Real exchange rate undervaluation and growth with credit constraints

Luigi Durand*

14th October 2019

Abstract

This paper argues that depreciating the real exchange rate can be welfare improving for an economy, through the impact that an undervaluation has on the tradable sector’s amount of available internal resources; specifically, a depreciated real exchange rate lowers the wage rate, which relaxes the credit constraints of firms. Firms can then devote more funds to investment in physical capital. Higher investment leads to higher future wages, which can benefit future generations of agents. I show that foreign reserves accumulation represents a viable policy instrument to reach this outcome, and that China’s significant increase in foreign reserves could be interpreted as also stemming from a willingness to maintain the real exchange rate undervalued. On a more general level, we argue that real exchange rate undervaluation can be beneficial in the long run for example when the economy is passing through phases of high productivity growth, as was the case for China.

Economists have long debated the association between economic growth and exchange rate undervaluation. For instance, real exchange rate distortions are at the center-stage of Dollar (1992) analysis on the relationship between the rate of devaluation and the degree of tradable

*Johns Hopkins University. I thank my advisor Olivier Jeanne for his guidance and support. I also thank Greg Duffee, Andrew Levin, Alessandro Rebucci and Jonathan Wright, together with seminar participants at the Johns Hopkins University for helpful comments. All errors are my own. WORK IN PROGRESS. PLEASE DO NOT CIRCULATE OR CITE WITHOUT PERMISSION.
sector development and they appear as a significant explanatory variable in Fischer (1993), among other key macroeconomic policy variables. More recently, the debate has centered on the aggregate welfare impact of real exchange rate policies with some studies going one step further by addressing the distributional impact of these policies (see Agénor (2002)). There are however few studies that combine the study of how the real exchange rate can affect investments, a key component of long term growth, and how, in turn, this can influence the welfare of current and future generations.

In this paper I therefore propose to analyze a channel through which real exchange rate management can affect the amount of investment undertaken by an economy experiencing a period of sustained growth, within a general equilibrium framework. In order to do so, I develop an Overlapping Generation Model (OLG) of a small open economy framework in which low levels of financial development limit the domestic firms' ability to invest in capital. Within this setup, I show that a more depreciated real exchange rate can help relax the credit constraints of firms operating in the tradable sector, which can then devote more resources to investments. I then go on to argue that even though this type of outcome entails some welfare costs in terms of depressed current levels of aggregate consumption, it can nevertheless be highly beneficial in the long run, especially so when looking at future generations of households.

This mechanism arises because an undervalued real exchange rate is tantamount to a compression of the real wage rate of workers, which unambiguously expands the tradable sector firm’s profits. When the firm is originally financially constrained, higher profits then lead to higher investments. When capital and labor are complementary factors in the production of consumption goods, a higher level of investment undertaken in the present translates in turn into a higher stock of capital that is available for production in the future and therefore into an increase in the future marginal product of labor, which directly benefits future cohorts of workers.

By embedding the framework within an OLG setup with financial exclusion of the younger agents I contribute to the literature on the welfare effects of real exchange rate under-
valuation by emphasizing the association between the level of the real exchange rate and intergenerational inequality. In the model, exchange rate fluctuations are transmitted into equity prices and therefore can generate significant wealth effects.

Exchange rate management represents a second best solution: In a first best world with a full set of instruments, the first order distortion in the tradable sector could be directly addressed by introducing a sectoral tax and subsidy scheme, with the objective of subsidizing the tradable sector firm’s investment. However, the general consensus is that this type of instruments is in practice hard to implement, hence the role of exchange rate management as a plausible, second-best, alternative. Specifically, the main concerns that are usually associated with a tax and subsidy instrument range from targeting problems to agency problems through rent extraction\(^1\). Furthermore, it is important to highlight that in the case of emerging market economies the introduction of sectoral subsidies would clash with WTO rules on “trade-distorting interventions”.

Even though the main arguments could in principle be tested across a broad sample of countries, we decide to focus our calibration and simulations on China, which we consider as an ideal laboratory for the theory. There are key reasons behind this choice: first, China was characterized by a closed capital account together with a resistance to exchange rate appreciation; second, despite being characterized by higher returns to real investment than most countries (see Bai and Qian (2006)) China has been exporting significant amounts of (financial) capital during the last decades. As a matter of fact, the Chinese economy has been buying foreign assets amounting to a value close to 4 trillion USD by the end of the first quarter in 2014.

In addition, and on a practical side, there are some properties of the Chinese economy which seem to fit particularly well with our theory; a first stylized fact is the high negative correlation between investment and the PPP exchange rate as shown in Figure 1. Specifically, the figure shows the series for the growth rate of the PPP exchange rate of China (an increase

\(^1\)For example, and as also highlighted by Korinek and Serven (2016), there have been several fraudulent schemes that tried to profit from the European Union’s Common Agricultural Policy.
corresponds to an appreciation) and the growth rate of investment in capital construction in
major tradable sectors. This evidence suggests that a more appreciated domestic currency is
associated with a decline in the growth rate of investment. Notice that while our theory is
well suited to analyze the relationship between changes in the exchange rate and investment
in the tradable sector, this is not the case when we turn to the non-tradable sector. In
general it is harder to unambiguously identify the qualitative impact of a real exchange rate
depreciation on investment undertaken by the non-tradable sector. For example, while an
decrease in the relative price of non-tradable is contractionary, non-tradable sector’s firms
may have relatively more monopoly power, which would make less immune to a change in the
relative price of their products; also, non-tradable output is often used as an intermediate
input in the tradable sector (Di Giovanni and Levchenko (2010)).

Figure 1: Exchange Rate and Investments in China

Notes: Continuous line: annual growth rates in investment; dotted line: PPP exchange rate. Investment in
capital construction data is from NBS; Exchange rate data is from Penn World tables.
Notes: Continuous line: growth rates in the real wage; dotted line: growth rate in the real exchange rate. Both the RER and wage rate series show the growth rates of 5 years moving averages.

The rest of the paper is organized as follows. In section 1 I present a review of influential works on the topics of real exchange rate undervaluation and welfare together with a review of works most closely related to this paper. In section 2 I then describe the model, while in section 3 I present an analysis. Finally, In section 5 I present several simulations and I study the welfare implications of foreign reserves accumulation together with those from other alternative policies.

1 Literature Review

The literature on the real exchange rate and growth nexus can be described as broadly divided between studies focused on overvaluation and undervaluation. In this section we briefly review the former category before delving into a more in-depth analysis of the latter. Starting with the former, the idea that overvaluation hampers macroeconomic stability is easily backed out from classic theoretical frameworks: because a balance of payment crisis mirrors the expansion of a current account deficit, then a simple monetary model á la Dornbusch and Fischer (1980) is all it takes to link exchange rate appreciations to current account deficits and eventually to crises. On the empirical front, several authors have emphasized a mercantilist view of exchange rate overvaluation; Rajan and Subramanian (2011) for example discusses the “dutch-disease” of foreign aid: inflows of foreign aids, by appreciating the real exchange rate, lower the competitiveness of the tradable sector; following a similar argument Prasad et al. (2007) show that “uphill” flows of capital from developing to advanced economies have a positive impact on the growth rate of the former group of economies, contrary to what the prevailing paradigm would seem to suggest. Also, Johnson et al. (2007) find a significant negative association between the degree of overvaluation, exports and the growth rate in a sample of african countries. When turning to the second category, the evidence and theoretical support of the nexus are more uncertain. Economists are finding increasing empirical support for the idea that an undervalued exchange rate can contribute
to the convergence of developing economies with advanced economies' level of income. For example, Rodrik (2008) highlights the beneficial effect that real undervaluation can have in correcting market failures and institutional weaknesses of the sort that are especially pervasive in developing economies. In this respect, there is an underlying assumption that the tradable sector suffers from some sort of inefficiency whose effects can, in the absence of a first best mechanism, be counterbalanced by manipulating the relative price of tradable goods. Correcting these effects can in turn propel developing countries toward a frontier close to that of advanced economies.

A parallel stream of literature places the study of the relationship between real exchange rate dynamics and growth into the broader question of the role of financial globalization and foreign finance. According to Rodrik and Subramanian (2009) the commonly held presumption that developing economies are savings-constrained and hence benefit from inflow of foreign capital does not stand empirical scrutiny; in particular, and according to the authors, the stylized fact of no correlation between financial globalization and long-run economic growth can be rationalized on the grounds that most developing economies are investment constrained, so that foreign capital flows are mainly directed toward consumption. In Rodrik and Subramanian (2009) this is welfare reducing since higher consumption will raise the relative price of non-tradable goods, or appreciate the real exchange rate, which will lower the relative profitability of the tradable sector. This stream of literature favors the view that it is only through a persistent undervaluation of the real exchange rate that the economy can bring about sustained long term growth. For instance suppose, following Rodrik and Subramanian (2009), that the tradable sector is disproportionately plagued by institutional failures that are hard to overcome in the medium run; then it is only by committing to a step devaluation that the authority can level out the playing field by raising the relative price of tradable goods and hence incentivizing tradable firms' production.

Another stream of literature, while recognizing the welfare improving effects that a real undervaluation can engender for a developing economy, argues that it is not necessary to assume a special role for the tradable sector nor to assume a persistent depreciation in order
to appreciate the welfare benefits from undervaluation. After all, the argument posited by Rodrik (2008), which is that tradable sector firms are disproportionately hurt by institutional weaknesses is debatable\textsuperscript{2}. With regards to the persistence aspect of undervaluations, Berg et al. (2012) show that even a temporary exchange rate depreciation can lengthen the duration of sustained growth in developing economies. Levy-Yeyati et al. (2013) emphasizes an "aggregate savings" channel in which a more depreciated exchange rate increases the economy’s “savings” and ultimately investments. In this sense depreciating the real exchange rates entails a redistribution of income towards financially constrained firms, and the expenses of workers. While this has generally been seen as contractionary, (see for example, Diaz-Alejandro (1965)), Levy-Yeyati et al. (2013) argument is that firms’ investment will increase output and through an aggregate demand channel, ultimately welfare. There is subtle although critical difference between Rodrik and Subramanian (2009) and Levy-Yeyati et al. (2013); while in both stories undervaluation boost investments, in the former case this happens through the effects that a real exchange rate depreciation has on the relative profitability of the tradable sector (which is assumed to be investment constrained) while in the latter case this happens because a real exchange depreciation boosts the capitalists’ savings that are in turn channeled toward profitable investment projects.

Within this broad literature, my paper is closely related to Dao et al. (2017), who espouse the view that a real exchange rate depreciations can benefit a constrained corporate sector through an “internal finance” channel. Specifically, real depreciation boosts the domestic value of internal resources available to the firm through a fall in the domestic wage rate and as such, reduces the corporate sector reliance on debt, for a given level of planned investment. Interestingly, Dao et al. (2017) find weak evidence for the existence of an aggregate savings channel in a large cross-sample of economies. In my work, and differently from Dao et al. (2017), who build a stylized two period model and focus on a cross country empirical evaluation of the channel, I develop a micro-founded dynamic (partial) equilibrium

\textsuperscript{2}Even though it is plausible to argue that a manufacturing firm has a higher degree of reliance on the contractual environment when compared to, say a barber, it is hard to think why this would still hold true when comparing that same manufacturing firm with non-tradable firms operating in sectors such as communications, utilities, construction.
model that can be calibrated and used to derive welfare carry out a welfare analysis of the mechanism at play.

