Divestment Dynamics in U.S. and Canadian Asbestos Mining Industry

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Abstract

This paper empirically studies U.S. and Canadian asbestos mining firms’ divestment process when the asbestos industry faced a long-run decline in demand. I develop a dynamic oligopoly model of capacity adjustment and exit based on Ericson and Pakes (1995). I calibrate my model to a panel data of the U.S. and Canadian asbestos mining industry’s capacities and exit/stay status over 1968-2002, and evaluate the welfare effect of The Asbestos Mines and Mills Release Regulations enacted in Canada in 1990. I find that the 1990 Regulations decreased producers’ surplus by 0.37% and decreased consumers’ surplus by 0.22%, besides positive health effects. Specifically, through the channel of changing market structure, the Regulations led to a welfare gain of asbestos producers because it hastened the divestment process of the industry, resulting in an increase of 0.39% in industry’s profits.

Keywords: Declining industry, Divestment process, Dynamic oligopoly, Capacity-constrained Cournot games

JEL Code: L13, L72, D24, D43

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1 Introduction

When facing a long-run decline in demand, it is beneficial for the industry to reduce capacity and remove excess capital stock in order to save on fixed costs associated with running relevant production facilities to some extent. However, according to Ghemawat and Nalebuff (1990), capacity reduction is a public good because part of the business created by one firm’s capacity reduction will benefit all other firms in the market. Therefore, there is an incentive to free ride on other firms’ divestment, which can delay the divestment process of the industry compared to the solution that maximizes industry’s total profits.

The asbestos industry provides a relevant case to study firms’ divestment behavior when they experience a secular decline in demand. Asbestos used to be an important construction material but scientific research in the 20th century identified asbestos as a major health and environmental hazard, and leading cause of serious and fatal illnesses including lung cancer, asbestosis etc. Since the second half of the 20th century, demand for asbestos plummeted in U.S. and many other countries in the world.

In this paper, I study long-run firms’ divestment process in the North America asbestos mining industry. U.S. was a large consumer and importer of asbestos worldwide, with more than 90% of its consumption imported from Canada during 1950-2002. Canada was a large producer and a net exporter of asbestos worldwide; more than 90% of its production was exported after 1960s. The main destinations of its exports were America, Europe and Asia. Because of the decline in demand, export volume to U.S. decreased sharply since late 1970s and export volume to Europe declined significantly after 1980s. In the 1970s, Canadian firms started divestment by decreasing their capacity, closing mines or exiting the market. U.S. firms started to exit the market in the 1960s and by 2002 they had all exited the market.

This paper evaluates the welfare effect of the Asbestos Mines and Mills Release Regulations on the North America asbestos mining industry. In 1990, Canada enacted the Asbestos Mines and Mills Release Regulations that limited discharge of asbestos fibers during the mining and milling process. The regulation required asbestos mines and mills to install environmental control technologies, which increased the operational cost of asbestos firms in Canada. To study the effect of the 1990 regulation, I set up a dynamic oligopoly model of capacity adjustment and exit based on the framework of Ericson and Pakes (1995). In each period, firms engage in Cournot competition with capacity constraints. To obtain the period payoff, I numerically compute the Nash equilibrium of the period games under all possible states of the industry and based on my demand and marginal cost estimates. I embed the period payoffs in the dynamic game and calibrate the model to data on all Canadian and U.S. mining firms’ capacities and exit/stay status during 1968-2002.
the calibrated parameters, I conduct a counterfactual experiment to evaluate the effects of the 1990 regulation. Apart from the positive health effects of the 1990 regulation, I find that the regulation decreased producers’ surplus by 0.37% and consumers’ surplus by 0.22% and product market surplus by 0.23%. I further decompose the effect on producers’ surplus into two components: effect of increasing firms’ marginal costs; and the effect of changing market structure. By speeding up the divestment and exit process of firms thereby changing market structure, the 1990 regulation resulted in an increase in total industry profits, and thus alleviated the welfare loss resulted from the delay in the divestment process of the industry.

In my model, within the dynamic game of divestment and exit, each period firms engage in quantity competition subject to their capacity constraints. Hence, I need to solve for the period game equilibrium analytically or numerically in order to solve for the dynamic game. Because of the nonlinearity of the demand curve, it is not possible to solve for the equilibrium analytically. A novelty of this paper is that I numerically compute the Nash equilibrium of Cournot games with capacity constraints within the dynamic game, given the specific estimates of demand and marginal cost. While solving for the equilibrium of Cournot games is straightforward, it is not clear what the Nash equilibrium of Cournot games when capacity constraints present is, and how to obtain it numerically. I derive some results that enable me to numerically compute the Nash equilibrium of the period games under all possible states of the industry.

I first discuss two cases where the market consists of only one type of firms and where there are two types of firms. For the first case, I compare the solution to firms’ first order condition to their capacity to obtain the equilibrium. For the second case, I compare the Nash equilibrium of Cournot games without constraints to firms’ capacities and discuss three scenarios. The first scenario is where the solutions for both types lie within their capacities. In this case, the equilibrium of the capacity-constrained Cournot game is the same as the equilibrium of the Cournot game without constraints. The second scenario is where one type (denote it by type I) of firms’ unconstrained solution exceeds its capacity and the other type (denote it by type II) does not. For this case, I prove that the equilibrium happens at type I firms’ capacity and the best response of type II firms given that type I firms are producing at their capacity (if the best response of type II firms exceeds their capacity, type II firms’ equilibrium production is at their capacity). The third scenario is where both types’ unconstrained equilibrium solutions exceed their capacities. For this scenario, I compare the marginal revenues of both type of firms given that they are producing at their capacities, to their respective marginal costs. I show it is impossible that the marginal revenues of both types of firms are smaller than their respective marginal costs. If the marginal revenues of both types of firms exceed their marginal costs, it is straightforward that the equilibrium
happens at both types producing at their capacities. If one type (denote it by type I) of firms’ marginal revenue exceeds its marginal cost and the other type (denote it by type II) does not, then I first compute type II’s best response given that type I is producing at its capacity. Given that type II firms are producing at the best response quantity, if the marginal revenue for type I firms exceeds their marginal cost, then the equilibrium happens when type I firms produce at their capacity and type II firms produce at the best response quantity. If the marginal revenue for type I firms is smaller than their marginal cost, then I compute for the best response quantity for type I firms given that type II firms are producing at their capacity. I prove that the equilibrium then happens when type I firms produce at this best response quantity and type II firms produce at their capacity. To the best of my knowledge, this is the first paper that have done this procedure in order to obtain period payoffs within a dynamic game. It is an essential step to obtain these results and solve for the equilibrium of Cournot games with capacity constraints so that I can embed period payoffs into the dynamic game and solve for the dynamic game of capacity adjustment and exit given any set of parameter values.

2 Related Literature

My research is mainly related to literatures on the devolution of declining industries, both theoretically and empirically, dynamic games of capacity adjustment, and industry consolidation. In this section below I discuss the existing work in these areas and how my research is related to them.

There are several theoretical papers that analyze the devolution process of declining industries. [Ghemawat and Nalebuff (1985)](#) looks into the exit decisions of oligopolists with asymmetric market shares and focuses on how relative size affects the order of exit. The paper’s main conclusion is that, given firms choose to continue to operate at their capacities or exit the market, there is a unique subgame-perfect equilibrium that the largest firms exit first. [Ghemawat and Nalebuff (1990)](#) considers another setting where firms can continuously adjust their capacities as demand declines. The paper shows that in this game there is a unique subgame-perfect outcome where large firms reduce their capacities first and continue to do so until they have shrunk to the size of their formerly smaller rivals. [Fudenberg and Tirole (1986)](#) investigates a duopoly game of incomplete information where each firm enters the market knowing its own cost but not that of its opponent and at each instant firms decide whether to exit or remain active. Assuming that there is some probability that the market can accommodate both firms forever, the paper shows that there is a unique equilibrium of firms’ exit times. In my model, each firm each period decides to stay or exit the market;
and conditional on staying in the market, each firm further chooses discretely to increase or decrease its capacity or keep it the same as the last period. Hence, the results of these papers are not applicable to my model.

The first category of empirical literature that this paper is related to is about dynamic games that involve capacity adjustment choices of firms each period. Ryan (2012) evaluates the welfare effect of the 1990 Amendments to the Clean Air Act on the U.S. Portland cement industry, from a dynamic game of oligopoly point of view. The Amendments increased the sunk cost of entry that a firm needed to pay to enter the industry. The paper finds that as a result incumbent firms gained from reduced competition, but overall product market surplus had decreased. Though Ryan (2012) assumes that firms play a capacity-constrained Cournot quantity game in each period, based on the fact that production costs increase as the square of the percentage of capacity utilization, the paper adds a penalty cost term when production exceeds a certain percentage of capacity and estimates both the penalty parameter and the threshold at which this cost binds. Collard-Wexler (2013) investigates the effect of demand shocks on the market structure and industry composition of the ready-mix concrete industry from 1976 to 1999. The paper estimates a model of entry, discrete investment in oligopolistic markets and simulates the effect of a government intervention that smooths out short-term fluctuations in demand. The paper’s main finding is that number of plants will increase and the size distribution of the industry will shift toward large plants. Collard-Wexler (2013) divides plants’ sizes into three discrete groups and estimates firms’ transition costs to change their plant sizes. While these two papers also model firms’ dynamic decisions of capacity adjustment, continuously or discretely each period, they do not solve for capacity-constrained Cournot games.

The current study is closely related to the strand of empirical literature that looks into the divestment/exit process of firms in declining industries using structural models. There have been few papers on this subject and they emphasize the inefficiencies associated with the divestment/exit process. Takahashi (2015) studies firms’ strategic exit decisions in the U.S. movie theater industry in the 1950s and evaluates the economic cost that arises due to strategic interactions during the exit process. The paper modifies Fudenberg and Tirole (1986) model of exit in duopoly with incomplete information and estimates a dynamic game of theaters choosing to exit or stay in the market each instant and quantifies the effect of strategic interaction on the process of consolidation. An important difference from the current study is that Takahashi (2015) focuses on firms’ decisions to exit or stay at each instant and firms have private information regarding their fixed costs while I consider firms’ discrete choices to change capacities or exit in a discrete time setting and there is no information asymmetry between firms. Nishiwaki (2016) looks into Japanese cement industry
during 1998-2009 when cement consumption in Japan declined substantially. This paper analyzes how mergers affect firms’ incentives for divesting cement distribution centers, service stations and whether merger-induced divestment improves welfare since merger can partly internalize the business-stealing effect. The main finding of the paper is that merged firms closed distribution centers more actively and total welfare improved even though there was a decline in consumer surplus. While this paper also analyzes firms’ divestment decisions, my paper is different in that this paper does not look into capacity reduction decision. In the paper, more service stations increase fixed maintenance cost but decrease marginal cost of production.

There are a few papers, including Lieberman (1990), Deily (1991), Gibson and Harris (1996), Bernard and Jensen (2007) and Nishiwaki and Kwon (2013), that also look into declining industries. The focus of these papers is how firm characteristics, such as plant size, productivity, affect firms’ exit and capacity reduction decisions.

