Macroprudential Policies in a Heterogeneous Agent Model of Housing Default

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(Link to the latest version)

Abstract

This paper uses a heterogeneous agent model of housing default to study how the effectiveness of macroprudential policies changes under different income and house price specifications. When calibrated to match the observed default choices of households during the financial crisis, the model has clear implications for the kind of macroprudential policies that will be more effective in different circumstances. When income shocks are large, restrictions on the loan-to-value ratio are more effective in reducing defaults, while when house price shocks are large, the default rate is more responsive to changes in payment-to-income limits. These results are an implication, filtered through the model, of the well-known double trigger fact: In the Great Recession, defaulting households tended to be those who were both seriously underwater and had experienced a substantial shock to income.

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1 Introduction

In the wake of the housing crisis of 2007-08, several countries have started implementing a variety of macroprudential policies to mitigate systemic risks arising from the housing sector (Lim et al. (2011), Claessens and Kodres (2014))\(^1\). During the crisis, these risks culminated in the form of higher defaults that had the potential of bringing down the entire financial system (Blinder (2013)). Consequently, a growing number of countries have mandated their central banks, or other regulatory authorities, to design and implement macroprudential policies. However, in spite of their increasing popularity, there is very limited quantitative theory to back the choice of macroprudential policies. This paper’s contribution towards filling this gap in the literature is twofold. Firstly, it builds a structural heterogeneous agent model with micro-foundations that can be used to study the outcomes of implementing different macroprudential policies. Secondly, the paper provides analytical results on how the effectiveness of macroprudential policies in reducing defaults changes under different income and house price specifications.

For policymakers to design effective macroprudential policies, it is imperative to have a good understanding of how these policies perform under different income and house price specifications. Depending on the kind of macroprudential policy in place, not only do income and house prices play a central role in determining a household’s borrowing decision, they also have an impact on a household’s default decision. The macroprudential policies studied in this paper are the loan-to-value (LTV) and payment-to-income (PTI) rules. Both these rules are the predominant macroprudential policies employed by countries to manage overall housing credit in the economy (IMF et al. (2016)); however, each rule operates through different channels (Greenwald (2018)). An LTV rule sets minimum requirements on down payments that depend on the prevalent house prices. In contrast, a PTI rule limits borrowing based on the burden that mortgage payments put on a household’s income level.

This paper finds that both the degree of income heterogeneity and house price fluctuations play a non-trivial role in determining the effectiveness of macroprudential policies in reducing defaults. When the size of income shocks increases, the effectiveness of an LTV rule in filtering out households with the highest ex-post probability of default increases. Defaulting households in the model are the ones who are not just seriously

\(^1\)IMF (2018), Mitra (2016), and Darbar and Wu (2015) provide an overview of the macroprudential policies being used by different countries.
underwater, but who also experience a substantial shock to their income. Under an LTV rule, the default risks primarily arise from low income households, who get access to large mortgage balances by meeting the LTV requirement, but the high debt burden makes them susceptible to a bad income and house price shock. Under this household behavior, when the size of income shocks is large, the effectiveness of an LTV rule in reducing the default rate increases. This is because an LTV rule sets minimum down-payment requirements, which become increasingly difficult for low-income households to meet when the size of income shocks is large. The households who can afford the downpayment and become homeowners are better poised to absorb shocks to both their income and house prices.

If instead of the income shocks, house price shocks are large, the default rate is more responsive to changes in PTI limits. Under a PTI rule, the default risks primarily arise from high income households who get access to large mortgage balances by meeting the PTI requirement, but are subsequently hit with a bad income shock. A PTI rule operates by limiting the debt burden that periodic mortgage payments put on a household’s income. This means that when the size of the house price shocks is large, for a given income level, a household cannot borrow more than what they would in the baseline. However, they have to put up a larger downpayment if the prices are above average and vice versa. Since under a PTI rule the main risks arise from high income households leveraging up very high when houses are expensive, a tightening of credit conditions is more effective at reducing defaults by requiring these high risk households to build larger equity buffers.

The key to matching the observed default decisions is the model’s ability to reproduce the well-known double trigger fact, i.e. in the Great Recession, defaulting households tended to be those who were both seriously underwater and had experienced a substantial shock to income (Bhutta et al. (2010)). The model features that produce these outcomes are built into the micro foundations of homeowners. For homeowners, default is costly. Not only do defaulters face a utility cost upon defaulting, they are also forced out of the housing market for the remainder of their life. In addition, staying a homeowner is appealing to households for multiple reasons. An owner-occupied house provides households higher utility compared to the utility they derive from rental housing. Since households face uninsurable income risk and there is no unsecured borrowing, owner-occupied housing also smoothes a household’s housing consumption. Moreover, owned houses are assets that have a potential for capital gains. These factors combined prevent
a household from defaulting, even when they are underwater.

While the micro foundations of homeowners determine the characteristics of households with the highest likelihood of default, the micro foundations of renters, in conjunction with the macroprudential policies, determine the characteristics of households who become homeowners. Under different income and house price specifications, macroprudential policies differ from each other in the distribution of homeowners that they filter through into the housing market. These homeowners are consequently exposed to a varying degree of default risks, depending on the mortgage choices they made, the amount of liquid assets they have, and their income level. Renters who intend to buy a house have access to a multitude of long-term mortgages; however, macroprudential policies limit the size of the maximum mortgage balance. With an LTV rule, only the house prices determine the mortgage limit and with a PTI rule, the income level also becomes relevant.

In order to accurately solve for the housing decision rules in a computationally feasible manner, this paper also introduces a new modeling technique for solving optimization problems with both discrete and continuous choices (DC models). Since the housing tenure is a discrete state variable, and liquid resources and mortgage balance are continuous state variables, the model in this paper is a DC model. DC models are highly non-linear and the accuracy of the solution can depend on the density of the grid space on which the model is solved. The modeling technique, which relies on endogenizing housing search effort, minimizes the computational burden arising from the non-linearities in DC models. This allows for a dense grid to be set in regions of the state space with the highest degree of non-linearity, which is essential for accurately solving the homeowners’ default and selling decisions. Another positive outcome of the technique is that it allows the model to capture the buyers’ and sellers’ time-on-market. This is because the housing search effort that households make is endogenously determined by the additional value that they get from switching their housing tenure.

The analysis in this paper is partial equilibrium in nature. Literature uses pecuniary externalities that arise from financial frictions (Bianchi (2011), Davila and Korinek (2018)) to rationalize limits on borrowing. This paper, on the other hand, sets the utility cost from default exogenously and assumes that the macroprudential policymaker’s goal is to reduce the default rate. The policymaker can achieve this goal by tightening credit conditions through various macroprudential policies. The simplifying assumptions pro-
vide tractability, which leads to very clear implications for the effectiveness of macroprudential policies under various income and house price specifications. In building these insights, this is also the first study to highlight in a structural heterogeneous agent setting the importance of income heterogeneity and house prices in determining the effective of macroprudential policies. Calibrated to micro-level data, the model replicates aggregate household default and homeownership rates very well. This gives the model solid foundations upon which a more complex general equilibrium model can be built.

1.1 Related literature

Due to the lack of harmonized data on defaults and limited time-series data on the outcomes of macroprudential policies, the empirical literature has found mixed results on the performance of macroprudential policies. While some studies find LTV and PTI rules to be effective macroprudential tools, others find the opposite or mixed results. Carreras et al. (2018), Akinci and Olmstead-Rumsey (2018), Cerutti et al. (2017b) use cross-country evidence to find that macroprudential policies have been effective in containing risks arising from rising housing credit and house prices. In contrast, Ono et al. (2016), using real estate registry data in Japan, find that caps on high LTV ratios are ineffective macroprudential tools in containing risks. Kuttner and Shim (2016), using data from 57 countries, find that PTI rules are effective macroprudential tools, but LTV rules are less effective in times of growing asset values. Adrian and Liang (2018) also note the mixed results for the effectiveness of these tools.

This paper is related to a growing literature that builds structural models of housing with micro foundations. Some of the main distinctive features of the model are that the mortgage contracts are long-term debt contracts and have an option to default. Berger et al. (2018) and Guerrieri and Lorenzoni (2017) construct heterogeneous agent housing models to study household responses to changes in house prices and credit conditions, respectively. Both these studies, however, do not have a default option. Guren et al. (2018) also construct a housing model with long-term mortgages and an option to default to study the housing wealth effect, but the utility cost from default in their model is set so high that the households never actually default. It is also important for mortgages to be long-term contracts, unlike the short-term contracts in Guerrieri and Lorenzoni (2017). This is because short-term mortgage contracts can lead to forced deleveraging in response to short-run fluctuations in house price, which makes it hard for the model to match the observed household behavior.
An important feature of this paper is that it models the implications of household heterogeneity for contract selection. Household characteristics, like wealth and income, have implications for not just contract selection, but also the pool of risky borrowers. A closely related paper to this one is Campbell and Cocco (2015), which constructs a heterogeneous agent model to study the effect of differences in LTV and loan-to-income on households’ foreclosure decisions. However, they do not consider the implications of household heterogeneity for contract selection. Similarly, Ganong and Noel (2018) construct a partial equilibrium life-cycle model with housing to study whether a borrower’s short-term constrains govern their response to long-term obligations. Households in their model; however, start off as homeowners with a fixed amount of mortgage balance. Greenwald (2018) also builds a general equilibrium model of housing and uses it to study the performance of LTV and PTI rules. The model in that paper though is a representative agent model, without an option to default. Our experience through the housing crisis has shown that representative agent models provide us with a narrow understanding of the risks that could be brewing due to the behavior of households on the tail ends of the income and wealth distribution.