Because real undervaluation is the outcome of a policy choice, the literature has also studied the role of capital account policies and specifically of foreign reserves accumulation and capital control. We can distinguish between a first set of papers which emphasized how capital account policies can reduce inefficiency losses due to externalities, Benigno and Fornaro (2012), Korinek and Serven (2016), Michaud and Rothert (2014), and a second set of papers which instead emphasized the role of financial frictions, Bacchetta et al. (2014), Bacchetta et al. (2013), Cheng (2015), Song et al. (2011), Song et al. (2014), Itskhoki and Moll (2014).

In Benigno and Fornaro (2012) productivity growth is increased by foreign knowledge, which accrues through imported inputs in the tradable sector; because depreciating the real exchange rate through foreign reserves accumulation shifts production toward the tradable sector which in turn increases the consumption of imported inputs used for production, foreign reserves accumulation raises welfare. In Korinek and Serven (2016) the economy is characterized by learning-by-investing externalities so that the level of productivity is proportional to the amount of capital accumulated. When the real exchange rate is depreciated a Stolper-Samuelson effect raises the private returns on the factor that is used more intensively, which is capital in the tradable sector, hence allowing for an alignment between private and social returns to capital, and higher welfare. Finally, Michaud and Rothert (2014) introduces capital control policies (as a borrowing constraint on households) in an environment characterized by learning-by-doing externalities and argue that repressing consumption can improve welfare since households will then find optimal to increase their labor supply. However the welfare benefits of capital controls is not monotonic and will in particular depend on the size of the externality.

In the second set of papers, the focus of research is mostly on financial frictions; for example Bacchetta et al. (2014) and Bacchetta et al. (2013) study economies where households alternate between periods of high and low income: when households experience a low income
phase a credit constraint limits their intertemporal consumption smoothing. This in turn generates excess savings during the high income periods. In a growing economy a social planner improves the savings opportunities by providing additional assets; moreover capital controls are optimal as they increase the return on savings, which benefit low income (savers) households. Cheng (2015) develops similar arguments by introducing credit constrained firms. My paper is related to Song et al. (2014), Itskoki and Moll (2014). In both papers real wages compression promotes capital accumulation and foster growth. In Itskoki and Moll (2014) the economy the authors argue argued that a labor tax that lowers the real wage of workers and that subsidizes entrepreneurs is pareto efficient when the economy is under capitalized. Increased profits leads to quicker wealth accumulation by credit constrained entrepreneurs, which in turn benefits workers too in the long run. Song et al. (2014) show that exchange rate policies generate an intertemporal substitution of consumption, which result in a trade surplus and an increase in the entrepreneurs’ savings, which eventually lead to capital accumulation.

While Song et al. (2014) distinguish between domestically privately owned firms and state owned firms, they do not emphasize the distinction between tradable and non-tradable goods which instead is at the core of this paper. As it is well known in the literature (see for example Uribe and Schmitt-Grohé (2016)) a model which does not account for non-tradable goods overstates the relationship between output, consumption and investments with terms of trade shocks. While this limitation may be minor when the emphasis is on the qualitative characterization of exchange rate policies, it may become more relevant when instead the objective is too also provide a quantitative evaluation of the channels.

2 Model

The deterministic model features infinite discrete periods $t = \{0, 1, 2, \ldots\}$ and two final consumption goods (tradables and nontradables).

The model includes the following agents: overlapping generations of domestic private agents
living for three periods, foreign lenders, price taking nontradable final goods producing firms and tradable final goods producing firms. Also, there is a consolidated government composed of a central bank and a treasury. The consolidated government has fiscal capacity and it sets an exchange rate policy.

I introduce some form of market segmentation: in particular I assume that the amount of foreign assets that the economy can hold is limited by financial frictions based on moral hazard considerations, which I further discuss below.

Finally, tradable goods firms in the model cannot borrow funds domestically (or equivalently do not issue new equity); this assumption can be interpreted as reflecting an underdeveloped domestic corporate bond/equity market.

The next sections describe in detail the problem faced by each agent of the model and characterize the competitive equilibrium that arises in the economy.

2.1 Households

The economy features overlapping households/consumers, indexed by \( i \). Each generation of agents lasts for three periods. An agent is born young at time \( t \), becomes middle aged at time \( t + 1 \) and old at time \( t + 2 \). In the following paragraph I describe the problem of household’s \( i \), but since each household faces the exactly same problem, and in order to streamline the presentation, I keep that notation implicit. I will maintain this approach also for the other agents described in the paper whenever indexation is not directly relevant to the description of the problem. I denote \( L_{\eta,t} \) the size of generation \( \eta \) and \( e_t \) the exogenous relative productivity of young consumers (\( e_t \leq 1 \)). Notice that \( L_{m,t} = L_{y,t-1} \), \( L_{m,t} = L_{o,t+1} \) and if each consecutive cohort grows a rate \( \Xi_{L,t} \) then \( L_{y,t} = (1 + \Xi_{L,t})L_{y,t-1} \).

A consumer born in period \( t \) earns a competitive wage \( w_{y,t} \) when young and \( w_{m,t+1} \) when middle age by supplying a total of 1 units of labor to the tradable and non tradable goods firms that populate the economy; a consumer born in time \( t \) of generation \( \eta = o \) does not work and consume all her available resources. Since labor is fully mobile between tradable and non-tradable sectors there is only one wage which clears both markets, \( w_t \). Her utility
derives from consumption of a final composite good \( c_{\eta,t} \) made of tradable goods \( (c^T_{\eta,t}) \) and nontradable goods \( (c^N_{\eta,t}) \).

The lifetime utility of a consumer born in period \( t \) is then given as a constant-relative-risk-aversion (CRRA) function of a composite good \( (c_{\eta,t}) \):

\[
U_t = v(c_{y,t}) + \beta v(c_{m,t+1}) + \beta^2 v(c_{o,t+2})
\]

\[
\text{with, } v(c_{\eta,t}) = \frac{c_{\eta,t}^{1-\sigma}}{1-\sigma}
\]

where \( 0 < \beta < 1 \) is the subjective discount factor and the parameter \( \sigma \) is the inverse of the intertemporal elasticity of substitution.

The composite consumption good is an Armington type Constant Elasticity of Substitution (CES) aggregator with elasticity of substitution \( 1/(1+\varphi) \) between tradables and nontradable consumption goods given by:

\[
c_{\eta,t} = \left[ \omega(c^T_{\eta,t})^{-\varphi} + (1-\omega)(c^N_{\eta,t})^{-\varphi} \right]^{-\frac{1}{\varphi}}, \varphi > -1, \omega \in (0, 1)
\]

where \( \varphi \) is the intratemporal elasticity of substitution between tradable and nontradable goods.

**Budget constraints:** The budget constraints for a consumer born in period \( t \), expressed in terms of tradable goods, are then given by:

\[
c^T_{y,t} + p_t c^N_{y,t} \leq w_{y,t} + \pi_{y,t}
\]

when young,

\[
c^T_{m,t+1} + p_{t+1} c^N_{m,t+1} + \frac{g_{m,t+2}}{1 + i_{t+1}} + s^T_{m,t+1} Q^T_{t+1} + s^N_{m,t+1} Q^N_{t+1} \leq w_{m,t+1} + s^T_{m,t+1} D^T_{t+1} + s^N_{m,t+1} D^N_{t+1} + \pi_{m,t+1}
\]

\[11\]
when middle-aged, and

\[ c_{o,t+2}^T + p_{t+2}c_{o,t+2}^N \leq p_{t+2}^c g_{m,t+2} + \pi_{o,t+2} + (1 - \xi_{t+2})s_{m,t+1}^T Q_{t+2}^T + \xi_{t+2}s_{m,t+1}^T Q_{t+2}^T + s_{m,t+1}^N Q_{t+2}^N \]  

when old.

where \( p_t \) is the relative price of nontradable goods in units of tradable goods, while the \( p^c_t \) is the relative price of the composite consumption good in units of tradable goods. As usually done in the literature (Obstfeld and Rogoff (1996)), I define an aggregate price index that corresponds to a minimum expenditure \( c_{\eta,t}^T + p_{t}c_{\eta,t}^N \) such that \( c_{\eta,t} = 1 \). Then given our specification we have that the consumption price index is given by:

\[ p^c_t = \left[ \omega \frac{1}{1+\varphi} + (1 - \omega) \frac{1}{1+\varphi} (p_t) \frac{1}{1+\varphi} \right]^{\frac{1}{1+\varphi}} \]  

(7)

The above expression defines a consumer price based measure of the real exchange rate (the foreign consumption price is assumed constant and normalized to 1). Also notice that this relative consumer based price is a monotonic increasing function of the relative price of nontradables.

I denote \( g_{\eta,t+1} \) the holdings of domestic assets at the end of period \( t \) of a consumer that belongs to generation \( \eta \). Notice that domestic assets are denominated in units of domestic consumption. The variable, \( \pi_{\eta,t} \) stands for the transfers that the consumer of generation \( \eta \) receives from the consolidated government, which can be positive, negative or equal to zero. The variable \( s_{\eta,t}^T, s_{\eta,t}^N \) denote the amount of shares owned by generation \( \eta \) and \( Q_{t}^T, Q_{t}^N \) is the domestic price of shares of tradable (T) and non-tradable (N) goods firms at the end of time \( t \). The variables \( i^*, i_t \) denote the exogenous foreign interest rate and the domestic interest rate. Finally, \( D_{t}^T, D_{t}^N \) represent the dividends from equity ownership in sectors T and N. Here we follow the assumption that equity is cum-dividend, which means that the equity share gives the buyers (middle aged agents) the right to receive the dividends.
**Financial markets:** Households of generations $\eta = y$ are credit constrained and excluded from financial markets. Differently, households of generations $\eta = \{m, o\}$ can trade non-state-contingent domestic assets (bonds); they can also hold one perfectly divisible share of equity associated with ownership of the tradable and non-tradable firms. The perfectly divisible share of equity of the tradable sector firm is traded between generations $\eta = \{m, o\}$, and it represents a claim to the dividends stream $\{D_t^T\}$ of the domestic representative tradable good firm. Similar arguments follow for the equity share of the non-tradable sector firm. We think of the equity market as an institution which allows the transmission of private (non-publicly listed) firms’ ownership across generations of agents. This assumption represents a valid characterization of developing and poor countries where the relevance of publicly listed firms is often of second order importance.

**Maximization Problem:** The representative household maximizes utility by choosing consumption of tradable and nontradable consumption goods and also decides how many assets to hold, subject to the budget constraints, credit constraints, equity ownership constraint and taking prices and returns, $p_T^T, p_T^N, w_t, i_t$ as given. The optimization program of the agent born at time $t$ can then be summarized as follows:

$$\max_{\Gamma} U_t = v(c_{y,t}) + \beta v(c_{m,t+1}) + \beta^2 v(c_{o,t+2})$$

s.t. (3)(4), (5) and (6)

where $\Gamma = \{c_{y,t}, c_{m,t}, c_{o,t}, c_{T}^T, c_{m,t+1}^T, c_{o,t+2}^T, c_{y,t}^N, c_{m,t+1}^N, c_{o,t+2}^N, s_{m,t+1}^T, s_{m,t+1}^N, g_{m,t+2}\}$.

