Monetary Backstop and Sovereign Default on Domestic Debt

Tongli Zhang†

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Abstract

Central banks can provide a monetary backstop for government debt by generating seigniorage at the cost of high inflation. This paper presents a sovereign default model to study how a monetary backstop affects the sustainable level of government debt and household welfare. The model features a government which adjusts fiscal policy infrequently. Uncertainty about fiscal adjustment reduces the government’s ability to roll over its debt and leads to the possibility of government default at relatively low debt-to-GDP ratios. The monetary backstop can lift the government’s debt limit, giving the government more room to roll over its debt until it can adjust fiscal policy. I calibrate the model to match data on government debt for a group of advanced economies. The model is able to explain the long-run cycles of government debt-to-GDP ratios and the behaviors of several macroeconomic variables within the cycle. The quantitative analysis shows that the benefit of avoiding a default outweighs the cost of inflation. A monetary backstop improves household welfare. It also suggests that if a central bank commits itself to providing a monetary backstop for government debt, it is better to choose a relatively high inflation target of around 15%.

Keywords: Monetary Backstop, Government Default, Fiscal Dominance, Seigniorage

JEL Classification: E58, E62, E63, H62, H63

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†Johns Hopkins University, Department of Economics, 3100 Wyman Park Drive, Baltimore, MD 21211. Email: tzhang31@jhu.edu
1 Introduction

The Covid-19 pandemic has led to large increases in government debt across advanced economies, generating renewed discussion about the sustainability of government debt. Most of the government debt of advanced economies is denominated in domestic currencies, which allows the central bank to support the government by raising inflation to generate seigniorage revenue for the government. This set of unconventional policies is often referred to as the monetary backstop for government debt.

The importance of a monetary backstop in reducing the risk of government default was revealed in the European debt crisis. After the announcement of the Outright Monetary Transactions programme (OMT) by the European Central Bank (ECB), the yield of the government bond of indebted peripheral European countries dropped significantly (Altavilla and Giannone, 2014).

Despite being a useful tool to prevent government default, the prospect of providing a monetary backstop for government debt has raised concerns about “Fiscal Dominance”. This term describes the situation where the central bank uses monetary policy to accommodate the borrowing needs of the government. The term was first introduced by Wallace and Sargent (1981). In their paper “Some Unpleasant Monetarist Arithmetic”, they showed that, in the state of fiscal dominance, when the government deficit can not be entirely financed by new borrowing, the central bank will be forced to tolerate excessive inflation in order to avoid government default.

The tension between the sustainability of government debt and the stability of prices indicates that there is a trade off for the central bank and for society at large. On the one hand, the central bank can prevent government default by providing a monetary backstop. Government default often causes disruptions of economic activity and significant income losses in the short run. On the other hand, commitment to the monetary backstop can pressure the central bank to increase the inflation to a costly high level.

In this paper, I study how a monetary backstop impacts the sustainable level of government debt and household welfare using a quantitative model of government default. The monetary backstop in the paper is defined as follows: in order to avoid the government default, the central bank raises the inflation rate to generate additional seigniorage for a sustained period of time. This is one of the channels through which the central bank can reduce government debt and avoid a default.

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1 Removing this monetary backstop can expose the government to larger risks of default. During the Covid-19 pandemic, ECB president Christine Lagarde made a speech suggesting that it was not the ECB’s role to “close the spread” in sovereign debt markets. The market interpreted it as ECB retreating from being a lender of last resort to Italy. Italian sovereign bond prices fell by a record daily amount (Arnold and Stubbington, 2020).
Figure 1: Debt-to-GDP ratios of selected advanced economies. Debt increase episodes are marked as red area and debt decrease episodes are marked as blue area.
One crucial feature of the model is that the government is slow in adjusting fiscal policy. This assumption is informed by the dynamics of government debt in advanced economies. As shown in Figure 1, the government debt-to-GDP ratio in advanced economies is marked by episodes of persistent debt increase and episodes of persistent debt decrease. The factors driving this very low frequency government debt cycle will be discussed in details in Section 4. To summarize, the debt cycle is linked to the fact that government revenues adjust much more quickly to a change in the trend growth rate than government expenditures as displayed in Figure 2. When the trend growth rate falls, government revenues fall immediately whereas expenditures tend to continue on their original path, leading to a persistent debt increase episode.

My model can reproduce debt increase and decrease episodes similar to those observed in the data by assuming that fiscal policy is adjusted infrequently. Debt increase episodes are triggered by falls in the trend growth rate and last until the government adjusts fiscal policy. The uncertainty about the fiscal adjustment reduces the government’s ability to roll over its debt and leads to government default. In my model, this type of rollover crises can happen even when the government debt-to-GDP ratio is relatively low if there is no monetary backstop.

The monetary backstop can remove the risk of rollover crisis and increases the debt limit of the government, giving the government more time to roll over its debt and adjust fiscal policy without default. Interestingly, the monetary backstop does not have to be actually enacted for a government to benefit from it. When the inflation target associated with the monetary backstop is sufficiently high, knowing that the central bank can provide that monetary backstop increases the debt limit of the government and allows the government to roll over its debt without actually receiving the monetary backstop. In equilibrium, the risk of high inflation is small.

In order to evaluate the impact of the monetary backstop quantitatively, I calibrate the model to match the data on the dynamics of government debt for advanced economies since 1980s. The calibrated model captures the long run cycle of the government debt-to-GDP ratio as well as the behavior of key economic variables such as the government primary balance, primary expenditure, and interest-growth differential. The simulation shows that a monetary backstop with mildly high level of inflation can significantly increase the debt limit of the government. Compared with an economy with no monetary backstop, allowing the central bank to raise inflation rate to 15 percent for 10 years on average can increase the debt limit by more than 200 percent of GDP and reduces the probability of government default to virtually zero. At the same time, the monetary backstop does not simply replace the risk of default with the risk of inflation. The monetary backstop is rarely enacted in equilibrium. The frequency of high inflation with the monetary backstop is much smaller than the frequency of default when there is no backstop.

Next, I used the calibrated model to study the optimal size of the monetary backstop which maximizes household welfare. My results indicate that, when the central bank provides a monetary backstop, it is better to choose a medium level of inflation target which is not
too low or too high. If the inflation rate is too low, the seigniorage is not large enough to
remove the risk of rollover crisis, the monetary backstop is triggered often in equilibrium,
and the government can still default even after receiving the monetary backstop. As a result,
both the risk of default and the risk of inflation are high. In this case, the welfare gain from
introducing the monetary backstop is small.

By contrast, when the inflation rate is sufficiently large, knowing that the central bank
can provide the monetary backstop increases the default threshold by a large amount. This
allows the government to roll over its debt and adjust fiscal policy later rather than defaulting
or actually receiving the monetary backstop. The monetary backstop is rarely enacted in
equilibrium. As a result, both the risk of default and the risk of inflation are low. However,
if the inflation rate is too high, enacting the monetary backstop will be too costly. In such
cases, the cost of high inflation reduces the welfare gain of the monetary backstop.

The quantitative analysis suggests that, when the inflation target of the monetary back-
stop is between 2 percent and 30 percent, the benefit of avoiding the default outweighs the
cost of inflation. In such cases, the monetary backstop improves household welfare. Under
the calibration of the model, the optimal inflation target is 15 percent and the welfare gain
is equivalent with 4 percent increase of the trend income.

1.1 Related Literature

The paper is related to several strands of literature.

First, this paper contributes to the large literature on fiscal dominance. The original
analysis of Sargent and Wallace (1992) was derived from a deterministic model. Several
papers have since extended the analysis to stochastic environments. Leeper (1991) introduced
the definition of “active” fiscal policies and “passive” monetary policies.\footnote{In Leeper (1991), an active authority pays no attention to the state of government debt and is free to set its control variable as it sees fit. A passive authority responds to government debt shocks.} He shows that
even when government debt is on average backed by direct taxes, innovations to debt may
be completely monetized. Davig et al. (2011) use a rational expectation framework to study
the implication of rising debt when there is a “fiscal limit”, which is defined as a point
where the government can not increase taxes to finance higher debt levels, so either a fiscal
adjustment or inflation must occur in order to stabilize debt. These papers, however, do not
allow for government default in their model.

Several other papers in this literature study the effect of monetary policy on the risk of
government default. Uribe (2006) shows that when both monetary policy and fiscal policy
are “active”, the probability of government default depends on the specifications of monetary
policy. Bi et al. (2018) show that the effect of monetary policy depends on its specification:
if the central bank targets the yield of risky government debt instead of the risk-free rate,
the monetary policy accommodates the rise of sovereign default risk by lifting the inflation
target even when the monetary policy is “active”. My paper differs from those contributions

\footnotetext[2]{In Leeper (1991), an active authority pays no attention to the state of government debt and is free to set its control variable as it sees fit. A passive authority responds to government debt shocks.}
in two ways. First, I assume that the government adjusts fiscal policy infrequently to capture the inertia of fiscal policy of advanced economies, and that the adjustment is optimal when it happens. Jeanne (2019) and Gourinchas and Jeanne (2012) present a mechanism similar to the one in this paper in the context of a two-period model. By contrast, in Uribe (2006), the fiscal policy is exogenous. The government is not able to adjust the fiscal policy. In Bi et al. (2018), the government adjusts the fiscal policy in every period based on reduced form relations between the tax rate and the government debt-to-GDP ratio. Second, in all these previous models, government debt is nominal and inflation reduces the real value of the debt. In this paper, I assume that the government debt is real. Inflation affects the path of government debt by generating seigniorage for the government.

Second, the modeling approach of this paper relates to the sovereign default framework developed by Aguiar and Gopinath (2006) and Arellano (2008). Specifically, this paper is related to literature on the default of domestic government debt (D’Erasmo and Mendoza, 2012, 2016; Casola and Sichlimiris, 2018; Mallucci, 2015; Niepelt, 2016; Pouzo and Presno, 2020). These papers, however, do not incorporate monetary policies in their model. More generally, this paper relates to recent research on the potential of using the monetary policy to avoid self-fulfilling debt crises (Aguiar et al., 2013; Bacchetta et al., 2015; Corsetti and Dedola, 2016; Camous and Cooper, 2019; Espino et al., 2020; Bianchi and Mondragon, 2018). In these papers, rollover crises happen when lenders believe that the government is more likely to default and bid down the prices of government bond. In my paper, rollover crises are caused by the inability of the government to adjust fiscal policy.