Another strand of literature that this paper is related to studies industry’s consolidation processes using a dynamic structural model. Jeziorski (2014) analyzes ownership consolidation in the U.S. radio industry using a dynamic oligopoly model with endogenous mergers and repositioning of products and finds that after the 1996 deregulation of U.S. radio industry, cost-side savings resulting from mergers amount to $1.2 billion per year. Calfee Stahl (2016) exploits the deregulation in the 1990s in the U.S. broadcast television industry as an exogenous event that led to consolidation to estimate the effects of consolidation on viewership and firm revenue. Igami and Uetake (2016) uses an alternating-move model of dynamic oligopoly to study endogenous decisions of mergers and innovation of the hard disk drive industry during 1996-2016.

3 Industry background

Asbestos is a kind of mineral that was widely used by manufacturers and builders in the 20th century for its desirable properties, such as high tensile strength, chemical and thermal stability, high flexibility, low electrical conductivity, and large surface area. However, starting from the early 20th century, many incidents of asbestos workers who suffered from lung cancer, asbestosis, etc were found and reported. Meanwhile, scientific research gradually established the link between workplace asbestos exposure and lung cancer, asbestosis, mesothelioma and identified asbestos as a major health and environmental hazard.
3.1 Industry knowledge

Asbestos deposits are found underground and the ore mined is extracted to the surface for processing. The ore contains only about 10% asbestos, which must be carefully separated from the rock to avoid fracturing the very thin fibers. After the asbestos ore has been mined, the material goes through milling operations: repetitive crushing operations and vacuum aspirating operations to separate the asbestos from the rock; screening operations to remove rock dust and other small debris. This process also separates the asbestos fibers by length. Special mill equipment is required for these operations, such as mobile jaw crushers, gyratory and cone crushers, trommel screens and so on.

According to Canadian Minerals Yearbook, the value of asbestos fiber depends mainly upon fiber length since this determines the applications where they may be used. The most common grading system for chrysotile asbestos fibers is the Quebec Standard dry classification method. This standard defines nine grades of fibers from Grade 1, which is the longest, to Grade 9, which is the shortest. At the upper end of the scale, Grades 1 through 3 are called long fibers and range from 0.74 in (19.0 mm) and longer down to 0.25 in (6.0 mm) in length. Grades 4 through 6 are called medium fibers, while Grades 7 through 9 are called short fibers. Grade 8 and 9 fibers are under 0.12 in (3.0 mm) long and are classified by their loose density rather than their length. Long fibers are used in the textile industry, as electrical insulation, as a filtration medium and as reinforcing fillers in asbestos-cement products. Medium-length fibers are used as reinforcing fillers in asbestos-cement products, friction materials such as brake linings and clutch facings, paper and pipe coverings. Short fibers are used as reinforcing fillers in plastics, floor tile, asphalt, and in paints and oil-well muds.

Unit prices for the long fibers are usually higher than those for medium-length fibers, and unit prices for the medium-length fibers are higher than those for short fibers. Though price for different grades of asbestos fibers differ substantially, the production procedures are pretty much the same and they use the same ingredients and machinery. Hence, I regard asbestos fibers as a homogeneous product.

3.2 Asbestos industry in U.S. and Canada

Canada was a large producer and a net exporter of asbestos worldwide. As shown in Figure 1, in 1970, Canada’s asbestos production was more than 40% of total world production. In 1980, its share was about 30%. In 1990, its share dropped to less than 20%. Moreover, more than 90% of its production was exported after 1960s. Figure 3 showed the dynamics of the composition of Canadian asbestos exports. America, Europe and Asia are the main
destinations of Canadian asbestos. Figure 3 showed that America was the destination that Canada exported the most asbestos before 1980s. In 1960, more than 50% of Canadian asbestos production was exported to United States. In 1970, about 40% of its production was exported to U.S.. U.S. was a large consumer for Canadian asbestos. But export volume to America as well as its share as a percentage of total Canada asbestos export decreased rapidly since late 1970s. Export volume to Europe also declined a lot after 1980. On the contrary, export volume to Asia increased in the 1980s. Export volume to Africa was always small during 1974-1992. Because U.S. demand plummeted since late 1970s and Europe’s demand also declined since 1980s, Canada’s asbestos exports shifted to Asia. In late 1980s, Asia was the largest consumer of Canadian asbestos. For these reasons, in the next subsection, when I discuss the important events that have happened and affected the demand for Canadian and U.S. asbestos, I need to take into considerations for the events happened in U.S. as well as in Europe and Asia during the period of 1968 and 2002.

U.S. was a large consumer and a net importer of asbestos. As shown in Figure 1, in 1960, U.S. asbestos consumption accounted for 30% of total world production. In 1970, its share was about 20%. In 1980, its share dropped to about 10%. For almost all years during 1950 and 2002, more than 90% of U.S. apparent consumption was dependent on imports from other countries. Among all U.S. asbestos imports, its imports volume from Canada accounted for more than 90% for all years during 1950 and 2002.

Figure 2 displays directly that U.S. asbestos production is negligible relative to Canadian asbestos production. U.S. production is also small relative to its apparent consumption. A considerable portion of U.S. consumption was supported by imports from Canada.

### 3.3 Declining trend in the industry

Starting from 1970s, U.S. government issued a series of policies and regulations that restricted, phased out or banned asbestos-related products. Since asbestos fibers are mainly used as an intermediate good to manufacture many construction and automobile goods, government regulations and laws that affect those asbestos products industries affect the demand of asbestos. Figure 1 showed that U.S. consumption of asbestos declined rapidly even though construction volume increased since late 1970s. According to Bureau of Mines Minerals Yearbook - Asbestos (1977), asbestos consumption had risen with rising construction activity and declined with each decrease in construction activity. However, in 1977, the paths of new

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1. Apparent consumption is production plus imports and minus exports.
2. According to Bureau of Mines Minerals Yearbook - Asbestos (1975), the dramatic decline in production in 1975 was a result of severe disruptions of productions in Quebec, Canada, including a seven-month strike from March 18 to October 12 that stopped most mining and milling activity.
construction and asbestos consumption diverged. They attributed this sudden divergence to lessened demand brought about by the environmental problems associated with asbestos and predicted that future asbestos consumption in the United States might have a nongrowth pattern, and that environmental considerations would be a causative factor.

In this subsection, I would like to enumerate the important events that have happened and had a substantial negative effect on the demand of U.S. and Canadian asbestos. Identifying these events is an essential step for estimating the demand curve.

According to Bureau of Mines Minerals Yearbook - Asbestos (1986), on January 29, 1986, Environmental Protection Agency (EPA) announced a proposed ruling that would immediately ban the manufacture, importation, and processing of certain asbestos construction materials, such as asbestos-cement pipe, roofing felt, vinyl asbestos floor tile under the 1976 Toxic Substances Control Act. In addition, the mining and importation of asbestos and the importation of asbestos products not directly banned would be placed under a permit system. According to Bureau of Mines Minerals Yearbook - Asbestos (1989), EPA issued a final rule banning the manufacture, importation, processing, and distribution of most asbestos-containing products on July 12, 1989. It is undoubtful that these events should have substantial negative effects on the demand for Canadian and U.S. asbestos, I include a dummy variable for 1986 and 1989 when estimating the demand curve.

According to Canadian Minerals Yearbook (1983-1984), shipments of asbestos in 1983-1984 was weak because of worldwide recessionary conditions particularly in the construction industry and foreign exchange shortages in the developing countries, uncertainties regarding future environmental regulations and adverse publicity associated with past exposure to asbestos dust in the workplace. I include a dummy variable for 1984 because of low demand from developing countries.

According to Canadian Minerals Yearbook (1993), asian countries were still the main markets for Canadian fibers, accounting for about 56% of Canadian exports in 1993. However, Japan’s share decreased due to the state of the Japanese economy then, which is expected to remain weak in comparison to other Asian countries in 1994. I include a dummy variable for 1994 to represent that demand from Japan decreased because of the state of Japanese economy.

According to Mineral and Metal Commodity Reviews - Chrysotile (1996), published by Natural Resources Canada, the French government announced on July 3, 1996, that it was banning the import, manufacture and sale of most asbestos products effective January 1, 1997. As a consequence of the France ban, chrysotile consumption in Europe in 1997 would be dramatically reduced, first because France was a major consumer, and second, because of the impact the French decision would have on chrysotile consumption in other European
consuming countries. I include a dummy variable for 1996 to represent the negative effects of France banning asbestos.

According to Mineral and Metal Commodity Reviews - Chrysotile (1997), as a consequence of the European ban movement, but foremost because of the Asian financial crisis, worldwide chrysotile consumption would be dramatically reduced in 1998. On June 18, 1997, the United Kingdoms Environment Minister, Angela Eagle, announced her governments intention to ban the use and importation of chrysotile asbestos products. I include a dummy variable for 1997 to because of Asian financial crisis.

According to Mineral and Metal Commodity Reviews - Chrysotile (1999), on August 6, 1999, the European Commission announced a ban of asbestos that member states of the European Union must phase out the placement on the market and the use of chrysotile asbestos and of products containing this fiber no later than January 1, 2005. In step with the European Commission ban announcement, the United Kingdom also announced on August 24, 1999 the implementation of The Asbestos Regulations 1999 to ban the use and import of chrysotile asbestos. I include a dummy variable for 1999 to represent European Commission and UK banned asbestos.

In response to the declined demand, Canadian firms started divestment by decreasing their capacity levels, closing mines or exiting the market in the 1970s. U.S. firms started to exit the market in 1960s and by 2002 they all exited.

4 Data

The data used to analyze the divestment dynamics of the U.S. and Canadian asbestos mining industry covers the period from 1968 to 2002. Late 1960s marked the beginning of industry decline. And by the end of 2002, all U.S. firms exited the market and only two Canadian firms were present. Hence, the period from 1968 to 2002 is appropriate for analyzing this declining industry.

There are three data elements. The first dataset I collected is for estimating the demand curve for U.S. and Canada asbestos fibers. To this end, I collected data on industry-average price, aggregate production of asbestos fibers of all U.S. and Canadian firms, volume of new construction put in place in the U.S., and electric and gasoline price in Canada, all from 1968 to 2002. Since asbestos in primarily used in construction industry, volume of new construction put in place in the U.S. is used as a demand shifter. Since the mining process of asbestos requires drilling and the separating process of asbestos fibers from ores requires the operation of jaw crushers and vibrating screens, electricity is an important input in the production of asbestos. Hence, I include it as an instrumental variable for the price of
asbestos. Since typical asbestos mines have daily truck runs that pickup deposit materials to be dropped off for processing before returning empty, and also transportation of asbestos is common, I include gasoline price as another instrumental variable.

Total production quantity data of all Canadian firms are obtained from Bureau of Mines Minerals Yearbook – Asbestos (1932-1993), Non-metallic Mineral Mining and Quarrying (26-226-X) and Canadian Mineral Yearbook. Total production quantity data of all U.S. firms are obtained from Bureau of Mines Minerals Yearbook – Asbestos (1932-2015). I collected Canadian asbestos unit value from Canadian Minerals Yearbook. Canadian Minerals Yearbook, published by Natural Resources Canada, provides data on Canada’s total production (shipment) volume in tons and the total value of shipment in the current Canadian dollars each year. I computed the average value of asbestos fibers by dividing the total value in Canadian dollars and total production volume in tons. Since most of U.S. consumption of asbestos were imported from Canada, I use the computed value of Canadian asbestos fibers to represent industry-average price each year. Value of new construction put in place in the U.S. are obtained from Census Bureau’s Construction Spending survey Historical C30 Value of Construction Put in Place Data (1964-2002). It provides data on annual value of construction put in place in the United States in millions of dollars in constant 1996 dollars. I collected Consumer Price Index for electricity and gasoline from Statistics Canada’s socio-economic database CANSIM, Table 326-0020. Based on the collected data, I constructed Canada’s electric price and gasoline price index each year 1968-2002 using 1998 as base year. Summary statistics of demand related variables are provided in Table 1.