### 2.2 Tradable Sector Firms

In the economy, there is a representative tradable consumption goods firm owned by the households, with a production function given by $f(A_t^T, K_t, N_t^T)$. The firm technology $f(\cdot)$, combines $N_t^T$ units of (efficiency adjusted) labor, supplied by
the households of generations $\eta \in \{y, m\}$ with $K_t$ units of aggregate physical capital into $f(A^T_t, K_t, N^T_t)$ contemporaneous units of tradable consumption goods that become available at the end of period $t$, where $A^T_t$ is the exogenous total factor productivity (TFP) level in the tradable sector.

The Cobb-Douglas production function is given by:

$$f(A^T_t, K_t, N^T_t) = A^T_t K_t^\alpha \left( N^T_t \right)^{1-\alpha}$$  \hspace{1cm} (8)

where $\alpha \in (0, 1)$ is the output elasticity of capital.

Capital depreciates each period, at rate $0 < \delta < 1$, after production.

At time $t$, the tradable sector firm employs a total of $N^T_t$ (efficiency adjusted) units of agents at wage rate $w_t$ (expressed in terms of tradable goods). Efficiency adjusted units of labor are defined as:

$$N^T_t = e_t N^T_{y,t} + N^T_{m,t}$$  \hspace{1cm} (9)

, where $N^T_{\eta,t}$ denotes the amount of labor employed from generations $\eta \in \{y, m\}$. At the end of period $t$ the firm distributes dividends to the middle aged agents ($D^T_t$) and chose the amount of investment and net foreign assets ($K_{t+1}, b^*_t + 1$). The firm has access to foreign capital markets and hence can issue foreign debt, at foreign borrowing rate $i^*$. Finally, I assume that domestic capital markets are underdeveloped and that therefore the firm cannot borrow in domestic currency and cannot issue new equity in order to finance investment expenditures.

Formally, the tradable goods firm can be described by looking in turn at the following aspects:

**International Financial Constraint:** The tradable good firm faces an international financial constraint on foreign currency borrowing along the lines of the literature on financial frictions (see for example Bernanke et al. (1999)) and more precisely it follows Buera and
Due to standard moral hazard arguments, a fraction \(0 \leq \rho \leq 1\) of end of period capital has to be posted as collateral for international borrowing. End of period financial constraint can then be expressed as a lower bound on the firm’s holding of net interest bearing foreign assets:

\[
\frac{b_{t+1}^*}{1 + i^*} \geq -\rho K_{t+1}
\]  

(Budget Constraint): The tradable good firm budget constraint is then given by:

\[
D_t^T + K_{t+1} - (1 - \delta)K_t + \frac{b_{t+1}^*}{1 + i^*} + w_t N_t^T \leq f(A_t^T, K_t, N_t^T) + b_t^*
\]  

(Equity payout Constraint): The tradable good firm faces a constraint such that she cannot finance new capital expenditures through additional equity issuance; therefore equity payout must be always weakly positive.

\[
D_t^T \geq 0
\]  

 Tradable goods firm program: It is convenient to write the tradable goods firm problem in recursive form.

\[
V_t(K_t, b_t^*) = \max_{D_t, N_t^T, N_{t+1}^T, K_{t+1}, b_{t+1}^*} \left[ D_t^T + m_{t+1} V_{t+1}(K_{t+1}, b_{t+1}^*) \right]
\]  

s.t. (9), (10), (11) and (12)

where \(m_{t+1}\) is the discount factor, which we will further describe below.
2.3 Non-Tradable Sector Firms

In the economy there is also a representative non-tradable firm that produce non-tradable goods with a production function given by \( g(A_t^N, N_t^N) \).

The firm’s technology \( g(\cdot) \), combines \( N_t^N \) units of current (efficiency adjusted) labor supplied by the households of generations \( \eta = \{y, m\} \) at wage rate \( w_t \) into \( g(A_t^N, N_t^N) \) contemporaneous units of non-tradable consumption goods that become available at the end of period \( t \).

The variable \( A_t^N \) is the exogenous total factor productivity level in the non-tradable sector.

Our choice to avoid modeling investment in the non-tradable sector is made for simplicity; as already pointed out in the introduction, while our theory has a straightforward application when it comes to predict the effects of a real exchange rate depreciation on investment undertaken by the tradable sector this is not so when instead we look at the N sector. Therefore, excluding investment in the N sector simplifies the model while maintaining the relevance of our main channel for the case of the T sector.

The function \( g(\cdot) \) is such that \( g(\cdot)' > 0, g(\cdot)'' < 0, g(0) = 0 \); I adopt the following functional form:

\[
g(A_t^N, N_t^N) = A_t^N \left( N_t^N \right)^{1-\gamma} \tag{14}
\]

where \( 1 - \gamma \in (0, 1) \) is the output elasticity of labor\(^3\).

Formally the firm problem is given by:

\[
\max_{N_t^N} p_t g(A_t^N, N_t^N) - w_t N_t^N \tag{15}
\]

\(^3\) Constant return to scale can be maintained in the non-tradable sector too by assuming that production combines both labor and land according to a constant return to scale Cobb-Douglas production function; when the supply of land is fixed and normalized to one we obtain the production function reported in the paper.
2.4 Foreign Investors

Outside of the domestic economy, there is a continuum of atomistic competitive foreign investors of mass 1, who own large endowments of tradable goods that can be lent to the domestic economy. I assume that the domestic economy is small relatively to foreign investors and that therefore the equilibrium international risk-free rate determined in international capital markets is exogenous to the domestic economy.

I assume that foreign lenders have access to both international bonds and domestic equity. The return on international bonds follows from the above equation and it is given by the exogenous interest rate $i^*$.

2.5 Consolidated Government and Foreign Reserves Accumulation

In the domestic economy there is a consolidated government composed of a central bank and a treasury.

Within a tradable-nontradable environment accumulating foreign reserves corresponds to granting credit to foreigners, in terms of tradable resources. In our environment loans to foreigners can be financed either through lump sum taxation, as in Korinek and Serven (2016) or by issuing domestic assets in the economy, as for example in Amador et al. (2017). The modeling of the consolidated government follows the tradition of Song et al. (2011) and Wen (2011) since it acts as an intermediary between the private sector and international capital markets.

The consolidated government issues domestic assets expressed in terms of consumption goods, $G_t$ and can hold foreign assets $F_t$ denominated in terms of traded goods; when the central bank accumulates foreign assets and finances this operation through an increase in the amount of domestic assets that are issued in the domestic economy\(^4\). When the economy

\(^4\)In reality, the accumulation of foreign reserves can happen either through unsterilized or sterilized interventions; in the former case the central bank finances the accumulation of foreign assets by issuing domestic currency, which in turn leads to higher inflation, while in the latter case it compensates for this effect by also contemporaneously issuing domestic assets. Even though our model does not contain money, we try to maintain the main essence of sterilization by assuming that foreign reserves accumulation is financed through domestic assets.
experiences an Interest Parity (IP) wedge, sterilized foreign exchange interventions generate revenues or losses, which are rebated to the households through lump sum transfers. The budget constraint of the consolidated government (expressed in terms of tradables) can then be summarized as

$$\frac{F_{t+1}}{1 + i^*} + p^c_t G_t + \pi_{\eta,t} = \frac{p^c_t G_{t+1}}{1 + i^*} + F_t$$

(16)

Where \( i_t \) is the domestic interest rate between period \( t \) and \( t + 1 \); in general the domestic interest rate can be different from the world interest rate (we will return to this point when discussing foreign reserves accumulation). I assume that the profits generated from the consolidated government balance sheet, are in turn transferred directly to the households of generation \( \eta \) on a period by period basis through the term \( \pi_{\eta,t} \). Profits at time \( t \) are given by the following expression:

$$\frac{F_t}{1 + i^*} \left( (1 + i^*) - 1 \right) - \frac{G_t}{1 + i_{t-1}} \left( p^c_t (1 + i_{t-1}) - p^c_{t-1} \right)$$

(17)

Substituting in the budget constraint through the lump sum transfer term we have that:

$$\frac{F_{t+1}}{1 + i^*} - \frac{F_t}{1 + i^*} = \frac{p^c_t G_{t+1}}{1 + i_t} - \frac{p^c_{t-1} G_t}{1 + i_{t-1}}$$

(18)

where subscript \(-1\) denotes the beginning of time. Then we impose that:

$$\frac{F_0}{1 + i^*} = \frac{p^c_{t-1} G_0}{1 + i_{t-1}}$$

(19)

we then have:

$$\frac{F_{t+1}}{1 + i^*} = \frac{p^c_t G_{t+1}}{1 + i_t}$$

(20)
Finally, we impose a no-ponzi condition for net assets of the consolidated government:

\[ \lim_{t \to \infty} \frac{F_t - p^G_{t-1} G_t}{(1 + i^*)^t} = 0 \]  

(21)

2.6 Households in Competitive Equilibrium

In competitive equilibrium, representative households solve their maximization problem. After restating their budget constraints in terms of an equality, and substituting the aggregate consumption good, given by (3), I solve the following maximization problem:

\[
\text{Max } U_t = v(c_{T,y,t}^T, c_{N,y,t}^N) + \beta v(c_{T,m,t+1}^T, c_{N,m,t+1}^N) + \beta^2 v(c_{T,o,t+2}^T, c_{N,o,t+2}^N)
\]

s.t. (4), (5) and (6)

where \( \Gamma = \{ c_{T,y,t}^T, c_{T,m,t+1}^T, c_{T,o,t+2}^T, c_{N,y,t}^N, c_{N,m,t+1}^N, c_{N,o,t+2}^N, s_{m,t+1}^T, s_{N,m,t+1}^N, g_{m,t+2} \} \).

Define \( \lambda_t > 0 \) as the lagrange multiplier associated with the budget constraints, \( \beta \lambda_{t+1} \nu_{\eta,t} \geq 0 \) as the (scaled) lagrange multiplier associated with the borrowing constraint on generation \( \eta \).