Third, this paper contributes to the literature studying the inertia in fiscal policy. Halleberg et al. (2001) provide empirical evidence for fiscal policy inertia in the event of negative economic shocks for European countries. Fatas and Mihov (2002) show that forecast errors or inability for the fiscal policy to response to shocks can cause excessive deficit. Larch and Salto (2005) show that, for countries in the Euro area, when the growth rate increase/decrease suddenly, the inertia of budgetary process leads to automatic increase/decrease in the government budget balance to surplus/deficit. All three papers above study the impact of the fiscal inertia on the cyclical behavior of the government budget balance. My paper makes a contribution by showing that the inertia in the fiscal policy can persist for long time and drive the long run dynamics of the government debt. The way the fiscal inertia is modeled in my paper is close to Jeanne (2021) in the sense that the opportunity to adjust fiscal policy follows a Poisson process. However, my paper differs from Jeanne (2021) by introducing shocks to the trend growth rate and explicitly modeling the monetary backstop.

\[^3\]D’Erasmo and Mendoza (2012, 2016) explore the sovereign default on domestic debt in a heterogeneous agent economy with wealth inequality. They argue that the government may choose to default on its domestic debt in order to achieve the desired wealth distribution. Casola and Sichlimiris (2018) examine a case of default on domestic debt where the government maintains monopoly in issuing debt by separating the external debt market and domestic debt market. Mallucci (2015) introduces the banking sector into a sovereign default model, and study how domestic holdings of sovereign debt affect the risk and cost of sovereign default. Niepelt (2016) presents an model with overlapping generations of agents where the government can partially default on its debt. Pouzo and Presno (2020) analyzes the optimal taxation in an economy where the government can default on its domestic debt.
Finally, the empirical work of this paper contributes to the literature on the long-run dynamics of the government debt for advanced economies. Early papers such as Bohn (1998) and Greiner et al. (2007) show that the primary balance-to-GDP ratio is a positive function of government debt-to-GDP ratio in US and EU countries, which suggests that the sovereign debt of advanced countries is sustainable. Using more recent data for OECD countries, Beqiraj et al. (2018) provide evidences that the primary balance decreases in response to increases in the debt-to-GDP ratio for advanced economies. Turner (2012) estimates the contribution of interest-growth differential to the increase of the government debt of advanced economies. Mauro et al. (2013) show that the response of fiscal policy to changes in government debt varies across countries and over time, and this variation is driven in part by unexpected changes in potential economic growth. This paper makes a contribution by quantifying the impact of both primary balance and interest-growth differential on the dynamics of the government debt of advanced economies, and shows that the slow adjustment of primary balance is the main cause for the large swing of the government debt-to-GDP ratio of advanced economies.

The remainder of this paper is structured as follows. Section 2 presents the model. Section 3 discusses the intuitions of the model using a special case where the income growth rate is constant. Section 4 calibrates the model to the long run cycles of the government debt of advanced economies. Section 5 and 6 contain the quantitative analysis of the effect of the monetary backstop on the sustainability of government debt and household welfare. Section 7 concludes.

2 Model

In this section, I present a model of government default on domestic debt. The model has two main assumptions. First, instead of having the government choose fiscal policy every period, the model assumes that fiscal policy is adjusted only when a window of opportunity arrives. Second, while the inflation rate in the model remains low in most periods, the central bank may raise inflation to increase seigniorage in order to avoid a government default. I use the model to explain the dynamics of government debt in advanced economies and examine the quantitative effect of the monetary backstop.

2.1 Environment

Time is discrete, with $t$ denoting the current period. The economy is populated by a large number of identical households and has a government and a central bank. In each period, the representative household receives an endowment $Y_t$ which grows with a stochastic growth factor $G_t$:

$$ Y_{t+1} = Y_t \cdot G_{t+1} $$

(1)
The growth factor $G_t$ can be either high or low and follows a two-state Markov process:

$$\begin{align*}
\text{Prob}(G_{t+1} = G^H | G_t = G^H) &= p_h \\
\text{Prob}(G_{t+1} = G^L | G_t = G^L) &= p_l
\end{align*}$$

### 2.2 Household

The household derives utility from private consumption goods $C_t$, public expenditure $E_t$, and real money balance $M_t = \frac{M_t}{P_t}$. The felicity function $U(\cdot)$ is additive separable,

$$U(C_t, E_t, M_t) = C_t + \alpha Y_t^{\psi} \frac{E_t^{1-\psi}}{1-\psi} + \mu Y_t^{\rho} \frac{(M_t/P_t)^{1-\rho}}{1-\rho}$$  \quad (2)

The scaling factors $Y_t^{\psi}$ and $Y_t^{\rho}$ ensure that the utility function is homothetic in $Y$.

The household discount future utility with factor $\beta$ and maximizes the infinite-horizon utility:

$$V_t = \mathbb{E}_0 \left[ \sum_{n=0}^{\infty} \beta^n U(C_{t+n}, E_{t+n}, M_{t+n}) \right]$$  \quad (3)

subject to the budget constraint:

$$C_t + q_t B_t (1 - \delta_t) + M_t = (1 - \tau_t - \eta \pi_t^2) (1 - \gamma \delta_t) Y_t + B_{t-1} (1 - \delta_t) + \frac{M_{t-1}}{1 + \pi_t} + F_t$$  \quad (4)

where $B_t$ is the value of one-period real government bond purchased in period $t$ at price $q_t$. The inflation rate $\pi_t$ is determined by the central bank one period in advance. I will describe the central bank’s policy in detail in Section 2.4. Given that in the model, government debt is short-term and the inflation rate is determined in advance, whether the government issues nominal or real debt does not change the ultimate outcome. I assume government debt is real to simplify the notation. These assumptions prevent the real value of the debt to be inflated away, and allow the analysis to focus on the impact of the seigniorage on the risk of default.

Binary variable $\delta_t$ denotes whether the government defaults or not, with $\delta_t$ equals 1 if the government defaults in the current period, 0 otherwise. If the government defaults on its debt, the household does not receive any payment on its government bond holdings. The government is unable to issue new bonds in that period. The government may resume debt issuance next period with zero debt. The default is associated with a temporary loss of income which equals $\gamma$ times the trend income.

The household pays tax at rate $\tau_t$. The tax is distortionary with a convex cost $\eta \pi_t^2$. The household receives a lump-sum transfer $F_t$ from the government.
The problem can be normalized by dividing the utility function and the budget constraint by $Y_t$. Hereafter I use lower case letters to denote normalized variables. (e.g. $c_t = C_t / Y_t$).

The normalized form of the household problem is

$$v_t(b_{t-1}, m_{t-1}) = \max_{c_t, m_t} \{ u(c_t, e_t, m_t) + \beta E_t [G_{t+1} v_{t+1}(b_t, m_t)] \}$$

s.t

$$c_t + q b_t (1 - \delta_t) + m_t = (1 - \tau_t - \eta \tau_t^2) (1 - \gamma \delta_t) + \frac{b_{t-1} (1 - \delta_t)}{G_t} + \frac{m_{t-1}}{(1 + \pi_t) G_t} + f_t \quad (5)$$

The normalized felicity function is

$$u(c_t, e_t, m_t) = c_t + \alpha \frac{e_t^{1-\psi}}{1-\psi} + \mu \frac{m_t^{1-\rho}}{1-\rho} \quad (6)$$

where $e_t = \frac{E_t}{Y_t}$ is the primary expenditure-to-GDP ratio, and $m_t = \frac{M_t}{Y_t}$ is the money to GDP ratio.

### 2.3 Government

The government chooses fiscal policy and whether to default. Fiscal policy includes the tax rate $\tau_t$ and the primary expenditure $E_t$.

The government can only make infrequent decisions on fiscal policy. In each period $t$, whether the government adjusts fiscal policy is determined by a random variable $\xi_t$ which takes the value 1 if the government adjusts its policy, and 0 otherwise.

$$\xi_t = \begin{cases} 
1 & \text{with probability } 1 - \phi \\
0 & \text{with probability } \phi 
\end{cases} \quad (7)$$

The intuition here is that the government budget decision is constrained by the political process and opportunities to reset the budget policy arrive infrequently. This assumption is consistent with the findings of literature on the political economy of fiscal reforms, which shows that fiscal reforms are often delayed due to election cycles or political gridlock (Alesina and Drazen (1991), Alt and Lassen (2006), Ma (2014), Yared (2018)).

When the output growth rate changes, the inability for the government to adjust fiscal policy produces inertia in government primary expenditure. Because the tax rate does not change, government revenue falls proportionately to output.

In addition, I assume that the government adjusts fiscal policy when it re-enters the debt market after a default.

#### 2.3.1 Fiscal Policy

At the beginning of each period $t$, the government observes $\xi_t$, and decides whether to default or not. Conditional on not defaulting, if $\xi_t = 1$, the government chooses tax rate $\tau_t$ and the
level of primary expenditure $E_t$ to maximize its value. The primary expenditure will grow at constant growth rate $g_t$ until the next adjustment of fiscal policy. When an adjustment happens, $g_t$ is adjusted to the income growth rate of that period.\(^4\)

$$g_t = G_t$$  \(8\)

If $\xi_t = 0$, the government keeps the existing fiscal policy. The tax rate $\tau_t$ and the growth rate of primary expenditure $g_t$ remain the same as the last time when the government adjusts the policy. Let $t_0$ denote the time for the most recent policy adjustment.

$$\tau_t = \tau_{t-1} = \tau_{t_0}$$  \(9\)

$$g_t = g_{t-1} = G_{t_0}$$  \(10\)

The level of primary expenditure grows with the planned growth rate

$$E_t = E_{t-1}g_t$$  \(11\)

Consequently, the primary expenditure-to-GDP ratio evolves as follows

$$e_t = e_{t-1}g_t$$  \(12\)

$$= e_{t-1}\frac{G_t}{G_{t_0}}$$

The process above does not guarantee that the expenditure-to-GDP ratio is bounded and remains below 100 percent. To ensure that the expenditure-to-GDP ratio fluctuates within a plausible interval, I assume that it is bounded

$$e_t \leq e \leq e_h$$  \(13\)

For example, if the expenditure-to-GDP ratio reaches its upper bound $e_h$, $e$ stays constant at $e_h$ instead of following equation \((12)\). When the government adjusts the fiscal policy, government choice of $e$ is also constrained by condition \((13)\).

### 2.3.2 Continuation

If the government adjusts fiscal policy to maximize household welfare (I refer to this case as state ‘a’), let $V^a$ be the value of continuation.\(^5\) The value of the government normalized by income $v^a = V^a/Y$ is as follows

$$v^a_t(b_{t-1}, m_{t-1}) = \max_{\tau_t, e_t} u(c_t, e_t, m_t) + \beta \mathbb{E}_t \left[ G_{t+1}v^a_{t+1}(b_t, m_t, e_t, g_t, \tau_t) \right]$$  \(14\)

\(^4\) Ideally, the government would choose the path of primary expenditures in all future periods $\{E_{t+n}\}_{n=0}^{+\infty}$. However, this would imply an infinite numbers of choice variables, which makes the model difficult to compute numerically.