This paper is closely related and complementary to Kaplan et al. (2017) and Garriga and Hedlund (2018). Both these papers are general equilibrium models of housing that are used to study the boom-bust cycle during the housing crisis. Kaplan et al. (2017) focus on the role played by households’ expectations during the boom-bust episode. Garriga and Hedlund (2018), on the other hand, study how arrangements in the mortgage market impact the dynamics of the housing boom-bust episode and the economy. Garriga and Hedlund (2018) also study the implications of macroprudential policies on the boom-bust cycle. This study, although partial equilibrium in nature, complements these two studies by providing new insights into how heterogeneity of households could impact the effectiveness of macroprudential policies in reducing default. Moreover, the qualitative results are more broadly applicable, not just to the boom-bust cycle.

The remainder of this paper is organized as follows. Section 2 discusses the institutional background that provides the guidelines for building a model of housing default. Section 3 describes the key components of the model. Section 4 highlights the computational innovations made to solve discrete-continuous choice models efficiently. Section 5 outlines the calibration and model fit. Section 6 discusses the housing decision rules. Section 7 analyzes the performance of alternative macroprudential policies under differ-
ent income and house price specifications. Section 8 concludes. The appendix contains the detailed housing problem and describes the perfect foresight solution.

2 Background

This section highlights some of the key features of the housing market and provides insights into the behavior of homeowners during the housing crisis. These insights are used to build the model. This section also discusses the institutional arrangements that could give rise to different income and house price specifications.

The housing crisis of 2007-08 impacted different regions within the US with a varying degree of intensity (Holly et al. (2010)). Some regions, such as California and Florida, experienced high house price volatility, while others regions, such as Indiana and Montana, did not. These diverse behaviors of house prices were even more pronounced on an international scale (Figure 1a). In countries like the US, the UK, and Spain, house prices experienced large swings around 2007. In contrast, in other countries, like Japan and Germany, house prices have been relatively stable over the last two decades. The countries that saw large swings in house prices also witnessed increased default rates. Just looking at the house price dynamics, though, only provides us with a partial picture of the factors that led to the increased default rates.

A boom-bust in house prices alone is not enough to explain the increase in default rates that was observed during the crisis. High leverage is also needed to drive foreclosures in the housing markets when house prices fall (Mian and Sufi (2018), Mian et al. (2017)). If a household is not highly levered, they can always sell their house rather than default. As households observe a rise in house prices, extrapolated expectations (Bordalo et al. (2018)) lead them to believe that house prices would rise even further. This means that the expected capital gains from homeownership increases, which results in a higher demand for houses. Since houses are expensive, households take up mortgages to buy a house. Eventually, when house prices fall, the highly levered households default. These features can be seen in Figure 1b, which shows that a drop in house prices under elevated levels of leverage lead to a rise in foreclosure rates. The foreclosure rates fall when leverage recedes or when house prices begin to rise.

Empirical evidence, however, suggests that being underwater is not a sufficient condition for households to default, they also need to be hit with a bad income shock (Foote
et al. (2008), Herkenhoff (2012)). This is referred to as the “double-trigger” that is needed for households to default. Bhutta et al. (2010), using mortgage data for households who purchased homes in 4 different states in 2006, find that 80% of households who default in their sample, default because of negative equity combined with a bad income shock. Thus, income heterogeneity is an essential component that is needed to understand households’ default behavior. Studies find that there is a high degree of heterogeneity in income variability across different countries (Acemoglu (1997)). A country with poor social insurance mechanisms in place would lead to a high degree of income heterogeneity and vice versa. Amongst the developed world, the US would correspond to a country with a high degree of income heterogeneity, compared to a country like Denmark, which has a low degree of income heterogeneity.

This study focuses on LTV and PTI rules as alternative macroprudential policies. These are two of the main macroprudential policies that are observed in countries around the world and they hold particular relevance for the US. In order to reform the financial regulation in the US, the Congress passed the Dodd-Frank Act, which became effective in 2010. The law instituted “Ability-to-Repay (ATR)” rules, which were rules that required mortgage lenders to make a good-faith effort to determine that the borrower was likely to be able to pay back the loan. Operationally, the ATR rules imposed limits on loan-to-value (LTV) and payments-to-income (PTI), and also included other measures to reduce the likelihood of a borrower defaulting.
3 Model

3.1 Households

A home purchase makes up the biggest investment for most households. This is in spite of the fact that a house is an illiquid asset due to both the transaction and search costs associated with buying or selling a house. In addition, house prices, particularly at the individual level, can be highly volatile (Case and Shiller (1989)) and unlike most financial assets, this idiosyncratic risk cannot be diversified, as a house is indivisible. High house price volatility, combined with illiquidity, implies that individual houses as an investment are not very attractive (Piazzesi and Schneider (2016)). Yet, roughly two-thirds of households in the US are homeowners. This can be explained by certain features of an owner-occupied house that make it an appealing asset. Owner-occupied housing provides housing services in excess of those provided by a rental house of similar size and, as an asset, it has the potential of capital gains. For households concerned about income risk, homeownership also allows households to smooth their housing consumption. These features are incorporated into the model.

In the model, a household’s housing tenure can take up four different states: renters, homeowners, tenants, and defaulters. Households in each tenure state derive utility from consuming non-durable/non-housing consumption goods, \(c_t\), and housing services, \(s_t\). The aggregate consumption bundle has a Cobb-Douglas form, \(\tilde{c}_t = c_t^\alpha s_t^{1-\alpha}\). This form is supported by a variety of micro-oriented studies (Berger et al. (2018)). The utility that a household derives from consuming this bundle has a CRRA form

\[
    u(\tilde{c}_t) = \frac{(c_t^\alpha s_t^{1-\alpha})^{1-\rho}}{1-\rho}.
\]

Households start each period with liquid assets, \(m_t\). Renters, in addition to non-housing consumption, also pay for rental services. Rental services can be adjusted costlessly and are assumed to have the same unit cost as a unit of non-housing consumption. This simplifying assumption does not affect our main results, since during most of the pre-crisis years the house price-to-rent ratio has primarily been driven by variations in house prices and rental prices have roughly grown at the same pace as the prices of non-durable consumption goods. Each period, renters decide whether they will stay as renters in the next period (rr) or if they will become homeowners (rh). If they choose
to become homeowners in the next period, the house must be purchased in the current period; however, it is only made available to the household in the next period. At the time of purchase, buyers choose how much mortgage debt, $b_t$, to take up and, in the baseline model, face a loan-to-value (LTV), or equivalently, a downpayment constraint. Homebuyers also incur lump-sum transaction costs $\kappa_p$.

Homeowners, in contrast to renters, only pay for non-durable/non-housing consumption and derive housing services that are proportional to the size of the house, $h_t$:

$$s_t = \zeta h_t. \quad (2)$$

Homeowners in each period decide whether they will stay as homeowners in the next period (hh), sell their house and become tenants in the next period (ht), or default on their mortgage (hd). The reason homeowners who sell their house are called tenants and not renters is because tenancy is a self-absorbing state. Once homeowners leave the housing market, they are not allowed to buy a house again. Similar to tenancy, defaulting is also a self-absorbing state. This assumption is made to simplify the numerical solution (explained in section 4). The simplifying assumption, however, does not significantly affect the relevant outcomes of the model, since the average duration of a household’s life is 30 years and they are replaced by renters who have the option to become homeowners.

Houses for purchase are only available in one size, $h$, and their depreciation is offset by maintenance costs, $\delta_m$, which are paid by homeowners who decide to continue being homeowners (hh). Continuing homeowners also have to make mortgage payments, which are determined by the size of their mortgage balance at the beginning of the period. Households who sell the house pay off their mortgage balance and receive proceeds from the sale of the house. Even though the household receives the proceeds from the sale of the house in the current period, the house is only made unavailable in the next period. While selling the house, the household also incurs lump-sum transaction cost $\kappa_s$. In case the household defaults, they walk away from the house and the mortgage balance, but they incur a utility cost, $\chi$, in the period in which they default. This cost captures the non-financial costs that inhibit a household from defaulting. As noted earlier, if a homeowner decides to discontinue being a homeowner (ht or hd), they are not allowed to participate in the housing market again in the future.
There are also search frictions in the housing market, which means that renters and homeowners stay in their original housing tenure state, unless they exert an effort to switch the state. Search effort entails convex utility costs and the amount of effort exerted depends on the excess utility flow that the household would receive from successfully making the switch.

Denoting the discount factor by $\beta$ and the probability of survival by $D$, households maximize their infinite horizon expected discounted utility:

$$u(\tilde{c}_t) + \mathbb{E}_t \left\{ \sum_{n=1}^{\infty} (\beta D)^n u(\tilde{c}_{t+n}) \right\}.$$  \hfill (3)

The infinite horizon problem can also be thought of as a finite horizon problem in which the agents have perfect altruism towards their descendants. In this context, the aggregate utility is called the dynastic utility. The choice of an infinite horizon problem means that the model abstracting away from its life-cycle features, which are studied in detail by Berger et al. (2018), Oswald et al. (2017), Wong (2017), and Yao et al. (2015), among others. The detailed households’ problem in Bellman form is outlined in Appendix A.

### 3.2 Income and House Prices

Each period, households also face an idiosyncratic risk of getting unemployed. While unemployed, households receive unemployment benefits $\nu$. The transition matrix for the employment state is given by,

$$\pi_{i,t} = \begin{pmatrix} \pi_{e,c} & \pi_{e,u} \\ \pi_{u,c} & \pi_{u,u} \end{pmatrix}.$$  \hfill (4)

Conditional on being employed, households face uninsurable income risk. Income, $Y_{i,t}$, follows a process with both a persistent and a transitory component. The process is specified as

$$Y_{i,t} = \exp \{ y_{i,t}^p + \theta_{i,t} \},$$  \hfill (5)

$$y_{i,t}^p = \gamma_y y_{i,t-1}^p + \psi_{i,t}.$$  \hfill (6)

where $\gamma_y$ is the persistence parameter of the persistent component of income, and $\psi_{i,t}$
and $\theta_{i,t}$ represent the persistent and transitory shocks to income, respectively. The variances associated with these shocks are denoted by $\sigma_{\psi}^{2}$ and $\sigma_{\theta}^{2}$.