---

5Because \( U \) is strictly increasing in at least one of its arguments it is always optimal to satisfy the budget constraints with equality.
The first order conditions are then given by:

\[ c_{y,t}^T : v'_{c_{y,t}} = \lambda_t \]  
(22)

\[ c_{m,t+1}^T : v'_{c_{m,t+1}} = \lambda_{t+1} \]  
(23)

\[ c_{o,t+2}^T : v'_{c_{o,t+2}} = \lambda_{t+2} \]  
(24)

\[ c_{y,t}^N : v'_{c_{y,t}} = \lambda_t p_t \]  
(25)

\[ c_{m,t+1}^N : v'_{c_{m,t+1}} = \lambda_{t+1} p_{t+1} \]  
(26)

\[ c_{o,t+2}^N : v'_{c_{o,t+2}} = \lambda_{t+2} p_{t+2} \]  
(27)

\[ g_{m,t+2} : \lambda_{t+1} p_{t+1} \frac{1}{1+i_{t+1}} = \beta p_{t+2} \lambda_{t+2} \]  
(28)

\[ s_{m,t+1}^j : D_{t+1}^j + Q_{t+2}^j \beta \frac{\lambda_{t+2}}{\lambda_{t+1}} = Q_{t+1}^j, j \in \{T,N\} \]  
(29)

By combining the optimality conditions with respect to domestic bonds and tradable consumption goods we can derive the following Euler:

\[ \beta \frac{p_{t+2}^c}{p_{t+1}^c} (1 + i_{t+1}) = \frac{v'_{c_{m,t+1}^T}}{v'_{c_{o,t+2}^T}} \]  
(30)

Combining the first order conditions of the household we have that:

\[ p_t = \frac{v'_{c_{y,t}^N}}{v'_{c_{y,t}^T}} = \frac{v'_{c_{m,t}^N}}{v'_{c_{m,t}^T}} = \frac{v'_{c_{o,t}^N}}{v'_{c_{o,t}^T}} \]  
(31)

Notice that since in equilibrium:

\[ g(A_t^N, N_t^N) = L_{y,t} c_{y,t}^N + L_{m,t} c_{m,t}^N + L_{o,t} c_{o,t}^N \]  
(32)

the real exchange rate is given by:

\[ p_t = \kappa \left( \frac{L_{y,t} c_{y,t}^T + L_{m,t} c_{m,t}^T + L_{o,t} c_{o,t}^T}{g(A_t^N, N_t^N)} \right)^{1+\varphi} \]  
(33)
where $\kappa = (1 - \omega)/\omega$. In this economy the real exchange rate depends on the ratio between tradable goods consumption and nontradable output\(^6\). Intuitively, if consumption of T goods increases, demand for NT goods increases, which requires an increase in their relative price for the market to clear, ceteris paribus.

Arbitrage in asset markets implies equalized returns between bonds and equity:

$$\frac{p_t^{c+1}}{p_t^c}(1 + \delta_t) = \frac{Q_t^j + D_t^j p_t^{c+1} (1 + \delta_t)}{Q_t^j}$$  \tag{34}

for $j \in \{T, N\}$. This implies that the price of equity satisfies:

$$Q_t^j = D_t^j + \sum_{k=1}^{\infty} \frac{1}{\Pi_{l=0}^{k-1} \frac{p_{t+l}^c}{p_{t+l}^c}(1 + \delta_{t+l})} D_{t+i}^j$$  \tag{35}

### 2.7 Non-Tradable Sector Firms in Competitive Equilibrium

In a competitive equilibrium, the non-tradable sector firm solves the maximization problem (15); in the decentralized equilibrium the individual firm takes all prices, $w_t, p_t^N$ as given. Formally, the firm problem is given by:

$$\max_{N_t^N} p_t A_t^N \left( N_t^N \right)^{1-\gamma} - w_t N_t^N$$  \tag{36}

The first order condition is then given by:

$$FOC(N_t^N) : p_t (1 - \gamma) A_t^N \left( N_t^N \right)^{-\gamma} = w_t$$  \tag{37}
2.8 Tradable Sector Firms in Competitive Equilibrium

In a competitive (decentralized) equilibrium, the tradable sector firm solves the maximization problem (13); in the decentralized equilibrium the individual firm takes all prices, \( w_{\eta,t}, p_t \) and returns \( i_t, i^{*} \), as given. As for the case of households, the firm’s budget constraint holds with an equality.

Formally, after substituting the efficiency adjusted labor term, the firm optimization program can then be summarized as follows:

\[
V_t(K_t, b^*_t) = \max_{D^T_t, N^T_t, K_{t+1}, b^*_{t+1}} \left[ D^T_t + m_{t+1} V_{t+1}(K_{t+1}, b^*_{t+1}) \right]
\]

s.t. (9), (10), (11) and (12)

where

\[
m_{t+1} \equiv \frac{v^{T}_{e_{t+1}}}{v^{T}_{e_{m,t}}}
\]

is the discount factor. Denoting \( \Lambda_t > 0, \Lambda_{t+1} \phi_t \geq 0, \mu_t \geq 0 \) the Lagrange multipliers associated with the budget constraint, the international financial constraint and the non-negativity constraint on equity, respectively\(^7\), we obtain the following Khun-Tucker necessary conditions, which include the following FOCs:

\[
N^T_t : \frac{\partial f(\cdot)}{\partial N^T_t} - w_t = 0
\]

\[
K_{t+1} : m_{t+1} \frac{\partial V_{t+1}(\cdot)}{\partial K_{t+1}} - \Lambda_t + \rho \phi_t \Lambda_{t+1} = 0
\]

\[
b^*_{t+1} : m_{t+1} \frac{\partial V_{t+1}(\cdot)}{\partial b^*_{t+1}} + \frac{1}{1 + i^{*}} \left( \phi_t \Lambda_{t+1} - \Lambda_t \right) = 0
\]

\[
D_t : 1 + \mu_t - \Lambda_t = 0
\]

\(^7\)Notice that the Lagrange multiplier on the international borrowing constraint has been scaled by \( \Lambda_{t+1} > 0 \) for convenience.
together with the envelope conditions:

\[
\frac{\partial V_t(\cdot)}{\partial K_t} = \Lambda_t \left( \frac{\partial f(\cdot)}{\partial K_t} + (1 - \delta) \right) \\
\frac{\partial V_t(\cdot)}{\partial b_t^*} = \Lambda_t
\] (42)

and with the complementary slackness conditions

\[
\Lambda_{t+1} \phi_t \left( \frac{b_{t+1}^*}{1 + i^*} + \rho K_{t+1} \right) = 0 , \quad \Lambda_{t+1} \phi_t \geq 0
\] (44)

\[
\mu_t D_t^T = 0 , \quad \mu_t \geq 0
\] (45)

Rearranging the optimality condition with respect to \( K_{t+1} \):

\[
m_{t+1} (1 + \mu_{t+1}) \left( \frac{\partial f(\cdot)}{\partial K_{t+1}} + (1 - \delta) \right) + (1 + \mu_{t+1}) \rho \phi_t = (1 + \mu_t)
\] (46)

The Lagrange multipliers \( \mu_t, \mu_{t+1}, \phi_t \) introduce wedges in the optimality condition for investment; a positive \( \mu_t \) increases the marginal costs of capital accumulation; when \( \mu_t > 0 \) the firm has extinguished her retained earnings and hence cannot equate marginal benefits with marginal costs so that the level of capital is inefficiently low; similarly, when next period dividend constraint is binding, \( \mu_{t+1} > 0 \) and an additional unit of investment today relaxes next period budget constraint through higher output and stock of available capital. Finally, when \( \phi_t > 0 \), one additional unit of capital relaxes the international borrowing constraint.

We can further combine the multipliers \( \phi_t, \mu_t, \mu_{t+1} \):

\[
\phi_t = \frac{1 + \mu_t}{1 + \mu_{t+1}} - \frac{1 + i^*}{\frac{p_{t+1}}{p_t} (1 + i_t)}
\] (47)

This expression highlights that even when \( D_t > 0, D_{t+1} > 0 \) so that \( \mu_t = \mu_{t+1} = 0 \) the international borrowing constraint is binding whenever the UIP gap is positive; the intuition being that foreign currency borrowing increases net present value of (real) dividends.
extent that the firm uses end of period capital as collateral for international borrowing, the higher the UIP gap the greater the marginal benefit of investment.

It is then useful to re-write the equation describing the optimal level of investment as:

\[
(1 + \mu_{t+1}) m_{t+1} \left( \frac{\partial f(\cdot)}{\partial K_{t+1}} + (1 - \delta) \right) = (1 + \mu_t) + \rho \left( \frac{1 + i^*}{p_{t+1}(1 + i_t)} - (1 + \mu_t) \right)
\]

(48)

Notice that the marginal cost collapses to 1 when the UIP gap is zero and \( \mu_t = \mu_{t+1} = 0 \) and that when \( \rho = 0 \) the marginal cost is naturally increasing in \( \mu_t \). When \( \rho > 0, \mu_t > 0 \), investing one extra unit of capital today provides additional collateral for borrowing which lowers the marginal cost by \( \rho(1 + \mu_t) \) but also leads to additional future repayments the discounted value of which is decreasing in the UIP gap.

An important aspect to highlight is the impact of the financial friction on investment; notice that when \( \mu_t > 0 \), the amount of investment undertaken by the T sector firm is given by (assuming no pre-existing stock of debt):

\[
K_{t+1} = \frac{1}{1 - \rho} \left( \alpha A^T_k K_t^{\alpha} (N^T_t)^{1-\alpha} + (1 - \delta) K_t \right)
\]

(49)

This equation highlights the role of the financial multiplier which indicates the units of investments that can be realized for each unit of the T sector firm retained earnings. One extra unit of internal wealth allows the T sector firm to invest one additional unit in physical capital, which allows the T sector to borrow and raise additional funds that can be invested, in the amount of \( \rho \). This in turn allows for further borrowing of \( \rho^2 \) additional units that can further be invested in capital. And so on. Therefore one unit of additional wealth allows the entrepreneur to have \( 1 + \rho + \rho^2 + ..... = \frac{1}{1-\rho} \) units of additional capital.
2.9 Definition of Competitive Equilibrium

An equilibrium in the small open economy is then summarized by:

- a consumption allocation for the domestic households of generation $\eta = \{y, m, o\}$, $(c^T_{\eta, t}, c^N_{\eta, t})$
- a saving allocation for the domestic households of generation $\eta = \{m, o\}$, $(g_{m, t}, s_{m, t})$
- an allocation for the tradable goods firms $(D^T_t, N^T_t, b^*_t, K_{t+1})$
- an allocation for the non-tradable goods firms, $(N^N_t)$
- prices and returns $(p_t, p^-_t, w_t, i_t)$
- which satisfy households optimization problem
- which satisfy the tradable goods firms optimization problem
- which satisfy non tradable goods firms optimization problem
- which clear goods, domestic bonds, equity and labor markets for all $t$
- given a foreign exchange reserves policy $(F_{t+1}, G_{t+1})$ and a transfer policy $\pi_{\eta, t}$, which satisfy the consolidated government budget constraint and the no-ponzi condition and an exogenous world interest rate ($i^*$).