\(^5\) To differentiate the value of the government from the value of the household, I use symbol $V$ to denote the value of the government. The superscript “a” stands for “adjust.”
Here \( v \) (with no superscript) denotes the value of the government that has the option to default. The government’s optimization problem is subject to

\[
e_t + \frac{b_{t-1}}{G_t} + f_t = \tau_t + q_t b_t + m_t - \frac{m_{t-1}}{(1 + \pi_t)G_t} \tag{15}
\]

Seigniorage

If there is no policy adjustment (I refer to this case as state 'n'), let \( V^n \) be the value of continuation.\(^6\) The value normalized by the income \( v^n = V^n / Y \) is

\[
v^n_t(b_{t-1}, m_{t-1}, e_{t-1}, g_{t-1}, \tau_{t-1}) = u(c_t, e_t, m_t) + \beta \mathbb{E}_t [G_{t+1}v_{t+1}(b_t, m_t, e_t, g_t, \tau_t)] \tag{16}
\]

In this case, the government does not optimize. The current fiscal policy is determined by the last time the government adjusted its policy

\[
\begin{align*}
\tau_t &= \tau_{t-1} = \tau_0 \\
e_t &= e_{t-1} \frac{g_t}{G_t} \\
g_t &= g_{t-1} = G_{t_0}
\end{align*} \tag{17}
\]

Here \( t_0 \) denotes the time of the most recent adjustment in fiscal policy. The government debt-to-GDP ratio \( b_t \) evolves according to the government budget constraint.

In addition, I assume that the government debt-to-GDP ratio can not be negative. Once the debt-to-GDP ratio reaches 0, the government rebates all surpluses to the household. This assumption allows the model to match the data where all advanced economies have positive debt through out the history.

### 2.3.3 Default

A defaulting government is excluded from the bond market for one period and resumes bond issuance in the next period. When the government reenters into the bond market, it starts with zero debt and adjusts fiscal policy. The value of default is:

\[
v^d_t(m_{t-1}) = \max_{\tau_t, e_t} u(c_t, e_t, m_t) + \beta \mathbb{E}_t [G_{t+1}v^a_{t+1}(0, m_t, \tau_t)] \tag{18}
\]

The tax rate \( \tau_t \) and the expenditure-to-income ratio \( e_t \) satisfy the government budget constraint:

\[
e_t + f_t = \tau_t (1 - \gamma) + m_t - \frac{m_{t-1}}{(1 + \pi_t)G_t} \tag{19}
\]
Default can happen for two different reasons. First, the government defaults if the value of default is greater than the value of continuation. I refer to this type of default as a strategic default. In this case, the default indicator $\delta_t$ is determined as follows:

$$
\delta_t(b_{t-1}, m_{t-1}, e_{t-1}, g_{t-1}, \tau_{t-1}, \pi_{t+1}) = \begin{cases} 1 & \text{if } \xi = 1, v^a < v^d \text{ or if } \xi = 0, v^n < v^d \\ 0 & \text{otherwise} \end{cases}
$$

(20)

Second, in periods when the government cannot adjust fiscal policy, a default happens if the government cannot cover its financing needs with new borrowing. This is the case if:

$$
\frac{b_{t-1}}{G_t} + e_t - \tau_t - s_t > \max\{q_ib_t\}
$$

(21)

Here $s_t$ is the seigniorage. In this case, the government will be forced to default due to lack of liquidity. I refer to this type of default as a rollover crisis.

The value of the government that has the option to default $v$ satisfies:

$$
v = \begin{cases} \max\{v^a, v^d\} & \text{if } \xi = 1 \\ \max\{v^n, v^d\} & \text{if } \xi = 0 \end{cases}
$$

(22)

### 2.4 Central Bank

In each period $t$, the central bank determines the inflation rate one period ahead: $\pi_{t+1}$. This assumption allows the inflation rate to be pre-determined which simplifies the state of money balance $m_t$. I discuss the relationship between money balance and inflation target in Section 2.5.

I assume that the inflation rate is either low or high: $\pi \in \{\pi^{Low}, \pi^{High}\}$. The central bank can provide a monetary backstop for the government by raising the inflation rate for a sustained period of time. Here instead of having the central bank optimize the inflation target, the inflation targets of the central bank are exogenous. The idea is to examine the impact of the monetary backstop by comparing models with different $\pi^{High}$ and with a model where there is no monetary backstop. In the following paragraphs, I describe the central bank’s policy rule and explain the intuition behind the rule.

When the current inflation rate is low: $\pi_t = \pi^{Low}$, the policy rule for the next period inflation target $\pi_{t+1}(x_{t-1}, \tau_{t-1}, G_t, \pi_t)$ is as follows.

$$
\pi_{t+1}(x_{t-1}, G_t, \pi^{Low}) = \begin{cases} \pi^{High} & \text{if } v^d(\pi^{Low}) > v^c(\pi^{Low}) \\ \pi^{Low} & \text{otherwise} \end{cases}
$$

(23)

### Footnote

For roll-over crises, the value of continuation is not well-defined. I assign a very small number for the value of continuation in the case of roll-over crises, so that equation (22) can be applied to both strategic defaults and roll-over crises.
Here \( x_{t-1} = \{b_{t-1}, e_{t-1}, g_{t-1}, \tau_{t-1}\} \) denotes the fiscal state of the government. \( v^c \) is the value of continuation, which equals to \( v^a \) when the government adjusts the policy and to \( v^n \) otherwise.

The inequality \( v^d(\pi^{Low}) > v^c(\pi^{Low}) \) captures the intuition that the central bank raises the inflation rate if the government will default under the low inflation rate. If the government does not default under the low inflation, the central bank keeps the inflation rate low.

The policy rule implies that the central bank is committed to raise inflation rate to avoid a government default, even though letting the government default can yield higher values in some circumstances. Particularly, this assumption on the central bank’s commitment allows the model to examine the effect of the monetary backstop when \( \pi^{High} \) is large. Alternatively, if the inflation policy is discretionary such that the central bank raises inflation only when it increases the welfare, the central bank will choose to never raise inflation when \( \pi^{High} \) is above certain value. This is because when \( \pi^{High} \) is large, the cost of raising the inflation is larger than the cost of default. In that case, there is effectively no monetary backstop in the model.

Once the economy is in the high inflation regime, \( \pi_t = \pi^{High} \), the central bank exits this regime with a constant probability \( p_\pi \), as long as the exit does not cause immediate government default. This assumption ensures that when a monetary backstop is triggered, it is expected to last for a sustained period of time.

### 2.5 Optimality Conditions

**Bond Price**

The bond price \( q_t \) satisfies the Euler Equation. Since the household is risk-neutral in private consumption, the bond price equals to the expected payoff of the government bond multiplies the time discount factor.

\[
q_t = \beta E_t [1 - \delta_{t+1}] \tag{24}
\]

**Money**

The no-arbitrage condition between money and bond ensures that \( m_t \) decreases with the expected next period inflation rate.

\[
m_t = \mu^{\frac{1}{p}} \left[ 1 - \beta \left( \frac{1}{1 + \pi_{t+1}} \right) \right]^{-\frac{1}{p}} \tag{25}
\]
Seigniorage

The seigniorage is
\[ s_t = m_t - \frac{m_{t-1}}{1 + \pi_t} \] (26)

When the current inflation rate is low, the seigniorage revenue is rebated to households as a transfer. When the central bank raises the inflation to provide the monetary backstop, the government receives the seigniorage revenue.

\[ f_t = \begin{cases} 
  s_t & \text{if } \pi_t = \pi^{Low} \\
  0 & \text{if } \pi_t = \pi^{High} 
\end{cases} \]

Resource Constraint

The government internalizes the impact of fiscal policy on the household’s consumption choices. As a result, the government recognizes the aggregate resource constraint
\[ c_t + e_t = (1 - \eta \tau_t^2)(1 - \gamma \delta_t) \] (27)

2.6 Timing

The timing of decisions in the model is the following:

1. The exogenous shocks \( G_t \) and \( \xi_t \) are realized.
2. Given the state variables \((b_{t-1}, m_{t-1}, e_{t-1}, g_{t-1}, \tau_{t-1})\) and the value of shocks, the central bank decides the inflation rate \( \pi \).
3. The government decides whether to default or not.
4. If the government defaults, it sets the fiscal policy \( \tau^d \) and \( e^d \) for the period in default. The government resets the fiscal policy when it re-enters the debt market.
5. Conditional on no default, if \( \xi = 1 \), the government adjusts the fiscal policy \((e_t, \tau_t)\). If \( \xi = 0 \), the government keeps the existing fiscal policy.
6. The household chooses his private consumption \( c_t \) and real money demand \( m_t \).
2.7 Recursive Equilibrium

**DEFINITION.** A Recursive Equilibrium is defined by: (i) a set of value functions: $v$, $v^a$, and $v^n$; (ii) the government default rule $\delta$; (iii) policy functions for the primary expenditure to income ratio and the tax rate when adjusting: $(e^{\xi=1}, \tau^{\xi=1})$; (iv) a policy function for inflation $\pi$; (v) private consumption $c$; (vi) real money balance $m$; and (vii) a real bond price $q$ such that

1. Given $v^a$, $v^n$ and $v^d$, $v$ satisfies equation (22), $\delta$ satisfies equation (20)
2. Given the value function $v$, and the bond pricing function $q$, $v^a$ and $(e^{\xi=1}, \tau^{\xi=1})$ solve the government optimizing problem (14).
3. $v^n$ satisfies equation (16).
4. $v^d$ satisfies equation (18)
5. The inflation rate $\pi$ satisfies the central bank policy rule (23).
6. Private consumption $c$ and money demand $m$ solve the household problem.
7. The real bond price $q$ satisfies optimality condition (24)

3 Constant Growth Rate

Before starting the quantitative analysis, it is useful to first understand how the sustainability of government debt is influenced by the slow adjustment of fiscal policy and the monetary backstop in the model. In this section, I discuss the intuitions of the model in a special case where the income growth rate is constant i.e. $G^H = G^L$. I show that when the government has a fiscal deficit, the slow adjustment of fiscal policy can significantly reduce the government default threshold (defined in next paragraph). The monetary backstop can lift the default threshold by turning the government deficit into surplus. The propositions in this section formalize the intuition.

3.1 Default Threshold

To study the impact of a monetary backstop on the sustainable level of government debt, I define the default threshold as the maximum debt-to-GDP ratio that the government can carry without default. In state ‘a’, the default threshold is a function of the GDP growth rate and the current inflation rate. In state ‘n’, the default threshold also depends on the existing fiscal policy.

$$\bar{b}_t = \begin{cases} 
\bar{b}^a(G_t, \pi_t) & \text{if } \xi = 1 \\
\bar{b}^n(G_t, e_{t-1}, g_{t-1}, \tau_{t-1}, \pi_t) & \text{if } \xi = 0
\end{cases}$$

(28)
As discussed in the Model section, the government can default due either to a rollover crisis or a strategic default. In state 'a', because the government can adjust fiscal policy, there is no rollover crisis. The default threshold $\bar{b}^a$ is the strategic default threshold. In state 'n', both rollover crisis and strategic default can happen. The default threshold $\bar{b}^n$ is the minimum between the threshold of strategic default and the threshold of rollover crisis.

Lemma 1 shows that the default threshold is smaller in state 'n' than it is in state 'a'.

**Lemma 1.** For any given $(e_t, g_t, \tau_t, \pi_t)$, the default threshold in state 'n' is smaller than the default threshold in state 'a'.

\[ \bar{b}^n(e_t, \tau_t, g_t, \pi_t) \leq \bar{b}^a(\pi_t) \]  

(29)

**Proof:** See Appendix A.1.

The intuition is the following. When the government can adjust fiscal policy, default can only happen strategically. When the government cannot adjust fiscal policy, default can happen due either to a rollover crisis or a strategic default, depending on which threshold is lower.

