Uncertainty, Financial Frictions, and Investment Sensitivity to Cash Flow

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November 7, 2017

Abstract

This paper introduces investment timing options into the long-standing discussion regarding the relationship between corporate investment sensitivity to cash flow (ISCF) and financial frictions. I develop a dynamic, stochastic model in which firms possess an option to delay investment. My model predicts that a higher degree of financial frictions decreases ISCF by reducing firms’ borrowing capacity. Meanwhile, more financial frictions also diminish the option value of delaying, encouraging firms to invest sooner rather than later; as a result, ISCF increases. The relative strength of these two countervailing forces depends on the distribution of heterogeneous agents. Furthermore, I introduce uncertainty as an additional determinant of ISCF. Heightened uncertainty decreases ISCF as it enhances the option value of delaying. Empirical work studying U.S. listed firms over the past 30 years confirms these predictions. In addition, I investigate the Bonus Depreciation policy (which boosts after-tax cash flow); results suggest that reducing uncertainty improves policy effectiveness.

Keywords: investment sensitivity to cash flow, uncertainty, real options, financial frictions, bonus depreciation

JEL Classification: E32, E44, E62, G31, G32, G33

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1 Introduction

Firms in the world of Modigliani and Miller (1958) can invest without having any cash on hand. However, in the real world, as emphasized by Fazzari et al. (1988), financial frictions give rise to positive “Investment Sensitivity of Cash Flow” – hereafter ISCF – which measures a firm’s marginal propensity to invest an additional dollar of cash. In this paper, I study the determinants of a firm’s ISCF both theoretically and empirically. Characterizing the determinants of ISCF matters at both micro and macro levels. At the micro level, it helps us better understand how firms make financing and investment decisions. At the macro level, it relates to the effectiveness of liquidity-provision policies designed to spur aggregate investment. In response to the Great Recession, policymakers considered numerous actions to provide liquidity to firms in an effort to overcome financial frictions. As Hubbard (1997) emphasizes, these policies can stimulate investment by increasing internal cash flow. The ultimate effects of these policies on investment therefore depend on ISCF.

I develop a dynamic, stochastic model featuring financial frictions and uncertainty about future profitability. These two shocks correspond to what Stock and Watson (2012) write about the causes of the Great Recession. Well-developed literature explains why such shocks reduce investment levels. My model further predicts that they affect a firm’s ISCF. First, more financial frictions can either increase or decrease ISCF, with the relative strength of the two countervailing effects depending on the distribution of heterogeneous plants within the firm. Moreover, greater uncertainty tends to reduce ISCF. I empirically confirm these theoretical predictions using U.S. firm-level data over the past 30 years. In addition, I evaluate the stimulative effects of the Bonus Depreciation policy, which boosts after-tax cash flow, on investment. I observe that the investment of firms facing greater uncertainty responds less to a given policy shock. This finding is consistent with these firms having smaller ISCF. It suggests that, although liquidity-provision policy can ease financial constraints during downturns, heightened uncertainty could reduce its effectiveness in stimulating investment.

The existing literature has investigated the effects of financial frictions on ISCF, al-

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1 Examples include quantitative easing, which lowers firms’ borrowing costs, and tax incentives, which enhance after-tax profits. In these two examples, quantitative easing lowers firms’ debt service expenditures, while tax incentives reduce tax expenditures. Both types of policy increase the availability of firms’ disposable incomes. Besides this income effect, these policies affect investment through other channels, including the neoclassical effect influencing users’ cost of capital; the currency effect, which changes exchange rates; etc.

2 Stock and Watson (2012) write “shocks that produced the recession primarily were associated with financial disruptions and heightened uncertainty.”

3 Kiyotaki and Moore (1997) and Bernanke et al. (1999) develop canonical frameworks to explore the effect of financial frictions on investment. The effect of uncertainty on investment in a real-options framework is studied in a long series of studies beginning with McDonald and Siegel (1986).
though it rarely mentions the role of uncertainty. In a frictionless world described by Modigliani and Miller (1958), ISCF is zero as long as cash flow contains no information about investment opportunities. Once frictions are introduced, ISCF becomes positive as more cash flow eases financial constraints and in turn enhances investment. Following this logic of comparing the cases with and without frictions, it is natural to conjecture that the magnitude of ISCF should increase with the degree of financial frictions. Fazzari et al. (1988) (henceforth, FHP) find empirical results consistent with this conjecture – ISCF is larger for more financially constrained firms. Many later papers reach similar conclusions while many others disagree, notably Kaplan and Zingales (1997) (henceforth, KZ). KZ provide a static model and empirical evidence supporting that ISCF instead decreases with the degree of financial frictions. My dynamic model can help understand these two opposing forces of financial frictions and further introduce uncertainty as an additional determinant of ISCF.

In Section 2 of this paper, I develop a model featuring exogenous cash flow that is uncorrelated with future investment opportunities and introduce financial frictions in the form of collateral constraints as in Kiyotaki and Moore (1997). In this setup, ISCF is positive if and only if collateral constraints bind. Following Jermann and Quadrini (2012), I model more financial frictions as collateral constraints being tighter – when there is a higher haircut or stricter down-payment requirements in collateralized borrowing.

A change in financial frictions has two opposing impacts on ISCF in my model. First, more frictions can decrease ISCF. This is because, when down-payment requirements become higher, an additional unit of cash flow can lever up less external funding and therefore adds less to investment. Henceforth, I refer to this channel as the “capacity channel,” which could be well interpreted in a static setup – more financial frictions limit firms’ ability to borrow and therefore negatively impact their capacity to invest in response to cash flow shocks.

In a dynamic setup, more financial frictions can also enhance ISCF. This is based on firms having multiple margins to adjust investment – besides the capacity channel, which works through the intensive margin, there is an extensive margin in which firms can choose to invest immediately or delay investment to the future. Suppose investing immediately and delaying investment if firms expect investment returns to grow over time. This mechanism can be implemented in many different ways. My model uses a temporary late-payment structure to achieve it; Boyle and Guthrie (2003), and Bolton et al. (2014) use other setups but develop a similar argument. See Section 2 for details.


6 Specifically in my model, firms want to delay investment if they expect investment returns to grow over time. This mechanism can be implemented in many different ways. My model uses a temporary late-payment structure to achieve it; Boyle and Guthrie (2003), and Bolton et al. (2014) use other setups but develop a similar argument. See Section 2 for details.
investing in the future are mutually exclusive. Abstracting from uncertainty (discussed in the next paragraph), firms choose by comparing the net present values (NPVs) of the investing-immediately choice and the investing-later choice. In my model, an increase in financial frictions can act as an effective hike in discount rates, reducing the value of investing later. More financial frictions therefore encourage firms to invest immediately and hence enlarge ISCF. I refer to this channel as the “timing channel” because financial frictions accelerate the timing of investment.

The dynamic setup also reveals that uncertainty affects ISCF via a “real-options channel,” again through the extensive margin. In particular, firms have an incentive to postpone irreversible investment when facing uncertainty. This is to wait for uncertainty to resolve in order to avoid regret. This incentive of delaying becomes stronger as uncertainty grows. Then, when given a marginal dollar, firms are less likely to invest that dollar immediately, hence ISCF decreases. As a model extension, I further discuss possible precautionary motives caused by uncertainty interacting with bankruptcy costs, and I discover qualitatively the same lessening impacts of uncertainty on ISCF. In the empirical part of this paper, I present evidence that the real-options effect is more important for my sample.

To the best of my knowledge, only a few previous studies concentrate on uncertainty’s impact on ISCF, but none of them investigate real options’ implications in a structural model. Baum et al. (2009) introduce financial constraints into an otherwise-standard neoclassical model without explicitly suggesting how and why uncertainty affects ISCF. Moreover, based on the convex adjustment costs in the neoclassical setup, their model is unable to address the role of real options. Li et al. (2015) document an empirical pattern whereby the ISCF of corporations in emerging markets is smaller when measured uncertainty in the U.S. is higher. In that paper, we link this phenomenon to the real-options effect without offering a structural explanation.

Section 3 confronts my model’s predictions about ISCF with a large panel of firm-level data from 1984 through 2015. My empirical method falls within a panel regression framework. I find that firms display larger ISCF when facing more financial frictions. This suggests that the timing channel, associated with the extensive margin, outweighs the capacity channel of the intensive margin. I also find ISCF to be smaller when uncertainty is higher,

7 Another strand of literature documents the time-series variation of ISCF but has not discussed the role of uncertainty. For example, Chen and Chen (2012) find that ISCF has become smaller in recent years. McLean and Zhao (2014) point out that ISCF tends to be bigger during recession years and when consumer sentiments are low. Also, see Allayannis and Mozumdar (2004), Ascioghi et al. (2008), A˘ gca and Mozumdar (2008), and Brown and Petersen (2009). Ai et al. (2016) instead propose that moral hazard and managerial compensation are crucial factors driving ISCF.
consistent with the model. Both results are economically significant. Point estimates imply that when financial frictions increase by one standard deviation, the median ISCF increases by 30 percent. When the measure of uncertainty increases by one standard deviation, the median ISCF decreases by 62 percent.

In the existing empirical literature, the sign of the effect of financial frictions on ISCF is unclear. Many papers agree with FHP that the sign is positive, while many others agree with KZ that the sign is negative. Besides the underlying theoretical causes, one reason for these divergent results is the choice of proxy for financial frictions. Section 3 discusses why I use the “excess bond premium” constructed by Gilchrist and Zakrajšek (2012) and firm size as proxies. Another reason for this divergence is substantial endogeneity in cash flow. Unlike in a model, real-life cash flow is typically not exogenous but correlated with investment opportunities; furthermore, controls for investment opportunities are imperfect. I follow much of the literature by using Tobins Q, adjusting for measurement errors with the method of Erickson et al. (2014) and Erickson et al. (2016). Section 3 discusses other methods I use to exploit exogenous variation in uncertainty and financial frictions.