House prices, $P_t$, are assumed to follow a first order autoregressive process:

\[
P_t = \exp\{p_t\},
\]

\[
p_t = \gamma_p p_{t-1} + \xi_t,
\]

where $\gamma_p$ governs the persistence of house prices and $\sigma_{\xi}^{2}$ denotes the variance of the house price shocks.

### 3.3 Financial Markets

Both renters and homeowners can deposit their liquid assets at banks at a fixed interest rate $r$. Neither renters nor homeowners have access to short-term borrowing, which means that their end of period liquid assets, $a_t$, cannot fall below 0. Homeowners, however, have access to long-term mortgage debt at a fixed interest rate $r_m$. Unlike studies that have short term mortgage debt and forced deleveraging (Guerrieri and Lorenzoni (2017), Berger et al. (2018)), households in this model are not forced to delever in response to negative house price shocks. Mortgage balance follows a constant geometric amortization schedule, with a half-life of 15 years (details in Appendix C). A constant geometric amortization schedule, instead of a constant amortization schedule, is used since the optimization problem is an infinite horizon problem. Each period’s mortgage payments, as a function of the beginning-of-period mortgage balance $b_t$, are given by

\[
\lambda(b_t) = \left(1 - \frac{(1/2)^{1/n}}{1 + r_m}\right) b_t,
\]

where $n = 15$ at the annual frequency.

At the time of purchase, households face an LTV constraint, which limits the maximum mortgage debt at origination, and is given by

\[
b_t \leq \eta^{LTV} p_t h_t.
\]
An LTV constraint can equivalently be thought of as a downpayment constraint. The minimum downpayment at the time of purchase is max \( \{0, (1 - \eta^{LTV}) p_i h\}\).

The baseline model uses an LTV constraint; however, later in the paper it is replace by a payment-to-income, or PTI, constraint. The PTI constraint imposes a borrowing limit at the time of home purchase based on the debt burden that periodic mortgage payments put on a household’s income. The burden is assessed using the household’s persistent income in that period, rather than the total income. Lenders typically adjust a loan applicant’s income for “special factors”, which can be interpreted as an adjustment towards the persistent income (FannieMae (2018), FreddieMac (2016)). The PTI constraint is given by

\[ \lambda(b_{t+1}) \leq \eta^{PTI}.y^p_t \] (11)

### 3.4 Search frictions

Households have to exert search effort to change their housing tenure. This search effort entails convex utility costs. Normalizing the search effort to equal the probability of a successful search, denote this normalized effort by \( \varepsilon^r_t \) and the utility cost associated with the effort by \( \sigma^r(\varepsilon^r_t) \). For renters choosing to become homeowners, denote the search effort and the utility cost associated with making that search effort by \( \varepsilon^h_t \) and \( \sigma^h(\varepsilon^h_t) \), respectively. The value function and the consumption function for the renter are given by:

\[ V^r_t(m_t, y^p_t, p_t) = \max_{\varepsilon^r_t \in [0,1]} (1 - \varepsilon^r_t)V^{rr}_t(m_t, y^p_t, p_t) + \varepsilon^r_t V^{rh}_t(m_t, y^p_t, p_t) - \sigma^r(\varepsilon^r_t) \] (12)

and

\[ c^r_t(m_t, y^p_t, p_t) = (1 - \varepsilon^r_t)c^{rr}_t(m_t, y^p_t, p_t) + \varepsilon^r_t c^{rh}_t(m_t, y^p_t, p_t). \] (13)

The cost function \( \sigma^h \) associated with the renter’s search effort \( \varepsilon^h_t \) is specified as

\[ \sigma^h(\varepsilon^h_t) = S^h \left( \varepsilon^h_t - \log \left( \frac{1}{1 - \varepsilon^h_t} \right)^{(1 - \varepsilon^h_t)} \right), \] (14)

where \( S^h \) is a smoothness parameter that captures the degree of frictions in the home purchase market. This cost function implies that \( \sigma^h(0) = 0 \) and \( \lim_{\varepsilon^h_t \to 1} \sigma^h(\varepsilon^h_t) = S^h. \)
The effort function associated with this cost function (details in Appendix D) is given by

$$
\varepsilon^h_t(m_t, y^p_t, p_t) = 1 - e^{-\frac{V^{rh} - V^{rr}}{S^h}}. (15)
$$

This implies that the degree of search effort exerted by the renters is a function of the additional utility flow that the renter gets from successfully making the switch. The greater the utility flow that a renter gets from becoming a homeowner, the greater the effort they exert in the housing search market. Since effort is non-negative, when the value of staying a renter exceeds the value of becoming a homeowner, the effort that a renter exerts is zero.

Unlike renters, homeowners can switch their housing tenure into two states: they can either sell their house and become tenants, or they can default. For better exposition, the homeowner’s search problem is split into two sequential search problems. In the first stage, the homeowner faces a choice between staying a homeowner or selling the house. The search effort that a homeowner makes to sell the house, rather than continue occupying it, is denoted by $\varepsilon^s_t$. The utility cost, $\sigma^d(\varepsilon^s_t)$, associated with this effort has the same functional form as the search cost function for the renter, but here the search friction parameter is given by $S^d$. In the second stage of the search problem for the homeowner, households have to choose between defaulting or facing the search problem described in the first stage of housing search. The search effort is denoted by $\varepsilon^d_t$ and the associate search cost and search friction parameter are denoted by $\sigma^d(\varepsilon^d_t)$ and $S^d$, respectively.

The value function and the consumption function for the homeowner are given by

$$
V^h_t(m_t, b_t, h_t, y^p_t, p_t) = \max_{\varepsilon^d_t, \varepsilon^s_t \in [0,1]^2} -\sigma^d(\varepsilon^d_t) + \varepsilon^d_t V^{hd}_t(m_t, b_t, h_t, y^p_t) 
+ (1 - \varepsilon^d_t) \left[ \varepsilon^s_t V^{ht}_t(m_t, b_t, h_t, y^p_t, p_t) + (1 - \varepsilon^s_t) V^{hh}_t(m_t, b_t, h_t, y^p_t, p_t) - \sigma^d(\varepsilon^d_t) \right] 
$$

and

$$
\text{(16)}
$$
respectively.

The effort functions for the homeowners are given by:

$$
\epsilon^t_t = 1 - e^{-\frac{\psi^h_t - \psi^{hh}_t}{s^t}}
$$

(18)

and

$$
\epsilon^d_t = 1 - e^{-\frac{\psi^{hd}_t - \psi^{hh}_t + (1 - \epsilon^t_t)\psi^{hh}_t - \sigma_t(\psi^h_t)}{s^d}}
$$

(19)

4 Computation

In the housing model outlined above, the beginning-of-period liquid assets \( m_t \) and mortgage balance \( b_t \) are treated as continuous state variables. This makes it possible to use the Endogenous Gridpoint Method (EGM) (Carroll (2006)), which offers greater computational efficiency compared to traditional root-finding solution methods. Using the EGM, one can feasibly construct a dense grid in the regions of the state space where the degree of non-linearity is the highest. These are also the regions where households typically switch their housing tenure states. Having a dense state space in these regions is particularly important for accurately capturing the households’ default behavior. However, the default decision, like the decision to buy or sell a house, is a discrete choice and in models with both discrete and continuous choices (DC models), standard EGM can produce suboptimal solution points. These points need to be identified and removed from the final solution and this additional step deteriorates the computational efficiency of the standard EGM.

To reduce the severity of this problem, Iskhakov et al. (2017) use exogenous taste shocks to smooth out the marginal value functions in their DC problem. In contrast, the model in this paper achieves the smoothness of the marginal value functions through the endogenous housing search mechanism. In the context of housing, this is more economi-
cally plausible compared to exogenous taste shocks. The search effort gets endogenously
determined by the severity of the kinks in the value function. This makes it possible to
smooth out the expected marginal value function enough that the EGM does not pro-
duce any suboptimal points at all. Moreover, in contrast to taste shocks, housing search
captures a very significant feature of the housing market: the \textit{time on market}. In addition
to the transaction costs, the time on market associated with buying and selling a house
is another feature of houses that makes them illiquid.

Details of the computational methods used to solve the model are provided in Appendix
E.

5 Calibration

Following Kaplan et al. (2017) and Garriga and Hedlund (2018) the model is calibrated
to match cross-sectional features of the U.S. housing market prior to the housing boom.
The model is calibrated at an annual frequency to data from 1998, as it also aligns with
a Survey of Consumer Finance release that year. Some of the parameters are calibrated
externally, while others are jointly calibrated internally to match key housing data mo-
ments.

As is standard in the literature, the coefficient of relative risk aversion $\rho$ is set to 2. In
the default model estimated by Campbell and Cocco (2015), the lifespan of a household
is set at 20 years. However, in this model the average lifespan of a household is set to
30 years. For the purpose of the modeling exercise in this paper, a lifespan of 30 years
is befitting, since 30 years is also the prevalent duration of a mortgage. A lifespan of 30
years implies a survival probability of 0.975.

Following Krueger et al. (2016), the income process is calibrated using annual PSID after-
tax earnings data, after removing age, education, and time effects. This yields estimates
of 0.9695 for the persistence parameter $\gamma_y$, 0.0384 for $\sigma_{\hat{\psi}}^2$, and 0.0522 for $\sigma_{\hat{\theta}}^2$. The transition
probabilities of unemployment are calibrated by converting Shimer (2005)’s quarterly
estimates to annualized values. This leads to an employment-to-unemployment transition
probability, $\pi_{e,u}$, of 0.1 and an unemployment-to-employment transition probability,$\pi_{u,e}$, of 0.99. Also following Shimer (2005), the unemployment benefits, $v$, are set
to 40% of the average labor income. The housing search parameters, $S_h$, $S^l$, and $S^d$ are set
to 0.25, 0.65, and 0.25, respectively. These values provide appropriate smoothing of the
expected marginal value functions in the problem\(^2\).