In either case - strategic default or rollover crisis - the inequality holds. In the case of strategic default, because the government optimizes fiscal policy in state 'a' but cannot do so in state 'n', the value of repaying the debt is smaller in state 'n' than in state 'a'. As a result, the threshold for strategic default is lower in state 'n' than state 'a'. In the case of a rollover crisis, which only happens in state 'n', the government defaults before the debt-to-GDP ratio reaches the threshold for a strategic default. The default threshold for state 'n' is even lower in this case.

In addition, because the government does not adjust fiscal policy in state 'n', the projected path for government debt-to-GDP ratio plays an important role in determining the risk of government default. In particular, a rollover occurs only if the government debt-to-GDP ratio is on an increasing path. When a rollover crisis occurs, the default threshold tends to be much smaller than the threshold for a strategic default, especially if the probability for adjusting fiscal policy $\phi$ is small. Proposition 1 characterizes the default threshold in case of rollover crises.

**Proposition 1.** A rollover crisis happens only if the government is in state 'n' and the debt-to-GDP ratio is increasing over time. In this case

\[ \bar{b}^n \approx \phi \bar{b}^a \]  

(30)

**Proof:** See Appendix A.2.

Proposition 1 states that the threshold for rollover crises is proportional to the probability of adjustment $\phi$. When $\phi = 1$, the government adjusts fiscal policy in every period, the model

---

8Here because the growth rate $G_t$ is constant, I drop it from the list of state variables.
reduces to a standard default model with no stickiness in government policy, \( \bar{b}^a = \bar{b}^a \). When \( \phi \) is small, the threshold for a rollover crisis is much lower than the threshold for a strategic default. The government may default with a relatively small amount of debt.

The intuition behind equation (30) is backwards induction. When the government cannot adjust fiscal policy and is running a fiscal deficit, the debt-to-GDP ratio is increasing. Once the government debt-to-GDP ratio reaches the default threshold for state 'n' \( \bar{b}^n \), the government will default in the next period unless it adjusts fiscal policy in the next period. The probability of repaying the debt is thus equal to the probability of adjustment \( \phi \). A small \( \phi \) would significantly decrease the bond price and reduces the amount of funds that the government receives from new borrowings. Given that the maximum amount of debt that the government can sustain in the next period is \( \bar{b}^a \), the maximum of debt that the government can roll over in the current period is close to \( \phi \bar{b}^a \).

Finally, if its debt-to-GDP ratio is on a downward path, the government can always roll over debt without the need to adjust fiscal policy. The default can only happen strategically. In this case, the default threshold in state 'n' will be close to the default threshold in state 'a' and significantly larger than \( \phi \bar{b}^a \).

3.2 Monetary Backstop

How does the monetary backstop affect the default threshold of the government? Depending on whether the default is a rollover crisis or a strategic default, the monetary backstop can have different effects on the default threshold.

Rollover Crisis If the government default is caused by a rollover crisis when there is no monetary backstop, the effect of the monetary backstop on the default threshold depends on whether the seigniorage is large enough to cover the financing gap of the government. Proposition 2 characterizes the result.

**Proposition 2.** If the government defaults due to a rollover crisis when there is no monetary backstop, the effect of introducing a monetary backstop on the default threshold depends on the size of the seigniorage. If the seigniorage satisfies condition

\[
s_t \geq \frac{1 - \beta G}{G} b_{t-1} + e_t - \tau_t
\]

(31)

the default threshold increases from the threshold of rollover crisis which is close to \( \phi \bar{b}^a \) to the threshold of strategic default which is close to \( \bar{b}^a \).

Proof: See Appendix A.3.

The intuition is the following. When a rollover crisis happens, Proposition 2 implies that the government must be in state 'n', and the debt-to-GDP ratio is on an increasing path. The monetary backstop creates a seigniorage revenue for the government, which improves
the government budget condition. The condition (31) ensures that the seigniorage is large enough to cover the financing gap of the government, which equals to the primary deficit \((e_t - \tau_t)\) plus the interest-growth differential \((\frac{1 - \beta G}{\phi}b_t)\). Once condition (31) is satisfied, the government debt-to-GDP ratio is on a downward path. As shown in Proposition 2, rollover crises do not happen when the government debt-to-GDP ratio is decreasing. In this case, the relevant default threshold is no longer the rollover crisis threshold, but the higher strategic default threshold.

Proposition 2 highlights that the monetary backstop can increase the default threshold of the government by removing the risk of rollover crises. This effect is particularly crucial for the sustainability of government debt when \(\phi\) is small. First, when \(\phi\) is small, the threshold for a rollover crisis \(\phi \theta^a\) is small. Removing the risk of rollover crisis increases the default threshold by a large amount. Second, when \(\phi\) is small, it is more likely that the government is in state ‘n’, and lifting the default threshold in state ‘n’ has a large impact on the sustainability of the government debt.

The benefit of monetary backstop exists only if the seigniorage is large enough to cover the financing gap of the government. If the seigniorage is too small to satisfy condition (31), rollover crises can happen even with the monetary backstop. In this case, the impact of the monetary backstop on the default threshold is quite small.\(^9\) The intuition is as follows. When rollover crisis happens, as I shown in Proposition 2, \(\theta^a \approx \phi \theta^a\). The monetary backstop can increase \(\theta^a\) by increasing \(\theta^a\). However, when \(\phi\) is small, the increase in \(\theta^a\) is much smaller than the increase of \(\theta^a\). As a result, the impact of the monetary backstop is small.

It is worth noting that the benefit of the monetary backstop does not require the monetary backstop to be actually enacted. As long as the seigniorage is large enough to cover the financing gap of the government, knowing that the central bank can provide the monetary backstop removes the risk of rollover crisis and increases the default threshold of the government. This gives government more room to rollover its debt and adjust fiscal policy later without actually implementing the monetary backstop.

**Strategic Default** For strategic default, the impact of a monetary backstop is ambiguous. The monetary backstop has two opposite effects on the default threshold. On the one hand, the monetary backstop reduces the cost of taxation by replacing the tax with seigniorage. Through this channel, the monetary backstop increases both the value of continuation and the value of default. However, the average tax rate in continuation is higher than the average tax rate in the case of default (and reentry). For the same amount of reduction in tax rate, because the cost of taxation is convex, the value of continuation increases more than the value of default. As a result, the monetary backstop increases the default threshold.

On the other hand, the high inflation reduces money demand which reduces the utility derived from money balances. Through this channel, the monetary backstop reduces both the value of continuation and the value of default. However, the monetary backstop tends to

\(^9\)See Appendix A.3 for details.
last longer in the case of continuation than in the case of default. This is because, in the case of continuation, the government debt-to-GDP ratio tends to stay high, which prevents the central bank from reducing the inflation target. As a result, the monetary backstop reduces the value of continuation more than the value of default, and therefore reduces the default threshold of the government. The total impact of the monetary backstop depends on the relative strengthen of these two effects. In the next section, I calibrate the model to evaluate the effect of the monetary backstop quantitatively.

4  Calibration

In order to analyze the impact of the monetary backstop beyond the special case of constant growth rate, I calibrate the parameters of the full model and show that the calibrated model is capable of explaining the dynamics of government debt in advanced economies. Using the calibrated model, I show that the monetary backstop increases the government’s default threshold. First, in Section 4.1, I show that the government debt-to-GDP ratio of advanced economies swings in long-run cycles and identify episodes of persistent debt increase and persistent debt decrease. In Section 4.2, I compute moments for these episodes and calibrate the model to match moments. Lastly, in Section 4.3, I show that the calibrated model captures the behavior of government debt and key economic variables in advanced economies, and I investigate the impact of a monetary backstop on the default threshold of the government.

4.1  Empirical Analysis

I obtain data on government debt for advanced economies from the IMF World Economic Outlook (WEO) database. Figure 1 shows the government debt-to-GDP ratio for a group of selected countries. We can see that the government debt-to-GDP ratio swings in long cycles.

To characterize the dynamics of the government debt-to-GDP ratio in the data, I introduce a set of criteria to identify persistent debt increase episodes and persistent debt decrease episodes.

A debt increase episode is identified if this period satisfies the following conditions:

1. starts from a year with ∆Debt-to-GDP ≥ 1%.
2. ends with one of the two following conditions:
   1. ∆Debt-to-GDP < 1% for three years in a row

10 See Appendix B.1 for details of the data
11 For the other countries, see Figure 16 in Appendix B.4
Debt-to-GDP \(< -4.5\%\) for one year

(3) lasts for more than 10 years. This separates the long-run government debt cycle from fluctuations due to the business cycle. (Beginning in 2008, this length is shortened to 7 years).\(^{12}\)

Conversely, I define a period as a debt decrease episode if it:

(1) starts from a year with the \(\Delta\text{Debt-to-GDP} < 0\%\).

(2) ends with one of the two following conditions:

i \(\Delta\text{Debt-to-GDP} \geq 0\%\) for next two years

ii \(\Delta\text{Debt-to-GDP} > 6\%\) for next year

(3) lasts for more than 10 years (Beginning in 2008, this length is shortened to 7 years).

Figure 1 shows debt increase episodes (red) and debt decrease episodes (blue) for selected economies.

In order to reveal the factors that drive the government debt cycle, I decompose the change of Debt-to-GDP ratio of each country into the contribution from the primary fiscal balance, the interest rate-growth differential, and the residual.\(^{13}\) Table 1 reports the summary statistics for each component.

For each component of the decomposition, I compute the difference between the average level of the variable during debt increase episodes and decrease episodes. I refer to this difference as the inter-episode difference. I compute inter-episode differences for each country and then take the average of this difference across countries.\(^{14}\)

The second column of Table 1 reports the inter-episode difference for each variable. We can see that the annual increase in the government debt-to-GDP ratio is 5.7 percent higher in debt increase episodes than in debt decrease episodes. The majority of the difference in the debt dynamics can be attributed to the primary balance. After decomposing the primary balance into revenue and primary expenditure, we can see that higher primary expenditure is the main factor that causes debt to go up in debt increase episodes.

It is worth noticing that the average real GDP growth rate is lower in debt increase episodes than in debt decrease episodes. I plot the level of output, fiscal revenue, and primary expenditure in the first eight years of debt increase episodes in Figure 2 (shown

\(^{12}\)Because the data series end at 2017, I shortened the length requirement to be 7 years for debt increase episodes after 2008 in order to capture debt increase episodes after the Global Financial Crisis.

\(^{13}\)See Appendix B.2 for details of the decomposition.

\(^{14}\)For countries with only debt increase episodes or debt decrease episodes, I calculate the difference between the mean of debt increase/decrease episodes and the rest of years. Countries that have neither debt increase episodes nor debt decrease are excluded from the calculation.
in the introduction). The graph shows that fiscal revenue falls in pace with GDP, whereas the primary expenditure continues growing in its original path. The inertia in primary expenditure causes the persistent increase of the government debt-to-GDP ratio.