Finally, Section 4 uses the Bonus Depreciation policy in the periods from 2001 to 2004 and from 2008 to 2011 to study a real-world implication of uncertainty decreasing ISCF. The policy reduces firms’ tax payments, boosting after-tax cash flow. I follow Zwick and Mahon (2017) by identifying the effects of the policy through industry variation and further allow the effects to vary with firms’ individual uncertainty. As my model predicts, the effects of the policy on investment are smaller for firms facing greater uncertainty. Point estimates imply that a one-standard-deviation increase in the measure of uncertainty decreases the investment response by 23 percent. This result highlights the importance of uncertainty in determining the effects of government policies to stimulate investment.

2 A Model

In this section, I present a dynamic, stochastic model in which ISCF is positive if and only if firms face financial frictions, specifically binding collateral constraints. I use the model to study analytically how the magnitude of ISCF changes with the degree of financial frictions.

8The measurement errors refer to the difference between measured Q and underlying investment opportunities. This is documented by, among others, Erickson and Whited (2000), Erickson and Whited (2002), Erickson and Whited (2012), Erickson et al. (2014), and Abel (2015).

9Bonus Depreciation has effects other than generating more cash flow to help firms ease financial constraints. See, among others, House and Shapiro (2008) and Zwick and Mahon (2017). I discuss this in more detail in Section 4.
and the uncertainty over future profitability. Studying this question structurally has, among others, two significant advantages. For one, being able to set cash flow uncorrelated to future investment opportunities makes financial frictions both sufficient and necessary for ISCF to be positive. Moreover, with a structural model, I can separately identify the potential two opposing impacts of more financial frictions on ISCF. In contrast, a reduced-form study can only detect the overall effect, depending on which single impact dominates.

2.1 Setup

Each firm $i$ consists of $J$ risk-neutral production units $\{i_j\}_{j=1}^J$ that exist for three periods. I consider a unit-level, rather than a firm-level, model to overcome a disconnection between theory and data. In theory, under certain circumstances, it is optimal for firms to choose zero investment. However, in the data, zero investment is almost non-existent for individual firms. Bloom (2009) documents that even though survey data demonstrate that zeros are common at the unit level, firms’ frequent reporting of total investment in consolidated financial statements makes zeros hard to detect at the firm level. Moreover, I present a three-period model to obtain a closed-form solution, while the model’s essence applies to a longer or infinite horizon.

Suppose production units produce using constant-returns-to-scale technology with capital as the single input. Units are heterogeneous both ex ante and ex post. At the beginning of period 1, unit $i_j$ (the $j$-th unit of firm $i$) is given a stochastic cash flow, $w_{ij}$, from the parent firm. At the same time, the unit learns about the distribution from which it will draw the marginal profitability of capital that will last in the following two periods; the realized profitability is referred to as $z_{ij}[10]$. Assume $z_{ij}$ is distributed log-normally and subject to a firm-specific mean, $\bar{z}_i$, but unit-specific variances: $\log z_{ij} \sim N(\log \bar{z}_i - \sigma_{ij}^2/2, \sigma_{ij}^2)[11]$. Furthermore, assume there is a firm-level common factor, $\sigma_i$, driving all the $\{\sigma_{ij}\}_{j=1}^J$ through a set of unit-specific factor-loading $\{\gamma_{ij}\}_{j=1}^J$, i.e., $\sigma_{ij} = \gamma_{ij} \cdot \sigma_i$ for all $j$. Call $\sigma_i$ the firm-level uncertainty. Any increase in $\sigma_i$ shifts the whole distribution of $\sigma_{ij}$ to the right[12] $\sigma_i$, $\{\gamma_{ij}\}_{j=1}^J$, and $\{w_{ij}\}_{j=1}^J$ are common knowledge after being realized. Denote the cross-sectional distri-

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[10] I follow Bloom (2009) to assume that $z_{ij}$ can represent either productivity or consumer-demand conditions, or both. Distinguishing whether supply or demand drives profitability of the corporate sector is not the focus of this paper.

[11] Whether two production units in the same firm have correlated or independent $z_{ij}$ does not affect the model’s results.

[12] This assumption is motivated by Bloom et al. (2014), who note that unit-level uncertainty (measured by TFP shocks) is significantly correlated with the parent firm’s stock volatility; the latter is a widely-adopted measure of firm-level uncertainty.
butions of $\{\gamma_{ij}\}_{j=1}^{J}$, $\{\sigma_{ij}\}_{j=1}^{J}$, and $\{w_{ij}\}_{j=1}^{J}$ by $F_{i,\gamma}$, $F_{i,\sigma}$, and $F_{i,w}$, respectively.\footnote{Here, I do not impose any restrictions on the relationship between $F_{i,\sigma}$ and $F_{i,w}$. In other words, this modeling setup is neutral to any internal competition for resources across different units. In particular, one could imagine that it is after the realization of $F_{i,\sigma}$ that the CFO of the parent firm redistributes resources so that $F_{i,w}$ is determined by uncertainty. While modelling the CFO’s optimization is beyond the scope of this paper, I discuss how it could qualitatively affect my empirical predictions at the end of this section.}

For brevity, in the following context, I suppress the subscript $i$—i.e., I use $j$ to denote $i_j$. I further employ the subscripts $(1j), (2j),$ and $(3j)$ to represent the variables in periods 1, 2, and 3.

Financial markets have frictions in the sense that (1) borrowings are subject to collateral constraints, as in Kiyotaki and Moore (1997), and (2) equity finance is disallowed.\footnote{Myers and Majluf (1984) predict that equity finance could be much more costly than debt finance. Here, I assume the cost to be infinite. Even though firms do use equity to fund investment projects in reality, this assumption does not affect my model’s predictions qualitatively.} The collateral constraints require the value of debt to be limited by a fraction $\theta$ (between 0 and 1) of the collateral value. $\theta$ is common to all production units within a single firm (subscript $i$ is suppressed). I assume $\theta$ to be exogenous, with lower $\theta$ indicating more exposure to financial frictions, namely, tighter collateral constraints. In this setup, $(1-\theta)$ represents the haircut ratio coming from the disagreement between borrowers and lenders in the valuation of collateral.\footnote{See Quadrini (2011) and Jermann and Quadrini (2012) for a similar setup.} Geanakoplos (2010) links this haircut ratio to lenders’ risk aversion, with more risk-averse lenders requiring a higher haircut ratio.\footnote{An alternative cause for the collateral constraints could pertain to agency problems. For instance, a firm’s manager can divert a proportion of the firm’s resources for private benefits, say $(1-\theta)$. Thus, only part of the firm’s value can be pledged to investors. See, among others, Lorenzoni et al. (2007) and Buera and Moll (2015) for motivating haircuts in this way. Another strand of the literature further utilizes the dynamic contracting theory to endogenize the haircut ratio; for example, see DeMarzo et al. (2012), Ai and Liu (2015), and Ai et al. (2016).}

Because of the collateral requirements, both savings and borrowings are risk-free at a gross interest rate, $R_f$, per period. Particularly, there exists a two-period bond in period 1 and a one-period bond in period 2 that each production unit can buy or sell. Capital goods and financial assets (i.e., savings) could serve as collateral, though only the former incurs a haircut. There is no haircut if financial assets are used as collateral.\footnote{For example, a production unit could save at $R_f^2$ in the first period and borrow against this two-period bond in the second period at $R_f$; effectively, the unit saves in a one-period bond in period 1 at a rate equal to $\frac{R_f^2}{R_f} = R_f$. Therefore, the setup where there is only a two-period bond in period 1 and a one-period bond in period 2 does not twist the optimal policy.} This is because lenders and borrowers have no disagreement on the value of risk-free savings.

Capital adjustment features an irreversibility constraint whereby disinvestment is not allowed unless the whole capital stock is sold at a discounted price. Suppose the purchasing price of capital is normalized to 1; the selling price (also called the “liquidation value”) is assumed to be exogenous, equal to $q$ between 0 and 1. This non-convex adjustment cost
generates a real-options effect when interacting with uncertainty, which encourages units to delay investment, waiting for uncertainty to resolve.\textsuperscript{18} In contrast, the convex adjustment costs commonly adopted in the neoclassical literature are neglected in this paper.\textsuperscript{19} Capital stock will thus spike to the constrained-optimal level during any adjustment process.

In addition, I make a specific assumption that production units anticipate the returns of capital investment to grow over time. This anticipation generates an additional waiting motive besides the real-options effect, if investing today and investing tomorrow are mutually exclusive – units delay investment today to wait for higher investment returns tomorrow.\textsuperscript{20} I further tune the model to let a higher degree of financial frictions reduce this waiting motive by decreasing the value of investing in the future relative to investing today; more frictions, therefore, accelerate exercising investment. This logic can be implemented in many different ways, but the exact form does not affect the logic’s implications. My model is designed in a specific way that simplifies the mathematics; see also Boyle and Guthrie (2003) and Bolton et al. (2014) for two other implementations.\textsuperscript{21}

In particular, I assume that the profits produced by the capital goods installed in period 1 cannot be delivered on time; the payment will be late by an extra period. This late payment reduces the returns of investing in period 1 directly because of a discounting effect. Based on financial frictions, the reduction could be even bigger. This is because, without late payment, a financially constrained unit would use the profits as a down payment to lever up external funds, which in turn can be used for dividends or reinvestment. With late payment, however, the value added by leveraging up disappears.