The second dataset I collected is a panel of Canadian asbestos mining firms (1968-2002) that contains information on their entry, exit decisions, as well as their capacity levels each year. This data element is also collected from Canadian Minerals Yearbook (CMY). Apart from data on aggregate production volume, CMY also provides the directory of all asbestos producers, their locations each year. In addition, CMY provides data about each asbestos producer’s milling capacity each year in tons of ore per day before 1977 and in both tons of ore per day and tons of fiber per year starting from 1977. For years before 1977, I transformed firms’ capacities in tons of ore per day to tons of fiber per year assuming that the relationship between the amount of fibers can be produced each year and the amount of ores that can be processed each day is constant before 1977 and 1977. Based on these
gathered data, I compile a dataset that contains each producer’s decision to exit, stay and doesn’t change its capacity, increase capacity, decrease capacity each year.

The third dataset I collected is a panel of U.S. asbestos mining firms (1968-2002) that contains information on their entry and exit decisions. This data element is collected from Bureau of Mines Minerals Yearbook – Asbestos (1932-2015). The minerals yearbook provides information on the directory of U.S. asbestos producers, their locations, name of their mines and type of asbestos they were producing each year. Unfortunately, the yearbook doesn’t provide producer-level data on milling capacities. However, since total production of U.S. firms is small compared to total production of Canadian firms, I treat all U.S. firms to be one firm when analyzing the divestment dynamics of U.S. and Canadian asbestos mining industry.

Summary statistics of Canadian firms’ capacities and U.S. total production is provided in Table 2.

5 Model

Time is discrete with finite horizon \( t = 1, 2, \ldots, 35 \) since all U.S. asbestos mining firms exited the market before 2003 and all Canada asbestos mining firms exited by 2013. During the period from 1968 \( (t=1) \) to 2002 \( (t=35) \), U.S. demand of asbestos plummets and so is the average unit value of Canadian asbestos. During 1968-2002, there is only one entry, United Asbestos Inc. in the year of 1975. It stayed in the market for only five years and kept its capacity at 100 thousand tons throughout. Since its capacity is relatively small (5.4% of the industry’s total capacity and there are only eight firms in total at that time) and according to the concentration ratio, the largest four firms account for 88.8% in 1972, I treat its entry and production to have a small effect on the industry and hence I neglect the entry element of this industry and focus on the exiting firms’ divestment process.

The U.S. - Canadian asbestos market is a concentrated market, given the number of firms in the market. As stated previously, the product can be viewed as a homogeneous good. As demand declines sharply, firms have to decrease their capacities to save fixed cost and maintain profitability. However, part of the business created by decreasing capacities will be stolen by competitors. Hence, strategic interaction among firms are important element that govern firms’ divestment choices and the state of the market.

To capture the above mentioned industry characteristics, I set up a model of a dynamic game of capacity divestment and exit of Canadian asbestos mining industry and U.S. asbestos mining industry in discrete time. The model is based on the dynamic oligopoly framework provided by [Ericson and Pakes (1995)](Ericson and Pakes (1995)) to capture industry dynamics. Since U.S. production...
accounts for only a very small proportion of its demand, I treat U.S. asbestos mining industry as one single firm that participates in the dynamic game.

In this model, each firm is described by its state and the whole market is summarized by the state vector that consists of all firm’s states, a demand shifter, demand shocks and marginal cost conditions. In each period, each firm makes a discrete choice to increase or decrease its capacity or keep it the same as previous period or exit permanently, given their beliefs regarding the future market states. After these ‘dynamic’ decisions take place, firms engage in Cournot competition subject to their respective capacity constraints. The state of the market evolves based on firms’ divestment decisions and the exogeneous evolution of demand and cost conditions. My exposition follows Pesendorfer and Schmidt-Dengler (2008).

5.1 States

At period 1, there are \( N_1 \) firms in total in the Canadian - U.S. asbestos market. In period \( t \), among the \( N_1 \) firms, there are \( N_t \) firms active in the market. The Canadian - U.S. asbestos market is characterized by all payoff relevant state variables. These state variables include each firm’s state variable, an exogenous demand shifter, negative demand shocks and marginal cost conditions. A firm’s state is fully captured in his capacity level since it affects each firm’s fixed cost of maintaining machines (jaw crushers, vibrating screens, etc) as well as marginal cost of production. The demand shifter is the amount of new construction taken place in U.S. each year. It is payoff relevant because firms’ profits rely on demand conditions and construction investment is the key determinant of asbestos demand.

Denote the capacity of firm \( i \) in year \( t \) by \( K_{it} \). Firm \( i \)’s state in year \( t \) is \( s_{it} = K_{it} \), and \( s_{-it} = (N_{-i1t}, N_{-i2t}) \), where \( N_{-ikt} \) counts the number of firms in the market except firm \( i \) that belong to capacity level group \( I_k \) in period \( t \). Denote \( s_t = (N_{1t}, N_{2t}) \) to be the state of the industry in period \( t \) that captures the endogenous evolvement of the number of firms in each group. Define two capacity level groups to be \( I_1 = (0, 200] \), \( I_2 = (200, +\infty) \). Group 1 consists of firms whose capacities don’t exceed 200 thousand tons of fibers; group 2 consists of firms whose capacities are above 200 thousand tons. In each year \( t \), each active firm \( i \) belongs to one of the two capacity level groups. Let \( N_{kt} \) counts number of firms in the market that belong to capacity level group \( I_k \) in period \( t \). Let \( MC_{kt} \) denote marginal cost of all firms that belong to capacity level group \( I_k \) in period \( t \), \( k \in [1, 2] \). Let \( Z_t \) denote U.S. value of new construction and \( D_t \) denote all demand shocks that have happened up to period \( t \). The state of the industry in period \( t \) is specified as

\[
\zeta_t = (N_{1t}, N_{2t}, MC_{1t}, MC_{2t}, Z_t, D_t).
\]
Number of firms that belong to each capacity group in period $t+1$, $\zeta_{t+1} = (N_{1t+1}, N_{2t+1})$, depend on $\zeta_t$ and all firms’ actions in period $t$. Demand shifter $Z_t$, demand shocks $D_t$ and marginal cost conditions $MC_{1t}$ and $MC_{2t}$ evolve exogenously.

Denote the set of the number of firms in each group to be $S = \{(N^1, N^2) : N^1 + N^2 \leq N_1, N^1, N^2 \in \mathbb{N}^0\}$. Hence, $s_t \in S$. The cardinality of $S$ is $(2 + 1)^{N_1} = 3^{N_1}$. To reduce the dimension of the state space, we can instead use the symmetric observable state space since firms that belong to the same capacity level group are regarded the same and firms’ identities don’t affect other firms’ payoff each period. Let $\delta(s_{it})$ be the function that counts the number of firms in each group except for firm $i$ in period $t$. An element of the symmetric observable state space is given by $(s_{it}, \delta(s_{it}))$. For firm $i$ in period $t$, he observes his own state, i.e., the capacity level group that he belongs in, and the number of rival firms in each group. The dimension of the symmetric observable state space is $m_s = (2 + 1) \cdot \binom{N_1 + 2}{2}$.

5.2 Private payoff shocks

Before making a decision in each period, each incumbent firm privately receives a four dimensional payoff shock $\epsilon_{it} = (\epsilon^0_{it}, \epsilon^1_{it}, \epsilon^{+1}_{it}, \epsilon^{-1}_{it}) \in \mathbb{R}^4$ associated with each action a firm possibly takes in that period. The pay-off relevant shocks can be viewed as any element of profits or costs that is not observed by other firms but is known to a firm itself, such as temporary productivity shocks, temporary adjustment costs to change capacity levels, temporary shocks to scrap value if the firm decides to exit the market, etc.

I assume that the cost shocks are identically and independently distributed across firms, actions and periods according to Type 1 generalized extreme value distribution.

5.3 Actions and state transitions

In each period after observing the state of the industry and its own payoff shocks $\epsilon_{it}$, each incumbent firm chooses to stay in the market or exit; and conditional on staying in the market, he chooses to adjust its capacity to a higher capacity level group or stay in the same group or divest to a lower capacity level group. Firms who have exited the market can’t re-enter the market.

I assume that adjusting capacity levels takes one year to realize. Hence, if a firm decides to decrease his capacity to a lower capacity group in period $t$, his capacity doesn’t change in period $t$ and changes in period $t + 1$.

Denote the action for firm $i \in \{1, 2, \cdots, N_1\}$ in period $t$ to be $a_{it}$ and the action set for each firm in the market to be $A_i = \{\text{exit, stay unchanged, increase, decrease} = \{0, 1, +1, -1\}$. Hence, $a_{it} \in A_i$. For a firm $j \in \{1, 2, \cdots, N_1\}$ who exited the mar-
ket in period $t_0$, he can’t re-enter the market and hence his action is ‘exit’. That is, $a_{jt} = 0$, $\forall t \geq t_0$. Denote the action profile for all firms except firm $i$ in period $t$ to be $a_{-it} = (a_{1t}, \cdots, a_{i-1t}, a_{i+1t}, \cdots, a_{Nt})$ and the action profile for all firms in period $t$ to be $a_t = (a_{1t}, \cdots, a_{Nt})$. Denote the set of all firms’ actions in period $t$ to be $A = \times_{i=1}^{N} A_i$. Hence, $a_t \in A$. Then define a state transition probability function to be $g : A \times S \times S \rightarrow [0, 1]$. Given that current action profile of all firms is $a_t$ and current state is $s$, $g(a_t, s, s')$ gives the probability that state $s'$ is reached next period.

5.4 Adjustment costs and fixed costs

If a firm chooses to increase or decrease its capacity to a different group, he needs to pay a one-time fixed cost of adjustment, $K_I$ and $K_D$ respectively in that period.

Each period, to stay in the market, a firm needs to pay a fixed operation cost that is dependent on his capacity level as well as a variable cost that is dependent on his marginal cost and the quantity he produces. The fixed cost of a firm is increasing in his capacity level. I make the following parametric assumption (linear) on the fixed costs:

$$FC(s_{it}) = \mathbb{1}(s_{it} \in I_1) \cdot [\delta_0 \cdot 100] + \mathbb{1}(s_{it} \in I_2) \cdot [\delta_0 \cdot 300].$$

5.5 Timing

In each period, the timing of events is summarized as follows:

1. Each period $t$, all firms $s_t$ in the market draw private cost shocks $\epsilon_{it} = (\epsilon_0^{it}, \epsilon_1^{it}, \epsilon_1^{it}, \epsilon_1^{it}, \epsilon_1^{it})$ of exiting the market, staying and keeping capacity in the same level group, increasing or decreasing capacity to different level groups. They also observe the one-time fixed cost of adjusting their capacities, $K_I$ and $K_D$ respectively, the fixed cost of operation for each capacity level group and the current demand and cost conditions.