4.2 Parameters

This section describes my calibration strategy. Each period in the model is one year. The model has seventeen parameters. I calibrate eight parameters \((G^H, G^L, p_h, p_l, \alpha, \Xi, \eta, \beta)\) to match the moments of the government debt cycle. The parameters related to money balance \((\mu, \psi)\) are calibrated separately to match the evidence on money demand. The value of the remaining seven parameters are either chosen exogenously or taken from the literature. Table 2 reports the parameter values.

Among the parameters targeting the moments of the government debt cycle, the income growth rate \(G^H\) and \(G^L\) are calibrated to match the average and the inter-episodes difference for real GDP growth rate in the data. The probability \(p_h\) and \(p_l\) are chosen to match the frequency of debt increase episodes and the frequency of debt decrease episodes. The coefficients \((\alpha, \Xi, \eta, \beta)\) are calibrated to match the average and the inter-episodes difference of the primary expenditure-to-GDP ratio, and the average of the revenue-to-GDP ratio in the data. The time discount factor \(\beta\) is calibrated to match the inter-episodes differences for the \(r - g\) component. Table 3 shows the value of targeted moments from the empirical data and the model counterparts.

The parameters \(\mu\) and \(\rho\) are calibrated to match the level and the interest elasticity of money demand. For the purpose of calibration, aggregate money is measured by Money Zero Maturity (MZM), which encompasses all non-term deposits, and money that can be accessed without notice and at par. Specifically,

\[
MZM = M2 - \text{Small-denomination Time Deposits} + \text{Institutional Money Market Funds}
\]

Motley (1988) first introduced MZM as a broad money aggregate which gives private agents immediate command over goods and services. By calibrating the demand for money to a broad money aggregate rather than just the monetary base, the seigniorage received by the government is large. In reality, this can be achieved when the higher inflation is associated with financial repression as shown by Reinhart and Sbrancia (2015). For the calibration of \(\rho\), I obtain the data from Teles and Zhou (2005), where the interest elasticity of MZM is estimated to be 0.24. This corresponds to \(\rho\) being almost equal to 4. For \(\mu\), I set \(\mu = 0.05\), so the money demand in the model under low inflation is 100 percent of GDP, which matches the current MZM-to-GDP ratio in the US.

For the remaining parameters, I set \(\psi = 2\), which is a standard value for the CRRA utility function. I assume the fraction of income loss in default is set to \(\gamma = 30\%\), which is in line with typical estimates in the sovereign default literature (D’Erasmo and Mendoza (2012) and Rogoff and Reinhart (2011)). I take the average minimum and maximum primary
expenditure-to-GDP ratio across countries from the data, hence, $e_l = 0.36$ and $e_h = 0.48$. I set $\pi^{Low}$ to be 2 percent which is a conventional inflation target for central banks of advanced economies. I assume that when the central bank provides the monetary backstop, the inflation rate increases to 15 percent and, on average, lasts for 10 years, hence $\pi^{High} = 15\%$, and $p_\pi = 0.1$. Later in Section 5, I relax the calibration for $\pi^{High}$, and show how the impact of monetary backstop varies with $\pi^{High}$.

### 4.3 Quantitative Results

In this section, I compare the model’s predictions with the data and examine the impact of a monetary backstop on the default threshold. Using the event study method, I show that the model can explain the behavior of the main variables of interest in the government debt cycle.

For the event study, I run 10000 simulations of the calibrated model. Each simulation lasts for 400 periods. For each simulation, I identify debt increase/decrease episodes using the same method as in the data and collect simulated data for variables including: change of debt-to-GDP ratio, primary balance, interest-growth differentials, revenue-to-GDP ratio, primary expenditure-to-GDP ratio, and income growth rate. The goal is to compare the behavior of each variable above in the simulation with its behavior in the data around the time period when debt increase episodes begin.\(^\text{15}\)

The event in the study is the beginning of debt increase episodes. For each debt increase episode, I first obtain the value of each variable during a time window starting three years before the event and ending eight years after the event. I then compute the average trend across all debt increase episodes. Figure 3 plots the path of $\Delta$Debt-to-GDP ratio and its components at the beginning of debt increase episodes. The simulated path is represented by the black dashed line and the path from the empirical data is represented by red solid line. The beginning of the debt increase episode is marked by a vertical black line at time 0.

The model simulation captures the behavior of all six variables. Specifically, at the beginning of the debt increase episode, the real growth rate drops sharply. While the revenue-to-GDP ratio stays roughly constant, the primary expenditure to GDP ratio increases significantly. The large increase in the primary expenditure to GDP ratio reduces the primary balance, and causes the increase of the debt-to-GDP ratio.

I then use the calibrated model to study the impact of the monetary backstop on the default threshold in state 'a' $\bar{b}^a$ and state 'n' $\bar{b}^n$. Figure 4 compares the default threshold in state 'a' of the calibrated model with an alternative model where there is no monetary backstop. We can see that the monetary backstop in the baseline model increases $\bar{b}^a$ by about 50 percent of GDP.

\(^{15}\text{The event study for the beginning of debt decrease episodes produces similar results. A brief analysis for debt decrease episodes can be found in Appendix B.3}\)
The monetary backstop has a larger impacts on the default threshold in state ‘n’. Figure 5 shows how the default threshold in state ‘n’ varies with the fiscal policy in the calibrated model (blue line) and in a model where there is no monetary backstop (red line). Here ε is fixed at 43 percent (which is the average in the data) and the default threshold is plotted as a function of tax rate τ.

First, we focus on the relationship between the default threshold and the tax rate τ. We can see that regardless of whether there is a monetary backstop, there are three different regimes in how the default threshold depends on τ. When the tax rate is low, the government has a fiscal deficit, and the projected path for government debt-to-GDP is increasing. As shown by Proposition 1, in this case, the default is caused by roll-over crisis. The default threshold is close to $\phi \tilde{b}^a$. As τ increases, the projected path for government debt-to-GDP ratio turns from an increasing path to a decreasing path. In this stage, the default threshold jumps from the threshold of roll-over crisis to the threshold of strategic default which is close to $\tilde{b}^a$. Finally, when τ is large, defaults only happen strategically. The default threshold stays close to $\tilde{b}^a$.

Second, we focus on the difference between the model with monetary backstop and the model with no monetary backstop. Compared with the case where there is no monetary backstop, the monetary backstop provides seigniorage revenue to the government. On the graph, this is equivalent with shifting the curve to the left. As a result, the largest impact of the monetary backstop happens in the middle part of the graph where τ is between 43 percent and 52 percent. The intuition is as follows. When τ is between 43 percent and 52 percent, the seigniorage from the monetary backstop is able to turn the path of government debt-to-GDP ratio from increasing into decreasing. Consistent with Proposition 2, once the central bank provides the monetary backstop, the government default threshold increases from the threshold of rollover crisis (which is close to $\phi \tilde{b}^a$) to the threshold of strategic default (which is close to $\tilde{b}^a$). The default threshold increases by more than 200 percent.

However, if τ is too small (below 43 percent) or too large (larger than 52 percent), the impact of the monetary backstop is small. This is because, when τ is too small, rollover crises happens regardless whether there is a monetary backstop or not. If τ is larger than 52 percent, the government has a large fiscal surplus. Defaults happen strategically even when there is no monetary backstop. In both of these cases, the monetary backstop does not change the nature of the default.

5 Debt Sustainability

In this section, I use the calibrated model to investigate how the size of the monetary backstop affects the sustainability of government debt by varying the calibration of $\pi^{High}$. I find that, as long as the seigniorage is large enough to cover the financing gap of the government, the monetary backstop significantly increases the default threshold of the government and reduces the frequency of default. In addition, I show that, when $\pi^{High}$ is sufficiently large,
the risk of inflation associated with the monetary backstop is low.

5.1 Debt Limit

Figure 6a shows how the default threshold in state ‘n’ changes with $\pi^{High}$, and compares it with the default threshold in the case when there is no monetary backstop. The state variables $(G, e, g, \tau, \pi)$ are chosen to match the empirical data for debt increase episodes.\(^\text{16}\)

The effect of the monetary backstop crucially depends on the value of $\pi^{High}$. When $\pi^{High}$ is below 8 percent, the impact of the monetary backstop on $\bar{b}^n$ is very small. This is because the seigniorage is too small to cover the financing gap of the government. The government can default due to rollover crisis even after receiving the seigniorage. When $\pi^{High}$ is larger than 8 percent, the seigniorage becomes large enough to cover the financing gap of the government. The monetary backstop increases the default threshold from the threshold of rollover crises to the threshold of strategic default. As a result, $\bar{b}^n$ increases from 40 percent of GDP to more than 200 percent of GDP. This result shows that, in order for the monetary backstop to have a large impact on the default threshold, the central bank needs to choose a sufficiently high inflation rate when providing the monetary backstop.

Figure 6b shows the default threshold in state ‘a’ in response to the variation of $\pi^{High}$.\(^\text{17}\) We can see that, compared with the case where there is no monetary backstop, the monetary backstop increases $\bar{b}^a$ by about 40 percent of GDP. As $\pi^{High}$ increases, $\bar{b}^a$ stays roughly constant, even though the present value of the seigniorage from the monetary backstop increases significantly. This is because, in state ‘a’, all government defaults are strategic defaults, and the cost of inflation offsets the positive effect of the seigniorage. As discussed in Section 3, the monetary backstop increases the default threshold in state ‘a’ by replacing the tax with seigniorage. However, the high inflation of the monetary backstop reduces the money demand and lowers the default threshold. As a result, even though the size of the seigniorage increases with $\pi^{High}$, the default threshold stays constant.

Lastly, given that the monetary backstop increases the debt limit of the government, I examine whether the higher debt limit leads to more government borrowing. Figure 7 plots the average government debt-to-GDP ratio as $\pi^{High}$ varies. We can see that when $\pi^{High}$ is below 15 percent, the government debt-to-GDP ratio increases with $\pi^{High}$. Having larger monetary backstop increases the debt limit of the government and gives the government more space to accumulate more debt and adjust later. When $\pi^{High}$ is larger than 15 percent, further increases of $\pi^{High}$ reduces the average government debt-to-GDP ratio. This is because when $\pi^{High}$ is large, the cost of inflation surpasses the cost of default. As concern for inflation

\(^\text{16}\)Based on the empirical data, in debt increase episodes: (i) the income growth is low $G = G^L$; (ii) the expenditure-to-GDP ratio increases faster than income $g = G^H$; (iii) the revenue-to-GDP ratio is about 42 percent of GDP $\tau = 42\%$; (iv) the primary expenditure-to-GDP ratio is about 44 percent of GDP $e = 44\%$.

\(^\text{17}\)The blue line in Figure 6b displays $\bar{b}^a(G^H, \pi^{Low})$ for the model with the monetary backstop. Default thresholds $\bar{b}^a$ for other states of $G_t$ and $\pi_t$ have similar relationships with the value of $\pi^{High}$. See Appendix D for details.
replaces concern for default, further increases in $\pi^{High}$ discourage government borrowing.