The generic house size, \( h \), is set to 3.4. This means that the house is 3.4 times the mean labor income (Mitman et al. (2017)). The persistence of house prices, \( \gamma_p \), is set to 0.988 using the Case-Shiller house price index, and the standard deviation of house prices is set to 0.162 following Campbell and Cocco (2015). The lump-sum transaction costs at the time of purchase, \( \kappa_p \), and sale, \( \kappa_s \), of a house are set to 1\% of the mean house price. Houses are assumed to not depreciate as homeowners have to incur maintenance costs that offset the depreciation rate. Following Kaplan et al. (2017), these maintenance costs, \( \kappa_m \), are set to 2\% of the mean house price.

The risk-free interest rate \( r \) is set to 3\%. The fixed mortgage rate \( r_m \) is set to 7\%, using the average 30-year fixed rate mortgage in the US over 1998. In the baseline model, the LTV limit \( \eta_{\text{LTV}} \) is set at 95\%, which is 10 percentage points higher than the average CLTV prior to the housing boom (UrbanInstitute (2017)). Since homeowners in this model are not allowed cash-out refinancing, or to borrow against their home equity, this adjustment is made to take into account the fact that in reality, cash-out refinancing can lead to many cases of new mortgages with LTVs greater than 100\% (Mitman et al. (2017)). This baseline LTV limit falls between the LTV limits of 90\% set by Campbell and Cocco (2015) and 125\% set by Mitman et al. (2017). In the baseline model, the PTI constraint is non-binding, however, in the alternate specification in which the LTV rule is replaced with a PTI rule, \( \eta_{\text{PTI}} \) is set to 45\%\(^3\), following the Seller/Servicer Guidelines provided by Freddie Mac (FreddieMac (2016)).

The remaining parameters of the model are calibrated jointly by targeting the annual foreclosure rate, the homeownership rate, and the median LTV. Using the National Delinquency Survey, the model targets an annual foreclosure rate of 1.6\%. The U.S. Census Bureau’s data for homeownership provides a target homeownership rate of 67.8\%. For the median LTV, the 1998 SCF provides a target of 0.62. Table 2 shows the resulting estimates for the discount factor, \( \beta \), the share of non-housing services in the aggregate consumption bundle, \( \alpha \), the housing services flow from owned housing, \( \zeta \), and the default utility cost, \( \chi \). All these estimates are within the range of estimates produced by

---

\(^2\)In the annual specification, these parameters do not result in time-on-market moments that match the data well. A quarterly specification would be needed for that.

\(^3\)This is almost equivalent to the PTI limit of 43\% set under the Qualifying Mortgage condition of the Ability-to-Repay Rules.
Table 1: Model parameters (external calibration)

<table>
<thead>
<tr>
<th>Preferences</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Coeff. of relative risk aversion</td>
<td>2</td>
<td>Standard in literature</td>
</tr>
<tr>
<td>$D$</td>
<td>Survival probability</td>
<td>0.975</td>
<td>30yrs lifespan</td>
</tr>
<tr>
<td>Income</td>
<td>Persistence of pers. income shock</td>
<td>0.9695</td>
<td>Krueger et al. (2016)</td>
</tr>
<tr>
<td>$\gamma_y$</td>
<td>Pers. income shock</td>
<td>0.9695</td>
<td>Krueger et al. (2016)</td>
</tr>
<tr>
<td>$\sigma^2_y$</td>
<td>Var. of pers. income shock</td>
<td>0.0384</td>
<td>Krueger et al. (2016)</td>
</tr>
<tr>
<td>$\sigma^2_\psi$</td>
<td>Var. of trans. income shock</td>
<td>0.0522</td>
<td>Krueger et al. (2016)</td>
</tr>
<tr>
<td>$\pi_{e,u}$</td>
<td>Probability of unemployment</td>
<td>0.1</td>
<td>Shimer (2005)</td>
</tr>
<tr>
<td>$\pi_{u,e}$</td>
<td>Probability of re-employment</td>
<td>0.99</td>
<td>Shimer (2005)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Unemp. benefit (rel. to $\bar{y}$)</td>
<td>0.4</td>
<td>Shimer (2005)</td>
</tr>
<tr>
<td>Housing</td>
<td>House size (rel. to $\bar{y}$)</td>
<td>3.4</td>
<td>Mitman et al. (2017)</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>Persistence house price shock</td>
<td>0.9880</td>
<td>Case-Shiller HPI</td>
</tr>
<tr>
<td>$\sigma^2_\zeta$</td>
<td>Var. of house price shock</td>
<td>0.0262</td>
<td>Campbell and Cocco (2015)</td>
</tr>
<tr>
<td>$\kappa_p$</td>
<td>Purch. trans. cost (rel. to $\bar{p}$)</td>
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<td></td>
</tr>
<tr>
<td>$\kappa_s$</td>
<td>Sale trans. cost (rel. to $\bar{p}$)</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>$\kappa_m$</td>
<td>Maint. cost (rel. to $\bar{p}$)</td>
<td>0.02</td>
<td>Kaplan et al. (2017)</td>
</tr>
<tr>
<td>$S^h$</td>
<td>Search param. for rh</td>
<td>0.25</td>
<td>Author’s calculations</td>
</tr>
<tr>
<td>$S^t$</td>
<td>Search param. for ht</td>
<td>0.65</td>
<td>Author’s calculations</td>
</tr>
<tr>
<td>$S^d$</td>
<td>Search param. for hd</td>
<td>0.25</td>
<td>Author’s calculations</td>
</tr>
<tr>
<td>Fin. conditions</td>
<td>Risk-free rate</td>
<td>3%</td>
<td>Av. 1yr Treasury</td>
</tr>
<tr>
<td>$r$</td>
<td>Mortgage rate</td>
<td>7%</td>
<td>Av. 30yr FRM in 1998</td>
</tr>
<tr>
<td>$r^{LTV}$</td>
<td>LTV limit</td>
<td>0.95</td>
<td>UrbanInstitute (2017)</td>
</tr>
<tr>
<td>$\eta^{PTI}$</td>
<td>PTI limit</td>
<td>0.45</td>
<td>FreddieMac (2016)</td>
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</table>

Table 2: Model parameters (joint calibration)

<table>
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<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.975</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Share of non-housing</td>
<td>0.70</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Housing services from owned house</td>
<td>1.03</td>
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<tr>
<td>$\chi$</td>
<td>Default utility cost</td>
<td>0.65</td>
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</table>

Table 3: Model fit

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreclosure rate (%)</td>
<td>1.60</td>
<td>1.62</td>
</tr>
<tr>
<td>Home ownership rate (%)</td>
<td>67.8</td>
<td>71.2</td>
</tr>
<tr>
<td>Median LTV</td>
<td>0.62</td>
<td>0.59</td>
</tr>
<tr>
<td>Median net worth (rel. to median after-tax income)</td>
<td>1.56</td>
<td>2.78</td>
</tr>
<tr>
<td>Median mortgage debt (rel. to median after-tax income)</td>
<td>2.14</td>
<td>2.62</td>
</tr>
</tbody>
</table>
various studies.

Table 3 shows that the model fits the key data moments quite well. The model does a very good job of matching the average annual foreclosure rate, the homeownership rate, and the median LTV. Matching the default rate so well is a result of the microfoundations of households and due to the solution technique employed, which allows the model to capture the behavior of low wealth households very accurately. Not only does the model generated foreclosure rate of 1.62% compare well with the empirical estimate of 1.60%, it also fares better than Kaplan et al. (2017) model-generated annual foreclosure rate of 0.4%. The model also does a fairly reasonable job matching the untargeted moments such as the median net worth (liquid assets + house value - mortgage debt), relative to median after-tax income, and the mean mortgage debt, relative to median after-tax income. The model partly produces a higher median net worth compared to the data, because it generates a slightly higher homeownership rate of 71.2% and a slightly lower median LTV of 0.59, compared to the data.

6 Housing decisions

Before analyzing the effectiveness of different macroprudential policies under various income and house price specifications, a discussion of the housing decision rules for renters and homeowners in the model is necessary. Figure 2 shows the renter’s converged housing decision rule under an LTV rule. On the y-axis, the zero-line indicates the renter’s decision to stay as a renter. Above the zero-line, a higher value indicates a larger mortgage balance at origination and below the zero-line indicates a renter’s decision to buy a house without any mortgage debt. Since the subplots in Fig 2 are made conditional on the house price levels, the y-axis normalized by the price level would give the LTV level at mortgage origination.

Due to the way in which an LTV rule operates, a household’s borrowing limit is entirely determined by the level of house prices. Under low house prices, households have access to lower mortgage levels, regardless of their income, and vice versa. An LTV limit is also equivalent to setting minimum requirements on downpayments that depend on the price of the house. When house prices are low (Fig 2a), households need to accumulate fewer liquid assets to buy a house. However, low house prices also limit the maximum available mortgage size at origination. In contrast, at higher house price levels (Fig 2b), renters have access to higher levels of mortgage balance. Households with higher liquid
assets opt for lower mortgage balance at origination and very rich households choose to buy a house by paying the full price, without any mortgage.

Renters’ housing decisions also vary with the level of persistent income. For a given level of house prices and liquid assets, low income homebuyers are levered at least as high as the high income homebuyers, which makes them more vulnerable to bad income or house price shocks. The LTV constraint implies that for a particular house price level, regardless of the income level, all renters have access to similar mortgage contracts. Since renters with low persistent income also have lower prospects of future income, they choose to lever up as much as they can and buy a house to smooth their housing consumption. Even at higher levels of liquid assets, low income households choose to lever up more than high income households, for a given level of liquid assets. This continues to hold even when house prices are high. High leverage, however, makes households a lot more vulnerable to default risks.