2. Firms simultaneously take actions to exit, stay unchanged, increase capacity or decrease capacity $a_{it} \in A_i$.

3. Firms who choose to exit obtain the realized scrap value and leave the market permanently; firms in the market compete with other firms in the market $s_t$ over quantity with capacity constraints.

4. Firms in the market obtain period profits from competition net of their variable cost of production as well as incurred respective fixed costs and the realized cost shocks.
5. Period $t$ ends and period $t + 1$ begins. The market structure then transits from $s_t$ to $s_{t+1}$ according to transition probability function after all firms take their actions in period $t$. Demand and cost conditions evolve exogeneously.

5.6 Period payoff

In each period, given the state of the industry $\zeta_t$, each firm in the market engages in product market competition. The profit for firm $i$ in period $t$ is given by

$$\pi_{it}(s_{it}, s_{-it}; MC_t, D_t, Z_t) = P(Q_t) \cdot q^{*}_{kt} \cdot 1(s_{it} \in I_k) - C(s_{it}, MC_t),$$

where $P(Q_t)$ is the demand function estimated from data. I assume an inverse demand function with constant price elasticity,

$$P(Q_t) = A \exp^{-\sum_{t'=1}^{t} r_{t' \cdot \text{dummy}_{t'}}} Z_{t'}^{\alpha_1} Q_{t'}^{\alpha_2}.$$

This equation is equivalent to

$$\ln Q_t = \ln A - \sum_{t'=1}^{t} r_{t' \cdot \text{dummy}_{t'}} - \beta_1 \ln P_t + \beta_2 \ln Z_t,$$

where dummy$_{t'}$ is a dummy variable for year $t'$ and $Z_t$ is the U.S. value of new construction in year $t$. I include dummy$_{t'}$ because there were major exogeneous demand shocks that affected the price of asbestos as well and we need to take them into consideration when estimating the relationship between price and quantity demanded. Otherwise, there will be endogeneity problem. Since asbestos is primarily used in construction industry, I include it as a demand shifter.

The total cost of operation of firm $i$ in period $t$ is given by

$$C(s_{it}, MC_t) = \sum_{k=1}^{2} MC_{kt} \cdot q^{*}_{kt} \cdot 1(s_{it} \in I_k) + FC(s_{it}),$$

where $q^{*}_{kt}, k \in \{1, 2\}$, are obtained as equilibrium strategies of a capacity-constrained Cournot game of the industry in period $t$. In section 7, I will discuss how to obtain the equilibrium quantities under all possible states of this industry using numerical methods.

I normalize the scrap value to be 0. Hence, for firms who exit in period $t$, their payoff is the private cost shock. Apart from profits from product market competition, firms in the market also receive private cost shocks associated with each action. Hence the total period
payoff of firm $i$ in period $t$ is given by
\[
\begin{cases}
  \epsilon_{it}^0, & \text{if } a_{it} = 0; \\
  \pi_{it}(s_{it}, s_{-it}; MC_t, D_t, Z_t) + \epsilon_{it}^1, & \text{if } a_{it} = 1; \\
  \pi_{it}(s_{it}, s_{-it}; MC_t, D_t, Z_t) - K^I + \epsilon_{it}^{+1}, & \text{if } a_{it} = +1; \\
  \pi_{it}(s_{it}, s_{-it}; MC_t, D_t, Z_t) - K^D + \epsilon_{it}^{-1}, & \text{if } a_{it} = -1.
\end{cases}
\]

5.7 Dynamic optimization

Based on period payoffs, assume all firms discount their future payoffs by a factor of $0 < \beta < 1$, the game payoff of firm $i$ is given by
\[
E\left[ \sum_{t=1}^{T} \beta^t u_{it}(s_{it}, s_{-it}; a_{it}; MC_t, D_t, Z_t) \right | s_1, \epsilon_{i1}].
\]

The expectation is taken over the realizations of the state and cost shocks.

Since firms’ decisions to exit, stay and capacity adjustment every period will affect market structure and future cash flows, firms make their discrete choices each period to maximize expected future payoffs given its beliefs regarding rival firms’ actions and the exogenous evolution of demand and marginal cost conditions. I assume firms perfectly know the evolution of all exogenous variables, including how demand shifter and marginal costs evolved and when each demand shock took place. They also know perfectly the shape of the demand curve.

I assume firms adopt pure Markovian strategies. In each period, firms choose one of the four actions by looking at current state and his private shocks. Past states affect current decision only through current state. Hence, firm $i$’s strategy each period is a function of the state variable and his private shocks to the probability simplex of four actions \( \{ x \in \mathbb{R}^4 : x_1 + x_2 + x_3 + x_4 = 1, x_i \geq 0, i = 1, \cdots, 4 \} \).

Let $\sigma_{it}(a_{-it}|s_{it})$ be firm $i$’s belief that all other firms are taking action profile $a_{-it}$ when the state is $s_{it}$ in period $t$, then firm $i$’s value function $W_{it}(s_{it}, \epsilon_{it}; \sigma_{it})$ is given by
\[
W_{it}(s_{it}, \epsilon_{it}; \sigma_{it}) = \max_{a_{it} \in A_i} \left\{ \sum_{a_{-it} \in A_{-i}} \sigma_{it}(a_{-it}|s_{it}) \left[ \pi_{it}(s_{it}; MC_t, D_t, Z_t) \cdot 1(a_{it} \in \{1, +1, -1\}) - K^I \cdot 1(a_{it} = +1) - K^D \cdot 1(a_{it} = -1) + \sum_{k} \epsilon_{it}^k \cdot 1(a_{it} = k) + \beta \sum_{s' \in S} g(s'|a_{it}, a_{-it}, s_{it}) E \epsilon_{it+1} W_{it+1}(s', \epsilon_{it+1}; \sigma_{it+1}) \right] \right\},
\]
where \( g(s'|a_it, a_{-it}, s_t) \) denotes the probability that the state of the industry in period \( t + 1 \) reaches \( s' \) given that action profile for period \( t \) is \((a_it, a_{-it})\) and industry state for period \( t \) is \( s_t \). It is derived from the state transition probability function. This is the Bellman equation that gives firm \( i \)'s dynamic programming problem.

### 5.8 Equilibrium

The equilibrium concept for this dynamic game of capacity adjustment is Markov Perfect Equilibrium. A Markov Perfect Equilibrium (MPE) of this game consists of a strategy profile \((a_1, a_2, \ldots, a_{N_1})\) of all firms in the market which specifies each firm’s strategy each period before he exits and a set of beliefs \((\sigma_1, \sigma_2, \ldots, \sigma_{N_1})\) that specifies each firm’s belief of all other firms’ strategies each period before he exits. An MPE requires that each firm’s strategy is optimal based on his belief of other firms’ strategies and his belief is also consistent with other firms’ strategies.

To formally define an MPE for this model, I first need to define firms’ ex ante value functions. An ex ante value function gives a firm’s expected payoff starting from a state before observing the private shock and choosing an action. Firm \( i \)'s ex ante value function in period \( t \) is given by \( V_{it}(s_t, \sigma_{it}) = \mathbb{E}_t[W_{it}(s_t, \epsilon_{it}; \sigma_{it})] \).

\[
V_{it}(s_t, \sigma_{it}) = \sum_{s \in S} \sigma_{it}(a_{it}|s_t)[E_t[u_i(s_t; a_{it}; MC_t, D_t, Z_t)] + \beta \sum_{s' \in S} g(s'|a_{it}, s_t)V_{it}(s', \sigma_{it})] \\
= \sum_{s \in S} \sigma_{it}(a_{it}|s_t)[\pi_{it}(s_t; MC_t, D_t, Z_t) \cdot \mathbb{I}(a_{it} \in \{1, +1, -1\}) - K^D \cdot \mathbb{I}(a_{it} = +1) \\
- K^D \cdot \mathbb{I}(a_{it} = -1) + \beta \sum_{s' \in S} g(s'|a_{it}, s_t)V_{it}(s', \sigma_{it})] \\
+ \sum_{k} \mathbb{E}[\epsilon_{it}^k|a_{it} = k] \cdot \sigma_{it}(a_{it} = k|s_t) \\
= \sum_{s \in S} \sigma_{it}(a_{it}|s_t)[\pi_{it}(s_t; MC_t, D_t, Z_t) \cdot \mathbb{I}(a_{it} \in \{1, +1, -1\}) - K^D \cdot \mathbb{I}(a_{it} = +1) \\
- K^D \cdot \mathbb{I}(a_{it} = -1) + \beta \sum_{s' \in S} g(s'|a_{it}, s_t)V_{it}(s', \sigma_{it})] \\
+ \sum_{k} (\gamma - \log(Pr(a_{it} = k|s_t))) \cdot \sigma_{it}(a_{it} = k|s_t) \\
= \sum_{s \in S} \sigma_{it}(a_{it}|s_t)[\pi_{it}(s_t; MC_t, D_t, Z_t) \cdot \mathbb{I}(a_{it} \in \{1, +1, -1\}) - K^D \cdot \mathbb{I}(a_{it} = +1) \\
- K^D \cdot \mathbb{I}(a_{it} = -1) + \beta \sum_{s' \in S} g(s'|a_{it}, s_t)V_{it}(s', \sigma_{it})] \\
+ \sum_{k} (\gamma - \log(\sigma_{it}(a_{it} = k|s_t))) \cdot \sigma_{it}(a_{it} = k|s_t),
\]
where \( \gamma \) is the Euler’s constant, 0.5772. The second to last equality is derived from the fact that the difference between two extreme value variables is distributed logistic, and for the logit case,

\[
E[\epsilon^k_{it} | a_{it} = k] = \gamma - \log \left( Pr(a_{it} = k | s_t) \right)
\]

where \( \gamma \) is Euler’s constant 0.5772... and \( Pr(a_{it} = k | s_t) \) is the choice probability of action \( k \) in state \( s_t \) in period \( t \).

Based on the ex ante value functions, to show the condition of an optimal action, I then define continuation value functions. A continuation value function describes a firm’s expected payoff when one action is taken but the privately-observable cost shocks are not realized. Given firm \( i \)’s belief of other firms’ action profile to be \( \sigma_{it}(a_{-it} | s_t) \) in period \( t \), the continuation value functions if firm \( i \) chooses action \( a_{it} = \{0, 1, +1, -1\} \) are given by:

\[
v_{it}(a_{it} = 0, s_t; \sigma_{it}) = \sum_{a_{-it} \in A_{-i}} \sigma_{it}(a_{-it} | s_t) \left[ \beta \sum_{s' \in S} g(s' | a_{it} = 0, a_{-it}, s_t) V_{it}(s', \sigma_{it}) \right];
\]

\[
v_{it}(a_{it} = 1, s_t; \sigma_{it}) = \sum_{a_{-it} \in A_{-i}} \sigma_{it}(a_{-it} | s_t) \left[ \pi_i(s_t; MC_t, D_t, Z_t) \right.

\left. + \beta \sum_{s' \in S} g(s' | a_{it} = 1, a_{-it}, s_t) V_{it}(s', \sigma_{it}) \right];
\]

\[
v_{it}(a_{it} = +1, s_t; \sigma_{it}) = \sum_{a_{-it} \in A_{-i}} \sigma_{it}(a_{-it} | s_t) \left[ \pi_i(s_t; MC_t, D_t, Z_t) - K^I \right.