2.10 Market Clearing and Aggregation

Production of non-tradable consumption goods equals aggregate consumption of non-tradables:

$$L_y t c^N_{y, t} + L_m t c^N_{m, t} + L_o t c^N_{o, t} = A^N_t \left( N^N_t \right)^{1-\gamma} \quad (50)$$

For each generation, labor demand equals labor supply:

$$N^T_{y, t} + N^N_{y, t} = L_y t \quad (51)$$
$$N^T_{m, t} + N^N_{m, t} = L_m t \quad (52)$$
Domestic bond and equity markets must clear so that:

\[ L_{m,t}g_{m,t+1} = G_{t+1} \]  \hspace{1cm} (53)

\[ L_{m,t}s_{m,t} = L_{m,t-1}s_{m,t-1} = 1 \]  \hspace{1cm} (54)

The aggregate resource constraints for each generation are given by:

\[ \omega^{1/v} (p_t^c)^{-1/(1+\varphi)} L_{y,t}c^T_{y,t} = w_{y,t} L_{y,t} \]  \hspace{1cm} (55)

\[ \omega^{1/(1+\varphi)} (p_t^c)^{1/v} L_{m,t}c^T_{m,t} + L_{m,t} \sum_{j \in \{T,N\}} s^j_m t Q^j_t + p_t^c G_{t+1} = \frac{w_{m,t} L_{m,t} + L_{m,t}}{1 + i_t} \sum_{j \in \{T,N\}} s^j_m t D^j_t \]  \hspace{1cm} (56)

\[ \omega^{1/(1+\varphi)} (p_t^c)^{1/v} L_{o,t}c^T_{o,t} = L_{o,t} s^T_{m,t-1} Q^T_t + L_{o,t} s^N_{m,t-1} Q^N_t + p_t^c G_t \]  \hspace{1cm} (57)

where we used the equilibrium condition \( c^N_{\eta,t} = c^T_{\eta,t} (p_t/k)^{-1/(1+\varphi)} \), so that:

\[ c^T_{\eta,t} + p_t c^N_{\eta,t} = \left( 1 + p_t^{1/v} \frac{1}{\kappa^{1+\varphi}} \right) c^T_{\eta,t} = \omega^{1/(1+\varphi)} (p_t^c)^{1/v} c^T_{\eta,t} \]  \hspace{1cm} (58)

### 3 Analysis

In this section we illustrate how changes in the real exchange rate affect the equilibrium in the labor market, the domestic bond market, households savings, and the level of investment that is undertaken by the T sector firm.

**Real exchange rate and Labor markets:** Combining the first order condition with respect to labor from the T and NT sectors firms optimization programs reveals how changes in the real exchange rate affect the amount of labor employed by each sector of the economy.

From the assumption of perfect labor mobility between sectors we have that:

\[ p_t \frac{\partial g(\cdot)}{\partial N^N_{\eta,t}} = \frac{\partial f(\cdot)}{\partial N^N_{\eta,t}}, \eta = \{y,m\} \]  \hspace{1cm} (59)
Intuitively, when the real exchange rate increases, the MPL in the NT sector increases relative to the MPL in the T sector. This in turn leads to an increase in labor demand from the NT sector. In equilibrium labor employed by the NT sector must then increase in order for the market to clear.

**Real exchange rate and Domestic bond market:** The economy features a domestic bond market, which in equilibrium must clear. The consolidated government chose an allocation of domestic assets and contemporaneously takes a position in the foreign bonds market, as shown in the Balance Sheet reported below. Notice that, because of capital controls, only the consolidated government has access to external assets, so that it has in practice a monopoly over the supply of domestic bonds. Specifically, the consolidated government (a) performs a credit operation involving domestic debt markets, (b) let agents chose their consumption of tradables and non-tradables competitively.

### Consolidated Government Balance Sheet

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign Assets: $F_{t+1}/(1+i)^t$</td>
<td>Domestic Liabilities: $p_t G_{t+1}/(1+i_t)$</td>
</tr>
</tbody>
</table>

In equilibrium foreign reserves accumulation affects the allocation of the economy because, by assumption, the capital account is closed and the consolidated government budget must balance every period. It follows that the amount of domestic assets in the economy determines the intertemporal allocation of consumption of middle aged agents, through the domestic interest rate. Formally, the Euler is given by:

$$\frac{v'^{t}_{c_{m,t}}}{v'^{t}_{c_{o,t+1}}} = \beta \frac{p_{t+1}}{p_t} (1 + i_t)$$  \hspace{1cm} \text{(60)}

We assume that profits/losses are transferred to the old generation only, through the lump
sum term $\pi_{o,t+1}$; rearranging equation (18) profits and losses are then given by:

$$
\left( 1 + i^* - (1 + i_t) \frac{p_{t+1}^c}{p_t^c} \right) F_{t+1}
$$

(61)

As a result of the consolidated government actions, *ceteris paribus*, the middle aged agents’ consumption falls by $\frac{F_{t+1}}{1+i^*}$ at time $t$, while the old aged agents’ consumption increases by $p_{t+1}^c G_{t+1} + \pi_{o,t} = F_{t+1}$ at time $t + 1$. Then, *ceteris paribus*, the larger is $F_{t+1}$, the greater is $v'_{c_{m,t}}$, and the smaller is $v'_{c_{o,t+1}}$, which is tantamount to an increase in the domestic interest rate. When the consolidated government can replicates the allocation which prevails when the capital account is open, $(1 + i_t) \frac{p_{t+1}^c}{p_t^c} = 1 + i^*$. In the following sections we will refer to such allocation as the “laissez-faire” allocation.

**Savings and real exchange rate:** In this paragraph we describe the relationship between savings and the real exchange rate.

The amount of savings of the middle aged generation at the end of period $t$ can be obtained by combining the euler condition and the budget constraints:

$$
p_t^c g_{m,t+1} + \sum_{j \in \{T,N\}} s^j_{m,t} \frac{Q_{t+1}^j}{p_{t+1}^c/p_t^c (1 + i_t)} = \frac{w_{m,t}}{1 + \frac{p_{t+1}^c}{p_t^c} (1 + i_t)^{\frac{1}{\sigma}} \beta^{-\frac{1}{\sigma}}}.
$$

(62)

Middle aged agents save through bonds and equity shares, which, are perfect substitutes. On aggregate, the amount of equity which is purchased by the middle aged agents must satisfy the domestic equity market clearing condition. It then follows that the amount of aggregate domestic bonds holding in the economy is decreasing in the market value of the firms, *ceteris paribus*. Finally, notice how in the expression, when the current period real exchange rate appreciates, savings increase.

**Real exchange rate and Investment:** When the T sector firm is borrowing constrained and the equity payout is binding, the amount of investment undertaken (assuming zero
previous debt repayments) is given by:

\[ K_{t+1} = \frac{1}{1 - \rho} \left( \alpha A_T^T K_t^\alpha (N_t^T)^{1-\alpha} + (1 - \delta) K_t \right) \]  \hspace{1cm} (63)

As we discussed above, a real exchange rate depreciation reallocates labor toward the T sector, which implies an increase in T sector output and hence an expansion of the internal resources that the T firm has available to finance investment. This positive relationship between real exchange rate depreciation and investment holds if and only if the firm’s borrowing and equity payout constraint are simultaneously binding. To appreciate this argument recall from equation (48) that the equilibrium efficient investment level is given by:

\[ m_{t+1} \left( \frac{\partial f(\cdot)}{\partial K_{t+1}} + (1 - \delta) \right) = 1 + \rho \left( \frac{1 + i^*}{\frac{p_{t+1}}{p_t}(1 + i_t)} - 1 \right) \]  \hspace{1cm} (64)

From which we have that:

\[ K_{t+1} = (\alpha A_T^{t+1}) \frac{1}{1 - \alpha} \left( \frac{p_{t+1}^c}{p_t^c}(1 + i_t)(1 + \rho \left( \frac{1 + i^*}{\frac{p_{t+1}}{p_t}(1 + i_t)} - 1 \right)) - (1 - \delta) \right) \frac{1}{\frac{\sigma}{\cdots}} N_{t+1}^T \]  \hspace{1cm} (65)

which highlights how foreign reserve accumulation, by increasing the domestic discount factor would in turn lower the efficient investment level chosen by the T sector firm. This argument holds both when the firm is financially unconstrained or when, starting from a laissez-faire allocation in which the T firms is financially constrained, the consolidated government’s accumulation of foreign assets induces a large enough increase in the discount factor, which eventually leads to a slack equity payout constraint. Then, any further accumulation of foreign assets will leads to a decrease in the level of investment.
4 Quantitative Exercise

Having established key qualitative predictions of the model we now evaluate the ability of the model to help explain the investment rate in China, over the period 1988-2008. We focus on a scenario in which, starting from a steady state, the productivity level is suddenly anticipated to permanently increases in the next period (2008). When this happens, the tradable goods firm desired amount of investment increases. When the next period anticipated increase in productivity is high enough, the firm’s borrowing constraint binds, which in turn leads the firm to draw from its own internal funds. In this simulation we are interested in an environment in which both the borrowing and equity payout constraints become binding leading to a laissez-faire allocation in which investment is inefficiently low.

We enrich the baseline model under several dimensions and we present a calibration exercise that can capture the Chinese experience. First, we introduce a pay-as-you-go pension scheme, which allows for further consumption of the old agents. Second, we allow for a bequest motive of the old agents. Third, we calibrate the demographic evolution, which allows us to incorporate the impact of an aging population on the overall dynamics. The calibration exercise’s objective is to match a zero initial stock of foreign reserves (consistently with the data as per 1988).

Preferences and bequests: We augment the theoretical framework with a bequest motive, following Abel (2001). The lifetime utility of an agent born at time $t$ is

$$\max_u U_t = v(c_{y,t}) + \beta v(c_{m,t+1}) + \beta^2 v(c_{o,t+2}) + \phi \beta^2 v(b_{t+1})$$  \hspace{1cm} (66)$$

where $b_{t+1}$ is the amount of bequest left at time $t+2$ by an agent born at time $t$ and $\phi$ defines the strength of the bequest motive. We assume that middle aged agents born at time $t+1$ receive the bequest.

Pay-as-you-go Pension Scheme: We introduce a pay-as-you-go pension scheme; the scheme is composed of a compulsory tax which, along the lines of China involves a contribution
rate of 10% of wages paid lump sum by all workers. The receipts are distributed to the old aged population.

**Equity Markets:** Until the "share split" reform of 2005, 2/3 of the Chinese stock market was composed of non-traded shares, which belonged to the government and were not publicly traded. As shown by Hubbard (2016), State Owned Enterprises (SOEs) are more prevalent in the non-tradable sector. In the model we replicate this characteristic and we assume that the N sector dividends, which accrue to the consolidated government from ownership of the N sector firms are rebated lump sum to the middle aged agents on a period by period basis. Regarding the T sector, we allow for 1/3 of the T sector equity shares to be traded among private agents.

### 4.1 Calibration

We now describe the calibration of the quantitative version of the model. Each period lasts 20 years and each agent lives for 3 periods. There are six age groups, which we map into the following age brackets: 15-34, 35-54 and above 55 years old. We follow a scenario, starting from time $t = -1$ (1988) to time $t = 1$ (2028). Foreign reserves accumulation above the laissez-faire levels happens, as before, at time $t = -1$. We calibrate the model so that under a laissez-faire allocation the stock of foreign reserves in 2008 is approximately equal to zero.

**Demographics:** While in our steady state we assume constant populations shares we try to match the sharp decline in the population growth rate between 1988 to 2008. We obtain the age distribution and evolution from the World Population Population Prospects data, sampled every decade since 1970. The growth rate of population is calibrated so as to offer a close match with the observed aged distributions from 1988 to 2008. During this period the share of young (as defined in our environment) in the total population declines from 44.6% to 31.1%. This corresponds to an average decline of 2% per year in the birth rate. We assume that after 2008 population growth is equal to zero.