Under a PTI rule, since borrowers are constrained by the burden that mortgage payments put on their income, for a given price level, households with lower income have
access to lower mortgage balances and vice versa. The behavior of renters is more interesting when we compare low income versus high income households. The y-axes of the plots in Fig 3 normalized by the income level are proportional to the PTI level at mortgage origination, since the interest rate is fixed. Fig 3a shows that under a PTI rule of $\eta^{PTI} = 0.45$, low income households, regardless of the price level, borrow up to the same PTI limit. This means that when house prices are high, low income households accumulate a significant equity buffer before buying a house. This is very different from the behavior of low income households under an LTV rule, in which case low income households lever up more when house prices rise. The behavior of high income renters remains roughly the same as it is under an LTV rule. The only difference is that high income households, when house prices are low, borrow slightly more under a PTI rule than they borrow under an LTV rule.

Homeowners’ housing decisions, in contrast to the renters’ decisions, are invariant to macroprudential policies, since macroprudential policies only operate through limiting prospective homebuyers’ borrowing. Nonetheless, understanding the homeowners’ decision rules is important to identify the environment under which households default.
Figure 4 shows the housing decision rule for low-income and high income homeowners when house prices are high. These results have two main outcomes. Firstly, when house prices are high, the housing decisions are unaffected by the household’s income level. Secondly, when homeowners have ample liquid resources, they choose to stay as homeowners. Homeowners sell their house if their liquid assets are too low. These liquidity constrained households sell rather than default because when house prices are high, homeowners have positive equity for the debt levels considered in this study. Thus, by selling their house, households can extract their equity and relax their budget constraint.

When house prices fall and the debt level is low, homeowners’ housing decision is still not sensitive to the income level. However, when the debt level is high, the decisions become highly sensitive to the income level. Fig 5 shows that when households are not highly levered, at low levels of liquidity, they sell their house. This decision does not change with the level of income. In contrast when debt level is high, under low house prices, the likelihood of default increases significantly when the household income is low. Fig. 5a shows that the range of liquid asset wealth along which households default, spreads out when the income level is low. Thus, a homeowner defaults if (i) they are are underwater and (ii) they receive a negative income shock that pushes their liquid assets very low. Moreover, the fewer the liquid resources that a household has, the smaller is the magnitude of the bad income shock that is needed to push the household below the
threshold beyond which they default. This is precisely the double-trigger fact that was observed during the Great Recession.

7 Effectiveness of macroprudential policies

The baseline income and house price processes are calibrated using the US data. Varying the LTV limit changes the default rate in the economy. This exercise provides the baseline effectiveness of the LTV rule in changing the default rate. In the counterfactuals, the size of the income and house prices are doubled by doubling the variances of these processes, separately, and the LTV limit is varied. The differences in the slopes of the default rate locus produced by this experiment transparently reveal how the effectiveness of an LTV rule changes as the size of the income and house price shocks increases. A similar experiment is repeated by replacing the LTV rule with a PTI rule to evaluate how the size of income and house price shocks affect the performance of a PTI rule.

7.1 Baseline performance of LTV rules

Although the focus of this study is defaults, this sub-section also evaluates how some of the other major housing moments change as the LTV limit is varied. This is to provide the reader with deeper insights into the functioning of the model. Figure 6 shows that a tightening of the LTV limit (i.e. moving left on the x-axis) leads to a decline in the default
rate. This is due to two main reasons. Firstly, a tightening of the LTV limits reduces the overall leverage in the economy and consequently the default rate as well. Secondly, since a reduction in the LTV limit is equivalent to requiring a higher downpayment, a tightening of the LTV rule makes it increasingly difficult for low income households to buy a house. These are households with the highest likelihood of default, as shown in the homeowners’ decision rule in Fig 5a, and filtering them out of the housing market reduces the default rate.

In the baseline setting, almost all the defaults are concentrated between the LTV limit of 90% and 100%. This is primarily due to the fact that there is only a single-sized house in the model and the households’ impatience factor, $\beta$, is homogenous. The impatience factor influences the households’ borrowing and default decisions. Since in this model it can only take up a single value, the calibration leads to an estimate of $\beta$ which generates a large density of households concentrated close to the borrowing limit. These are also the households who are the most likely to default if they are hit with bad income and house price shocks. A model with a variety of house sizes and heterogeneous $\beta$s would not only be able to match the distribution of LTV much better, it would also expand the range of LTV limits over which households default.

Although a tightening of credit conditions reduces the default rate, since houses become less affordable, the homeownership rate declines in a monotonic manner as well. Unlike the homeownership rate, the relationship between the LTV limit and the net worth of households is non-monotonic. When households default, they lose their home equity.
and, consequently, their net worth takes a hit. A tightening of the LTV limit means that households now have to accumulate higher levels of liquid assets to pay for the higher down-payment, before they can buy a house. As a result of this, households have a larger equity share in their house. This means that they have a larger capacity to absorb negative income and house price shocks, and their likelihood of default falls. Tightening the LTV limit from a very loose level leads to an increase in the median net worth of households. This is due to both a lower default rate and households accumulating greater liquid assets. A borrowing limit that is too tight, however, can lead to excessive reduction in the homeownership rate, which can lead to a negative impact on the households’ median net worth. Figure 6 illustrates this point. We can see that the median net worth of households as a function of the LTV limit produces a hump shape. A tightening of the LTV limit, from a very loose value of 100%, initially leads to an increase in the median net worth of households; however, further tightening results in a decline in the median net worth for the reasons outlined above. In the baseline calibration, the LTV limit of 95% is in the region where a moderate tightening of the LTV limit leads to an increase in the median net worth of households.

7.2 Performance of LTV rules under alternative income and house price shocks

To study the performance of the LTV rule under larger income shocks, the variance of persistent income, $\sigma^2_{\psi}$, is doubled. The blue line in Fig 7a plots the performance of the LTV rule in this economy. Fig 7a shows that in an economy with larger income shocks, the performance of an LTV rule in reducing the default rate increases. This is demonstrated by the steeper slope of the default rate line under more volatile income. To study the performance of the LTV rule under larger house price shocks, the variance of the house price shocks, $\sigma^2_{\xi}$, is doubled. The yellow line in Fig 7a shows that when house price shocks are large, the effectiveness of an LTV rule in reducing defaults declines, as demonstrated by the lower slope of the default rate line.

Under an LTV rule, the default risks primarily arise from low income households, who get access to large mortgage balances by meeting the LTV requirement, but are subsequently hit with a bad house price shock. When income shocks are large, the income distribution becomes wider, which means that low income households have smaller incomes compared to low income households when the shock size is small. Under these
circumstances, a tightening of the LTV rule is more effective at filtering out from the housing market households with the highest ex-post risk of defaulting. This is because it is relatively harder for low income households to accumulate the higher downpayment when income shocks are large. To put it differently, when income shocks are large, it becomes easier for a tightening in the LTV rule to identify and exclude from the housing market households who have the highest likelihood of defaulting. Consequently, the effectiveness of the LTV rule in reducing defaults improves.

When house price shocks are larger, the ability of a tightening in the LTV rule to exclude high risk households worsens, demonstrated by the more moderate slope of the yellow line in Fig 7a. This is because under large house price shocks, when house prices rise, they rise to higher levels, giving both the low and high income households access to much larger mortgage balances. Even though the rise in house prices is met with a proportionate rise in the downpayment, the fact that households have access to larger mortgages means that those households who do become homeowners have a much higher debt burden. Compared to high income households, the debt burden is higher for low income households who are able to meet the LTV requirement to become homeowners. Consequently, if house prices drop, even a moderate shock to income can lead them to default. Even though tightening of the LTV limit still reduces the default rate, because homebuyers have to put in additional downpayment, the reduction is more moderate under large house price shocks. The LTV rule does not consider the debt burden that buying a house puts on households and, therefore, cannot identify and exclude from the housing market the households who are able to meet the downpayment requirement but are nonetheless highly burdened by the mortgage payments. In this model, an LTV rule in which $\eta_{LTV}$ increases with the size of the house prices could potentially minimize this weakness of the standard LTV rule.

7.3 Performance of PTI rules

In this section, the LTV rule is replaced with a PTI rule. The dotted line in Fig 7b shows that under the baseline income and house price specifications, a tightening of credit conditions that is achieved by decreasing the PTI limit leads to a decline in the default rate. Compared to when an LTV rule is operational, the results show some salient differences under a PTI rule. Another thing to note is that since the income distribution is discretized to finite points, for extremely lax PTI limits, all households have access to the housing market without requiring any downpayment. Unless the PTI limit is tightened
aggressively, the default rate is unresponsive to the credit conditions.

Under a PTI rule, the default risks primarily arise from high income households who at the time of mortgage origination get access to large mortgage balances, but are subsequently hit with a bad income shock. When the size of house price shocks is large, the PTI rule becomes more effective at reducing defaults. This is demonstrated by the steeper slope of the yellow line in Fig 7b. Under a PTI rule, to maintain the debt burden for a given income level, households have to put up a larger downpayment if house prices are above average and vice versa. Since with a PTI rule in place, the default risks primarily arise from high income households buying expensive houses, when credit conditions are tightened, a PTI rule is more effective at forcing the high risk households to build larger equity buffers. This translates to a higher effectiveness of the PTI rule in reducing defaults when house prices shocks are large.

In contrast, the blue line in Fig 7b shows that when the size of income shocks is increased by doubling the variance of the persistence income process, $\sigma^2_{\phi}$, the effectiveness of the PTI rule in reducing the default rates deteriorates. A negative consequence of larger income shocks under a PTI rule is that now high income households have access to much larger mortgage balances, regardless of the price level, and there is no explicit requirement for higher downpayments. Even though a tightening of the PTI rule still reduces the defaults that could potentially arise from low income households entering the housing market, it is unable to effectively filter out risky high income households from the mortgage market. Consequently, a PTI rule’s effectiveness in reducing the default rate

Figure 7: Effectiveness of LTV and PTI rules in reducing defaults under different income and house price specifications.
deteriorates. In this model, a PTI rule in which $\eta^{PTI}$ increases with the size of the income level could potentially minimize this weakness of the standard PTI rule.