\left. + \beta \sum_{s' \in S} g(s' | a_{it} = +1, a_{-it}, s_t) V_{it}(s', \sigma_{it}) \right];
\]

\[
v_{it}(a_{it} = -1, s_t; \sigma_{it}) = \sum_{a_{-it} \in A_{-i}} \sigma_{it}(a_{-it} | s_t) \left[ \pi_i(s_t; MC_t, D_t, Z_t) - K^D \right.

\left. + \beta \sum_{s' \in S} g(s' | a_{it} = -1, a_{-it}, s_t) V_{it}(s', \sigma_{it}) \right].
\]

Then firm \( i \) chooses \( a_{it} = k \) under belief \( \sigma_{it}(a_{-it} | s_t) \) if and only if

\[
v_{it}(a_{it} = k, s_t; \sigma_{it}) + \epsilon^k_{it} \geq v_{it}(a_{it} = j, s_t; \sigma_{it}) + \epsilon^j_{it}, \forall j \in \{0, 1, +1, -1\} \quad \text{and} \quad j \neq k.
\]
The probability that firm $i$ chooses action $a_{it} = k \in \{0, 1, +1, -1\}$ in period $t$ is given by

$$p_{it}(a_{it} = k|s_t, \sigma_{it}) = Pr(v_{it}(a_{it} = k, s_t; \sigma_{it}) + \epsilon_{it}^k \geq v_{it}(a_{it} = j, s_t; \sigma_{it}) + \epsilon_{it}^j, \forall j \in \{0, 1, +1, -1\} \text{ and } j \neq k)$$

$$= \int \Pi_{j \neq k} 1(v_{it}(a_{it} = k, s_t; \sigma_{it}) - v_{it}(a_{it} = j, s_t; \sigma_{it}) \geq \epsilon_{it}^j - \epsilon_{it}^k, \forall j \in \{0, 1, +1, -1\}) \ dF(\epsilon)$$

$$= \Psi(a_{it}, s_t, \sigma_{it}).$$

This holds for each firm $i \in \{1, 2, \cdots N_t\}$ in each period $t$ and each state $s_t$.

Let $P$ denote the $N_1 \times m_s \times T$ dimensional vector of choice probabilities. An element of the vector is the probability simplex of a firm choosing four actions given a state in a period. Let $\sigma$ denote the $N_1 \times m_s \times T$ dimensional vector of beliefs. Likewise, an element of the belief vector gives the probability simplex of the beliefs of a firm choosing four actions given a state in a period. Hence, an MPE of pure Markov strategies for this game consists of a vector of choice probabilities $P$ such that

$$P = \Psi(\sigma).$$

Since consistency of beliefs require that

$$P = \sigma,$$

equilibrium choice probability vector $P$ is a fixed point to the equation system

$$P = \Psi(P).$$

Continuity of $\Psi$ guarantees that an MPE exists by applying Brouwer’s fixed point theorem.

6 Empirical Analysis

My empirical analysis consists of four steps. In the first step, I estimate the demand function for asbestos fibers using two stage least squares. Second, with the estimated demand curve and assuming homogeneous Cournot competition with capacity constraints each period, I recover the marginal costs of production for both groups of firms implied by the first-order conditions of firms’ period-profit maximization. In the third step, based on both demand
curve estimate and marginal cost estimates, I numerically compute for the Nash equilibrium of Cournot competition with capacity constraints each year and under any possible market state. The computation then permits the calculation of period profits for both groups of firms. In the fourth step, I embed these period profits into the dynamic discrete game of capacity adjustment and exit, which I use to match data on the process of divestment and exit of the industry.

6.1 Demand estimation

Taking into consideration of the exogenous demand shocks, the amount of construction activity in the U.S. as a demand shifter, I assume the following demand function with constant price elasticity:

\[ P(Q_t) = A (\exp^{-\sum_{t'=1}^{t} r_{t'} \cdot \text{dummy}_{t'}}) Z_t^{\alpha_1} Q_t^{\alpha_2}. \]

And I estimate the following log-linear demand function:

\[ \ln Q_t = -\frac{1}{\alpha_2} \log A - \frac{1}{\alpha_2} \sum r_{t'} \cdot \text{dummy}_{t'} - \frac{\alpha_1}{\alpha_2} \log Z_t + \frac{1}{\alpha_2} \log P_t, \]

where \( Q_t \) is summation of the total quantity produced by all Canada firms and total quantity produced by all U.S. firms, \( P_t \) is the average unit value of Canadian asbestos in constant 1998 U.S. dollars and \( Z_t \) is the value of new construction put in place in U.S., millions of dollars, in constant 1998 U.S. dollars. I include one dummy variable representing each of the following years: 1984, 1986, 1989, 1994, 1996, 1997, 1999. As I argued, there were events during each of these years that affected the demand for U.S. and Canadian asbestos substantially. Table 1 gives a summary of these events for each of the above mentioned years.

The parameters of the demand function are estimated using two-stage least squares. Since \( P_t \) is an endogenous variable, I employ two instrumental variables: electric price and gasoline price. Table 2 gives the estimates of demand function parameters. The estimated price elasticity of demand is -1.14.

6.2 Marginal costs estimation

Each period, firms engage in spot-market competition according to Cournot competition with capacity constraints. I assume that marginal costs for firms in the same capacity group are the same each period. And marginal costs are constant with respect to quantity. Firm
i’s maximizes period profit in period $t$

$$\max \ P(\sum \ N_{jt} \cdot q_{jt}) \cdot q_{it} - \MC_{it} \cdot q_{it},$$

with respect to its production $q_{it}$.

Because I only observe the aggregate production data each year - not firm level, and I observe each firm’s capacity each year, I assume that in each period firms are producing at the same percentage of their capacities and I maintain symmetry across firms in the same capacity group. Specifically, production quantity for a firm that belongs to capacity group $k$ is obtained from data in the following way:

$$q_{kt} = \frac{Q_t \cdot \sum \ K_{jt} \cdot \mathbb{1}(s_{jt} \in I_k)}{\sum_{i=1}^{N_k} K_{it} \cdot N_{kt}}.$$

For firms in capacity group $k$, their first order condition in period $t$ is given by

$$P(Q_t) + P'(Q_t) \cdot q_{kt} = \MC_{kt}.$$

Their optimal production is $q_{kt}^*$ if $q_{kt}^* \leq K_{it}$; and $q_{kt}^* = K_{it}$ if $q_{kt}^* > K_{it}$. Because I made the assumption that firms are utilizing their capacities at the same ratio, the quantities produced for both groups are smaller than their respective capacities. Hence, these quantities should satisfy firms’ first-order conditions. From data we know $q_{kt}^*$ and we can estimate $\MC_{kt}$.

Figure 11 displays the marginal costs estimates for both capacity groups each period. There was a dramatic decline in production in 1975 because a seven-month workers’ strike that stopped most mining and milling activities. As a result, unit value was high during several years after 1975 and the estimated marginal costs increased sharply around 1975 and reached high levels few years after 1975 and then declined substantially.

6.3 Characterization of equilibrium of period game

In each period, all firms in the market engage in Cournot quantity competition subject to their respective capacity constraints. Though solving for unconstrained Cournot equilibrium of different types of firms is straightfoward, it is not clear what is the Nash equilibrium of the Cournot game with two types of firms after introducing capacity constraints.

If the market consists of only one type of firms, then the Nash equilibrium is obtained by firstly solving for firms’ first order condition numerically, since the demand function is nonlinear. If the solution exceeds firms’ capacity, then equilibrium production is at the capacity, due to the fact that marginal revenue of a firm is decreasing in its own production.
If the solution lies within firms’ capacity, then equilibrium production is at the quantity suggested by the first order condition.

If the market consists of two types of firms, I derive some results regarding what are the Nash equilibria for the Cournot game with capacity constraints or how we can compute them(it) numerically.

I first solve for the Nash equilibrium for the Cournot game where firms don’t have capacity constraints. Then by comparing the solutions to the unconstrained game and the respective capacities of two types of firms, there could be three scenarios.

The first scenario is when the solutions for both types lie within the respective capacity levels. In this case, the equilibrium for the capacity constrained Cournot game is the same as the equilibrium for the Cournot game without constraints.

The second scenario is where one type (denote it by type I) of firms’ unconstrained solution exceeds its capacity and the other type (denote it by type II) does not. For this case, I proved that the equilibrium happens at type I firms’ capacity and the best response of type II firms given that type I firms are producing at their capacity (if the best response of type II firms exceeds their capacity, type II firms’ equilibrium production is at their capacity).

The third scenario is where both types’ unconstrained equilibrium solutions exceed their capacities. For this scenario, I compare the marginal revenues of both type of firms given that they are producing at the respective capacities, to their respective marginal costs. I show it is impossible that the marginal revenues of both types of firms are smaller than their respective marginal costs. If the marginal revenues of both firms exceed their marginal costs, it is straightforward that the equilibrium happens at both types producing at their capacities. If one type (denote it by type I) of firms’ marginal revenue exceeds its marginal cost and the other type (denote it by type II) does not, then I first compute type II’s best response given that type I is producing at its capacity. Given that type II firms are producing at the best response quantity, if the marginal revenue for type I firms exceeds their marginal cost, then the equilibrium happens when type I firms produce at their capacity and type II firms produce at the best response quantity. If the marginal revenue for type I firms is smaller than their marginal cost, then I compute for the best reponse quantity for type I firms given that type II firms are producing at their capacity. I prove that the equilibrium then happens when type I firms produce at this best response quantity and type II firms produce at their capacity.

For the details of analysis of the three scenarios and proofs, please see first section of the Appendix.
6.4 Calibration

Based on demand curve estimate and numerical computation of the equilibrium of period Cournot games, I calibrate the model in Section 5 to data on firms’ capacities as well as their exit/stay status.

I impose an assumption that $K_I = K^O = K_A$. I calibrate two parameters, capacity adjustment cost parameter $K_A$ and fixed operation cost parameter $\delta_0$ by matching the predictions of the model with their empirical counterparts. I use the demand elasticity reported in Table 4.

To capture the important features of industry dynamics, I consider these two moments: (1) Number of capacity changes, including both number of increasing capacity from group 1 to group 2 and number of decreasing capacity from group 2 to group 1 and (2) Number of exits.

I use data from 1968 to 2002 to calculate the empirical moments. To obtain moments implied by the model, for any parameter values, I simulate the game and compute an equilibrium and take averages over 200 simulated sample paths of industry over 35 time periods. The objective function is defined as the sum of the percentage deviation between the data and model moments. Table 5 reports the calibrated parameters and table 6 shows the model fit. The result shows that fixed operation cost ($500 for capacity group 1 and $1500 for capacity group 2) is very small relative to capacity adjustment cost ($275000). Though it might be costly to purchase additional equipments (jaw crushers, etc) to increase capacities, this appears to be unrealistic. However, the result may not be too surprising since I only divide firms into two groups according to whether their capacities exceeded 200 thousand metric tons of fibers each period and attempted to account for the complicated industry dynamics of divestment and exit by the relative simple model of firms choosing discretely among two capacity groups and exit each period.

7 Evaluating The 1990 Asbestos Mines and Mills Release Regulations

On June 14, 1990, Canada issued The Asbestos Mines and Mills Release Regulations, which applied to owners and operators of asbestos mines or mills. The Regulations limited concentration of asbestos fibres emitted into the ambient air at asbestos mines or mills. To comply with the Regulations, firms needed to install necessary environmental control technology and use dust collection systems to control for asbestos fibers. Therefore, the Regulations raised production standard and increased cost of operation of Canada asbestos producers.
From marginal cost estimates in section 6.2, there was a clear increasing trend in marginal costs for both groups’ firms from sometime shortly after 1990 onwards to about the year of 1996. I attribute this increase trend in marginal costs completely to the 1990 Regulations.