### 5.2 Frequency of Default

As the monetary backstop increases the default threshold of the government, it also reduces the frequency of government default. Figure 8 shows the frequency of government default as $\pi^{High}$ varies. We can see that when there is no monetary backstop, the government defaults once every 120 years on average. For the model with the monetary backstop, the frequency of default decreases significantly as $\pi^{High}$ increases. Specifically, for $\pi^{High}$ above 8 percent, the frequency of default reaches zero. There is no risk of government default in this case.

While the monetary backstop reduces the risk of government default, it does not simply replace defaults with episodes of high inflation. The red dashed line in Figure 8 plots the frequency of high inflation in response to the variation of $\pi^{High}$. We observe that, when $\pi^{High}$ is small, the monetary backstop is enacted often in equilibrium. The risk of high inflation is high. However, as $\pi^{High}$ increases, the frequency of high inflation drops significantly. Specifically, when $\pi^{High}$ is larger than 10 percent, as the risk of default reaches zero, the frequency of high inflation associated with the monetary backstop is much smaller than the frequency of default when there is no monetary backstop.

The result indicates that when the central bank provides a monetary backstop, it is better to choose a relatively high inflation target for the monetary backstop. If $\pi^{High}$ is too small, the seigniorage is not large enough to remove the risk of rollover crisis, the monetary backstop is triggered often in equilibrium, and the government can still default even after receiving the monetary backstop. In this case, both the risk of default and the risk of inflation are high. By contrast, when $\pi^{High}$ is sufficiently large, knowing that the central bank can provide the monetary backstop increases the default threshold by a large amount. This allows the government to rollover its debt and adjust later rather than defaulting or actually receiving the monetary backstop. The monetary backstop is rarely enacted in equilibrium. As a result, both the risk of default and the risk of inflation are low.

### 6 Welfare

The previous section showed that the monetary backstop increases the default threshold of the government and reduces the risk of default. At the same time, the monetary backstop may lead to costly high inflation and inefficient debt accumulation. In this section, I examine this trade-off by looking at how the monetary backstop affects household welfare.

To quantify the impact of the monetary backstop on household welfare, I perform two numerical exercises. In the first exercise, I examine the welfare benefit of unexpectedly introducing a monetary backstop. To do so, I first simulate an economy with no monetary backstop for $t$ periods. This initial period is chosen to be sufficiently long to ensure that, at
time \( t \), the state of the economy is random. I then introduce a monetary backstop at time \( t \). The welfare benefit of the monetary backstop is calculated as follows

\[
\Delta V = \frac{(V_t^\pi - V_t^f)}{V_t^f}
\]

(32)

Here \( V_t^\pi \) is household welfare after the introduction of monetary backstop at time \( t \) and \( V_t^f \) is household welfare if the monetary backstop is not introduced into the economy. Because the value function is homothetic in the trend income, by dividing the difference with \( V_t^f \), the welfare benefit is measured in equivalent changes of the trend income.\(^{18}\)

I run 10000 simulations. Figure 9 shows the average welfare gain of introducing the monetary backstop across simulations. We can see that for any \( \pi^{High} \) below 30 percent, introducing a monetary backstop increases household welfare. Under the calibration of the model, the welfare maximizing level of \( \pi^{High} \) is 15 percent. Introducing a monetary backstop of this size is equivalent with permanently increasing the trend income by about 4 percent. When \( \pi^{High} \) is too low or too high, the welfare gain is small. In the next paragraph, I discuss the intuition by decomposing the welfare impact of the monetary backstop.

The monetary backstop affects household welfare through three channels: (i) reducing the frequency of default; (ii) reducing the cost of taxation; (iii) reducing money demand. To analyze the contribution of each channel, I decompose the welfare benefit of the monetary backstop into these three effects.

\[
\Delta V = \Delta V_\delta + \Delta V_\tau + \Delta V_m
\]

(33)

Here \( \Delta V_\delta \) is the welfare benefit due to fewer defaults, \( \Delta V_\tau \) is the welfare benefit due to lower cost of taxation, and \( \Delta V_m \) is the welfare cost of high inflation. Figure 10 shows how each component varies with \( \pi^{High} \).

First, the black line in Figure 10 shows that the monetary backstop improves household welfare by reducing the risk of government default. When \( \pi^{High} \) is small, default happens frequently. As a result, the welfare gain of the monetary backstop is small.

Second, the blue line in Figure 10 shows that the monetary backstop reduces household welfare by reducing money demand. The magnitude of this effect depends on the frequency of high inflation as well as the cost of each high inflation episode. When \( \pi^{High} \) is small, the cost of each high inflation episode is small but the frequency of inflation is high. When \( \pi^{High} \) is large, each high inflation episode becomes costly, which reduces the welfare gain of the monetary backstop.

Lastly, the red line in Figure 10 shows the welfare gain from reducing the cost of taxation. This is due to (i) seigniorage directly replacing taxes and reducing the cost of taxation and (ii) the monetary backstop increasing the default threshold, which allows the government to

\(^{18}\)See Appendix A.4 for the derivation.
smooth the tax rate by accumulating debt. Under my calibration of the model, I found that the second effect dominates.

In the second exercise, I examine the welfare cost for an economy that suddenly loses a monetary backstop. An incident like this can occur either when the central bank retreats from its position of providing the monetary backstop for government debt or when the market suddenly realizes that its initial belief in the central bank’s commitment on providing the monetary backstop is false. This scenario is relevant for Euro countries, given that support from the ECB for government debt has been frequently questioned over the last decade.

In this exercise, I first simulate an economy with a monetary backstop for $t$ periods. Then I remove the monetary backstop at time $t$. The welfare cost of removing the monetary backstop in the model is computed as

$$\Delta V_t = \frac{(V^{\pi}_{t} - V^{d}_{t})}{V^{\pi}_{t}}$$

Here $V^{d}_{t}$ is household welfare after the monetary backstop is removed in period $t$, and $V^{\pi}_{t}$ is household welfare if the monetary backstop is not removed. The welfare cost of removing the monetary backstop is measured in equivalent changes of the trend income.

Figure 11 shows the average welfare cost of removing the monetary backstop over 10,000 simulations. Compared with Figure 9, the cost of removing the monetary backstop is larger than the benefit of introducing it. The largest welfare cost happens when $\pi^{High}$ is 15 percent, and is equivalent with permanently reducing the trend income by more than 5 percent.

Figure 12 decomposes the welfare cost of removing the monetary backstop into three effects: (i) Higher cost of taxation; (ii) More frequent default; (iii) Higher money demand. Compared with Figure 10, the cost of default has a larger welfare impact in removing the monetary backstop than it does in the introduction of the monetary backstop. This is because, in an economy which previously had a monetary backstop, the default threshold is high, and the government tends to accumulate a large amount of debt. A sudden removal of the monetary backstop abruptly reduces the government default threshold, exposing the government to large risks of default.

This intuition is confirmed by Figure 13, which shows the probability of default in the period of removing the monetary backstop. We can see that the government has a 40 percent chance of defaulting immediately after the removal of the monetary backstop. The large probability of default implies that removing the monetary backstop can be very costly to the economy. By contrast, in an economy where there is originally no monetary backstop, the government adopts a prudent fiscal policy, and the frequency of default is lower. Even though the introduction of a monetary backstop reduces the frequency of default, the welfare gain through this channel is not as large.

This result highlights the asymmetric nature of the monetary backstop. When agents in the economy do not expect the central bank to provide the monetary backstop, the welfare benefit for introducing it may be small. However, for an economy which previously had the
monetary backstop, indicating an intention to remove it can exposes the government to large risks of default. The asymmetric effect of the monetary backstop provides an explanation for the dynamics of the European Debt Crisis. Before the European Debt Crisis, investors implicitly expected the ECB to provide monetary backstop for Euro countries in time of crises. As a result, peripheral countries were able to accumulate large amounts of debt at relatively low cost. Later during the European Debt Crisis, investors started worrying that their initial belief of the monetary backstop might have not been correct, which leads to large increases in the yield of government bond of indebted peripheral countries. The panic subsided only after the ECB showed that it was committed to avoid government default by announcing the Outright Monetary Transactions programme (OMT).

7 Conclusion

This paper studies how a monetary backstop affects the sustainability of the government debt and household welfare. I present a model where the government adjusts fiscal policy infrequently and the central bank can raise the inflation rate for a sustained period of time in order to avoid government default. The model is able to explain the long-run dynamics of the government debt-to-GDP ratio of advanced economies. In particular, the model predicts that when the trend growth rate falls, the government is slow in adjusting primary expenditure, which continues along its original path and leads to a persistent increase in government debt. I test model predictions using the event study method and find consistent evidence. Finally, I calibrate the model and conduct quantitative analysis.

The results indicates that a monetary backstop can significantly increase the government’s debt limit. When the government has a fiscal deficit, uncertainty about fiscal adjustment exposes the government to risks of a rollover crises. The government can end up defaulting even with a relatively low debt-to-GDP ratio when there is no monetary backstop. The monetary backstop provides the government with seigniorage, which can remove the risk of a rollover crisis and increase the government’s debt limit.

The quantitative analysis highlights that the risk of inflation crucially depends on the size of the monetary backstop. When the monetary backstop is sufficiently large, knowing that the central bank can provide the monetary backstop lifts the debt limit of the government and allows the government to roll over its debt without actually enacting the monetary backstop. In equilibrium, the risk of inflation is low and the monetary backstop increases household welfare.

This paper suggests avenues for future research. Given that the inflation rate is determined one period in advance and the government issues short-term debt, the model addresses the impact of the seigniorage but avoids the possibility of using surprise inflation to reduce the real value of government debt. The model could be extended to include long-term nominal government debt, and thereby used to study the impact of surprise inflation on government debt dynamics.
Finally, the political economy of slow fiscal adjustment deserves further study. In particular, income inequality coincides with political gridlock in advanced economies (Ma, 2014; Voorheis and McCarty, 2015; Yared, 2018). One could extend the model in this paper to include income heterogeneity among households and study how income inequality mutually interacts with fiscal inertia and a monetary backstop.
Figure 2: The level of Output, Revenue, and Primary Expenditure around the beginning (set as time 0) of debt increase episode. The window covers from three years before the debt increase period to eight years into the debt increase period. All levels at year $-2$ are normalized to be 1.
Figure 3: Mean of selected macroeconomic variables for simulations (dashed line) and empirical data (solid line) at the beginning of a debt increase episodes. The debt increase episodes starts at time 0 which is marked with a black line. All variables are measured in percentage of GDP
Figure 4: The default threshold of government debt-to-GDP ratio in state ‘a’. From left to right, each blue bar indicates $\bar{b}^a$ of the calibrated model for different states of $(G_t, \pi_t)$. Each red bar shows the default threshold from an alternative model where all the calibrations are the same except there is no monetary backstop.

Figure 5: The default threshold of government debt-to-GDP ratio in state ‘n’ as a function of $\tau$. Here $\pi_t = \pi_{Low}$, $G_t = G_h$, and $e_t = 0.43$. 


Figure 6: The blue line shows the default threshold for different calibrations of $\pi^{High}$. The red line indicates the default threshold when there is no monetary backstop.
**Figure 7:** The average government debt-to-GDP ratio for an economy with a monetary backstop (blue line) and an economy with no monetary backstop (red line).