I further suppose that the late-payment friction only exists in period 1. In other words, by delaying investment to period 2, the unit can skip the late payment. It will get the profits produced by invested capital goods sooner. In this sense, investment returns grow upon

\textsuperscript{18}See Caggese (2007) for a similar setup. Other types of non-convex adjustment costs, such as partial irreversibility, will achieve the same goal. I stick to total irreversibility because disinvestment is not the focal point of this paper. Another alternative is fixed costs; including them in the model will further strengthen the effects of real options, but with no qualitative difference.

\textsuperscript{19}In fact, the structural estimation in Bloom (2009) shows that these costs are indeed small.

\textsuperscript{20}In contrast, the real-options effect makes units to wait for less-uncertain investment returns.

\textsuperscript{21}Boyle and Guthrie (2003) argue that more financially constrained firms are less willing to delay investment because they fear that, in future, they may be hit by negative shocks and lose the opportunity to fund the project. Bolton et al. (2014) propose that with multiple rounds of growth options, a financially constrained firm may choose to over-invest via accelerating investment timing in earlier stages in order to mitigate the under-investment problems in later stages. There is also empirical evidence supporting that more financial frictions might accelerate exercising investment. Whited (2006) documents that small firms (which potentially face more financial frictions) invest more often than big firms. Boyle and Guthrie (2003) document that it is the standard folklore that small firms are more aggressive about entering new markets or launching new products than big firms.
delaying, where the growth majorly comes from the capacity of using profits from invested capital as a down payment for borrowing. A disadvantage of delaying is that the unit will need to forego a whole period’s production – if it invests in period 1, it will produce in both period 2 (though the payment is delayed) and period 3; if it invests in period 2 instead, it can only produce in period 3. In order to decide whether to delay investment, the unit compares what it gives up by delaying to what it gains. When collateral constraints become tighter, what the unit gains from delaying will be reduced, as the unit’s capacity to borrow becomes more limited. The unit therefore has fewer motives to delay investment.

The late payment setup is a reduced-form assumption, which is qualitatively equivalent to more financial frictions effectively increasing firms’ discount rates, and therefore lowering the value of delaying investment. This assumption simplifies the mathematics and, as I will state later, contributes by mitigating a well-documented Oi-Hartman-Abel effect. On the other hand, the late payment setup per se can be linked to reality in many ways. For instance, it could be treated as unsecured accounts receivable that the unit cannot borrow against. However, over time, as the parent firm builds up reputation with lenders, the unsecured accounts receivable could become secured. Then, as long as the unit can borrow against the accounts receivable, the late payment effectively disappears. Another example is that the parent firm might need time (e.g., the first period) to set up the market (e.g., through advertising) before its production units can transfer production into profits, which leads profits to be paid late. In addition, the assumption could characterize the fact that units are able to utilize capital goods more efficiently as they receive technological spillovers from other units over time.

The model’s timeline is as follows. In period 1, unit $j$ has an initial capital stock, $k_{1j}$. It distributes its exogenous cash flow, $w_j$, between dividends, $d_{1j}$, and investment, $\iota_{1j}$. The unit is also able to save in a risk-free bond ($b^L_{1j} < 0$) or borrow external debt ($b^L_{1j} > 0$). The value of external debt has to be limited by a fraction, $\theta$, of the liquidation value of the capital that the unit possesses. The superscript “L” stands for “long,” indicating that the savings or borrowings are in two periods. There is no production or capital depreciation in the first period to simplify the algebra. Formally, see equations (1) to (5). Equations (1) and (5) are the budget constraint and the irreversibility constraint, respectively. Equation (3) is the collateral constraint for two-period debt. Equation (2) describes the capital accumulation

\footnote{In the COMPSTAT U.S. sample, the flow of accounts receivable takes up approximately 25\% of annual cash flow for the median firm, and the number of days’ sales is nearly one and a half years. Borrowing against accounts receivable requires high-quality invoices and short maturity (e.g., six months).

Both the collateral-constraint parameter $\theta$ and the liquidation price $q$ determine the value of collateral simultaneously. The product of the two, $\theta \times q$, corresponds to the fire-sale value of capital. Distinguishing or combining the two parameters makes no difference mathematically. I use two separate terms because}
process, and equation (4) rules out external equity finance.

\[
\begin{align*}
  w_j + b^L_j & = d_{1j} + \iota_{1j} \quad (1) \\
  k_{2j} & = k_{1j} + \iota_{1j} \quad (2) \\
  R_2^j b^L_j & \leq \theta q(1 - \delta)^2 k_{2j} \quad (3) \\
  d_{1j} & \geq 0 \quad (4) \\
  \iota_{1j} & \geq 0 \quad (5)
\end{align*}
\]

At the beginning of period 2, the marginal profitability of capital, \(z_j\), realizes. All the uncertainty resolves in the sense that \(z_j\) will be carried over to the last period. The unit generates profits, \(z_j k_{2j}\), but in the form of accounts receivable – it cannot obtain the proceeds until the last period. Besides leading to a waiting motive, as documented by the previous context, this late-payment setup is also a technical assumption to mitigate the Oi-Hartman-Abel effect, matching the model with the data.\(^{24}\) Unit \(j\) can decide whether to invest, but it is not allowed to disinvest, as expressed by (10). Capital accumulation is subject to depreciation in equation (7). Borrowings and savings are allowed in the one-period bond, \(b^S_j\), as long as the collateral constraint (8) is satisfied, where the superscripted “S” signifies short-term debt. Long-term debt accumulated from the previous period also needs to be taken into account. Equation (6) is the budget constraint, and (9) rules out equity finance.\(^{25}\)

\(\theta\) represents the risk appetite of marginal lenders in financial markets, while \(q\) reflects the preference of marginal buyers in the markets of used capital.

\(^{24}\) Specifically, without such a design, if the unit invests in period 1 and receives investment payoff in period 2, it may reinvest the proceeds when \(z_j\) is high, which leads to a convex term of \(z_j^2\) in the marginal value of capital. Such convexity will lead to a Jensen’s inequality effect when \(z_j\) is uncertain – the greater the variance of \(z_j\), the higher the expected marginal value of capital before investing; units are more willing to invest. Therefore, uncertainty encourages investment, opposing the commonly-accepted empirical evidence, e.g., Bloom (2009). This effect of convexity is in a similar spirit as Oi (1961), Hartman (1972), and Abel (1983), though in their work the convexity is in the production function.

\(^{25}\) Equations (6), (9), and (10), taken together, suggest that \(b^S_j\) is non-negative; however, this does not mean the production unit cannot save in period 2. In fact, with its two-period savings carried over from period 1, it effectively saves in period 2 if it chooses not to withdraw (borrow against) this amount of savings.
\[ b_j^S = d_{2j} + \nu_{2j} \]  \hfill (6)

\[ k_{3j} = (1 - \delta)k_{2j} + \nu_{2j} \]  \hfill (7)

\[ R_j^2 b_j^L + R_j b_j^S \leq \theta q(1 - \delta)k_{3j} \]  \hfill (8)

\[ d_{2j} \geq 0 \]  \hfill (9)

\[ \nu_{2j} \geq 0 \]  \hfill (10)

In period 3, \( z_j \) is carried over from period 2. Production unit \( j \) operates to generate profits, \( z_j k_{3j} \), and the payment is delivered immediately. As this is the last period, the unit no longer invests but instead liquidates the total capital stock at the price \( q \). It does not save or issue new debt. Accounts receivable, \( z_j k_{2j} \), and any savings from previous periods pay back. All proceeds less existing debt will be paid out as dividends. Formally,

\[ d_{3j} = z_j (k_{2j} + k_{3j}) + q(1 - \delta)k_{3j} - R_j^2 b_j^L - R_j b_j^S \]  \hfill (11)

\[ d_{3j} \geq 0 \]  \hfill (12)

Each production unit maximizes \( \mathbb{E}[d_{1j} + \beta d_{2j} + \beta^2 d_{3j}] \) subject to (1)–(12), where \( \beta \) denotes the time preference common to all units.

### 2.2 Analytical Solution

This section solves the model analytically through backward induction. The last period is trivial – the production unit pays all remaining resources out as dividends. Assumption 1 pins down the solution of period 2, as described by Proposition 1.

**Assumption 1.**

\[ \beta R_f < 1 \]

**Proposition 1.** Under Assumption 1, in period 2, if the realized \( z_j \) is sufficiently high, the production unit will invest all financial resources, including both internal funds and external credit; otherwise, it will maintain \( k_{3j} = (1 - \delta)k_{2j} \) and pay out other resources. Formally,

\[ \text{Letting accounts receivable earn interests or not does not affect the model’s predictions, because the value loss of getting accounts receivable instead of immediate payments majorly comes from being unable to use the proceeds as a down-payment to lever up external funds.} \]
If $z_j \leq \psi_1$, 
\[
\begin{align*}
k_{3j} &= (1 - \delta)k_{2j} \\
b_j &= \frac{1}{R_f} \left[ \theta q(1 - \delta) z_j - R_f^2 b_j^L \right] \\
d_{2j} &= b_j^S
\end{align*}
\]

If $z_j > \psi_1$, 
\[
\begin{align*}
k_{3j} &= \frac{(1 - \delta) k_{2j} - R_f b_j^L}{1 - \theta q(1 - \delta) R_f} \\
b_j &= \frac{1}{R_f} \left[ \theta q(1 - \delta) k_{3j} - R_f^2 b_j^L \right] \\
d_{2j} &= 0
\end{align*}
\]

where 
\[
\psi_1 = \frac{1}{\beta} \left[ 1 - \beta q(1 - \delta) - \frac{1}{R_f} \beta \theta q(1 - \delta) \right]
\]