What will happen without this regulation? According to Ghemawat and Nalebuff (1990), capacity reduction is a public good that must be provided privately in industries facing declining demand. If one firm decreases its capacity, then the increase in price resulted from the reduction benefits all firms in the market, unless this part of capacity is completely idle before being reduced. Hence firms have an incentive to free-ride on someone else’s divestment. As a result, there is a delay of firms’ divestment process from the outcome that maximizes industry total profits. Apart from the health benefits the Regulations brought, even though the Regulations increased firms’ operation cost, from the perspective of hastening the divestment process of firms, it could bring a welfare gain to the industry as a whole.

I evaluate the 1990 Regulations’ effect on producers’ surplus, consumers’ surplus and total welfare besides health effects. When I evaluate the effect on producers’ surplus, I decompose the effect into two parts. The first part is producers’ surplus loss due to the increase in firms’ marginal costs while market structure is kept the same as without the Regulations. The second part is the effect of the Regulations through changing market structure. The results are reported in Table 7 and Table 8. The results show that because of the 1990 Regulations, overall producers’ surplus has decreased by $2.06 \times 10^4$ in 1998 U.S. dollars, and 0.37% in percentage term. The effect of increasing marginal costs while market structure is the same as without the Regulations resulted in a loss of $4.22 \times 10^4$ 1998 U.S. dollars and 0.76% in percentage term to producers’ surplus. The isolated effect of the Regulations through changing market structure is an increase in producers’ surplus by $2.16 \times 10^4$ 1998 U.S. dollars and 0.39% in percentage term. Consumers’ surplus has decreased by 0.22% because of the Regulations and total welfare of producers and consumers has decreased by 0.23%, not taking into consideration of positive health effects of the Regulations.

8 Conclusion

In this paper, I look into strategic industry dynamics of divestment and exit of the North America asbestos mining industry when demand declined rapidly due to health problems brought about by asbestos. Using a dynamic model of capacity adjustment and exit, I evaluate the welfare effect of the 1990 Asbestos Mines and Mills Release Regulations enacted in Canada and found that though overall the Regulations resulted in a decrease in producers’ surplus as well as consumers’ surplus, it brought a welfare gain to the industry because it
hastened the divestment process.

A novel feature of my model is that in each period, all firms in the market engage in Cournot quantity competition subject to capacity constraints. Computation for the period game equilibrium is an important step to solve the dynamic game because it is essential for the calculation of period profits and period payoffs are embedded in the dynamic game. I derived some results regarding how to compute for the capacity-constrained Nash equilibrium for the period games, and numerically compute for the equilibrium under all possible market structures and demand and cost conditions.
9 Tables and Figures

Figure 1: Dynamics of U.S. asbestos consumption, Canadian asbestos production, U.S. asbestos imports, Canadian asbestos exports

Figure 2: Dynamics of U.S. production, U.S. apparent consumption and exports, Canada’s production, U.S. imports from Canada

Figure 3: Canadian asbestos exports, 1974-1992

Source: Natural Resources Canada; Statistics Canada.
Figure 4: Number of Canadian asbestos mining firms and mines, 1968-2014

Figure 5: Canadian asbestos production and total capacity, 1968-2012
Figure 6: Number of U.S. asbestos mining firms, 1950-2002

Sources: Bureau of Mines Minerals Yearbook – Asbestos.
Figure 7: U.S. apparent consumption of asbestos (tons) and value of new construction (millions of dollars, constant 1998 U.S. dollars)

Figure 8: U.S. and Canada asbestos production (thousand tons) and average unit value of Canadian asbestos (constant 1998 U.S. dollars)

Table 1: Summary statistics of Demand Related Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>US_production</td>
<td>55.914</td>
<td>43.885</td>
<td>3</td>
<td>136</td>
</tr>
<tr>
<td>Canada_production</td>
<td>918.257</td>
<td>469.838</td>
<td>242</td>
<td>1690</td>
</tr>
<tr>
<td>Canada_unit_value</td>
<td>414.724</td>
<td>126.35</td>
<td>251.891</td>
<td>695.341</td>
</tr>
<tr>
<td>US_value_new_construction</td>
<td>544818.514</td>
<td>83538.453</td>
<td>420297</td>
<td>747456</td>
</tr>
<tr>
<td>electric_price</td>
<td>81.491</td>
<td>7.245</td>
<td>69.5</td>
<td>93.45</td>
</tr>
<tr>
<td>gasoline_price</td>
<td>106.511</td>
<td>15.807</td>
<td>83.19</td>
<td>136.23</td>
</tr>
<tr>
<td>Num. of Observations</td>
<td>35</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


Table 2: Summary statistics of Canadian firms’ capacities and U.S. production

<table>
<thead>
<tr>
<th>Variable</th>
<th>Nums. of Obs.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>firm1</td>
<td>35</td>
<td>474</td>
<td>245</td>
<td>250</td>
<td>848</td>
</tr>
<tr>
<td>firm2</td>
<td>19</td>
<td>310</td>
<td>95</td>
<td>70</td>
<td>472</td>
</tr>
<tr>
<td>firm3</td>
<td>19</td>
<td>209</td>
<td>45</td>
<td>123</td>
<td>257</td>
</tr>
<tr>
<td>firm4</td>
<td>19</td>
<td>205</td>
<td>19</td>
<td>168</td>
<td>231</td>
</tr>
<tr>
<td>firm5</td>
<td>25</td>
<td>145</td>
<td>50</td>
<td>100</td>
<td>221</td>
</tr>
<tr>
<td>firm6</td>
<td>17</td>
<td>305</td>
<td>19</td>
<td>285</td>
<td>340</td>
</tr>
<tr>
<td>firm7</td>
<td>31</td>
<td>64</td>
<td>26</td>
<td>20</td>
<td>88</td>
</tr>
<tr>
<td>firm8</td>
<td>19</td>
<td>60</td>
<td>7</td>
<td>55</td>
<td>80</td>
</tr>
<tr>
<td>firm9</td>
<td>5</td>
<td>72</td>
<td>0</td>
<td>72</td>
<td>72</td>
</tr>
<tr>
<td>firm10</td>
<td>4</td>
<td>48</td>
<td>0</td>
<td>48</td>
<td>48</td>
</tr>
<tr>
<td>firm11</td>
<td>2</td>
<td>50</td>
<td>0</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>firm12</td>
<td>5</td>
<td>100</td>
<td>0</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>All firms</td>
<td>200</td>
<td>217</td>
<td>182</td>
<td>20</td>
<td>848</td>
</tr>
<tr>
<td>US_production</td>
<td>35</td>
<td>55</td>
<td>43</td>
<td>2</td>
<td>136</td>
</tr>
</tbody>
</table>

“Mean capacity”, “Min capacity” and “Max capacity” are measured as the amount of fibers in thousand metric tons that can be produced in a year.
<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984</td>
<td>Low demand from developing countries;</td>
</tr>
<tr>
<td>1986</td>
<td>EPA announced a proposed ruling that would immediately ban the manufacture, importation, and processing of certain asbestos construction materials;</td>
</tr>
<tr>
<td>1989</td>
<td>EPA issued a final rule banning the manufacture, importation, processing, and distribution of most asbestos-containing products;</td>
</tr>
<tr>
<td>1994</td>
<td>Demand from Japan has decreased due to the state of Japanese economy;</td>
</tr>
<tr>
<td>1996</td>
<td>The French government announced that it was banning the import, manufacture and sale of most asbestos products effective Jan 1, 1997;</td>
</tr>
<tr>
<td>1997</td>
<td>Asian financial crisis leads to reduced demand of asbestos;</td>
</tr>
<tr>
<td>1999</td>
<td>European Commission and UK announced intentions to ban the use and importation of asbestos products.</td>
</tr>
</tbody>
</table>

Table 4: Demand Function Estimation Result

<table>
<thead>
<tr>
<th></th>
<th>(1) log_consumption</th>
<th>(2) log_unitvalue</th>
</tr>
</thead>
<tbody>
<tr>
<td>log_unitvalue</td>
<td>-1.136*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.548)</td>
<td></td>
</tr>
<tr>
<td>log_construction</td>
<td>0.162</td>
<td>-0.668</td>
</tr>
<tr>
<td></td>
<td>(0.992)</td>
<td>(0.506)</td>
</tr>
<tr>
<td>dummy_1984</td>
<td>-1.021***</td>
<td>-0.338*</td>
</tr>
<tr>
<td></td>
<td>(0.286)</td>
<td>(0.159)</td>
</tr>
<tr>
<td>dummy_1986</td>
<td>-0.190</td>
<td>-0.102</td>
</tr>
<tr>
<td></td>
<td>(0.134)</td>
<td>(0.138)</td>
</tr>
<tr>
<td>dummy_1989</td>
<td>-0.214*</td>
<td>-0.296</td>
</tr>
<tr>
<td></td>
<td>(0.0943)</td>
<td>(0.143)</td>
</tr>
<tr>
<td>dummy_1994</td>
<td>-0.0637</td>
<td>0.121</td>
</tr>
<tr>
<td></td>
<td>(0.121)</td>
<td>(0.131)</td>
</tr>
<tr>
<td>dummy_1996</td>
<td>-0.167*</td>
<td>0.0924</td>
</tr>
<tr>
<td></td>
<td>(0.0750)</td>
<td>(0.186)</td>
</tr>
<tr>
<td>dummy_1997</td>
<td>-0.298***</td>
<td>0.0211</td>
</tr>
<tr>
<td></td>
<td>(0.0760)</td>
<td>(0.186)</td>
</tr>
<tr>
<td>dummy_1999</td>
<td>-0.370*</td>
<td>-0.115</td>
</tr>
<tr>
<td></td>
<td>(0.154)</td>
<td>(0.144)</td>
</tr>
<tr>
<td>log_electric</td>
<td></td>
<td>2.002**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.687)</td>
</tr>
<tr>
<td>log_gasoline</td>
<td></td>
<td>0.0712</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.228)</td>
</tr>
<tr>
<td>Constant</td>
<td>12.19</td>
<td>5.985</td>
</tr>
<tr>
<td></td>
<td>(13.69)</td>
<td>(6.000)</td>
</tr>
<tr>
<td>Observations</td>
<td>35</td>
<td>35</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.801</td>
<td>0.724</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Figure 9: when \( \frac{N_j t \cdot q_{jt}^*}{N_i t \cdot q_{it}^* + N_j t \cdot q_{jt}} > \frac{1}{2} \) and \( \frac{N_i t \cdot q_{it}^*}{N_i t \cdot q_{it}^* + N_j t \cdot q_{jt}} < \frac{1}{2} \)

Figure 10: when \( \frac{N_j t \cdot q_{jt}^*}{N_i t \cdot q_{it}^* + N_j t \cdot q_{jt}} \leq \frac{1}{2} \) and \( \frac{N_i t \cdot q_{it}^*}{N_i t \cdot q_{it}^* + N_j t \cdot q_{jt}} \geq \frac{1}{2} \)
Figure 11: Marginal costs estimates