**Age-Income profiles:** In the quantitative model we assume that old agents also work. We
calibrate the age income profile of agents following Coeurdacier et al. (2015); specifically \( e_{j,t} \) is calibrated using the Chinese Urban Households Survey (UHS) and it represents the relative income profile of generation \( \eta = \{m, o\} \) normalized by the income profile of generation \( m \), so that \( e_{m,t} = 1 \). I then calibrate \( e_{y,-1} = 0.79, e_{o,-1} = 0.10 \). These values are taken from the first Chinese UHS and pertain to 1992 data. For \( t = 0 \) I refer to the 2008 survey, which gives \( e_{y,0} = 0.41, e_{o,0} = 0.07 \). I assume that the values of the relative efficiency parameters remain constant after 2008.

**Other calibrated Parameters:** The degree of financial frictions for Chinese firms is calibrated following Bacchetta and Benhima (2015) and it is set to \( \rho = 0.1 \); this value is also in line with Song et al. (2014) who evaluate the share of investments financed through bank loans and government budgets in domestic private enterprises during the years going from 1997 to 2003\(^9\); the authors find that this share has ranged between 5\% and 12\%.

Furthermore, and following major literature, we set the share of capital in total tradable output \( \alpha = 0.5 \) and the share of labor in non-tradable output \( 1 - \gamma = 0.5 \). Notice that for the case of China, Bai and Qian (2006), finds that the share of capital in production is equal to 0.5 when referring to a production function for the whole economy. Concerning the share of non-tradable consumption we set a share \( 1 - \omega = 0.2 \), consistently with Han et al. (2016) who find that the share of non-traded goods varies between 18\% and 28\% across Chinese cities.

We then calibrate \( i^* \) using the three month U.S. treasury bill (deflated by the inflation rate)\(^{10}\). The average annual real treasury bill rate during the years going from 1988 to 2007 is approximately equal to \( i^* = 3.7\% \). The calibration of the sectoral TFP growth rate that

\(^8\)Income is calculated as the sum of wages and self business income net of taxes.

\(^9\)Also, Dollar and Wei (2007) use a survey to determine the sources of financing of firms during the years going from 2002-2004; in their study the authors focus on State Owned Entreprises (SOEs) and Domestic Private Entreprises (DPEs); as shown by the authors, SOE’s rely more heavily on bank loans than DPEs: bank financing accounts for 36.20\% of working capital financing for the former group of firms while only 22.46\% for the latter group. Similarly for the case of investment financing, the authors find that the share of bank financing equal 24.72\% and 18.26\%, respectively. As shown by Hubbard (2016), SOEs are more prevalent in the non-tradable sector. In principle the share of SOEs and DPEs within each sector can be recovered through the Chinese NBS, which reports wages per sector and ownership, by assuming that the wage rate is proportional to the average revenue product of labor.

\(^{10}\)Federal Reserve Bank of St. Louis.
we use as a benchmark is from Tyers and Golley (2008) and refers to the years from 1988 to 2005\textsuperscript{11}. However, for completeness we also report in table 5 the estimated values from a few other studies.

We assume that in steady state the firm does not hold any debt and that she only borrows (up to the amount allowed by value of the collateral) when $\phi_t > 0$.

The remaining parameters are the intertemporal elasticity of substitution, the intratemporal elasticity of substitution, the households’ discount factor, the parameter that defines the strength of the bequest motive and the initial levels of the T sector and N sector TFP. We normalize the T and N sectors initial TFP levels to 1. We then build a grid containing the remaining parameters $[\sigma, \varphi, \beta, \phi, ...]$ and chose the combination that minimizes the distance:

$$F_0/GDP_0 - 0$$

subject to

$$\text{Interest Parity Gap at time} - 1 = 0$$

In words, we chose the combination of parameters which creates a laissez-faire allocation characterized by a stock of foreign reserves to GDP in 2008 equal to zero. In table 1 below we summarize the calibrated parameters.

### 4.2 Threshold Productivity Growth

Table 2 summarizes the value of the threshold productivity growth, $\hat{A}_{t+1}$, which leads to a binding constraint. We also show how the threshold productivity level changes when we modify some of the parameter values. In block 1 of Table 2 I show the results for a

\textsuperscript{11}I assume that TFP growth remains equal to its 2005 values for the years up to 2008. The authors distinguish TFP growth in the food sector, industry and services. The former two pertain to the tradable sector while the last is inherently a non-tradable sector. We use World Bank data on shares of Agriculture, forestry and fishing industry value added as percentage of GDP and on shares of Industry value added as percentage of GDP and calculate a weighted average Tradable sector TFP growth rate for the years going from 1988 to 2008. We find that productivity growth has been equal to approximately an average of 5.12% per year in the tradable sector and to 2.24% in the non-tradable sector. Consistently with these findings we calibrate the productivity growth in the non-tradable sector as being equal to 52.74% of the productivity growth in the tradable sector.
Table 1: Calibration of parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output elasticity of capital in T sector</td>
<td>$\alpha$</td>
<td>0.5</td>
</tr>
<tr>
<td>Output elasticity of labor in NT sector</td>
<td>$1 - \gamma$</td>
<td>0.5</td>
</tr>
<tr>
<td>Depreciation rate of capital</td>
<td>$\delta$</td>
<td>0.1</td>
</tr>
<tr>
<td>Households discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Foreign interest rate(in %)</td>
<td>$i^*$</td>
<td>3.7</td>
</tr>
<tr>
<td>Households coefficient of risk aversion</td>
<td>$\sigma$</td>
<td>1.6</td>
</tr>
<tr>
<td>Intratemporal elasticity parameter</td>
<td>$\varphi$</td>
<td>0.205</td>
</tr>
<tr>
<td>Constraint parameter of tradable firm</td>
<td>$\rho$</td>
<td>0.1</td>
</tr>
<tr>
<td>Share of tradables in consumption</td>
<td>$\omega$</td>
<td>0.8</td>
</tr>
<tr>
<td>Constraint parameter of tradable firm</td>
<td>$\rho$</td>
<td>0.1</td>
</tr>
<tr>
<td>Relative efficiency of young</td>
<td>$e_{y,t}$</td>
<td>{0.79, 0.41}</td>
</tr>
<tr>
<td>Relative efficiency of old</td>
<td>$e_{o,t}$</td>
<td>{0.10, 0.07}</td>
</tr>
<tr>
<td>Bequest Parameter</td>
<td>$\phi$</td>
<td>0.23</td>
</tr>
<tr>
<td>Initial TFP T sector</td>
<td>$A^T$</td>
<td>1</td>
</tr>
<tr>
<td>Initial TFP N sector</td>
<td>$A^N$</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2: Threshold Productivity $\hat{A}^T_0$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\sigma$</th>
<th>$\omega$</th>
<th>$\rho$</th>
<th>$\hat{A}^T_0$</th>
<th>$g_{\hat{A}^T_0}$(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark Economy</td>
<td>0.50</td>
<td>1.60</td>
<td>0.80</td>
<td>0.10</td>
<td>1.32</td>
</tr>
<tr>
<td>Less financially constrained economy</td>
<td>0.50</td>
<td>1.60</td>
<td>0.80</td>
<td>0.15</td>
<td>1.36</td>
</tr>
<tr>
<td>Less capital intensive economy</td>
<td>0.30</td>
<td>1.60</td>
<td>0.80</td>
<td>0.10</td>
<td>1.45</td>
</tr>
<tr>
<td>Less service oriented economy</td>
<td>0.50</td>
<td>1.60</td>
<td>0.90</td>
<td>0.10</td>
<td>1.36</td>
</tr>
</tbody>
</table>

Notes: Initial level of productivity in T and N sector is normalized to 1. $\hat{A}^T_0$ refers to the minimum level of T sector TFP at time 0 that makes the firm financially constrained at time $-1$; similarly, $g_{\hat{A}^T_0}$ refers to the minimum growth rate of T sector TFP between time $-1$ and 0 that makes the T sector firm financially constrained at time $-1$.

"benchmark" calibration. In block 2 I present the results for an economy where firms are less financially constrained and in block 3 I show the case of an economy which is less capital intensive. In both cases the threshold productivity growth increases, as expected. Finally in block 4 I consider the case of an economy where consumption of non-tradable
goods as a share of total consumption is lower. In this case the threshold productivity level also increases compared to the benchmark; the reason is that at each level of tradable goods consumption there is now less associated consumption of non-tradables and hence less upward pressure on the real exchange rate.

4.3 How much Unrealized Investment?

We then turn to study the role of the financial constraint on investments in the laissez-faire economy, by comparing the level of investment undertaken at the time before the increase in the productivity level against its long run level\textsuperscript{12}. Specifically I ask what fraction of its long term investment level the firm achieves as of time $-1$, as a consequence of the borrowing friction. I show the results in 3. I find that in the benchmark economy undertaken investments at time $t = -1$ correspond to only 13% of their long run level. Consistently with the previous results on the threshold productivity growth I find that for the cases of a less financially constrained economy, less capital intensive economy and less service oriented economy the firm is able to achieve higher investments compared to the benchmark economy.

<table>
<thead>
<tr>
<th>Table 3: Unrealized Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
</tr>
<tr>
<td>Benchmark Economy</td>
</tr>
<tr>
<td>Less financially constrained economy</td>
</tr>
<tr>
<td>Less capital intensive economy</td>
</tr>
<tr>
<td>Less service oriented economy</td>
</tr>
</tbody>
</table>

Notes: $g_{AT}^{\alpha}$ refers to the calibrated growth rate of T sector TFP between time $-1$ and 0. $K_0/K^{ss}$ shows the fraction of realized investment at time $-1$ compared to its long run steady state.

\textsuperscript{12}Even though the analysis in this work is focused on the time period going from 1988 to 2007 (time $t = -1$), it should be stressed that, given the current calibration, the T sector firm remains financially constrained (i.e. $D_t^T = 0$) also at time $t = 0, 1$. 
4.4 Steady State and Transition

In this paragraph we describe the transition of a selection of key variables from the initial steady state to the steady state that prevails following the productivity shock. We define $t = -1$ as corresponding to 1988 and $t = 0$, which is the time at which productivity increases permanently, as corresponding to 2008 (hence one period corresponds to 20 years). It follows that at time -2 the economy is in steady state, while at time -1 it starts to transition toward the new steady state. The figure highlights that the dividend constraint binds up to time 0. Notice how investment remains relatively subdued at time $-1$ despite the anticipated raise in the future productivity level. The real exchange rate initially slightly appreciates due to the unanticipated capital gains from holding equity shares. Finally, the labor market experiences a shift at time $-1$, which is then partly offset once the productivity levels adjust as of time 0. Starting from time 0 the labor market adjust monotonically to reflect the differences in productivity levels.