The value function is given by:
\[
V_2(k_{2j}, b_j^L, z_j) = \begin{cases} 
\left[ \beta z_j + (1 - \delta)z_j + q(1 - \delta)^2 \left( \frac{1}{R_f} - \frac{1}{R_f} - \beta \right) (1 - \theta) \right] k_{2j} - R_f b_j^L & \text{if } z_j \leq \psi_1 \\
\beta z_j k_{2j} + \beta \left( \frac{z_j + q(1 - \delta)(1 - \theta)}{1 - \theta q(1 - \delta) R_f} \right) ((1 - \delta) k_{2j} - R_f b_j^L) & \text{if } z_j > \psi_1 
\end{cases}
\]

The production unit distributes limited resources among dividends, savings, and investment. The impatience condition in Assumption 1 states that the production unit prefers dividends to savings, so the borrowings constraint (8) maintains binding as the unit borrows to consume dividends. Comparing $z_j$ to $\psi_1$ is, in fact, comparing $[\beta z_j + \beta q(1 - \delta) + \left( \frac{1}{R_f} - \beta \right) \theta q(1 - \delta)]$ to 1; the latter represents the price of capital goods in terms of dividend consumption. The former summarizes the benefits of investing – the first term is the value of extra production from the invested capital, the second term is the liquidation value of the capital goods after depreciation, and the last term indicates the utility gains from impatience, where the production unit can borrow to consume dividends using the additional capital goods as collateral. Whether to invest therefore relies on whether this whole term exceeds 1. Conditional upon investing, the target capital stock is proportional to the production unit’s net worth, $((1 - \delta) k_{2j} - R_f b_j^L)$. This is because of the first-order homogeneity of the value function, caused by constant-returns-to-scale technology and lacking convex capital adjustment cost. The denominator, $(1 - \theta q(1 - \delta) R_f)$, indicates the down-payment requirement.\[27\]

If not investing, the production unit pays out all the available resources, letting capital goods depreciate.

Assumption 2.

$\bar{z} > \max \{ \psi_1, \psi_2 \}$

\[27\]This expression is common in the literature of collateral constraints. Specifically, for each unit of capital goods that is worth 1, only an amount equal to $\frac{\theta q(1 - \delta)}{R_f}$ is allowed to be paid by external funds; the rest has to be paid by internal funds as down payments.
\[\psi_2 = \psi_1 + \frac{1}{\beta} \left[ \frac{1}{\beta R_f} - 1 \right][1 - \frac{\theta q(1 - \delta)}{R_f}]\]

**Proposition 2.** Under Assumption 2, in period 1, if the expected marginal value of capital is greater than the expected marginal value of savings, the production unit will invest all resources, including both internal funds and external credit; otherwise, it will save internal funds in the two-period bond, i.e., \(b^L_j < 0\). Formally,

\[
\begin{align*}
\text{If } x_{kj} \leq -(x_{bj}), & \quad k_{2j} = k_{1j} \\
& \quad b^L_j = (-w_j) \\
& \quad d_{1j} = 0
\end{align*}
\]

\[
\begin{align*}
\text{If } x_{kj} > -(x_{bj}), & \quad k_{2j} = \frac{w_j + k_{1j}}{1 - \frac{\theta q(1 - \delta)^2}{R_f}} \\
& \quad b^L_j = \frac{\theta q(1 - \delta)^2 k_{2j}}{R_f} \\
& \quad d_{1j} = 0
\end{align*}
\]

where

\[
\begin{align*}
x_{kj} &= \mathbb{E}\left[ \frac{\partial V_2(j, k_{2j}, b^L_j, z_j)}{\partial k_{2j}} \right] \\
x_{bj} &= \mathbb{E}\left[ \frac{\partial V_2(j, k_{2j}, b^L_j, z_j)}{\partial b^L_j} \right]
\end{align*}
\]

Under Assumption 2, Proposition 2 summarizes the optimal policy in period 1. I use \(x_{kj}\) to denote the expected marginal value of capital. It is indeed comparable to the marginal \(q\) in the neoclassical literature. I use \(x_{bj}\) to denote the expected marginal value of debt. Conditional on debt being treated as negative savings, \(-x_{bj}\) is the expected marginal value of savings. When the unit saves in period 1, it effectively delays investment and will re-decide whether to invest in period 2. The incentives to delay come from the motives of waiting for investment returns to become either less volatile (as uncertainty resolves) or higher (as the unit skips the late payment) or both. I therefore also refer to \(-x_{bj}\) as the value of waiting.

Assumption 2 is derived from \(\beta(-x_{bj}) > 1\) for all \(j\), which is to compare saving (waiting) with dividend consumption. Specifically, as long as the expected profitability \(\bar{z}\) is high enough, the production unit is willing to save rather than consume dividends, conditional on not investing, so that it keeps the option open to invest later.

As a consequence, the optimal policy in period 1 is to choose between investing and saving (waiting), which is determined by comparing \(x_{kj}\) and \(-x_{bj}\) based on the first-order homogeneity of the value function. If \(x_{kj}\) is greater, the production unit will invest all financial resources, rendering the borrowing constraint binding. The amount of new capital equals the ratio between the production unit’s net worth \((w_j + k_{1j})\) and the down-payment.

\(^{28}\)For example, see [Hayashi (1982)](http://example.com) and [Abell (1983)](http://example.com).
requirement. Otherwise, the production unit will wait by saving all resources in the two-
period bond. Intuitively, the trade-off is, when investing in period 1, the unit expands capital
stock and therefore can produce extra output in period 2, but it receives late payment for
such proceeds. Moreover, as profitability is uncertain, this amount of extra production could
be too low to be efficient ex post (lower than the opportunity cost of investment). In contrast,
if a production unit chooses to delay investment, in period 2 it will invest only if it observes
that the realized profitability is high. As such, it foregoes the extra amount of production but
gains the option to avoid inefficient investment; at the same time, it skips the late payment.

**Proposition 3.** \((x_{kj} + x_{bj})\) is a decreasing function of \(\sigma_j\).

Proposition 3 shows that the investment determinant, \((x_{kj} + x_{bj})\), decreases with the
individual variance, \(\sigma_j\). The production unit tends to wait by saving when the variance is
large. This is because, by saving, the unit invests only after observing high \(z_j\) ex post. It
thereby generates a realized marginal value function of saving that is convex in \(z_j\). Greater
variance therefore raises the expected marginal value of savings \((-x_{bj})\) from a Jensen’s
inequality effect, which decreases \((x_{kj}+x_{bj})\). **Figure 1** and **Figure 2** depict the case graphically
under standard calibration in **Table 1**. Thanks to the closed-form solution, Proposition 3 does
not rely on these numerically set parameters as long as Assumptions 1 and 2 are satisfied.

**Assumption 3.**

\[
\max\{\psi_1, \psi_3\} < \zeta < \psi_4
\]

\[
\psi_3 = \frac{[R_f - (1 - \delta)](1 - \frac{\beta \psi_1 (1 - \delta)}{1 + \theta q (1 - \delta) R_f})}{\beta [1 + \frac{1}{1 - \theta q (1 - \delta) R_f} (1 - \delta - R_f)]}
\]

\[
\psi_4 = \frac{[R_f - (1 - \delta)] + \beta \psi_1 (1 - \delta)}{\beta [1 + \frac{1}{1 - \theta q (1 - \delta) R_f} (1 - \delta - R_f)]}
\]

\[
\beta > (1 - \delta)
\]

\[
R_f < (1 - \delta) + (1 - \frac{\theta q (1 - \delta)}{R_f})
\]

**Proposition 4.** Under Assumption 3, there exists a unique root, \(\sigma^*\), for \((x_{kj} + x_{bj})\)\((\sigma_j) = 0\).
For any production unit, \(j\), if \(\sigma_j < \sigma^*\), then \(x_{kj} > -x_{bj}\); if \(\sigma_j > \sigma^*\), then \(x_{kj} < -x_{bj}\).

Given that \((x_{kj} + x_{bj})\) continuously declines with \(\sigma_j\), the existence and uniqueness of
\(\sigma^*\) come directly from the Intermediate Value Theorem, which requires \((x_{kj} + x_{bj})\) to be

\[29\] is specific to a particular consolidated firm and is common to all production units within this firm.
Again, my notation suppresses the firm subscript \(i\) here.

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positive at $\sigma_j = 0$ and negative at $\sigma_j = +\infty$. Assumption 3 ensures this $-\overline{z}$ must be in a moderate region. If $\overline{z}$ is too small, $(x_{kj} + x_{bj})$ will always be negative, and the production unit will always opt to wait because of the expectation that the benefits of investing are too minimal to overcome the value of waiting. In contrast, if $\overline{z}$ is too large, $(x_{kj} + x_{bj})$ will always be positive (the production unit will always choose to invest immediately). The other two conditions in Assumption 3 are sufficient (not necessary) to make a choice of $\overline{z}$ exist, i.e., $\max\{\psi_1, \psi_2, \psi_3\} < \psi_4$\(^{30}\). If the production unit is too impatient, it will never choose to wait, because forgoing near-future profits is just too costly. Moreover, if saving is too profitable, the production unit will always choose to save (wait) but never invest. In the next subsection, I show that more financial frictions contribute to this decision process by increasing the root $\sigma^*$, therefore reducing the value of waiting.

### 2.3 Investment Sensitivity to Cash Flow

I construct ISCF analytically using the optimal investment policy in period 1. For a consolidated firm, consider all the production units that belong to this firm, $\{j\}_{j=1}^J$. ISCF at the unit level is defined as the partial derivative of investment to cash flow, with both terms normalized by the existing capital stock, namely, $\frac{\partial \iota_j}{\partial w_j/k_1}$. Taking Propositions 2 and 4 together,

$$ISCF_j = \begin{cases} 
0 & \text{if } \sigma_j > \sigma^* \\
\frac{1}{1 - \theta_j (1 - \delta)^2} \frac{\partial \iota_j}{\partial w_j/k_1} & \text{if } \sigma_j \leq \sigma^* 
\end{cases} \quad (14)$$

Depending on their individual variances, production units can be classified into two groups, in which they have different investment behavior and display distinct ISCF. For those units with variances greater than the firm-specific $\sigma^*$, investment is zero, regardless of cash flow; ISCF is therefore zero. For those possessing variances smaller than $\sigma^*$, in contrast, investment responds positively to cash flow because of binding collateral constraints; ISCF is positive. Moreover, it is straightforward from (14) that when $\theta$ is set to $+\infty$, namely when borrowing constraints are removed, ISCF is zero for both types of unit. This evidence proves that, in my model, ISCF’s being positive is a pure result of financial frictions.