8 Conclusion

This paper studies how the effectiveness of LTV and PTI rules in reducing defaults changes under various income and house price specifications. The results suggest that policymakers, when designing macroprudential tools, must also consider the household-level income and house price dynamics of the economy. In an economy with large income shocks, an LTV rule serves as a more effective macroprudential tool in reducing the default rates. In contrast, in an economy with large house price shocks, a PTI rule fares better. A prevalent macroprudential policy, which is not considered in this study, is the debt-to-income (DTI) rule. DTI rules are popular in the European countries, which predominantly have adjustable rate mortgages. A study of whether a DTI rule insulates better against interest rate shocks is left for future research. The modeling framework introduced in this paper can also be extended to address more complex questions, such as the welfare implications of macroprudential policy.
References


Appendices

A Detailed Problem

In this section, I solve the problem of the different sub-types of households separately.

A.1 Note on state variables

Since this paper employs the endogenous gridpoint method in solving our model, a distinction needs to be made between the pre-decision and post-decision state variables. I denote the pre-decision liquid assets by $m_t$ and the post-decision liquid assets by $a_t$. For homeowner, there are 2 more state variables: the level of housing stock, $h_t$, and the mortgage balance, $b_t$. Although there is no uncertainty involved in the dynamic equations linking these pre- and post-decision state variables, just for clarity of thought, I will denote the post-decision housing stock (mortgage debt) variable, which will also be the choice variables for new homeowners, by $h_t$ ($h_t$), and the pre-decision housing stock (mortgage debt) variable by $h_t$ ($b_t$).

The dynamic equation for liquid assets is given by

$$m_{t+1} = (1 + r)a_t + y_{t+1},$$

where $a_t$ is the end-of-period assets after all consumption and housing decisions have been made.

The dynamic equation for housing stock is given by

$$h_{t+1} = h_t,$$

where $h_t$ is a choice variable for new homeowners and thereafter it evolves according to

$$h_t = h_t,$$

unless the homeowner is moving out of the house ($h_t$, $h_t$), in which case $h_t = 0$.

The dynamic equations for mortgage balance is given by
\[ b_{t+1} = (1 + r^m) b_t, \] (23)

where \( b_t \) is a choice variable for new homeowners and thereafter it evolves according to

\[ b_t = b_t - \lambda(b_t), \] (24)

where \( \lambda(b_t) \) is the mortgage function that gives the mortgage payments for a given beginning-of-period mortgage balance.

Before I proceed to solving the detailed problem, I must note that I will denote the variables that are defined as a function of the end-of-period state variables using a gothic font. That is, the end-of-period value function is denoted by \( v \), and the end-of-period consumption function (or the consumed function) is denoted by \( c \).

A.2 Renter’s problem

A.2.1 rr

The intertemporal optimization problem for (rr), in Bellman form, is given by

\[
V^R_t(m_t, y^p_t, p_t) = \max_{c_t, s_t} u(\tilde{c}_t) + \beta \mathbb{E}_t \{ V^R_{t+1}(m_{t+1}, y^p_{t+1}, p_{t+1}) \}
\]

s.t.

\[
a_t = m_t - c_t - s_t \geq 0
\]

\[
m_{t+1} = (1 + r)a_t + y_{t+1}
\]

Defining total consumption expenditure, \( \hat{c}_t \), as \( \hat{c}_t = c_t + s_t \) and using the Cobb-Douglas functional form for \( \tilde{c}_t \), the intratemporal optimality conditions imply that

\[ c_t = \alpha \hat{c}_t \] (25)

and

\[ s_t = (1 - \alpha) \hat{c}_t. \] (26)

Moreover,

\[ \tilde{c}_t = \alpha^\alpha (1 - \alpha)^{1-\alpha} \hat{c}_t \] (27)
Hence, for renters, the maximization problem can be written in terms of the total consumption expenditure, $\hat{c}_t$ and the non-durable and housing expenditure can be determined by eqs. 25 and 26:

$$V_{r}^{rr}(m_t, y^p_t, p_t) = \max_{\hat{c}_t} u(\hat{c}_t) + \beta \mathbb{E}_t \{ V_{r}^{r}(m_{t+1}, y^p_{t+1}, p_{t+1}) \}$$

s.t.

$$a_t = m_t - \hat{c}_t \geq 0$$
$$m_{t+1} = (1+r)a_t + y_{t+1}$$

**A.2.2  rh**

The intertemporal optimization problem for (rh), in Bellman form, is given by

$$V_{r}^{rh}(m_t, y^p_t, p_t) = \max_{\hat{c}_t, h_t, b_t} u(\hat{c}_t) + \beta \mathbb{E}_t \{ V_{h}^{h}(m_{t+1}, h_{t+1}, b_{t+1}, y^p_{t+1}, p_{t+1}) \}$$

s.t.

$$a_t = m_t - \hat{c}_t - [p_t h_t - b_t] \geq 0$$
$$m_{t+1} = (1+r)a_t + y_{t+1}$$
$$h_{t+1} = h_t$$
$$b_t \leq \eta^{LTV} p_t h_t$$
$$\lambda_{t+1}(b_{t+1}) \leq \eta^{PTI} y^p_t$$
$$b_{t+1} = (1+r^m)b_t$$

The renters’ problem yields:

$$\frac{\partial V_{r}^{r}(m_{t+1}, y^p_{t+1}, p_{t+1})}{\partial m_{t+1}} = \alpha^\alpha (1-\alpha)^{1-\alpha} u'(\hat{c}_{t+1}). \quad (28)$$
A.3 Homeowner’s problem

There are 3 sub-types of homeowners: (hh), (ht), and (hd).

A.3.1 hh

The intertemporal optimization problem for (hh), in Bellman form, is given by

$$V_{hh}^t(m_t, h_t, b_t, y_t, p_t) = \max_{\tilde{c}_t} u(\tilde{c}_t) + \beta \mathbb{E}_{t+1} \{ V_{hh}^{t+1}(m_{t+1}, h_{t+1}, b_{t+1}, y_{t+1}, p_{t+1}) \}$$

s.t.

$$a_t = m_t - c_t - \lambda(b_t) \geq 0$$
$$m_{t+1} = (1 + r) a_t + y_{t+1}$$
$$h_{t+1} = h_t$$
$$b_{t+1} = b_t - \lambda(b_t)$$
$$s_t = \zeta h_t$$

A.3.2 ht

The intertemporal optimization problem for (ht), in Bellman form, is given by

$$V_{ht}^t(m_t, h_t, b_t, y_t, p_t) = \max_{\tilde{c}_t} u(\tilde{c}_t) + \beta \mathbb{E}_{t+1} \{ V_{ht}^{t+1}(m_{t+1}, y_{t+1}, p_{t+1}) \}$$

s.t.

$$a_t = m_t - c_t + [p_t h_t - b_t] \geq 0$$
$$m_{t+1} = (1 + r) a_t + y_{t+1}$$
$$s_t = \zeta h_t$$
A.3.3 hd

The intertemporal optimization problem for \((hd)\), in Bellman form, is given by

\[
V^h_{t}(m_t, h_t, b_t, y^p_t) = \max_{c_t} u(\tilde{c}_t) - \chi + \beta \mathbb{E}_t \{ V^d_{t+1}(m_{t+1}, y^p_{t+1}) \}
\]

s.t.

\[
a_t = m_t - c_t \geq 0
\]

\[
m_{t+1} = (1 + r)a_t + y_{t+1}
\]

\[
s_t = \zeta h_t
\]

The homeowners’ problem yields:

\[
\frac{\partial V^h_{t+1}(m_{t+1}, h_{t+1}, b_{t+1}, y^p_{t+1}, p_{t+1})}{\partial m_{t+1}} = u'(\tilde{c}_{t+1}) \frac{\partial \tilde{c}_{t+1}}{\partial c_{t+1}}. \tag{29}
\]

A.4 Tenant and Defaulter’s problem

Since the tenant or defaulter always remains a tenant or defaulter, respectively, \(V^t_{t,d} = V^{t,dd}_{t,d}\). Their intertemporal optimization problem, in Bellman form, is similar and is given by

\[
V^t_{t,d}(m_t, y^p_t) = \max_{\tilde{c}_t} u(\tilde{c}_t) + \beta \mathbb{E}_t \{ V^d_{t+1}(m_{t+1}, y^p_{t+1}) \}
\]

s.t.

\[
a_t = m_t - \tilde{c}_t \geq 0
\]

\[
m_{t+1} = (1 + r)a_t + y_{t+1}
\]

The tenants’ and defaulters’ problem yields:
\[ \frac{\partial V_{t+1}(m_{t+1}, y_{t+1}^p)}{\partial m_{t+1}} = \alpha^\alpha (1 - \alpha)^{1-\alpha} u' (\tilde{c}_{t+1}) \] (30)

### A.5 Consumption functions of sub-types

#### A.5.1 Tenant and Defaulter

It is possible to define a function

\[
\psi_{t}^{t,d}(a_t, y_t^p) = \beta \mathbb{E}_t \{ V_{t+1}^{t,d}((1+r)a_t + y_{t+1}, y_{t+1}^p) \} 
\] (31)

that returns the expected $t+1$ value for the tenant or defaulter associated with ending period $t$ with assets $a_t$, having received persistent income $y_t^p$.

The non-housing consumption function for a tenant or defaulter is then given by:

\[
c_t(a_t, y_t^p) = \alpha \left( \frac{\psi_{t}^{t,d}(a_t, y_t^p)}{(\alpha^\alpha (1 - \alpha)^{1-\alpha})^{1-\rho}} \right)^{-1/\rho} 
\] (32)

#### A.5.2 Renters

It is possible to define a function

\[
\psi_t(a_t, y_t^p, p_t) = \beta \mathbb{E}_t \{ V_t^{t,r}((1+r)a_t + y_{t+1}, y_{t+1}^p, p_{t+1}) \} 
\] (33)

that returns the expected $t+1$ value for a household ending period $t$ as a renter with assets $a_t$, having received persistent income $y_t^p$, and house price shock $p_t$. 