In the first exercise I study the deviations of key variables from the laissez-faire allocation (which is characterized by a stock of foreign reserves to GDP equal to 0, as of time 0) when the consolidated government implement an allocation characterized by a stock of foreign liabilities to GDP at time 0 equal to 1%:

$$\Delta \left( \frac{F_0}{Y_T^0} + p_0 Y_N^0 \right) = 1\%$$

Figure 3 presents the results. The model confirms that a real exchange rate depreciation is associated with an increase in the amount of investments at time $-1$. This outcome follows from a relocation of labor toward the tradable sector, which eventually leads to an expansion of the available internal funds that the T sector firm can access. Notice that, as a result of higher $N_T^1$ and higher $K_T^0$, the marginal product of capital initially increases at time $-1$, before falling at time 0. The model also predicts a significant improvement in the current account at time $-1$, driven by the decline in agents consumption and a small deterioration as of time 0 due to higher consumption and investments.
Notes: Each variable refers to the percentage deviation from laissez faire, with $\Delta \left( \frac{F_0}{1 + \rho Y_0^T} \right) = 1\%$. Productivity shock happens at time 0 and foreign reserves are accumulated at time $-1$. In the case of the interest rate $i$ the deviation is yearly.
4.5 Quantitative Evaluation:

In this section we compare how the model compare with the data for China.

**Investment Rate:** We analyze the allocation that replicates a stock of foreign reserves to (20 years) GDP of approximately 6.2% as observed in 2008. In Figure 4 we plot and compare the dynamics of the model in a laissez-faire allocation, an allocation with foreign exchange accumulation and the actual data. We are interested in the periods $t = -1$ (1988), and $t = 0$ (2008), but we also include the successive period (2028) so as to include a prediction for the future. The Figure shows that deviating from laissez faire generates an increase in the investment to (20 years) GDP ratio of approximately 1.25% compared to a laissez-faire economy. This value is economically significant as it is equivalent to 25% ($1.25 \times 20$) of the GDP produced in one year by the Chinese economy (on average). Subsequently, the model, in accordance with the data, predicts a gradual decline in the ratio of investments to GDP. Specifically, as of time 0 (which corresponds to year 2028) the ratio of investment to GDP has declined to approximately 28%.

![Figure 4: Quantitative Predictions- Investment](image)

Investment to GDP data source: National Statistics Office. Data retrieved from CEIC Latest actual data: 2018. Subsequent years are estimates.

**Current Account:** Before joining the WTO in 2001, China’s external balance had an initial

\[\text{This value corresponds to the stock of foreign reserves as of 2008 to the sum of GDP over the years going from 1988-2008.}\]
deficit followed by a surplus, starting after 1988. Since 2002 the trade surplus significantly expanded until 2007, after which, it tumbled and consistently declined: As of 2016, the trade surplus totaled approximately 2% of GDP\textsuperscript{14}.

Our theory falls short in accurately predicting the current account deficit at time $-1$; this is unsurprising given the assumption of an initial steady state without growth. Our theory predicts a significant improvement in the deficit by approximately 10% as a result of the intervention. Furthermore, our model is consistent with the current account surplus in 2008, as the surplus equals approximately 12%. Interestingly, the current account surplus is approximately 2% lower under the foreign reserves accumulation policy.

Figure 5: Quantitative Predictions- Current Account

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Current Account (Model) vs Current Account (Data) with Intervention and LF}
\end{figure}

\textbf{Sensitivity Analysis:} In this paragraph we further investigate how investment is affected when we change some of the parameters, keeping the interest parity gap fixed at the level prevailing in the benchmark economy.

In particular, we focus on the role of the intratemporal elasticity of substitution, $1/(1 + \varphi)$ which is a key factor behind the elasticity of $p_t$ to changes in the sectoral consumption allocation\textsuperscript{15}, on the relative share of tradables in the agents consumption basket, $\omega$, which determines how much N goods consumption changes for a given change in T consumption,

\textsuperscript{14}However the fall in external surplus is hiding a consistently surplus in the manufacturing sector, which remained well over 10 percent of GDP; this manufacturing surplus is hidden by factors including prices commodity imports and the increase in tourism.

\textsuperscript{15}More precisely, for a given decline in the relative consumption of tradables, a higher elasticity implies a smaller change in the price of nontradables.
on the relative output elasticity of labor in the N sector and $1 - \gamma$, which influences how much labor is relocated toward the T sector for a given real depreciation.

Table 4: Sensitivity of Investment to Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\Delta(I^T_0/GDP_0)(%)$</th>
<th>$\Delta I^T_0(%)$</th>
<th>$F_0/GDP_0(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi = 0.205$</td>
<td>+1.2</td>
<td>+1.6</td>
<td>6.2</td>
</tr>
<tr>
<td>$\varphi = 0.505$</td>
<td>+1.2</td>
<td>+1.6</td>
<td>5.6</td>
</tr>
<tr>
<td>$\omega = 0.8$</td>
<td>+1.2</td>
<td>+1.6</td>
<td>6.2</td>
</tr>
<tr>
<td>$\omega = 0.5$</td>
<td>+2.3</td>
<td>+5.9</td>
<td>22.1</td>
</tr>
<tr>
<td>$\gamma = 0.5$</td>
<td>+1.2</td>
<td>+1.6</td>
<td>6.2</td>
</tr>
<tr>
<td>$\gamma = 0.4$</td>
<td>+1.2</td>
<td>+1.8</td>
<td>5.5</td>
</tr>
</tbody>
</table>

In Table 4 we compare the amount of additional investment that the economy achieves when we modify a parameter of choice. In the first row of each block of the table we show the results for the benchmark calibration, while in the second row of each block we report the results when the parameter is modified.

While changes in the intratemporal elasticity do not have a significant impact, a reduction in the share of tradable goods in the consumption basket significantly increases the percentage change of investment. The reason is that for a given reduction in consumption of tradables, the decline in nontradables consumption needed to satisfy the intratemporal optimality condition is now higher. Finally, turning to $\gamma$, we find that a higher elasticity of output to labor, increases the impact that a given IP deviation has on investment. The reason is that for a given initial change on consumption of tradable goods, the fall in N sector output is larger, which leads to a more depreciated exchange rate at time $-1$.

5 Welfare

In this section we study the impact of foreign exchange intervention on welfare. First we review how consumption is affected across the different cohorts and second we evaluate the welfare maximizing foreign reserves accumulation policy.
5.1 Impact of Foreign Reserves Accumulation on Cohorts’ Consumption

In Figure 6 we illustrate the impact on consumption of an allocation that targets a stock of foreign reserves to GDP equal to 6.2% at time 0. The results are presented so as to show the percentage change in consumption for households born at time -3 (“Cohort -3”), born at time -2 (“Cohort -2”), born at time -1 (“Cohort -1”) and born at time 0 (“Cohort 0”), and by distinguishing between consumption of tradable (T) nontradables (N) and total consumption.

Notice that “Cohort -3” welfare deteriorates following the intervention, due to the decline in the firm’s equity price, which must now reflects a more appreciated future exchange rate together with a higher domestic interest rate. Differently, “Cohort-2”, ratchets significant gains from the intervention. Specifically, “Cohort-2” benefits from wealth gains at time at time 0 when she is selling non-tradable firm’s equity at a higher price, while also receiving a higher wage. Turning to “Cohort-1”, her aggregate consumption unambiguously increase.

Finally, looking at “Cohort0” we notice that consumption of tradable goods benefits from a higher real wage, while the consumption of non-tradables is negatively impacted as a consequence of a more appreciated exchange rate. Overall, given the predominance of tradables in her consumption basket, the welfare gains are positive.

5.2 Welfare Maximizing Foreign Reserves Accumulation

In order to study how different policies affect aggregate welfare we first need to specify how a social planner weights different cohorts of agents. We therefore define an aggregate welfare function ($\mathcal{W}(\cdot)$) that incorporates past present and future cohorts; the social welfare of the economy can be then established by the following expression:

$$\mathcal{W} = \sum_{t=j-2}^{\infty} \vartheta_t U_t$$

(67)
Figure 6: Welfare of Cohorts

Notes: Each variable refers to the percentage deviation from laissez faire. The allocation is such that \( \frac{p_0}{\nu_0' + p_0 \nu_0} = 6.2\% \). Productivity shock happens at time 0 and foreign reserves are accumulated at time \(-1\).

where as before \( U_t \) is the lifetime utility function of the cohort born at time \( t \). We assume that the planner weights the lifetime utilities of different generations, where \( \{\vartheta_t\}_{t=j-2}^\infty \) are the Pareto weights on different generations, which satisfy \( \vartheta_t \geq 0 \). Since lifetime utility of each cohort is bounded, so will be the social welfare function provided that \( \sum_{t=j-2}^\infty \vartheta_t < \infty \).

The planner chose a foreign reserves accumulation policy \( F_{t+1}, G_{t+1} \) where \( t = -1 \) that maximizes the aggregate welfare function.

We compute the welfare gains from the interventions as the proportional increase in consumption in the laissez-faire equilibrium (without foreign reserves accumulation) that would make agents indifferent between the decentralized allocation (without deviation from the laissez-faire) and the distorted allocation that follows from the consolidated government accumulation policy. The welfare gain for cohort born at time \( t \), \( \Omega \) is given by:

\[
(1 + \Omega)^{1-\sigma} U_t^{lf} = U_t^{sp}
\]

(68)

where the superscript "lf" stands for “laissez-faire” while the superscript "sp" stands for
“social planner”. Similarly, we can define aggregate welfare gains as

\[ (1 + \Omega)^{1-\sigma} W^{1f} = W^{sp} \]  

We first show in Figure 7 the evolution, in terms of percentage deviations from the laissez-faire allocation, of welfare gains for a range of IP gaps. As expected, “Cohort-3’ suffer from lower equity prices, while “Cohort-2” achieves high welfare gains. “Cohort 0” welfare’s improves while “Cohort1” welfare slightly declines.

We highlight with a vertical line the level of foreign reserves to GDP, consistent with China. Notice that the IP gap that maximizes aggregate welfare is below the level associated with a 6.2% increase in FX to GDP. Nevertheless the current calibration still documents a positive aggregate welfare increase from the intervention.

**Impact of Pay-as-you-go on welfare:** The pay-as-you-go policy consists in a revenue neutral lump sum transfer equal to 10% of workers’ salary to old aged agents. In order to determine the impact of the policy on welfare, in Figure 8 we compare the benchmark result with a pay-as-you-system (continuous line) with those from an economy without a social security system (dotted lines). Overall, the pay-as-you system reduces aggregate welfare through a significant reduction in “Cohort-2” and “Cohort1” welfare. We then compare these results with a counterfactual scenario without a population decline between time \( t = -1 \) and time \( t = 0 \). As expected, removing demographic growth significantly increases “Cohort-1” welfare gains, as a result of larger pension benefits (and which reflect the higher contributions at time \( t = 2 \) relative to their own contributions).

### 6 Conclusion

In this paper we presented a simple mechanism through which foreign exchange accumulation promotes capital accumulation, within a financially constrained economy. We emphasized how this mechanism leads to an environment where the increase in growth and investment
Figure 7: Impact of Foreign Reserves Accumulation on Welfare

Notes: Each variable refers to the percentage deviation from laissez faire. Productivity shock happens at time 0 and foreign reserves are accumulated at time $-1$. We assume that the social planner weights past present and future generations with $\theta_t = \beta_t$.

is coupled with net capital outflows, contrary to what the standard intertemporal model predicts. We applied the main theoretical predictions of the framework by calibrating a quantitative version of the model, with a focus on the case of China.