Marginal Cost Estimates of Each Capacity Group
Table 5: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibrated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_A$: Capacity adjustment cost</td>
<td>275000</td>
</tr>
<tr>
<td>$\delta_0$: Fixed operation cost parameter</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 6: Model fit

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of capacity changes</td>
<td>6</td>
<td>7.22</td>
</tr>
<tr>
<td>Number of exits</td>
<td>6</td>
<td>5.04</td>
</tr>
</tbody>
</table>
Table 7: Effects of The 1990 Regulations on Producers’ Surplus, Consumers’ Surplus and Total Welfare

<table>
<thead>
<tr>
<th></th>
<th>Producers’ Surplus</th>
<th>Consumers’ Surplus</th>
<th>Product Market Surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td>With regulation</td>
<td>5.5532 × 10^6</td>
<td>4.4808 × 10^7</td>
<td>5.0361 × 10^7</td>
</tr>
<tr>
<td>Without regulation</td>
<td>5.5738 × 10^6</td>
<td>4.4906 × 10^7</td>
<td>5.0480 × 10^7</td>
</tr>
<tr>
<td>Market structure as without regulation; actual marginal costs</td>
<td>5.5316 × 10^6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Effects of The 1990 Regulations on Producers’ Surplus, Consumers’ Surplus and Total Welfare, in percentage terms

<table>
<thead>
<tr>
<th></th>
<th>Producers’ Surplus</th>
<th>Consumers’ Surplus</th>
<th>Product Market Surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall effect</td>
<td>−0.37%</td>
<td>−0.22%</td>
<td>−0.23%</td>
</tr>
<tr>
<td>Effect of increasing marginal costs</td>
<td>−0.76%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effect of changing market structure</td>
<td>0.39%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
10 Appendix

Characterization of equilibrium of Cournot game with capacity constraints

Given that all firms know the shape of demand curve \( P(Q) \), for a firm that belongs to group \( i \) with marginal cost \( MC_{it} \), his profit maximization problem in period \( t \) is given by

\[
\max \ P(\sum_j N_{jt} \cdot q_{jt}) \cdot q_{it} - MC_{it} \cdot q_{it}.
\]

From demand estimation, \( \alpha_2 = -\frac{1}{\alpha_1} \). The marginal revenue (MR) for group \( i \) is given by

\[
MR_i(q_{it}) = P'(Q_t) \cdot N_{it} \cdot q_{it} + P(Q_t) \]
\[
= \frac{\alpha_2 \cdot P(Q_t)}{Q_t} \cdot N_{it} \cdot q_{it} + P(Q_t) \]
\[
= \frac{\alpha_2 \cdot N_{it} \cdot q_{it} + Q_t}{Q_t} \cdot P(Q_t) > 0.
\]

Marginal revenue is declining in one firm’s own production since

\[
\frac{\partial MR_i(q_{it}, q_{jt})}{\partial q_{it}} = P''(Q_t) \cdot N_{it}^2 \cdot q_{it} + P'(Q_t) \cdot N_{it} + P'(Q_t) \cdot N_{it} \]
\[
= P''(Q_t) \cdot N_{it}^2 \cdot q_{it} + 2P'(Q_t) \cdot N_{it} \]
\[
= \frac{(\alpha_2 - 1) \cdot \alpha_2 \cdot P(Q_t)}{Q_t^2} \cdot N_{it}^2 \cdot q_{it} + 2 \cdot \alpha_2 \cdot P(Q_t) \]
\[
= \frac{\alpha_2 \cdot P(Q_t) \cdot N_{it} \cdot [(\alpha_2 - 1) \cdot N_{it} \cdot q_{it} + 2 \cdot Q_t]}{Q_t^2} < 0.
\]

However, whether marginal revenue for one firm is increasing or decreasing in another firm’s production is ambiguous. We have

\[
\frac{\partial MR_i(q_{it}, q_{jt})}{\partial q_{jt}} = P''(Q_t) \cdot N_{it} \cdot N_{jt} \cdot q_{it} + P'(Q_t) \cdot N_{jt} \]
\[
= \frac{(\alpha_2 - 1) \cdot \alpha_2 \cdot P(Q_t)}{Q_t^2} \cdot N_{it} \cdot N_{jt} \cdot q_{it} + \frac{\alpha_2 \cdot P(Q_t)}{Q_t} \cdot N_{jt} \]
\[
= \frac{\alpha_2 \cdot P(Q_t) \cdot N_{jt} \cdot [(\alpha_2 - 1) \cdot N_{it} \cdot q_{it} + Q_t]}{Q_t^2}.
\]

If \( (\alpha_2 - 1) \cdot N_{it} \cdot q_{it} + Q_t > 0 \), \( \frac{dMR_i(q_{it}, q_{jt})}{dq_{jt}} < 0 \). Hence, if we have \( \frac{N_{it} \cdot q_{it}}{Q_t} < -\frac{1}{\alpha_2 - 1} \approx \frac{1}{2} \), \( \frac{dMR_i(q_{it}, q_{jt})}{dq_{jt}} < 0 \); if we have \( \frac{N_{it} \cdot q_{it}}{Q_t} > -\frac{1}{\alpha_2 - 1} \approx \frac{1}{2} \), \( \frac{dMR_i(q_{it}, q_{jt})}{dq_{jt}} > 0 \).

10.1 Cournot Equilibrium for the Case of One Group of Firms

When there is only one group of firms present in the market, first order condition is given by \( MR(q_{it}) = MC_{it} \). Since \( MR \) is declining as \( q_{it} \) increases, F.O.C is also sufficient for optimality. If the market only consists of group \( i \) firms, then Cournot equilibrium production for each firm is given by \( q_{it}^* \) such that

\[
MR_i(q_{it}^*) = (\alpha_2 + 1) \cdot P(N_{it} \cdot q_{it}^*) = MC_{it}
\]
if \( q_{it}^* \leq K_i \); and \( q_{jt}^* = K_j \) if \( q_{jt}^* > K_j \).

### 10.2 An Algorithm to Compute Capacity Constrained Cournot Equilibrium for the Case of Two Groups of Firms

1. Compute the unconstrained Cournot equilibrium \((q_{it}^*, q_{jt}^*)\);

2. If \( q_{it}^* \leq K_i \) and \( q_{jt}^* \leq K_j \), then the Cournot Equilibrium with capacity constraints is given by \((q_{it}^*, q_{jt}^*)\);

3. If \( q_{it}^* \leq K_i \) and \( q_{jt}^* > K_j \), then we solve for firm \( i \)'s best response production \( q_{it}^{**} \) given that firm \( j \) is producing at its capacity \( K_j \). If \( q_{it}^{**} > K_i \) then let \( q_{it}^{**} = K_i \). I claim that the Cournot Equilibrium with capacity constraints is given by \((q_{it}^{**}, K_j)\).

**Claim 1.** If the unconstrained Cournot equilibrium satisfies \( q_{it}^* \leq K_i \) and \( q_{jt}^* > K_j \), then the Cournot Equilibrium with capacity constraints for the case of two types of firms is given by \((q_{it}^{**}, K_j)\).

**Proof.** By the above result on the partial derivative of one group's marginal revenue with respect to another group's production, I prove this claim by discussing the following two cases: if \( \frac{N_i q_{it}^*}{N_i q_{it}^* + N_j q_{jt}^*} > \frac{1}{2} \) and \( \frac{N_i q_{it}^*}{N_i q_{it}^* + N_j q_{jt}^*} < \frac{1}{2} \); if \( \frac{N_i q_{it}^*}{N_i q_{it}^* + N_j q_{jt}^*} \leq \frac{1}{2} \) and \( \frac{N_i q_{it}^*}{N_i q_{it}^* + N_j q_{jt}^*} \geq \frac{1}{2} \).

(a) If \( \frac{N_i q_{it}^*}{N_i q_{it}^* + N_j q_{jt}^*} > \frac{1}{2} \) and \( \frac{N_i q_{it}^*}{N_i q_{it}^* + N_j q_{jt}^*} < \frac{1}{2} \)

We prove by discussing two cases. The first case is where as \( q_{jt} \) decreases to \( K_j \) and \( q_{it} \) increases to \( q_{it}^{**} \) as a best response (if \( q_{jt}^{**} > K_j \) holds, then let \( q_{jt}^{**} = K_j \)), and the ratio \( \frac{N_i q_{it}^*}{N_i q_{it}^* + N_j q_{jt}^*} \) always holds. In this case, \( MR_j(q_{jt}, q_{it}) \) increases as \( q_{jt} \) decreases to \( K_j \) and \( q_{it} \) increases to \( q_{it}^{**} \). Hence, at \( q_{jt} = K_j \), \( MR_j(K_j, q_{it}^{**}) > MC_{jt} \). If \( q_{it}^{**} \leq K_i \), by producing \( q_{it}^{**} \), firm \( i \)'s marginal revenue equals its marginal cost; if \( q_{it}^{**} > K_i \), by producing at \( q_{it}^{**} = K_i \), firm \( i \)'s marginal revenue exceeds its marginal cost. In both cases, firm \( i \) will not deviate. At production profile \((q_{it}^{**}, K_j)\), neither firm \( i \) nor firm \( j \) will deviate; thus the Cournot Equilibrium with capacity constraints in this case is given by \((q_{it}^{**}, K_j)\).

The second case is where as \( q_{jt} \) decreases to \( K_j \) and \( q_{it} \) increases to \( q_{it}^{**} \) as a best response, and the ratio \( \frac{N_i q_{it}^*}{N_i q_{it}^* + N_j q_{jt}^*} > \frac{1}{2} \) fails to hold at some quantity \( \tilde{q}_{jt} \) such that \( K_j \leq \tilde{q}_{jt} < q_{it}^* \). At \( q_{jt} = \tilde{q}_{jt} \), \( \frac{N_i q_{it}^*}{N_i q_{it}^* + N_j \tilde{q}_{jt}} = \frac{1}{2} \). Let \( \tilde{q}_{it} \) denote firm \( i \)'s best response given that firm \( j \) is producing \( \tilde{q}_{jt} \) (if \( \tilde{q}_{it} > K_i \) holds, then let \( \tilde{q}_{it} = K_i \)). Then as \( q_{jt} \) keeps decreasing toward \( K_j \) from \( \tilde{q}_{jt} \), firm \( i \)'s marginal revenue becomes increasing in firm \( j \)'s production and hence it decreases. Firm \( i \) then decreases its production from \( \tilde{q}_{it} \). Since firm \( j \)'s marginal revenue is now decreasing in firm \( i \)'s production hence its marginal revenue increases both because it decreases its own production and firm \( i \) is producing less. It is possible that as firm \( i \) decreases its production optimally the ratio \( \frac{N_i q_{it}^*}{N_i q_{it}^* + N_j q_{jt}^*} \) becomes larger than \( \frac{1}{2} \) again. In that case, firm \( i \) then increases its production and firm \( j \)'s marginal revenue still increases both because itself is producing less and firm \( i \) is increasing its production. In conclusion, at \( K_j \), \( MR_j(q_{it}^{**}, K_j) > MC_{jt} \). Firm \( j \) will not deviate. If \( q_{it}^{**} \leq K_i \), by producing at \( q_{it}^{**} \), firm \( i \)'s marginal revenue equals its marginal cost; if \( q_{it}^{**} > K_i \), by producing at \( q_{it}^{**} = K_i \), firm \( i \)'s marginal revenue exceeds its marginal cost. In both cases, firm \( i \) will not deviate. Therefore, \((q_{it}^{**}, K_j)\) is the unique Cournot Equilibrium with capacity constraints in this case.
(b) If \( \frac{N_{it}q_{it}^*}{N_{it}q_{it}^* + N_{jt}q_{jt}^*} \leq \frac{1}{2} \) and \( \frac{N_{jt}q_{jt}^*}{N_{it}q_{it}^* + N_{jt}q_{jt}^*} \geq \frac{1}{2} \)

We also prove by discussing two cases. The first case is where as \( q_{jt} \) decreases to \( K_j \) and \( q_{it} \) decreases to \( q_{it}^* > 0 \) as a best response, the ratio \( \frac{N_{it}q_{it}^*}{N_{it}q_{it}^* + N_{jt}q_{jt}^*} \) always holds. In this case, \( MR_i(q_{jt}, q_{it}) \) increases as \( q_{jt} \) decreases to \( K_j \) and \( q_{it} \) decreases to \( q_{it}^* \). Hence, at \( q_{jt} = K_j \), \( MR_i(K_j, q_{it}^*) > MC_{it} \). Also, \( MR_i(q_{it}^*, K_j) = MC_{it} \). At production profile \( (q_{it}^*, K_j) \), neither firm \( i \) nor firm \( j \) will deviate; thus the Cournot Equilibrium for this case is given by \( (q_{it}^*, K_j) \).