**Figure 8:** The frequency of defaults (blue solid line) vs the frequency of high inflation episodes (red line). The blue dashed line indicates the frequency of defaults when there is no monetary backstop.
Figure 9: The welfare gain from introducing the monetary backstop. The welfare gain is measured as fraction of permanent income.

Figure 10: Decomposition of the welfare gain from introducing a monetary backstop. Here $\Delta V_m$ is the welfare decrease due to a lower demand for money, $\Delta V_T$ is the welfare gain due to lower cost of tax, and $\Delta V_\delta$ is the welfare gain due to lower cost of default.
Figure 11: The welfare loss from removing the monetary backstop. The welfare loss is measured as fraction of permanent income.

Figure 12: Decomposition of the welfare loss from removing a monetary backstop. Here $\Delta V_\tau$ is the welfare cost due to higher cost of tax, $\Delta V_\delta$ is the welfare cost due to more frequent default, and $\Delta V_m$ is the welfare increase due to a higher demand for money.
Figure 13: Frequency of government default in the period immediately when the monetary backstop is removed
Table 1: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Difference</th>
<th>Median</th>
<th>Std.Dev</th>
<th>Max</th>
<th>Min</th>
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<tr>
<td>ΔDebt-to-GDP</td>
<td>1.394</td>
<td>5.703</td>
<td>0.920</td>
<td>4.498</td>
<td>20.864</td>
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<td>Primary Balance</td>
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<td>-0.466</td>
<td>3.787</td>
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<td>( r - g ) Component</td>
<td>-0.274</td>
<td>1.048</td>
<td>-0.379</td>
<td>2.467</td>
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<td>Stock Flow</td>
<td>1.554</td>
<td>0.519</td>
<td>0.757</td>
<td>3.900</td>
<td>32.637</td>
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<tr>
<td>Revenue</td>
<td>41.083</td>
<td>-0.457</td>
<td>40.445</td>
<td>8.351</td>
<td>58.279</td>
<td>26.225</td>
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<tr>
<td>Primary Expenditure</td>
<td>41.206</td>
<td>3.678</td>
<td>41.063</td>
<td>7.452</td>
<td>63.998</td>
<td>28.257</td>
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<tr>
<td>Real Growth (( g ))</td>
<td>2.176</td>
<td>-1.980</td>
<td>2.335</td>
<td>1.996</td>
<td>7.237</td>
<td>-8.269</td>
</tr>
<tr>
<td>Interest Rate(( r ))</td>
<td>0.543</td>
<td>-0.682</td>
<td>1.553</td>
<td>4.905</td>
<td>12.923</td>
<td>-21.129</td>
</tr>
</tbody>
</table>

Note: Table 1 reports the summary statistics for advanced economies during 1981-2017 using data from World Economic Outlook (October 2018). Countries include: Australia, Austria, Belgium, Canada, Chile, Denmark, Finland, France, Germany, Italy, Japan, Netherland, Norway, Portugal, Spain, UK, and USA. The second column reports the difference between the mean for debt increase episodes and the mean for debt decrease episodes. ΔDebt-to-GDP, Primary Balance, \( r - g \) Component (interest-growth differential), stock flow, revenue, and primary expenditure are measured as percent of nominal GDP. Real growth rate and Interest rate are reported in percentage.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td>High Growth Rate</td>
<td>( G^H )</td>
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</tr>
<tr>
<td>Low Growth Rate</td>
<td>( G^L )</td>
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<tr>
<td>Probability of High Growth</td>
<td>( p_h )</td>
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<tr>
<td>Probability of Low Growth</td>
<td>( p_l )</td>
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<td>Coefficient of Money in Utility</td>
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<tr>
<td>Coefficient of Expenditure in Utility</td>
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</tr>
<tr>
<td>Relative Risk Aversion (money)</td>
<td>( \rho )</td>
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</tr>
<tr>
<td>Relative Risk Aversion (expenditure)</td>
<td>( \psi )</td>
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<tr>
<td>Time Discount Factor</td>
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<td>Probability of Adjustment</td>
<td>( \Xi )</td>
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<tr>
<td>Cost of Taxation</td>
<td>( \eta )</td>
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<td>Loss of Income in Default</td>
<td>( \gamma )</td>
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<td>Low Inflation Target</td>
<td>( \pi^{\text{Low}} )</td>
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<td>High Inflation Target</td>
<td>( \pi^{\text{High}} )</td>
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<td>Probability of Inflation Restoration</td>
<td>( p_\pi )</td>
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<td>Lower Bound of Primary Expenditure</td>
<td>( e_l )</td>
<td>36%</td>
</tr>
<tr>
<td>Upper Bound of Primary Expenditure</td>
<td>( e_h )</td>
<td>48.6%</td>
</tr>
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<td></td>
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<td>Baseline Model</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>---------</td>
<td>----------------</td>
</tr>
<tr>
<td>Ave. Revenue</td>
<td>41.08</td>
<td>41.19</td>
</tr>
<tr>
<td>Ave. Primary Expenditure</td>
<td>41.21</td>
<td>40.78</td>
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<tr>
<td>Ave. Growth Rate</td>
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<tr>
<td>Probability of Debt Increase</td>
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<tr>
<td>Probability of Debt Decrease</td>
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<td>0.30</td>
</tr>
<tr>
<td>Diff. Primary Expenditure</td>
<td>3.68</td>
<td>3.81</td>
</tr>
<tr>
<td>Diff. $r - g$ Component</td>
<td>1.05</td>
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</tr>
<tr>
<td>Diff. Real Growth ($g$)</td>
<td>-1.98</td>
<td>-1.40</td>
</tr>
</tbody>
</table>

**Note:** Table 3 compares the empirical moments with simulation results from the calibrated model. The first column reports the moments of the empirical data. The second column shows the same moments calculated using the simulation result of the calibrated model.
References


Appendix A  Theory

A.1 Proof of Lemma 1

In state 'a', for any \((e, \tau, \pi)\), let \(v^a(b, e, \tau, \pi)\) be the value of the government when it chooses fiscal policy \((e, \tau)\). The value is the same if the government does not adjust fiscal policy and inherits \((e, \tau)\) from the previous period

\[
v^a(b, e, \tau, \pi) = v^n(b, e, \tau, \pi) \tag{A.1}
\]

In state 'a', the government optimizes fiscal policy

\[
v^a(b, \pi) \geq v^a(b, e, \tau, \pi) \quad \forall \ e, \tau, \pi \tag{A.2}
\]

Combining condition (A.1) and (A.2)

\[
v^n(b, e, \tau, \pi) \leq v^a(b, \pi) \quad \forall \ e, \tau, \pi \tag{A.3}
\]

Because in state 'a', defaults only happen strategically (See Section A.2 for proof), the default threshold \(\bar{b}^a(\pi)\) satisfies

\[
v^a(\bar{b}^a, \pi) = v^d(\pi) \tag{A.4}
\]

Combining (A.3) and (A.4),

\[
v^n(\bar{b}^a, e, \tau, \pi) \leq v^d(\pi) \quad \forall \ e, \tau, \pi \tag{A.5}
\]

In addition, both \(v^n\) and \(v^a\) are decreasing in \(b\), so \(\forall \ b > \bar{b}^a\)

\[
v^n(b, e, \tau, \pi) < v^n(\bar{b}^a, e, \tau, \pi) \leq v^d(\pi) \quad \forall \ e, \tau, \pi
\]

Hence the default threshold in state 'n' \(\bar{b}^n(e, \tau, \pi)\) satisfies

\[
\bar{b}^n(e, \tau, \pi) \leq \bar{b}^a(\pi) \quad \forall \ e, \tau, \pi \tag{A.5}
\]

A.2 Proof of Proposition 1

Let \(b\) denote government debt-to-GDP ratio of the current period, and \(b'\) be the debt ratio of next period.

First, I show that in state 'n', if government debt-to-GDP ratio is decreasing \(b' < b\), then there is no rollover crises.

In state 'n', if the government does not default in current period

\[
v^n(b, e, \tau, \pi) > v^d(\pi) \tag{A.6}
\]
Because $v^n(b, e, \tau, \pi)$ is decreasing in $b$, if $b' < b$, the government will not default strategically in next period either
\[
v^n(b', e, \tau, \pi) \geq v^n(b, e, \tau, \pi) = v^d(\pi) \tag{A.7}
\]
If the government can rollover its debt in current period, the government budget constraint implies that
\[
\frac{b}{G} + e - \tau - S \geq \max_{b'} \{q(b', e, \tau, \pi)b'\} \tag{A.8}
\]
Because $b' < b$, the government will be able to rollover its debt next period. In this case, the bond price is
\[
q(b', e, \tau, \pi) = \beta \tag{A.9}
\]
Combining condition (A.8) with equation (A.9),
\[
\frac{b}{G} + e - \tau - S \geq \max_{b'} \{\beta b'\} \tag{A.10}
\]
Given that $b' < b$, $\max_{b'} \{\beta b'\} = \beta b$.
\[
\frac{b}{G} + e - \tau - S \geq \beta b \tag{A.11}
\]
Here I assume the risk-free interest rate is larger than the income growth rate: $\frac{1}{\beta} > G$. We can solve $b$ as
\[
b \leq \frac{(\tau + s - e)G}{1 - \beta_G} \tag{A.12}
\]
As long as $b$ satisfies condition (A.12), and $b' < b$, the government will not default due to rollover crisis.

Note when $b > \hat{b}$, government debt-to-GDP ratio is increasing which contradicts the assumption that $b' < b$. Hence, there is no rollover crises when $b' < b$.

Second, I show that rollover crises cannot happen in state 'a'. This is because, in state 'a', the government can choose fiscal policy $(e, \tau)$ to put government debt-to-GDP ratio on a decreasing path. As I show in previous paragraph, the rollover crisis will not happen in this case.

Third, I show that in state 'n', if government debt-to-GDP ratio is increasing $b' > b$, a rollover crisis can happen, and the threshold for the rollover crisis $\bar{b}^n \approx \phi \hat{b}^a$.