---

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The non-housing consumption function for \( rr \) is given by:

\[
t_t(a_t, y^p_t, p_t) = \alpha \left( \frac{\psi_t'(a_t, y^p_t, p_t)}{(\alpha^\alpha (1 - \alpha)^{1-\alpha})^{1-\rho}} \right)^{-1/\rho}
\]

(34)

\((rh)\)

It is possible to define a function

\[
\psi_t^h(a_t, h_t, b_t, y^p_t, p_t) = \beta \mathbb{E}_t \{ V_{t+1}^h((1+r)a_t + y_{t+1}, h_{t+1}, b_{t+1}, y^p_{t+1}, p_{t+1}) \}
\]

(35)

that returns the expected \( t + 1 \) value for a household ending period \( t \) as a homeowner with assets, \( a_t \), housing stock, \( h_t \), mortgage balance, \( b_t \), having received persistent income \( y^p_t \), and house price shock \( p_t \).

The non-housing consumption function for \( rh \) is given by:

\[
c_t(a_t, h_t, b_t, y^p_t, p_t) = \alpha \left( \frac{\psi_t^h(a_t, h_t, b_t, y^p_t, p_t)}{(\alpha^\alpha (1 - \alpha)^{1-\alpha})^{1-\rho}} \right)^{-1/\rho}
\]

(36)

\((hh)\)

A.5.3 Homeowners

The non-housing consumption function for \( hh \) is given by:

\[
c_t(a_t, h_t, b_t, y^p_t, p_t) = \left( \frac{\psi_t^h(a_t, h_t, b_t, y^p_t, p_t)}{(\zeta h_t(h_t))^{(1-\alpha)(1-\rho)}} \right)^{-\frac{1}{\alpha-\alpha p-1}}
\]

(37)
where \(\varrho = \alpha^{a_\rho - 1} (1 - \alpha)^{(1-a)(1-\rho)}\).

\((ht, hd)\)

The non-housing consumption function for \(ht\) or \(hd\) is given by:

\[
\zeta_t(a_t, h_t(h_t), b_t(b_t), y_t^p, p_t) = \left( \frac{\psi_t^{dt}(a_t, y_t^p)}{\left(\zeta h_t(1-a)(1-\rho)\right)} \right)^{\frac{1}{1-\alpha}}.
\] (38)

**A.6 The method of endogenous gridpoints**

Denote by \(\bar{a}_i^t\) the grid of end-of-period assets (greater than their lower bound). Each element \(i\) of the grid is denoted by \(a_{t,i}\). Similarly, a grid is set over persistent income \((\bar{y}_t^i)\), transitory income \((\bar{b}_t)\), house prices \((\bar{p}_t)\), housing \((\bar{h}_t)\), and mortgage debt \((\bar{b}_t)\).

Each \(\{a_{t,i}, y_{t,j}^p\}\) pair is associated with some marginal valuation as of the end of period \(t\) for tenants, and defaulters, i.e. \(\psi_t^{dt}(a_{t,i}, y_{t,j}^p)\). Each \(\{a_{t,i}, y_{t,j}^p, p_{t,k}\}\) pair is associated with some marginal valuation as of the end of period \(t\) for renters, i.e. \(\psi_t^r(a_{t,i}, y_{t,j}^p, p_{t,k})\). Similarly, each \(\{a_{t,i}, h_{t,j}, b_{t,k}, y_{t,l}^p, p_{t,m}\}\) pair is associated with some marginal valuation as of the end-of-period \(t\) for homeowners, i.e. \(\psi_t^h(a_{t,i}, h_{t,j}, b_{t,k}, y_{t,l}^p, p_{t,m})\). Using the expressions for the end-of-period consumption functions solved above, it is then trivial to solve for the value of \(\tilde{c}\) that yields the appropriate marginal valuation for the 7 sub-types of households, \(s\).

With mutually consistent values of \(\tilde{c}_{t,i,j}^s\) and \(\{a_{t,i}, y_{t,j}^p\}\) for \(s \in \{tt, dd\}\), we can find the \(m_{t,i,j}\) that corresponds to them. The \(m_t^i\) gridpoints are endogenous and we can generate a set of \(m_{t,i,j}\) and \(c_{t,i,j}^s\) pairs that can be interpolated in order to yield the consumption interpolation function \(c^s(m_t^i, \bar{y}_t^i)\).

For \(s \in \{rr\}\), with mutually consistent values of \(c_{t,i,k}^s\) and \(\{a_{t,i}, y_{t,j}^p, p_{t,k}\}\), we can find the \(m_{t,i,k}\) that corresponds to them. This results in the consumption function for continuing renters, given by \(c^{rr}(\bar{m}_t, y_{t}^i, p_t)\). For \(s \in \{rh\}\), with mutually consistent values of \(c_{t,i,k,l,m}^h\) and \(\{a_{t,i}, h_{t,j}, b_{t,k}, y_{t,l}^p, p_{t,m}\}\), we can find the \(m_{t,i,k,l,m}\) that corresponds to them. \(c^{rh}(m_t, \bar{y}_t^i, \bar{p}_t)\) then is the consumption function that forms an upper-envelope over
the different \{h_{t,i},b_{t,k}\} pairs.

Similarly, with mutually consistent values of \(c^{hh}_{t,i,j,k,l,m}\) and \(\{a_{t,i},h_{t,j},b_{t,k},y^p_{l,i,j},p_{t,m}\}\), we can find \(\{m_{t,i,j,k,l,m},h_{t,i,j,k,l,m},b_{t,i,j,k,l,m}\}\) that correspond to them. This results in the consumption function for continuing homeowners, given by \(c^{hh}(\vec{m}_{t,i}, \vec{h}_{t,i}, \vec{b}_{t,i}, \vec{y}^p_{l,i}, \vec{p}_{t,i})\). Finally, for households ceasing to be homeowners, \(s \in \{ht,hd\}\), with mutually consistent values of \(c^s_{t,i,j,k,l,m}\) and \(\{a_{t,i},h_{t,j}(h_{t,j}),b_{t,k}(b_{t,k}),y^p_{l,i,j},p_{t,m}\}\), we can find the \(m_{t,i,j,k,l,m}\) that corresponds to them. Note that in this case, the exogenous grid is over \(h_t\) and \(b_t\) rather than \(h_{ht}\) and \(b_{ht}\). This gives us the consumption function \(c^s(\vec{m}_{t,i}, \vec{h}_{t,i}, \vec{b}_{t,i}, \vec{y}^p_{l,i}, \vec{p}_{t,i})\).

For each of the 7 sub-types, \(m_t\) is calculated using

\[
m_t = \begin{cases} 
  a_{t,i} + a^{-1}q^d_t(a_{t,i},y^p_{l,i,j}) & t, d \\
  a_{t,i} + a^{-1}q^r_t(a_{t,i},y^p_{l,i,j},p_{t,k}) & rr \\
  a_{t,i} + a^{-1}q^r_t(a_{t,i},y^p_{l,i,j},p_{t,k}) + \left(p_{t,m}h_{t,i,j} - b_{t,k}\right) + \kappa_p & rh \\
  a_{t,i} + a^{-1}q^h_t(a_{t,i},h_{t,j},b_{t,k},y^p_{l,i,j},p_{t,m}) + \left(p_{t,m}h_{t,i,j} - b_{t,k}\right) + \kappa_m & hh \\
  a_{t,i} + a^{-1}q^h_t(a_{t,i},h_{t,j},b_{t,k},y^p_{l,i,j},p_{t,m}) - \left[p_{t,m}h_{t,i,j} - b_{t,k}\right] + \kappa_s & ht \\
  a_{t,i} + a^{-1}q^d_t(a_{t,i},h_{t,j},b_{t,k},y^p_{l,i,j},p_{t,m}) & hd
\end{cases}
\]

For \(hh\)

\[
b_{t,i} = \frac{1 + r_m}{(1/2)^{1/n}} b_{t,i}
\]

\[
h_{t,i} = h_{t,i}.
\]

### A.7 Conditional value functions of sub-types

Each of the gridpoint pairs are also associated with some valuation as of the end of period \(t\), i.e. \(v^d_t(a_{t,i},y^p_{l,i,j})\), \(v^r_t(a_{t,i},y^p_{l,i,j},p_{t,k})\), and \(v^h_t(a_{t,i},h_{t,j},b_{t,k},y^p_{l,i,j},p_{t,m})\). Given the consumption functions, we can also calculate the conditional value functions \(v_t\). Denoting the \(\{h_{t,i},b_{t}\}\) pair associated with the upper-envelope for the \(rh\) problem by \(\{h, b\}\), the expressions for the conditional value functions are given by

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For interpolation, however, we use the transformed value functions, \( \Lambda^s_i(\ldots) \), which is given by

\[
\Lambda^s_i(\ldots) = u^{-1}(v^s_i(\ldots)).
\]

### A.8 Period-T solution and T-1 adjustment

For faster convergence of the solution, we use the converged tenant’s solution as the terminal period T’s solution. Denote the converged consumption function as \( c^t_{\infty}(m_t, y^p_t) \) and the converged value function as \( V^t_{\infty}(m_t, y^p_t) \). Since we are using the converged tenant’s solution as the period T solution for all types of households, we have to make an adjustment to the liquid assets of households who are homeowners at the end of period T-1 to compensate them for the house that will be revoked from them. One way of making this compensation is by giving them funds that are equivalent to the home equity they would have had at the beginning of period T. This adjustment is made between the end of period T-1 and the beginning of period T.
The T-1 problem for \( rh \) is given by:

\[
V^{rh}_{T-1}(m_{T-1}, y_{T-1}^p, p_{T-1}) = \max_{c_{T-1}, s_{T-1}, y_{T-1}, h_{T-1}} u(c_{T-1}, s_{T-1}) + \psi^h_{T-1}(a_{T-1}, y_{T-1}^p),
\]