The second case is where as \( q_{jt} \) decreases to \( K_j \) and \( q_{it} \) decreases to \( q_{it}^* \) as a best response, and the ratio \( \frac{N_{it}q_{it}^*}{N_{it}q_{it}^* + N_{jt}q_{jt}^*} \leq \frac{1}{2} \) fails to hold at some quantities. Since firm \( i \)'s best response \( q_{it}^* \) is continuous in firm \( j \)'s production \( q_{jt} \), the ratio \( \frac{N_{jt}q_{jt}^*}{N_{it}q_{it}^* + N_{jt}q_{jt}^*} \) is continuous in \( q_{jt} \). There must exist a \( \tilde{q}_{jt} \) such that \( K_j \leq \tilde{q}_{jt} \leq q_{jt}^* \) and at \( q_{jt} = \tilde{q}_{jt} \), \( \frac{N_{it}q_{it}^*}{N_{it}q_{it}^* + N_{jt}q_{jt}^*} = \frac{1}{2} \). And for all \( q_{jt} < \tilde{q}_{jt} \), \( \frac{N_{jt}q_{jt}^*}{N_{it}q_{it}^* + N_{jt}q_{jt}^*} \leq \frac{1}{2} \). Let \( \tilde{q}_{jt} \) denote firm \( i \)'s best response given that firm \( j \) is producing \( \tilde{q}_{jt} \). Then before \( q_{jt} \) decreases to \( \tilde{q}_{jt} \), firm \( i \) decreases its production because its marginal revenue is increasing in firm \( j \)'s production and firm \( j \)'s marginal revenue keeps increasing because on one hand its own production is decreasing and on the other hand firm \( i \) is producing less. When \( q_{jt} \) begins to decrease from \( \tilde{q}_{jt} \), firm \( i \)'s marginal revenue becomes decreasing in firm \( j \)'s production and it increases. Firm \( i \) then increases its production from \( \tilde{q}_{jt} \). Firm \( j \)'s marginal revenue increases because itself is reducing production and firm \( i \) is increasing its production. It is possible that as firm \( i \) increases his production optimally the ratio \( \frac{N_{it}q_{it}^*}{N_{it}q_{it}^* + N_{jt}q_{jt}^*} \) becomes smaller than \( \frac{1}{2} \) again. In that case, firm \( i \) then decreases its production and firm \( j \)'s marginal revenue still increases both because itself is producing less and firm \( i \) is decreasing its production. Regardless of how the ratio \( \frac{N_{it}q_{it}^*}{N_{it}q_{it}^* + N_{jt}q_{jt}^*} \) changes, firm \( i \)'s best response always makes firm \( j \)'s marginal revenue increase. In addition, firm \( j \) is decreasing its production from \( q_{jt}^* \) to \( K_j \). Denote firm \( i \)'s best response given that firm \( j \) is producing at \( K_j \) to be \( q_{it}^* \). If \( q_{it}^* = 0 \), then firm \( i \)'s marginal revenue is smaller than marginal cost. If \( q_{it}^* > K_i \), then let \( q_{it}^* = K_i \). Firm \( i \)'s marginal revenue is larger than marginal cost. Firm \( j \)'s marginal revenue exceeds its marginal cost at \( (q_{it}^*, K_j) \). Firm \( i \) will not deviate as well. Therefore, \( (q_{it}^*, K_j) \) is the Cournot Equilibrium for this case.

4. If \( q_{it}^* > K_i \) and \( q_{jt}^* > K_j \),

(a) if \( MR_i(K_i, K_j) \geq MC_{it} \) and \( MR_j(K_j, K_i) \geq MC_{jt} \), then the Cournot Equilibrium with capacity constraints is given by \( (K_i, K_j) \);

(b) if \( MR_i(K_i, K_j) \leq MC_{it} \) and \( MR_j(K_j, K_i) \geq MC_{jt} \), I show that it has to be at \( (K_i, K_j) \), \( \frac{N_{it}K_i}{N_{it}K_i + N_{jt}K_j} < \frac{1}{2} \) and \( \frac{N_{jt}K_j}{N_{it}K_i + N_{jt}K_j} > \frac{1}{2} \).

In this case, firm \( i \) decreases its production and firm \( j \)'s marginal revenue at \( K_j \) increases, at least at first. Solve for firm \( i \)'s best response production given that firm \( j \) is producing at \( K_j \) and denote it by \( q_{it}^{**} \). If \( MR_i(q_{it}^{**}, K_j) = MC_{it} \) and \( MR_j(K_j, q_{it}^{**}) \geq MC_{jt} \) then \( (q_{it}^{**}, K_j) \) is the Cournot equilibrium. If \( MR_i(q_{it}^{**}, K_j) = MC_{it} \) and \( MR_j(K_j, q_{it}^{**}) < MC_{jt} \) then solve for \( q_{it}^{***} \) firm \( j \)'s best response production \( q_{jt}^{**} \) given firm \( i \) is producing at \( K_i \). I argue that the Cournot equilibrium is \( (K_i, q_{jt}^{**}) \).

\textbf{Claim 2.} If \( MR_i(K_i, K_j) < MC_{it} \) and \( MR_j(K_j, K_i) \geq MC_{jt} \), then it has to be that at \( (K_i, K_j) \), \( \frac{N_{it}K_i}{N_{it}K_i + N_{jt}K_j} < \frac{1}{2} \) and \( \frac{N_{jt}K_j}{N_{it}K_i + N_{jt}K_j} > \frac{1}{2} \).
Proof. Proof by contradiction. Suppose at \((K_i, K_j)\), \(\frac{N_{it} \cdot K_j}{N_{it} \cdot K_i + N_{it} \cdot K_j} \geq \frac{1}{2}\) and \(\frac{N_{it} \cdot K_i}{N_{it} \cdot K_i + N_{it} \cdot K_j} \leq \frac{1}{2}\). Then \(MR_i(K_i, q_{jt}^*) \leq MR_i(K_i, K_j) < MC_{it}\) and \(MR_i(q_{it}^*, q_{jt}^*) < MR_i(K_i, q_{jt}^*) < MC_{it}\). This contradicts with the fact that the unconstrained Cournot equilibrium be at \(q_{it}^* > K_i\) and \(q_{jt}^* > K_j\).

Claim 3. If \(MR_i(K_i, K_j) < MC_{it}\), \(MR_j(K_j, K_i) \geq MC_{jt}\) and \(MR_i(q_{it}^*, K_j) = MC_{it}\), \(MR_j(K_j, q_{it}^*) < MC_{jt}\), then solve for firm \(j\)'s best response production \(q_{jt}^*\) given firm \(i\) is producing at \(K_i\). The capacity-constrained Cournot equilibrium in this case is \((K_i, q_{jt}^*)\).

Proof. From Claim 2 we know that at \((K_i, K_j)\), \(\frac{N_{it} \cdot K_j}{N_{it} \cdot K_i + N_{it} \cdot K_j} < \frac{1}{2}\) and \(\frac{N_{it} \cdot K_i}{N_{it} \cdot K_i + N_{it} \cdot K_j} > \frac{1}{2}\). Then firm \(i\) decreases its production and firm \(j\)'s marginal revenue of producing at \(K_j\) increases. Denote the best response of firm \(i\) given firm \(j\) is producing at \(K_j\) to be \(q_{it}^*\). Hence, if \(MR_j(K_i, q_{it}^*) > MC_{jt}\), the capacity-constrained Cournot equilibrium is \((q_{it}^*, K_j)\). However, if instead \(MR_j(q_{it}^*, K_j) = MC_{it}\), \(MR_j(K_j, q_{it}^*) < MC_{jt}\), that means as \(q_{it}\) decreases from \(K_i\) to \(q_{it}^*\), \(\frac{N_{it} \cdot K_j}{N_{it} \cdot q_{it}^* + N_{it} \cdot K_j}\) becomes larger than \(\frac{1}{2}\) so that firm \(j\)'s marginal revenue becomes increasing in firm \(i\)'s production. As \(q_{it}\) decreases further, \(MR_j(K_j, q_{it})\) decreases to some value smaller than \(MC_{jt}\). Then firm \(j\) will decrease from \(K_j\) and firm \(i\) will increase its production because its marginal revenue is decreasing in firm \(j\)'s production. Since the only unconstrained Cournot equilibrium consists of firm \(i\) and firm \(j\) both producing more than their respective capacity levels, there is no interior equilibrium starting from \((q_{it}^*, K_j)\). The only possible Cournot equilibrium for this case is at \((K_i, q_{jt}^*)\), where \(q_{jt}^*\) the best response of firm \(j\) given firm \(i\) is producing at \(K_i\). And at \((K_i, q_{jt}^*)\), \(MR_j(q_{it}^*, K_i) <= MC_{jt}\) and \(MR_i(K_i, q_{it}^*) >= MC_{it}\) with at most one equality holding.

(c) I show it is impossible that \(MR_i(K_i, K_j) < MC_{it}\) and \(MR_j(K_j, K_i) < MC_{jt}\).

Claim 4. It is impossible that \(MR_i(K_i, K_j) < MC_{it}\) and \(MR_j(K_j, K_i) < MC_{jt}\).

Proof. Proof by contradiction. Without loss of generality, suppose at \((K_i, K_j)\), \(\frac{N_{it} \cdot K_j}{N_{it} \cdot K_i + N_{it} \cdot K_j} \geq \frac{1}{2}\) and \(\frac{N_{it} \cdot K_i}{N_{it} \cdot K_i + N_{it} \cdot K_j} \leq \frac{1}{2}\). Then \(MR_i(K_i, q_{it}^*) \leq MR_i(K_i, K_j) < MC_{it}\) and \(MR_i(q_{it}^*, q_{jt}^*) < MR_i(K_i, q_{jt}^*) < MC_{it}\). This contradicts with the fact that the unconstrained Cournot equilibrium be at \(q_{it}^* > K_i\) and \(q_{jt}^* > K_j\).
References