In state 'n', the rollover crisis happens when
\[
\frac{b}{G} + e - \tau - S > \max_{b'} \{q(b', e, \tau, \pi)b'\} \tag{A.13}
\]
Depends on whether the government defaults next period, the bond price

\[
q(b', e, \tau, \pi) = \begin{cases} 
\beta & \text{if } b' \leq \bar{b}^n(e, \tau, \pi) \\
\phi \beta & \text{if } \bar{b}^n(e, \tau, \pi) < b' \leq \bar{b}^a(\pi) \\
0 & \text{if } b' > \bar{b}^a(\pi)
\end{cases}
\]  (A.14)

As a result

\[
\max_{b'} \{q(b', e, \tau, \pi) b'\} = \max \{\beta \bar{b}^n(e, \tau, \pi), \beta \phi \bar{b}^a\} 
\]  (A.15)

When government debt-to-GDP ratio reaches the default threshold in current period \( b = \bar{b}^n(e, \tau, \pi) \). Because \( b' > b > \bar{b}^n(e, \tau, \pi) \), the government can roll over its debt only if it adjusts fiscal policy in next period:

\[
\frac{\bar{b}^n}{G} + e - \tau - S = \beta \phi \bar{b}^a 
\]  (A.16)

Solve \( \bar{b}^n(e, \tau, \pi) \) as

\[
\bar{b}^n = \beta G \phi \bar{b}^a + (\tau + S - e) G 
\]  (A.17)

Here \( \beta G \) is close to 1, and the primary balance-to GDP ratio \( \tau + S - e \) is much smaller than \( \bar{b}^a \)

\[
\bar{b}^a \approx \phi \bar{b}^a 
\]  (A.18)

A.3 Proof of Proposition 2

Given that the default threshold \( \bar{b}^n \) is determined by a rollover crisis,

\[
\frac{\bar{b}^n}{G} + e_t - \tau_t = \beta \phi \bar{b}^a 
\]  (A.19)

we can solve \( \bar{b}^n(e_t, \tau_t, \pi^{Low}) \)

\[
\bar{b}^n(e_t, \tau_t, \pi^{Low}) = G \left[ \beta \phi \bar{b}^a(\pi^{Low}) + \tau_t - e_t \right] 
\]  (A.20)

If the seigniorage \( s_t \) satisfies

\[
s_t \geq \frac{1 - \beta G}{G} + e_t - \tau_t 
\]  (A.21)

We can rearrange (A.21)

\[
\beta b_{t-1} \geq \frac{b_{t-1}}{G} + e_t - \tau_t = \beta b_t \\
\Rightarrow b_{t-1} \geq b_t 
\]  (A.22)
The government debt-to-GDP ratio is on a decreasing path. Proposition 1 ensures that the default threshold \( \bar{b}^n \) is determined by strategic default where

\[
v^n(\bar{b}^n, \cdot) = v^d(\cdot) \tag{A.23}
\]

In this case, \( \bar{b}^n \) is close to \( \bar{b}^a \). If the seigniorage does not satisfy condition (A.21), the government debt-to-GDP ratio is on an increasing path \( b_{t-1} < b_t \). The threshold for rollover crisis \( \bar{b}^n \) is

\[
\bar{b}^n(e_t, \tau_t, \pi^{High}) = G \left[ \beta \phi \bar{b}^a(\pi^{High}) + \tau_t + s_t - e_t \right] \tag{A.24}
\]

Combining equation (A.20) and (A.24), we can solve the increase in the default threshold due to monetary backstop as

\[
\frac{\bar{b}^n(e_t, \tau_t, \pi^{High}) - \bar{b}^n(e_t, \tau_t, \pi^{Low})}{\Delta \bar{b}^n} = G \beta \phi \left[ \frac{\bar{b}^a(\pi^{High}) - \bar{b}^a(\pi^{Low})}{\Delta \bar{b}^a} \right] + G s_t \tag{A.25}
\]

\[\Rightarrow \Delta \bar{b}^n = G \beta \phi \Delta \bar{b}^a + G s_t \tag{A.26}\]

The first part on the right hand side (RHS) of equation (A.25) indicates that the monetary backstop increases the threshold of rollover crisis by increasing the debt limit in state 'a'. The second part on the RHS indicates that the monetary backstop also directly relaxes the government budget constraint producing a seigniorage in current period. The seigniorage \( s_t \) tends to be much smaller than the existing debt stock \( \bar{b} \). When \( \phi \) is small, the first part on the RHS is also small. As a result, when the seigniorage is not large enough to put the government debt on a downward path, the impact of the monetary backstop on the default threshold is small.

### A.4 Derivation for equation (32) and (34)

**Equation (32)**

We can show that equation (32) measures the benefit of introducing a monetary backstop as equivalent changes in trend income.

Because the value function can be normalized by the level of trend income,

\[
V_t^\tau = v_t^\tau Y_t \tag{A.27}
\]

I assume that, after introducing the monetary backstop, the value of the government is equal to the value in the case when there is no monetary backstop but the trend income increases to \( \hat{Y} \)

\[
V_t^\pi = v_t^\hat{\pi} \hat{Y} \tag{A.28}
\]
The equation (32) can be written as

\[ \Delta V = \frac{V_t^\pi - v_t^\pi Y_t}{V_t^\pi} = \frac{v_t^\pi \hat{Y}_t - v_t^\pi Y_t}{v_t^\pi Y_t} = \frac{\hat{Y}_t - Y_t}{Y_t} \]  

(A.29)

Hence, equation (32) measures the welfare benefit of introducing the monetary backstop as an equivalent increase in the trend income.

**Equation (34)**

Similarly, we can show that equation (34) measures the welfare cost of removing the monetary backstop in fraction changes in trend income. Because the value function is homothetic in \(Y\)

\[ V_t^\pi = v_t^\pi Y_t \]  

(A.30)

Here \(v_t^\pi\) is the value of the household normalized by the level of trend income \(Y_t\).

I assume that the value of the government after removing the monetary backstop is equal to the value in the case where the central bank continues providing the monetary backstop, but the trend income is reduced to \(\tilde{Y}_t\)

\[ V_t^{\#} = v_t^\pi \tilde{Y}_t \]  

(A.31)

We can rewrite equation (34) as

\[ \Delta V = \frac{V_t^\pi - V_t^{\#}}{V_t^\pi} = \frac{v_t^\pi Y_t - v_t^\pi \tilde{Y}_t}{v_t^\pi Y_t} = \frac{Y_t - \tilde{Y}_t}{Y_t} \]  

(A.32)

Hence, equation (34) measures the welfare cost of removing the monetary backstop as an equivalent reduction in the trend income.
Appendix B  Empirical

B.1 Data

I obtain data on the government budget for advanced economies from the IMF World Economic Outlook (WEO) database. The definition of advanced economies follows the WEO categorization. To capture the long-run dynamics of government debt, I select countries that have data since 1992. 19 For each country, the data set includes yearly data on general government budget, such as debt-to-GDP ratio, revenue, and primary expenditure, and data on macroeconomic variables such as GDP, inflation, and real growth rates.

B.2 Decomposition of The Change of Debt-to-GDP

This appendix shows how to decompose the change of debt-to-GDP ratio into the contribution of primary balance, interest-growth differential, and stock flow residuals.

The change of debt-to-GDP ratio:

\[
\Delta d_t = d_t - d_{t-1} = \frac{D_t}{Y_t} - \frac{D_{t-1}}{Y_{t-1}}
\]  

(B.1)

Here \( D_t \) is the nominal amount of government debt outstanding at the end of time \( t \). \( Y_t \) is the nominal GDP of period \( t \). Let \( TR_t \) be the amount of total revenue of the government, \( TE_t \) be the amount of total expenditure. The nominal amount of government debt evolves according to:

\[
D_t = D_{t-1} - (TR_t - TE_t) + SF_t
\]  

(B.2)

\( SF_t \) is the difference between the annual change in the amount of debt outstanding and the budget deficit, here we refer to it as the nominal amount of the stock flow. The stock flow is usually the result of net acquisition of financial assets, debt adjustment effects, and statistical discrepancies.

The total government expenditure \( TE_t \) is the sum of the primary expenditure \( PE_t \) and the interest payment on the existing debt. Let \( i_t \) be the average nominal interest rate on the government bond.: \( TE_t = PE_t + i_t \times D_{t-1} \). We can rewrite equation (B.2) as:

\[
D_t = D_{t-1} - (TR_t - (PE_t + i_t \times D_{t-1})) + SF_t
\]  

\[
= (1 + i_t)D_{t-1} - (TR_t - PE_t) + SF_t
\]  

\[
= (1 + i_t)D_{t-1} - PB_t + SF_t
\]  

(B.3)

19 These countries are: Australia, Austria, Belgium, Canada, Chile, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Portugal, Spain, UK, and USA. See Appendix A for the detail of data.
Here $PB_t = TR_t - PE_t$ is the amount of the primary balance.

Let $g^n_t$ be the nominal growth rate: $Y_t = (1 + g^n_t) Y_{t-1}$. We can substitute $D_t$ with equation (B.3):

$$
\Delta d_t = \left(1 + i_t\right) \frac{D_{t-1} - PB_t + SF_t}{Y_t} Y_{t-1} - \frac{D_{t-1}}{Y_{t-1}} Y_{t-1}
$$

$$
= \left(1 + i_t\right) \frac{(1 + g^n_t) Y_{t-1}}{1 + g^n_t} + \frac{(-PB_t + SF_t)}{Y_t} Y_{t-1} - \frac{D_{t-1}}{Y_{t-1}} Y_{t-1}
$$

$$
= \left[\left(1 + i_t\right) \frac{(1 + g^n_t)}{(1 + g^n_t) - 1}\right] d_{t-1} - pb_t + sf_t
$$

The first term on the right hand side of equation (B.3) represents the contribution of interest-growth differential to the change of debt-to-GDP. The interest rate and the growth rate here are both measured in nominal terms. In the $r - g$ part that I present in the main text, both the interest rate and the growth rate are in real terms.

To convert nominal value into real value, let $Z_t$ be real GDP at time $t$, and $P_t$ be the price index of all output at time $t$, the growth rate of nominal GDP is:

$$
1 + g^n_t = \frac{Y_t}{Y_{t-1}} = \frac{P_t Z_t}{P_{t-1} Z_{t-1}} = (1 + \pi_t)(1 + g_t)
$$

Here $g_t$ is the growth rate of real GDP. $\pi_t$ is the inflation rate calculated with the price of all output.\(^{20}\)

Define the real interest rate $r_t$ as:

$$
1 + r_t = \frac{1 + i_t}{1 + \pi_t}
$$

we prove the decomposition in the main text:

$$
d_t = \left[\frac{(1 + r_t)}{(1 + g_t)} - 1\right] d_{t-1} - pb_t + sf_t
$$

### B.3 Event Study for Debt Decrease Episodes

Figure 14 and 15 show the result for debt decrease episodes.

\(^{20}\) $P_t$ is calculated by dividing the GDP deflator with 100. \(\frac{1 + \pi_t}{\text{GDP deflator of time } t} = \text{GDP deflator of time } t + 1\)
At the beginning of debt decrease episodes, the real growth rate rises substantially, reduces the \( r - g \) component. In addition, when the growth rate increases, the primary expenditure continue growing with the original low growth rate while the output and the revenue grow faster. The inertia of the primary expenditure causes the primary balance-to-GDP to increase, eventually reduces the \( \Delta \text{Debt-to-GDP} \).

**Figure 14:** The level of Output, Revenue, and Primary Expenditure around the beginning (set as time 0) of debt decrease period. The window covers from three years before the debt increase period to eight years into the debt increase period. All levels at year \(-2\) are normalized to be 1.

### B.4 Additional Results

Figure 16 shows the dynamics of government debt-to-GDP ratio for additional countries in the data set.
Figure 15: Mean of selected macroeconomic variables for simulations (dashed line) and empirical data (solid line) at the beginning of a debt decrease episodes. The debt decrease episodes starts at time 0 which is marked with a black line. All variables are measured in percentage of GDP.
Figure 16: Debt-to-GDP ratios for selected advanced economies. Debt increase episodes are marked as red area, debt decrease episodes are marked as blue area.