Firm-level policy rules are obtained through extensive-margin aggregation across production units. Employ $\sigma$ to denote the firm-level uncertainty that drives all the unit-level variances, $\sigma_j = \gamma_j \sigma$. Based on Propositions 2 and 4, define the set of investing units by

\(^{30}\)Assumption 2 has to be satisfied simultaneously; hence, there is a $\psi_2$. \(15\)
\( I = \{ j | \gamma_j < \frac{\sigma^*}{\sigma} \} \). Firm-level investment is thus \( \tau_1 = \sum_{I \tau_1 j} \); firm-level cash flow is \( w = \sum_{j=1}^J w_j \); firm-level initial capital stock is \( k_1 = \sum_{j=1}^J k_{1j} \). It is straightforward from Proposition 2 that investment is linear in cash flow, as shown by equation (15). The firm-level ISCF, \( \frac{\partial k_1/k_1}{\partial w/k_1} \), is thus given by the following equation (16) – it is equal to a ratio between the share of cash flow in the hands of investing production units and the down-payment requirement imposed by the collateral constraint in period 1. Effectively, firm-level ISCF is the weighted average of the two distinct unit-level ISCF in (14), with unit-level cash flow serving as the weight.

\[
\frac{\tau_1}{k_1} = \frac{\sum_{\tau_1} w_j/w}{1 - \frac{\theta q(1 - \delta)^2}{R_j^2}} \frac{\sum_{\tau_1} w_j/1}{k_1} + \frac{\theta q(1 - \delta)^2}{R_j^2} \sum_{\tau_1 \tau_1 j} \frac{\tau_1 k_{1j}/k_1}{w_1} (15)
\]

\[
ISCF = \frac{\sum_{\tau_1} w_j/w}{1 - \frac{\theta q(1 - \delta)^2}{R_j^2}} = \sum_j ISCF_j \frac{w_j}{w} (16)
\]

Next, I show how an increase in the firm-level uncertainty (\( \sigma \)) could alter (16). Suppose \( \sigma \) rises, and suppose temporarily that the distribution of cash flow across production units \( F_w \) – or explicitly, \( \{ w_j \}_{j=1}^J \) – remains unchanged after this increase in uncertainty. From equation (16), it is clear that higher \( \sigma \) reduces the numerator by shrinking the set of investing units, \( \tau_1 \), while leaving the denominator unchanged; firm-level ISCF decreases as a result. Intuitively, when the parent firm’s uncertainty increases, it drives up the individual uncertainty (the variances of \( z_j \)) of all production units through \( \sigma_j = \gamma_j \sigma, \forall j \). Facing greater uncertainty gives production units more incentives to delay investment, so there will be fewer investing production units displaying positive ISCF and more waiting units showing zero ISCF. Firm-level ISCF decreases, as it equals the weighted average of unit-level ISCF. This is named the real-options channel because uncertainty increasing the value of waiting corresponds to the real-options effect in the literature. To the best of my knowledge, previous studies have not applied the real-options theory to study ISCF formally.

A higher degree of financial frictions (lower \( \theta \)) instead has two competing impacts on ISCF. On the one hand, a lower \( \theta \) enlarges the denominator in (16); if the numerator holds constant, ISCF will be smaller. This effect acts through the intensive margin – as showed by (14), smaller \( \theta \) makes investing production units have lower, positive ISCF. The firm-level ISCF decreases as a result. Intuitively, as higher haircut ratio increases the down-payment requirement, extra cash flow can purchase fewer additional capital goods. I call it the “capacity channel” because more frictions reduce firms’ capacity to borrow and invest.

On the other hand, a lower \( \theta \) has an additional extensive-margin effect via increasing \( \sigma^* \),
leading the numerator in (16) to become larger. This effect can make more financial frictions increase ISCF. The mechanism is that tighter collateral constraints negatively impact the value of waiting and, as a result, encourage units to exercise investment sooner. As a greater proportion of production units invest and display positive unit-level ISCF, the firm-level ISCF could increase. In my specific model, this channel is a direct result of the late-payment assumption. Before the collateral constraints become tighter, some units delay investment to skip the late payment so that they can get the proceeds from production sooner to lever up external funds. More financial frictions reduce the value of delaying by limiting the ability to lever up. As a result, these waiting units may no longer want to wait after collateral constraints tighten. I call this the “timing channel” because it comes from production units choosing the optimal time to exercise investment based on favorable market conditions. In my specific model, whether market conditions are favorable refer to whether units can receive payment for production immediately. However, the essence of this channel does not rely on my specific assumption. Any setup leading to the lessening impact of financial frictions on the value of waiting can cause an increase in ISCF upon more frictions.

Even though $\sigma^*$ does not have an explicit expression, how it changes with $\theta$ can be shown via the Implicit Function Theorem. Given the downward slope of $(x_{kj} + x_{bj})$ in $\sigma_j$, lower $\theta$ leads to higher $\sigma^*$ if and only if it generates bigger $(x_{kj} + x_{bj})$. I describe the analytical conditions under which this timing channel exists in the Appendix. The simplest sufficient (not necessary) condition is to have $\beta$ close to $1/R_f$. Mathematically, this channel depends on how tighter collateral constraints influence the expected benefits from investment $(x_{kj})$ and the value of waiting $(-x_{bj})$ differently. According to Proposition 1, there exist two stochastic states in period 2: a high state with $z_j$ being greater than $\psi_1$, in which units will choose to invest, and a low state when $z_j$ is smaller than $\psi_1$, in which units will choose to consume. Tighter collateral constraints affect $x_{kj}$ and $(-x_{bj})$ by influencing the actual marginal value added by capital and saving in the two states, respectively, and the probability of each state.

First, as lower $\theta$ induces larger $\psi_1$, the probability of high state becomes smaller when units face tighter collateral constraints; the probability of low state increases as a result. Second, conditional on high state, lower $\theta$ reduces the production unit’s capacity to borrow and invest. Taking the two points together, the portions in $x_{kj}$ and $(-x_{bj})$ regarding the expected value added from investing become smaller. However, the magnitude is different for $x_{kj}$ and $(-x_{bj})$. This is because the value added from investing depends also on net worth acting as a down-payment on investment expenditure. An additional unit of capital goods (that is worth $1$) installed in period 1 is worth $(1-\delta)$ in period 2 after depreciation; in

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31 See [https://sites.google.com/site/delonglihopkins/home/research](https://sites.google.com/site/delonglihopkins/home/research).
contrast, saving one extra dollar in the bond leads to an increment of $R_f$ in net worth. Based on $R_f$ being greater than $(1 - \delta)$, these two effects cause a reduction in $-x_{bj}$ bigger than that in $x_{kj}$, rendering $(x_{kj} + x_{bj})$ larger. This is a direct result of late payment; without late payment, the extra net worth added by an additional unit of capital should be $(z_j + 1 - \delta)$, which can be bigger than $R_f$.

Besides, the third effect of tighter collateral constraints is to influence the marginal value added conditional on low state. If the unit chooses to save an extra dollar in period 1, it consumes the resulting $R_f$ as a dividend when encountering low $z_j$ in period 2. Tighter financial constraints have no effect in this case because the unit does not incur collateralized borrowing. In contrast, when the unit chooses to install additional capital in period 1, it can borrow against it in period 2 to increase dividend consumption. Lower $\theta$ limits the value gained in this borrow-to-consume effect of extra capital. Therefore, the third effect reduces only $x_{kj}$ but not $-x_{bj}$.

Taking all the three aforementioned impacts together, how $(x_{kj} + x_{bj})$ changes with $\theta$ relies on their relative strength. When the production unit has limited impatience (e.g., when $\beta$ is close to $1/R_f$), the last one is dominated by the first two. $(x_{kj} + x_{bj})$ will become larger, and the unit prefers immediate investment. Figure 3 illustrates this channel graphically under the calibration in Table 1. It plots the corresponding $\sigma^*$ against various values of $\theta$, indicating that as $\theta$ is reduced, the value of $\sigma^*$ rises.

Based on the two competing forces, the net effect of tighter collateral constraints on ISCF depends on the distribution of production units, $\{\gamma_j\}_{j=1}^J$. For example, if many production units are clustered at the bottom of the waiting region (i.e., their values of $\gamma_j$ are only slightly higher than that of $\sigma^*/\sigma$), a lower $\theta$ that increases $\sigma^*$ could change these units from waiting units to investing units, and the firm-level ISCF will become larger. On the contrary, if the distribution of production units is relatively far from the threshold, changing the threshold will have a limited impact via the extensive margin, and the firm-level ISCF will decline upon lower $\theta$ because of the intensive-margin effect through capacity channel.

Lastly, I discuss the potential influences of ignoring the redistribution of cash flow by the parent firm’s CFO. This is actually about how the distribution of $\{w_j\}_{j=1}^J$ ($F_w$) changes with $\sigma$ and $\theta$. Consider the following three possibilities:

1. There is no strategic redistribution, i.e., $F_w$ does not change.

2. The CFO solves an optimization to adjust $F_w$. The optimal policy always puts all liquid resources in the active units, i.e., $\Sigma z^j w_j = w$. 

