p_{j_2} \mid b, x^n, p_A, p_B, \omega_H(j_1)) - \theta,$$

if and only if

$$\theta < (=, >) \theta^*(b, r(x^n, p), p, \omega_H(j_1)).^{23}$$

Consequently,

$$\sigma_1^*(\omega_H(j_1) \mid \theta, v, b, r(x^n, p), p))(a) =$$

$$\left| \begin{array}{lllll} 1 & \text{if} & \theta > \theta^*(b, r(x^n, p), p, \omega_H(j_1))) & \text{and} & v > \omega_H p_{j_1} \\ 0 & \text{if} & \theta < \theta^*(b, r(x^n, p, \omega_H(j_1)), p) & \text{or} & v < \omega_H p_{j_1} \\ [0,1] & \text{if} & \theta = \theta^*(b, , r(x^n, p), p, \omega_H(j_1))) & \text{and} & v > \omega_H p_{j_1} \end{array} \right|. \tag{12}$$

Hence, $\sigma_1^*(\omega_H(j_1) \mid \theta, v, b, r(x^n, p), p)$ is nonempty upper hemicontinuous.

Consider next $\sigma_2(\omega \mid \theta, v, b, r(x^n, p), p, \omega(j_1))$. Obviously, if the second provider prescribes $\omega_L(j_2)$, then $\sigma_2^*(\omega_L(j_2) \mid \theta, v, b, r(x^n, p), p, \omega(j_1)(a) = 1$. If the second provider prescribes $\omega_H(j_2)$, and $v > \omega_H p_{j_2}$ then $\sigma_2^*(\omega_H(j_2) \mid \theta, v, b, r(x^n, p), p, \omega(j_1)(a) =$

[23] By definition $\theta^*(b, r(x^n, p), p, \omega_H(j_1)) = \Psi_{j_2}(p_{j_2} \mid b, r(x^n, p), p, \omega_H(j_1)) - \omega_H p_{j_1}$.

1, and if $v < \omega_H p_{j_2}$, then $\sigma_2^*\left(\omega_H(j_2) \mid \theta, v, b, r(x^n,p), p, \omega(j_1)\right)(a) = 0$ and if $v = \omega_H p_{j_2}$ then $\sigma_2^*\left(\omega_H \mid \theta, v, b, r(x^n,p), p, \omega(j_1) = [0,1]\right.$.

Thus, the customer's best-response strategy,

$$\sigma^* = \left(\sigma_1^*\left(\theta, v, b, r(x^n,p), p\right), \sigma_2^*\left(\theta, v, b, r(x^n,p), p, \omega_k(j_1)\right)\right)$$

is upper hemicontinuous correspondence with nonempty convex range.[24]

Providers' mixed strategies in the prescription subgame are correspondences $\alpha_j : 1/2, 1] \times [0, \bar{v}]^2 \times [0,1] \rightrightarrows \Delta(\Omega)^S$, $j \in \{A, B\}$, whose domains and ranges are compact and convex.

Consider the providers' best-response correspondences contingent on receiving the signal $s \in S$ are:

$$\alpha_j^*\left(e_{j_1}, r(x^n,p), p\right)(s) = \begin{vmatrix} \delta_{\omega_L} & \text{if} & P_j\left(\omega_H \mid s\right) < P_j\left(\omega_L \mid s\right) \\ \delta_{\omega_H} & \text{if} & P_j\left(\omega_H \mid s\right) > P_j\left(\omega_L \mid s\right) \\ [0,1] & \text{if} & P_j\left(\omega_L \mid s\right) = P_j\left(\omega_H \mid s\right) \end{vmatrix} \tag{13}$$

$j \in \{A, B\}$. Hence, the providers' best-response correspondences have bounded closed graphs bounded. Thus, they are upper hemicontinuous.

To sum up. The best-response correspondences of both providers and the customer are upper hemicontinuous with convex, nonempty, ranges. Hence, by Kakutani's (1941) fixed point theorem, for every given $r(x^n)$ and p there exists strat-

[24]Note that under the assumption on the distributions of θ and v, the probability that the customer's best response takes the form of randomized choice is of measure 0. That is, for all practical purposes, the best response correspondence consists of pure strategies.

egy profile $\vartheta^*(r(x^n,p),p) \in \Sigma \times \Delta(\Omega)^S \times \Delta(\Omega)^S$ such that $\Xi(\vartheta^*(r(x^n,p),p)) = \vartheta^*(r(x^n,p),p)$.

The customers' system of beliefs is: $\zeta_c^*(\omega_H) = (\eta(\cdot \mid r(x^n),p), \mu, \mu(\omega_H \mid r(x^n),p,\omega_k(j_1))))$. The providers' systems of beliefs is: $\zeta_j^*(\omega_H) := (F_\theta, F_v, F_b, \mu, \mu_j(\omega_H \mid s_H), \mu_j(\omega_H \mid s_H))$, $j \in \{A,B\}$, where $\mu_j(\omega_H \mid s_H)$ and $\mu_j(\omega_H \mid s_H)$ are given in (1) and (2). In all of these case, it was shown in the body of the paper,the posterior beliefs were obtained by the application of Bayes' rule on the equilibrium paths. Hence, $(\vartheta^*, \eta_c^*, \eta_A^*, \eta_B^*)$ constitutes a PBE. ■

Proof of Proposition 2. For every given p_A let $p_B^*(p_A)$ be the solutions to the following equation:

$$Q_B^*(p_A,p_B \mid r(x^n,p)) + (p_B - c)\frac{\partial Q_B^*(p_A,p_B \mid r(x^n,p))}{\partial p_B} = 0, \tag{14}$$

where $Q_B^*(p_A,p_B \mid r(x^n,p))$ is the demand for provider B's service induced by the customers' and the providers' equilibrium strategies in the prescription subgame .

Given B's reaction function, $p_B^*(\cdot)$, define p_A^* by the solution to the equation

$$Q_A^*(p_A, p_B^*(p_A) \mid r(x^n,p)) =$$

$$(c - p_A)\left[\frac{\partial Q_A^*(p_A,p_B^*(p_A) \mid r(x^n,p))}{\partial p_A} + \frac{\partial Q_A^*(p_A,p_B^*(p_A) \mid r(x^n,p))}{\partial p_B}\frac{dp_B^*(p_A)}{dp_A}\right].$$

Then $(p_A^*, p_B^*(p_A))$ is an equilibrium of the pricing game that is consistent with the equilibrium strategies of the prescription subgame. ■

The strategies profile $(p_A^*, p_B^*, \sigma^*, \alpha_A^*, \alpha_B^*)$ together with the customers' and providers' beliefs system constitute an equilibrium of the stage game.

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