Compared to the previous literature, our research highlights that a policy of real exchange rate undervaluation, by depressing current levels of consumption, raises intergenerational concerns that may require further ancillary policies, such as lump sum taxes, which can redistribute the future welfare gains across generations of agents living in the economy.

A major limitation of the current model is the absence of frictions in the labor markets. A natural extension of the model would investigate how accounting for the process of structural change that an economy undergoes during its development phase, influences the baseline results presented in the paper.
Figure 8: Impact of Pay-as-you-Go on Welfare

Notes: The continuous and dashed lines refer to the benchmark model; the dotted lines refers to the case without lump sum transfers; the dashed line refers to the scenario without decline in population growth between time $t = -1$ and time $t = 0$. The vertical line denotes the level of Interest Parity gap which achieves a stock of foreign assets to GDP equal to 6.02% a time $t = 0$. 
References


A Appendix

A.1 Tables

Table 5: Productivity Growth in China

<table>
<thead>
<tr>
<th>Sector</th>
<th>( g_{AT} )</th>
<th>Years</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tradable Sector Productivity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other non metallic minerals</td>
<td>4.7%</td>
<td>1995 – 2007</td>
<td>Hsieh and Ossa (2016)</td>
</tr>
<tr>
<td>Non-Tradable Sector Productivity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-tradables</td>
<td>3.5%</td>
<td>1995 – 2007</td>
<td>Hsieh and Ossa (2016)</td>
</tr>
</tbody>
</table>

A.2 Derivation of Bond Holdings Equation:

In this paragraph we show the algebraic steps necessary to express the amount of bonds holdings as a function of the household income and the equity holdings. Starting from the Euler equation:

\[
\beta \frac{p_{t+1}^e}{p_t^e} (1 + i_t) = \frac{(\omega + (1 - \omega)(pt/\kappa)^{1+\varphi})^{1+\varphi-\sigma}}{(\omega + (1 - \omega)(p_{t+1}/\kappa)^{1+\varphi})^{1+\varphi-\sigma}} \cdot \frac{(c_{m,t}^T)^{-\sigma}}{(c_{o,t+1}^T)^{-\sigma}}
\]

We can then use the equilibrium condition \( c_{o,t}^N = c_{o,t}^T (p_t/\kappa)^{-1/(1+\varphi)} \) together with the definition of \( p_t^e \):

\[
\beta \frac{p_{t+1}^e}{p_t^e} (1 + i_t) = \frac{(p_t^e)^{1+\varphi-\sigma}}{(p_{t+1}^e)^{1+\varphi-\sigma}} \cdot \frac{(c_{m,t}^T)^{-\sigma}}{(c_{o,t+1}^T)^{-\sigma}}
\]
The above expression can then be simplified:

\[
\beta^{1/\sigma} \left( \frac{p_{c+1}^e}{(p_{c}^e)^{1+\varphi}} \right) (1 + i_t)^{1/\sigma} = \frac{c_{o,t+1}^T}{c_{m,t}^T}
\]  

(70)

We multiply and divide equation (70) by \(p_{c+1}^e/p_t^e\):

\[
(p_t^e)^{\varphi} c_{m,t}^T = \frac{1}{\frac{p_{c+1}^e}{p_t^e} \beta^{1/\sigma} (1 + i_t)^{1/\sigma}} \left( \frac{p_{c+1}^e}{(p_{c}^e)^{1+\varphi}} c_{o,t+1}^T \right)
\]

The above expression can then be substituted in the middle aged budget constraint, which is given by:

\[
\omega^{-1/\varphi} (p_t^e)^{\varphi} L_{m,t} c_{m,t}^T + L_{m,t} \sum_{j \in \{T,N\}} s_{m,t}^j Q_t^j + p_t^e \frac{G_{t+1}}{1 + i_t} = w_{m,t} L_{m,t} + L_{m,t} \sum_{j \in \{T,N\}} s_{m,t}^j D_t^j
\]

By substituting in the old agent budget constraint, and rearranging using the asset price equation, we can finally derive the following expression as per the main text:

\[
p_t^e g_{m,t+1} \frac{1 + i_t}{1 + \frac{p_{t+1}^c}{p_t^c} (1 + i_t)} = \frac{w_{m,t}}{1 + \frac{p_{t+1}^c}{p_t^c} (1 + i_t) \frac{\sigma}{\sigma - \beta}} - \sum_{j \in \{T,N\}} s_{m,t}^j (Q_t^j - D_t^j)
\]

A.3 Decomposition of Equity Prices Dynamics:

Equity price dynamics play a major role in the economy, when foreign reserves are accumulated. In this paragraph we propose to highlight the contribution of macroeconomic factors in the evolution of the equity prices through a simple decomposition exercise. The equity price is given by:

\[
Q_t^j = D_t^j + \frac{D_{t+1}^j}{\frac{p_{t+1}^c}{p_t^c} (1 + i_t)} + \frac{D_{t+2}^j}{\frac{p_{t+2}^c}{p_t^c} (1 + i_t)(1 + i_{t+1})} + ... 
\]
Denoting with a “hat”, variables that correspond to the laissez-faire equilibrium we can approximate the percentage deviations as:

$$\frac{1}{\hat{Q}_t} (Q^j_t - \hat{Q}^j_t) \approx \frac{1}{\hat{Q}_t} \left( (D^j_t - \hat{D}^j_t) + \frac{1}{\hat{p}_{t+1}} (D^j_{t+1} - \hat{D}^j_{t+1}) - \frac{\hat{D}^j_{t+1}}{\hat{p}_{t+1}} (\hat{i}_t - \hat{i}_t) - \frac{\hat{p}_{t+1}}{\hat{p}_t} (\hat{p}_{t+1}^c - \hat{p}_t^c) (1 + \hat{i}_t^*) \right)$$

where \( \hat{p}_t^c (1 + \hat{i}_t^*) = 1 + i^* \). In turn we can further decompose dividends into the various components. Notice that the first few terms on the right hand side could be equal to zero when the T sector firm is financially constrained (\( D^j_t = 0 \)). We show in Figure 9 that equity price dynamics are driven by changes in the domestic discount factor\(^\text{16}\). The stacked bar decomposes the influence of each variable in the observed percentage deviation in the T sector equity price at time \(-1\). At time 0 the model predicts zero dividends. The stacked bar at time 1 illustrates the impact coming from the percentage change in the discounted dividends received at time 1.

**Figure 9: Decomposition of T sector equity price deviation**

---

Notes: The stacked bar decomposes the observed percentage deviation of time \(-1\) T sector equity price when the consolidated government deviates from a laissez faire equilibrium. The dark colored stacked bar refers to the equity price drop observed at time \(-1\); the light color stacked bar highlights the impact of the dividends received at time 1.

\(^{16}\)For the purpose of this exercise we chose a small deviation from laissez-faire, corresponding to 1% decrease in the T sector equity price, so as to maintain the linear approximation accurate.
A.4 Alternative Policies

This paper focused on a very specific instrument through which an economy can promote investment. In this section we compare the outcomes from foreign reserves accumulation with those from alternative policies. Specifically we review the role of sectoral taxes and subsidy and of foreign aid. What are the implications of these policies for the real exchange rate, investments and welfare?

A.4.1 Sector-Specific Taxes and Subsidies

We analyze the role of sectoral taxes and subsidies. This policy represents a first best choice for the consolidated government since it does not involve any intertemporal distortion of the agents’ consumption, and it directly targets the source of inefficiency in the tradable sector. We consider a sector specific tax (τ_N^t > 0) on the labor hired by the N sector and we assume that the proceeds from this same tax are then rebated as a subsidy (τ_T^t < 0) to labor hired by the T sector.

The first order conditions of the non-tradable good firm becomes

\[ p_t(1 - \gamma)A_t^N \left( N_t^N \right)^{-\gamma} = (1 + \tau_t^N)w_t \]

Similarly, in the case of the tradable good firm:

\[ (1 - \alpha)A_t^T K_t^\alpha \left( N_t^T \right)^{-\alpha} = (1 + \tau_t^T)w_t \]

We assume that the pair of taxes/subsidies (τ_T^t, τ_N^t) is revenue-neutral so that

\[ \tau_t^T w_t N_t^T + \tau_t^N w_t N_t^N = 0, \quad \text{or} \]

\[ \frac{\tau_t^N}{\tau_t^T} = - \frac{N_t^T}{N_t^N} \]
The tax on the N sector is levied one period before the realization of the TFP shock; the receipts from the sectoral tax are then used to subsidize labor in the T sector during that same period.

In Figure 10 we show the results for the case of a 10% tax rate on the non-tradable sector firm. The main difference from a foreign reserves accumulation policy is that the real exchange rate initially appreciates as a consequence of the tax on the N sector firm. In terms of welfare, a more appreciated real exchange rate has in general a positive impact on the consumption of tradables and a negative impact on the consumption of non-tradables. A tax/subsidy scheme raises a trade-off for cohorts “-3,-2,-1” between higher current and future consumption of tradables, due to significant initial gains in wages and a lower current level of consumption of non-tradables, which follows from the initial appreciation.

**A.4.2 Foreign Aid**

We analyze the macroeconomic and welfare effects of foreign aid. This extension is particularly interesting for the case of China, which was a recipient of World Bank foreign aid until the year 2000, when it was eventually deemed credit worthy and therefore not anymore eligible. As already described in the introduction, recent research emphasizes the “dutch-disease” of foreign aid, which naturally arises from the decrease in the relative profitability of the tradable sector that follows from an exchange rate appreciation. However, there is no overall consensus; for example, Galiani et al. (2014) finds a positive causal effect from foreign aid on growth using a large sample of countries beneficiaries of the World Bank foreign aid program.

We consider an environment where the consolidated government is receiving foreign aid as a one period loan, in the amount of $A_{t+1}$ at the exogenous interest rate $i^*$. The economy is initially in a laissez-faire equilibrium and we allow the consolidated government to contemporaneously rebate the amount lump sum to the workers in equal size (young and middle aged private agents). The consolidated government then repays the foreign loans using a lump sum tax on workers (both young and middle aged in proportion to their
Notes: Each variable refers to the percentage deviation from laissez faire. Productivity shock happens at time 0. The tax rate $\tau_t^N = 0.1$.

population share), which is levied one period after the loan.

In Figure 11 we show the results. As expected foreign aid appreciates the real exchange rate at time $-1$ and leads to lower production of tradable goods. Also, foreign aid reduces the amount of investments at time $-1$. In terms of welfare, a foreign aid policy seems to generally benefit current generations of workers at the expense of future generations: while “Cohort-2” achieves higher welfare throughout their lifetime, “Cohort-1” only achieves higher welfare when young and “Cohort0” largely suffer throughout his lifetime due to lower wages and repayments.
Figure 11: Impact of Foreign Aid (subsidies to firms)

Notes: Each variable refers to the percentage deviation from laissez faire. Productivity shock happens at time 0 and foreign aid is set to 5% of GDP.