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3. The CFO solves an optimization to adjust $F_w$. The optimal policy places partial resources in the active units ($I$), and this distribution of resources changes with uncertainty and the degree of financial frictions.

Previous arguments assume case 1 is true. Under case 2 instead, the extensive-margin effects, specifically the real-options channel and the timing channel, no longer matter, because the CFO’s redistribution keeps the numerator of (16) constant (though the set $I$ still changes upon uncertainty and financial shocks). In other words, an increase in uncertainty will not have any effect on the firm-level ISCF, while more financial frictions will unambiguously diminish ISCF.

In case 3, what matters is whether the CFO optimally chooses to inject money into the group of active units ($I$) or to take money out of them upon heightened uncertainty or more financial frictions. The former counter-balances the extensive-margin effect of uncertainty and reinforces that of financial frictions, and vice versa. While solving this optimal redistribution is beyond the scope of this paper, I contend that, in reality, the CFO’s optimal choice would be to take money out of the group of active units upon heightened uncertainty as a precautionary motive and to inject money into the group of active units when facing tighter constraints to ease financing conditions. Under this conjecture, compared to case 1, an increase in uncertainty should have stronger effects in reducing ISCF, and more financial frictions should have more profound impacts with regards to increasing ISCF. I will return to this discussion in the empirical portion later. My estimation results are actually informative in distinguishing the three scenarios.

### 2.4 An Extension: Endogenous Default and Credit Spread

The baseline model described assumes that all borrowings are risk-free because of collateral constraints. Default therefore does not take place in equilibrium. Now, I drop this assumption by letting production units borrow beyond collateral constraints. The extra debt will hence become risky, and default will be unavoidable if the realized $z_j$ is not high enough to service the debt. In particular, suppose the units can borrow risky debt after the collateral constraints bind if they are willing to pay credit spreads to compensate for the possible loss of lenders in default. The risky debt is assumed to be less senior than the collateralized debt. This extension seeks to incorporate the potential precautionary effects of uncertainty shocks that were neglected before. I show that this incorporation further decreases the firm-level

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32 Bates et al. (2009) documents that precautionary motives lead firms to hoard cash when facing heightened uncertainty. Hoarding cash means taking money away from actively investing production units.
ISCF beyond the real-options channel.

Following Bernanke et al. (1999), I model the bankruptcy process by employing the costly state-verification setup in Townsend (1979). Specifically, production units in default are taken over by lenders and liquidated. During this process, a fraction of value, $\xi < 1$, is destroyed as dead-weight loss. Lenders need to break even, so they require higher compensation from production units, rendering risky borrowing costly. Such costs stimulate a precautionary motive in production units – as uncertainty increases the probability of default, production units tend to reduce borrowing (risky leverage) and investment ex ante to avoid bankruptcy. This motive bestows uncertainty shocks with a new intensive-margin effect – ISCF becomes smaller for the investing units while remaining unchanged for the waiting units because they do not incur risky debt. Consequently, the firm-level ISCF declines, as it is the average of the unit-level ISCF. The solution to this extension is no longer in closed form; numerical work with calibration is instead provided. See details in the Appendix. In the empirical part, I further show that subsample estimation can distinguish this precautionary channel from the real-options channel.

3 Firm-Level Empirical Work

This section examines the model’s predictions using data from more than 10,000 U.S. listed firms over the past 30 years. As suggested by the linear investment rule in equation (15), I perform a linear regression of investment on cash flow and control variables, calling the coefficient of cash flow the empirical ISCF. I test how ISCF changes with uncertainty and financial frictions by including their interaction terms with cash flow into the regression. After carefully taking into account the potential endogeneity problems caused by measurement errors and reverse causality, I find that the empirical ISCF is smaller when firms face greater uncertainty. On the contrary, empirical ISCF rises with the degree of financial frictions. Both results are consistent with my model’s predictions.

3.1 Data and Empirical Specifications

In equation (15), firm-level investment is linear in cash flow, with both terms normalized by the existing capital stock. I thus start with a linear investment regression in (17), where

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33 See also Hennessy and Whited (2007), Arellano et al. (2012), Christiano et al. (2014), and Gilchrist et al. (2014) for a similar setup.

34 See https://sites.google.com/site/delonglihopkins/home/research.
ι_{i,t} refers to the investment of firm \( i \) in year \( t \). \( w_{i,t} \) is the corresponding cash flow; \( k_{i,t-1} \) is the existing capital stock; \( X_{i,t-1} \) is a group of control variables; \( \rho_i \) and \( \eta_t \) refer to firm- and year-fixed effects, respectively, while \( \epsilon_{i,t} \) is the disturbance term. Each firm-level variable is recorded in consolidated accounts corresponding to those in the theoretical model after the extensive-margin aggregation. The time frequency is annual. For \( X_{i,t-1} \), I utilize Tobin’s Q and real sales growth to control for expected future profitability (investment opportunities), i.e., \( \bar{z}_i \) in the theoretical model. Moreover, previous studies indicate that existing debt could crowd out current investment, known as the debt-overhang problem. Therefore, I therefore include the existing net leverage to control for it, treating cash stock as negative debt.

\[
\frac{\iota_{i,t}}{k_{i,t-1}} = (\rho_i + \eta_t) + \zeta_{i,t}(w_{i,t}/k_{i,t-1}) + X'_{i,t-1}\phi + \epsilon_{i,t} 
\]  

(17)

The coefficient of cash flow, \( \zeta_{i,t} \), is the empirical ISCF. My theoretical model suggests that \( \zeta_{i,t} \) depends on firm-level uncertainty (\( \sigma_{i,t} \)) and the degree of financial frictions (\( ff_{i,t} \)).

Approximating the relationship with a linear equation, \( \zeta_{i,t} = \zeta + \tau_1\sigma_{i,t} + \tau_2 ff_{i,t} \), and substituting it into (17) generates a regression with interaction terms in equation (18). I include the levels of \( \sigma_{i,t} \) and \( ff_{i,t} \) into the regression to isolate the effects of interaction terms. The real-options channel suggests that \( \tau_1 \) should be negative; the capacity channel infers that \( \tau_2 \) should be negative, while the timing channel predicts \( \tau_2 \) to be positive.

\[
\frac{\iota_{i,t}}{k_{i,t-1}} = (\rho_i + \eta_t) + \zeta(w_{i,t}/k_{i,t-1}) + X'_{i,t-1}\phi + \mu \sigma_{i,t} + \chi ff_{i,t} + \tau_1(w_{i,t}/k_{i,t-1}) \times \sigma_{i,t} + \tau_2(w_{i,t}/k_{i,t-1}) \times ff_{i,t} + \epsilon_{i,t} 
\]

(18)

I employ the annual data from the U.S. listed firms in the COMPUSTAT North America database from 1984 to 2015. To focus on spontaneous capital investment decisions, I drop firms in the financial, utility, and public sectors, as they either do not invest majorly in capital goods or are highly regulated. After a standard selection procedure following the literature, the sample consists of more than 10,000 individual firms in an unbalanced panel. On average, the sample contains more than 3,000 firms cross-sectionally in each year; each firm stays in the panel for approximately 15 years. I use capital expenditure to measure investment (\( \iota \)) and employ property, plant, and equipment as measures of capital stock.

\[35\text{See, among others, Myers (1977) and Hennessy et al. (2007).}\]

\[36\text{Corresponding to the theoretical model, } ff_{i,t} \text{ measures } (1 - \theta_{i,t}), \text{ representing the haircut ratio, with a higher value indicating a tighter collateral constraint (lower } \theta_{i,t}).\]

\[37\text{Observations are required to have positive values of the following variables: total assets; property, plant and equipment; book value of common equity; sales; capital expenditure; and cash and cash equivalents. Moreover, daily stock prices have to be higher than$1 and lower than$2,000. Firms also need to have daily data on stock returns for at least 150 trading days in each year.}\]
Cash flow ($w$) is measured by income before extraordinary items plus depreciation and amortization. Tobin’s Q is the macroeconomic Q defined by Erickson and Whited (2006). It equals the sum of debt and market capitalization less total inventories, divided by capital stock. Real sales growth is the growth rate of sales (net) adjusted to the 1982 dollar using the Producer Price Index. Net leverage is calculated as the sum of debt and market capitalization less the stock of cash, divided by market capitalization.

I construct the measure of firm-level uncertainty ($\sigma$) in two steps. Initially, I obtain the realized equity volatility from daily stock returns for each individual firm. As an alternative, I estimate the underlying conditional volatility using the Exponential GARCH model in Nelson (1991). Next, I divide the obtained equity volatility by capital structure, with the latter measured by one plus net leverage. This is in an effort to follow Leahy and Whited (1995); the resulting variable captures the volatility of the enterprise values, namely, the present values of all the firm’s future projects. Without this adjustment, equity volatility is not a reliable measure of uncertainty (over business prospects) for this study – two firms with the same volatility of enterprise values, thus equally volatile business prospects, could have different equity volatilities because of distinct leverage. In addition, Bloom et al. (2014) observe that this measure of uncertainty constructed by listed firms’ daily stock returns (adjusted or unadjusted by capital structure) is highly correlated with the uncertainty faced by production units (measured by TFP shocks) that belong to the corresponding firm. This empirical evidence is in agreement with my theoretical model, wherein firm-level uncertainty drives the dynamics of all unit-level uncertainty (called “variances” in the theoretical model).