(39)

where

\[
\psi^h_{T-1}(a_{T-1}, y_{T-1}^p) = \beta \mathbb{E}_{T-1}\{V^l_{\infty}(m_T(a_{T-1}), y_T^p)\}
\]

\[
a_{T-1} = m_{T-1} - c_{T-1} - s_{T-1} - [p_{T-1}.h_{T-1} - b_{T-1}] - \kappa_p, \quad a_{T-1} \geq 0
\]

\[
m_T = R.a_{T-1} + y_T + [p_T.h_T - b_T]
\]

\[
h_T = h_{T-1}
\]

\[
b_{T-1} = \eta^{TV}.p_{T-1}.h_{T-1}
\]

\[
b_T = R^{m}.b_{T-1}
\]

The T-1 problem for \( hh \) is given by:

\[
V^{hh}_{T-1}(m_{T-1}, h_{T-1}, b_{T-1}, y_{T-1}^p, p_{T-1}) = \max_{c_{T-1}, \zeta, h_{T-1}} u(c_{T-1}, \zeta h_{T-1}) + \psi^h_{T-1}(a_{T-1}, y_{T-1}^p),
\]

(40)

where

\[
\psi^h_{T-1}(a_{T-1}, y_{T-1}^p) = \beta \mathbb{E}_{T-1}\{V^l_{\infty}(m_T(a_{T-1}), y_T^p)\}
\]

\[
a_{T-1} = m_{T-1} - c_{T-1} - \lambda_{T-1}(b_{T-1}) - \kappa_m, \quad a_{T-1} \geq 0
\]

\[
m_T = R.a_{T-1} + y_T + [p_T.h_T - b_T]
\]

\[
h_T = h_{T-1}
\]

\[
b_{T-1} = b_{T-1} - \lambda_{T-1}(b_{T-1})
\]

\[
b_T = R^{m}.b_{T-1}
\]

B  Perfect Foresight Solution and Method for Extrapolation

B.1  Tenant’s perfect foresight consumption function

I solve for the perfect foresight solution of the tenant’s problem without a liquidity constraint. The intertemporal optimization problem for \((tt)\), in Bellman form, is given by
\[ V_t^t(m_t, y_t^p) = \max_{\hat{c}_t} u(\hat{c}_t) + \beta \mathbb{E}_{t}\{ V_{t+1}(m_{t+1}, y_{t+1}^p) \} \]

s.t.

\[ a_t = m_t - \hat{c}_t \]
\[ m_{t+1} = R a_t + y_{t+1} \]

The Euler equation for this problem is given by

\[ u'(\hat{c}_t) = R \beta \mathbb{E}_{t}\{ u'(\hat{c}_{t+1}) \} \quad (41) \]

With perfect foresight and CRRA utility, this becomes

\[ \hat{c}_t^{-\rho} = R \beta \hat{c}_{t+1}^{-\rho} \quad (42) \]

This implies that

\[ \hat{c}_{t+1} = (R \beta)^{1/\rho} \hat{c}_t \quad (43) \]

Define, \( \mathbb{P} \equiv (R \beta)^{1/\rho} \) and \( \mathbb{P}_R \equiv \frac{(R \beta)^{1/\rho}}{R} \).

Then, the present discounted value of total consumption is given by

\[ PDV(\hat{c}_t) = \hat{c}_t + \frac{\hat{c}_{t+1}}{R} + \frac{\hat{c}_{t+2}}{R^2} + ... \]
\[ = \hat{c}_t + \mathbb{P}_R \hat{c}_t + \mathbb{P}_R^2 \hat{c}_t + ... \quad (44) \]

For the finite horizon problem, ending at period \( T \),

\[ PDV(\hat{c}_t) = \left( \frac{1 - \mathbb{P}_R^{T-t+1}}{1 - \mathbb{P}_R} \right) \hat{c}_t \quad (45) \]

The lifetime human wealth of the perfect foresight household, \( h_t \), is given by

\[ h_t = PDV(y_t) = PDV(\bar{y}) = \left( \frac{1 - (1/R)^{T-t+1}}{1 - (1/R)} \right) \bar{y} \quad (46) \]

From the intertemporal budget constraint (IBC) we have that,
\[ PDV(\hat{c}_t) = PDV(y_t) + m_t - y_t \]
\[ = PDV(\bar{y}) + m_t - \bar{y} \]
\[ = \bar{h}_t + m_t - \bar{y} \]  

(47)

Substituting from 45, the IBC yields

\[ \left( \frac{1 - \beta R^{T-t+1}}{1 - \beta R} \right) \hat{c}_t = \bar{h}_t + m_t - \bar{y} \]

This gives the **perfect foresight consumption function** for the tenant,

\[ \bar{c}_t(m_t) = \alpha \kappa_t \left( \bar{h}_t + (m_t - \bar{y}) \right), \]

(48)

where \( \alpha \) is the fraction of the total expenditure spent on non-durable consumption and \( \kappa_t = \left( \frac{1 - \beta R}{1 - \beta R^{T-t+1}} \right) \).

### B.2 Tenant’s perfect foresight value function

For tenants

\[ V_t(m_t) = \sum_{j=0}^{T-t} \beta^j u \left( \bar{c}_{t+j} \right) \]
\[ = \sum_{j=0}^{T-t} \beta^j u \left( \bar{P}^j \bar{c}_t \right) \]
\[ = \bar{c}_t^{1-\rho} \sum_{j=0}^{T-t} \beta^j u \left( \bar{P}^j \right) \]
\[ = \bar{c}_t^{1-\rho} \sum_{j=0}^{T-t} \left( \frac{\beta \bar{p}^{1-\rho}}{1-\rho} \right)^j \]
\[ = \left( \frac{1 - \left( \frac{\beta \bar{p}^{1-\rho}}{1-\rho} \right)^{T-t+1}}{1 - \left( \frac{\beta \bar{p}^{1-\rho}}{1-\rho} \right) } \right) u \left( \bar{c}_t(m_t) \right) = \frac{1}{\kappa_t^V} u \left( \bar{c}_t(m_t) \right), \]

where \( \kappa_t^V = \left( \frac{1 - \left( \frac{\beta \bar{p}^{1-\rho}}{1-\rho} \right)^{T-t+1}}{1 - \left( \frac{\beta \bar{p}^{1-\rho}}{1-\rho} \right)^{T-t+1}} \right). \)
Applying the $u^{-1}(.)$ operator to this yields:

$$\text{invV}_t(m_t) = u^{-1}(V_t(m_t))$$

$$= ((1 - \rho)V_t(m_t))^{\frac{1}{1 - \rho}}$$

### B.3 Transformation for extrapolation

In the EGM, we need to evaluate the gothic-\(v\) values for all points on the a-grid. For most of the points on the a-grid interpolation is used; however, for high values of a-grid, extrapolation might be needed. There are two problems involved with extrapolation at high values of a-grid that need to be addressed. Firstly, extrapolation at the highest values of a-grid can possibly lead to inaccurate results. Secondly, certain multidimensional interpolators do not extrapolate. In order to avoid these problems, I will transform the consumption and value functions in a way that allows us to get as close as possible to the true solution at large values of $m_t$.

In our model, at very high values of liquid assets, $m_t$, renters choose to stay as renters and homeowners choose to sell their house and become tenants. This is because homeowners only have access to single sized house and at very high values of $m_t$ that house does not provide enough housing services. In contrast to homeowners, renters and tenants can adjust their housing services freely without an upper-bound on the rental services. I use this feature of the model and the fact that the consumption functions are bounded above by the perfection foresight solution to extrapolate a transformed consumption function.

For clarity, I will suppress all the arguments of the consumption functions except for $m_t$. Denote the consumption function from the optimization problem by $c_t(m_t)$ and the perfect foresight consumption function by $\overline{c}_t(m_t)$. Now define the ratio of the two as

$$c_t(m_t) = \frac{c_t(m_t)}{\overline{c}_t(m_t)}.$$

We know that

$$\lim_{m_t \to \infty} c_t(m_t) = 1.$$ 

Define

$$\gamma_t(m_t) = \frac{1}{1 + e^{-m_t}}.$$
This implies that
\[ m_t = \log \left( \frac{y_t}{1 - y_t} \right). \]

\[ m_t \in (-\infty, \infty) \] and \( y_t \in (0, 1). \)

Now define
\[ \tilde{c}_t(y_t) = c_t(m_t(y_t)). \]

\[ \lim_{y_t \to 1} \tilde{c}_t(y_t) = 1. \]

Calling the interpolated version \( \tilde{c}_t \), we can get the consumption function \( c_t(m_t) \) by using
\[ c_t(m_t) = \tilde{c}_t(y_t(m_t)) \tilde{c}_t(m_t). \]

### C Geometric mortgage payments

I assume a geometric mortgage payment schedule in which each period’s mortgage payment, \( \pi_t \), is a fixed proportion \( \rho \) of the mortgage balance at the beginning of the period, i.e.
\[ \pi_t = \rho b_t \]  \tag{49} \]

Mortgage balance evolves according to
\[ b_{t+1} = (b_t - \pi_t)(1 + r_m). \]  \tag{50} \]

We need to determine the value of \( \rho \) such that the half-life of the mortgage balance is 15 years, i.e.
\[ b_{t+n} = \frac{1}{2} b_t \]  \tag{51} \]

where \( n = 15 \) at an annual frequency and \( n = 60 \) at a quarterly frequency.

We first substitute eq.(49) into eq.(50), which yields
\[ b_{t+1} = (b_t - \rho b_t)(1 + r_m) \]
Figure 8: Geometric mortgage payment schedule with a half-life of 15 years.

or

$$\frac{b_{t+1}}{b_t} = (1 - \rho)(1 + r_m)$$

(52)

This can be iterated forward to get

$$\frac{b_{t+n}}{b_t} = [(1 - \rho)(1 + r_m)]^n$$

(53)

Substituting eq.(51) into eq.(53), we get

$$\rho = 1 - \frac{(1/2)^{1/n}}{1 + r_m}.$$  

(54)

This expression does not depend on the time since mortgage origination, which means that we do not need an extra state variable that tracks the age of the mortgage. As can be seen in Figure.1, the mortgage balance has a half life of 15-years and unlike constant amortization, the annual mortgage payments decline with time.
D Detailed search problem

E Computation