To measure the degree of financial frictions, $ff$, I first employ the excess bond premium (EBP) in Gilchrist and Zakrajšek (2012), constructed by using the data on publicly

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38See Kaplan and Zingales (1997) for a similar choice of variables.
39Three points are worth noting regarding the macroeconomic Q. First, it corresponds to the average Q in theory rather than the marginal Q. This is because Erickson and Whited (2006) state that average Q is more empirically relevant when there exist financial frictions. Second, except for the market capitalization, all terms used here in constructing the macroeconomic Q are measured in book values. Erickson and Whited (2006) provide a detailed survey of alternative methods in the literature, such as estimating the replacement costs of capital stock as in Salinger and Summers (1983). By comparing these different methods, Erickson and Whited (2006) determine that using book values does not underperform other alternatives but serves as the most convenient way. In addition, besides the macroeconomic Q, another widely-adopted measure is the market-to-book ratio of assets (called “financial Q”). Erickson and Whited (2006) show that using this alternative definition introduces unnecessary measurement errors into the regression, as the denominator of Q (total assets) is different from that of the investment rate (capital stock). My baseline findings remain robust if I replace the macroeconomic Q with the financial Q. However, using macroeconomic Q does lead to a better fit, indicated by a higher R-squared value.
40The EGARCH model is selected by the Akaike Information Criterion independently for each individual firm. The resulting conditional volatility is on a daily basis. I take the average across all trading days within the corresponding year. Based on the consistency of EGARCH and the large number of daily observations, the estimation process is quite precise. I therefore ignore the standard errors associated with this step.
traded corporate bonds. Gilchrist and Zakrajšek (2012) first predict each corporate bond’s default loss by a forecasting model; then they subtract the expected default loss from the corporate-Treasury spread of the bond. Lastly, they average the residuals across all the available corporate bonds in the market for any given year. The obtained EBP is therefore a market-wide indicator. As suggested by the existing literature, this portion of variation in credit spreads appears to reflect the time-varying liquidity premium, the tax treatment of corporate bonds, and most importantly, investors’ risk aversion. A widened EBP indicates a reduction in the risk-bearing capacity of investors, which could lead to tighter borrowing constraints. This is supported by the theoretical work of Geanakoplos (2010), which highlights that increasing investors’ risk aversion could worsen their disagreement with borrowers over the value of collateral, thus laying a theoretical foundation for using EBP as an empirical counterpart proportional to the haircut ratio \( (1 - \theta) \) in Section 3.

Despite many strong features, EBP is not a perfect measure of financial frictions. For one, it is a generated variable and thus could contain measurement errors. Further, it is constructed using a small group of firms with publicly traded debt available. Even though Gilchrist and Zakrajšek (2012) argue that EBP has implications far beyond influencing firms that are used in the construction process, this procedure introduces a sample-selection bias. In particular, I have more than 10,000 listed firms in my sample, but only around 1,000 of them were used to construct EBP. What worsens the situation is that firms that possess publicly traded debt are in general bigger than those that do not. In addition, EBP is a market-wide measure common to all firms in a single year; it ignores that firms could have individual exposure to financial frictions.

To mitigate this concern, I include firms’ sizes (the market value of assets in 1982 dollars adjusted by the Producer Price Index) as an additional individual proxy, assuming that smaller firms potentially face tighter collateral constraints. This could be rooted in a lack of information, reputation, economies of scale, or other reasons. Besides size, existing literature has put forward many other choices for individual measures of frictions, for example, whether the firm pays dividends (Fazzari et al. (1988)), the Kaplan-Zingales index (Kaplan and Zingales (1997)), the Cleary index (Cleary (1999)), and the Whited-Wu index (Whited and Wu (2006)). Hennessy and Whited (2007) employ structural estimation to show that size is superior to these alternatives when capturing the “deep” characteristic of facing more financial frictions. The latter instead represent the needs for external financing.

\[41\] Data are available from the website of the Board of Governors of the Federal Reserve System.

\[42\] For example, see Collin-Dufresn et al. (2001), Houweling et al. (2005), and Driessen (2004).

\[43\] For example, EBP performs well in forecasting macroeconomic fluctuations.

\[44\] Needing external financing does not necessarily mean facing significant frictions. For instance, a cred-
addition, the existence of bond rating has also been used widely as an indicator of financial constraints, for example in Whited (1992). As a consequence of the lack of access to data, I leave examining the role of bond rating to future research.

Table 2 presents the summary statistics of the corporate-level variables. Two points are worth discussing. For one, a major fraction of observations have values of macroeconomic Q greater than one; the latter acts as the investment threshold in neoclassical theory. This phenomenon, however, is consistent with real options raising the conventional investment hurdle denominated in Q as in Dixit and Pindyck (1994). It therefore justifies my modelling of non-convex adjustment costs (that generate real options). Erickson and Whited (2006) further stipulate that, besides this structural reason, there could be measurement errors in the macroeconomic Q leading to its large values. Specifically, the numerator is the firm’s market value, including both tangible and intangible assets; in contrast, the denominator is the book value of tangible assets (capital goods) solely. If the firm holds valuable intangible assets, the macroeconomic Q for measuring investment opportunities in tangible assets could be overvalued. Seeing that roughly 25% of my sample consists of firms in the service sector, which potentially hold significant amounts of intangible assets, this measurement issue could be significant. To mitigate such a concern, I conduct robustness checks in three different ways. First, I drop observations with values of macroeconomic Q greater than 40. Second, I replace the macroeconomic Q with the financial Q, putting the book value of total assets (both tangible and intangible) in the denominator. Alternatively, I admit the existence of the measurement errors and employ the measurement-error-correcting estimator in Erickson et al. (2014) and Erickson et al. (2016) to correct for the impacts.

Another point worth considering is regarding some observations presenting large, negative numbers for the cash-flow ratio. Besides operating losses, a potential reason could be certain firms writing off serious amounts of expenses, such as research and development (R&D) expenses, leading to negative income. However, such expenses could be regarded as part of their internal funds, and R&D investment itself has become more and more crucial in recent years. To account for this, I add back the R&D expenditure to both cash flow and capital expenditure to obtain the “total cash-flow ratio” and “total investment ratio.”

worthy international corporation based in the U.S. but with a huge amount of foreign income might need to issue corporate bonds in the U.S. against its foreign cash holdings to pay dividends while avoiding repatriation costs. On the other hand, a firm with more frictions when seeking external financing may have the incentive to hoard cash and end up with few needs for external funds. However, hoarding cash does not resolve any fundamental reason that led to such frictions ex ante, like agency costs or information asymmetry. To remove the impacts of possible outliers, I winsorize each variable to its top and bottom 2%. This is even stricter, and thus more conservative, than the criterion in Erickson and Whited (2006), which is 60.

I check the robustness of my baseline results in terms of these two alternative definitions of variables.

### 3.2 Measurement-Errors-Correcting Estimation

Estimating equation (18) could incur two potential endogeneity problems. First, there could be reverse causality that more investment leads to firm values being more volatile. For instance, investing aggressively may incur certain riskier projects, generating greater volatility. It can also enlarge the firm’s size. To mitigate such a concern, I utilize exogenous variation to obtain identification. First, I replace firm-level volatility with the cross-sectional weighted-average volatility year-by-year, using firms’ market value of assets as the weight. The resulting market-wide, weighted-average volatility is assumed to be exogenous to any individual firm’s investment decisions. I also temporarily drop the measure of size and only use EBP as an exogenous measure of financial frictions.

Another potential endogeneity problem is with respect to an omitted-variable bias caused by measurement errors in Tobin’s Q. If Q cannot fully capture underlying investment opportunities (call the difference “measurement errors”), cash flow will be correlated with the regression’s error term. Erickson and Whited (2000) point out that such measurement errors can bias the estimated coefficients of not only Q but also other variables, such as $\tau_1$ and $\tau_2$ in equation (18), even when the weighted-average volatility and EBP are exogenous by construction. Erickson et al. (2014) and Erickson et al. (2016) further put forward an estimator that utilizes higher-order cumulants to correct for measurement errors. The identification requires the data to have at least non-zero skewness and a sufficient sample size to ensure valid estimates of higher-order cumulants. My sample meets these requirements.

Combining the measurement-errors-correcting method with the use of weighted-average volatility and EBP as exogenous variables should mitigate the two aforementioned endogeneity concerns simultaneously. Before showing the results, I quickly summarize the sources

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48 Alfaro et al. (2016) instead utilize industry-level variation, specifically the different exposure of firms towards political uncertainty and oil price shocks, as instruments to obtain identification.

49 See also, among others, Erickson and Whited (2002), Erickson and Whited (2012), and Abel (2015).

50 Erickson and Whited (2012) demonstrate that the estimator performs well in simulations when the skewness is around one, the cross-sectional ($N$) is equal to 1,500, and the time-series length ($T$) equals 10. My sample has on average $N > 3,000$ and $T > 15$, and all my variables have skewness (in absolute values) greater than one.

51 Effectively, the identification is achieved through differences in differences. The periods with soaring volatility and widened EBP act as the treatment group, while those with low volatility and EBP are the control group. Cash flow, after measurement errors in Q are corrected for, performs as the continuous-treatment variable. Investment responses upon cash-flow shocks in the two classified periods are compared. See McLean and Zhao (2014) for a similar method; their way to distinguish the treatment and control groups...
of measurement errors and the identification strategy (details found in Erickson and Whited (2000), Erickson et al. (2014), and Erickson et al. (2016)). The measurement errors come from the differences between measured Q and the true future investment opportunities that are unobservable. Such differences could be rooted in either theoretical reasons, like Q being an insufficient statistic for investment, or measurement issues, such as the challenges in estimating replacement costs and the overestimation caused by neglecting intangible assets. The measured Q is assumed to follow a classical errors-in-variables (EIV) model. In particular, it equals the sum of an unobserved variable (that stands for the true investment opportunities) and a mean-zero error term; the unit slope and zero intercept in this measurement equation are crucial for identification.

The estimation process takes place via two steps. The first step separately regresses investment and the measured Q on all other perfectly measured explanatory variables. The two regression residuals depend on the unobserved investment opportunities, the perfectly measured variables, the measurement error, and the error term in the original regression equation. The second step takes the powers of these two residual equations, multiplies the results together, and takes expectations on both sides. Under a set of assumptions on the error terms, this leads to moment conditions expressed by the regression coefficients and the higher-order moments of both the data and the unobserved investment opportunities. A moment estimator could be constructed by using subsets of these moment conditions (i.e., based on moments up to a certain order); however, the numerical optimization required to minimize the approximated moment conditions could render the estimation process sensitive to starting points. Based on Geary (1941), Erickson et al. (2014) put forward cumulant estimators that are asymptotically equivalent to the moment estimators, but with a convenient closed form.

Table 3 lists the estimation results. Columns (1), (2), and (3) utilize cumulants up to orders five, six, and seven for estimation, respectively. Regarding these choices, Erickson et al. (2016) suggest that the order-five estimator is a worthwhile start, and the more observations one has, the higher the order of cumulants that could be used. Given the relatively large sample size I have, I select the order-seven estimator to be my preferred method and show how the results can be altered if I replace the realized means of calculating volatility with the EGARCH-inferred values in column (4).

\[ \text{is through recession dummies.} \]
\[ ^{54}\text{Reasons could be imperfect competition, non-linear production functions, non-convex adjustment costs, financial frictions, and other deviations from the neoclassical framework.} \]
\[ ^{53}\text{Cumulants are polynomial functions of moments.} \]
\[ ^{54}\text{Indeed, for both the realized and the EGARCH-inferred volatility, results are similar for all the specifications from order four through order eight. Results are available upon request.} \]
demeaned and year dummies are included to account for both firm- and year-fixed effects; standard errors are clustered at the firm level. The levels of weighted-average volatility and EBP are omitted to avoid multi-collinearity.

Estimation results indicate that cash flow, lagged Q, and sales growth contribute positively to investment, as expected, while the existing net leverage has a negative impact, suggesting a potential debt-overhang problem; all these estimates are highly significant from a statistical perspective. More importantly, cash flow interacting with volatility has a negative coefficient ($\tau_1$), which is significant at the 1% statistical level. In contrast, the coefficient of its interaction with EBP is positive ($\tau_2$), with a 1% statistical significance. These estimates are equivalent to firms exhibiting smaller ISCF values when there is heightened market-wide uncertainty (represented by spikes in the weighted-average volatility) and greater ISCF when firms in general face more financial frictions (indicated by the widened EBP). Further, they are in line with the real-options channel and the timing channel put forward by the model in the previous section, respectively. Last but not least, the R-squared value is equivalent to 0.3, which is indicative of a decent fit.

### 3.3 Generalized Method of Moment Estimation

Despite its various strong aspects, the measurement-errors-correcting estimator has some unappealing features. First and foremost, the identification based on the exogeneity of the market weighted-average volatility and EBP ignores the cross-sectional variation in the firm-specific volatility and exposure to financial frictions. In other words, results in the previous section should be interpreted as ISCF being less during periods of heightened uncertainty and larger when financial markets are generally tighter. However, they have little to offer regarding whether a more volatile (or financially constrained) firm should have a larger or a smaller ISCF compared to others. Introducing firm-level volatility or an individual proxy for financial frictions (size), however, could cause reverse causality that the measurement-errors-correcting estimator cannot fix.

Second, the moment conditions in the measurement-errors-correcting estimator are based on a group of orthogonality assumptions imposed by Erickson et al. (2014) and Erickson et al. (2016). They can be examined by a Hansen-J over-identifying test. The test, however, strongly rejects the null hypothesis for all the specifications in Table 3 (p-value smaller than 1%). Such rejections are also present in Erickson et al. (2014) and Erickson et al. (2016). The order must be at least four to obtain overidentification; orders higher than eight are not advised because of the computational burden.

55Specifically, see Assumption 1 in Erickson et al. (2016).
et al., (2016) for different datasets and empirical specifications. The test could suggest a violation of one of the conditions imposed by those assumptions, such as the strong exogeneity of the perfectly measured corporate-level variables (e.g., sales growth, net leverage). If these variables are contemporaneously correlated with the error term, then the demeaning process implicit in the inclusion of fixed effects induces a well-known correlation between regressors and the error term, leading to an endogeneity problem discovered by Nickell (1981). Lastly, only firm-level clustered standard errors are allowed in the current algorithm of the measurement-errors-correcting estimator. However, firms within the same sector might receive common investment shocks so that their error terms could be correlated. Adopting sector-clustered standard errors would thus be more robust.

To deal with these issues, I estimate equation (18) by the Generalized Method of Moments (GMM) from Arellano and Bond (1991) instead, treating all firm-level variables as endogenous variables. Based on the inclusion of firm-specific volatility, size, and their interaction terms with cash flow, this estimation investigates not only via a time-series but also cross-sectionally how uncertainty and financial frictions influence ISCF. I transform equation (18) by taking the first difference to remove firm-fixed effects. Unlike demeaning, first differencing guarantees that lagged variables are valid instruments for the transformed equation, conditional on the error term having limited serial correlations. The latter is examined by the Arellano-Bond autocorrelation test (AR test) and is cross-checked by the Hansen-J over-identifying test. Particularly with respect to my sample and specification, the AR test provides evidence for order-two but not order-three serial correlations in the error term. I therefore use the lagged three-year and earlier firm-level variables as instruments. I estimate the GMM using the two-step method with standard errors clustered at the sector level.

Table 4 shows the estimation results. Individual volatility serves as a proxy for firm-level uncertainty, and the size represents the potential financial frictions an individual firm might face. The element of EBP is considered simultaneously, as it captures the general tightness of financial markets. Year dummies are employed to control for other macroeconomic factors common to all firms; as a result, the level of EBP is again omitted. Column (1) includes the estimate adopting lagged three-year and four-year corporate variables as instruments for the transformed regression equation. In theory, earlier lags are valid instruments as well; however, when the number of lags rises, the relevance of instruments becomes weaker. In column (2), I check the robustness for earlier lags by utilizing lagged three- to six-year

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56 The relevance of instruments is guaranteed by the autocorrelation nature of firm-level variables. All the first-stage regressions have big F statistics.

57 For clustered standard errors, sectors are classified by the four-digit SIC codes.

58 See Brown et al. (2009) for the same choice of lags as instruments.
corporate variables as instruments. In column (3), I instead portray the estimate when the EGARCH-inferred volatility measures individual uncertainty. Column (4) replaces the year dummies with a full set of sector-year-interacting dummies to control for potential industry-specific, time-varying shocks.\[59\]

The estimation results confirm previous conclusions for all specifications. Specifically, cash flow interacting with individual-specific volatility has a negative coefficient; also, that of its interaction with EBP remains positive; both terms are statistically significant. Moreover, the coefficient of size’s interaction with cash flow is significantly negative. Based on the smaller size representing more exposure to financial frictions, such findings are in agreement with those of EBP, where more financial frictions enlarge ISCF.\[60\] In summary, introducing cross-sectional variation and GMM estimators supplies further evidence to support the real-options channel (such that uncertainty diminishes ISCF) and the timing channel (such that financial frictions elevate ISCF). Besides this, the negative coefficient of individual volatility itself illustrates the depressing role of uncertainty at the levels of investment, which is consistent with previous studies (e.g., Bloom [2009]). The AR tests ensure the validity of the instruments used, which are also cross-checked by the Hansen-J over-identifying test.

To gauge the economic significance of the estimated channels, I numerically fit the values of ISCF conditional on different levels of uncertainty and financial frictions. This procedure utilizes the linear combinations of regression coefficients in column (1) of Table 4. In particular, I differentiate investment in equation (18) with respect to cash flow, leading to the following linear representation of ISCF:

$$\hat{\text{ISCF}}_{i,t} = \frac{\partial \left( \frac{\iota_{i,t} - k_{i,t-1}}{w_{i,t} - k_{i,t-1}} \right)}{\partial \left( \frac{\iota_{i,t} - k_{i,t-1}}{w_{i,t} - k_{i,t-1}} \right)} = \xi + \hat{\tau}_1 \sigma_{i,t} + \hat{\tau}_{21} EBP_t + \hat{\tau}_{22} Size_{i,t} \quad (19)$$

Next, I substitute different sample percentile values for $\sigma_{i,t}$, $EBP_t$, and $Size_{i,t}$, respectively; when changing one of these, I fix the other two at their sample median values to isolate the impacts. The fitted ISCF is listed in Table 5. The first column is based on when $\sigma_{i,t}$ is valued at its 10th, 20th, 30th, ... and 90th percentile values, respectively, while at the same time fixing $EBP_t$ and $Size_{i,t}$ at the corresponding sample median values. The second column changes $EBP_t$ while fixing $\sigma_{i,t}$ and $Size_{i,t}$. The last column changes $Size_{i,t}$ but fixes $\sigma_{i,t}$ and $EBP_t$. The results show that the estimated impacts are, in fact, large in economic terms. First, when volatility moves from its bottom 10th percentile to the top 10th percentile, the fitted ISCF decreases by 81%. Moreover, the same movement of EBP

\[59\] For dummy variables, sectors are classified by the 10 major SIC divisions.
\[60\] Regarding the impacts of size, see also Gilchrist and Himmelberg [1995] and Abel and Eberly [2011].
increases the fitted ISCF by more than 100%. In contrast, the influence of size on the fitted ISCF is not as prominent – when size changes from the bottom to the top 10th percentile, the fitted ISCF only drops by around 8%.

Another point regarding the fitted ISCF is worth noting. In Table 5, when all three determinants of ISCF are valued at their sample medians, the fitted ISCF is 0.0157. This number is much smaller than what the existing literature has documented (usually between 0.1 to 0.6). Besides the inclusion of more control variables, such as sales growth, that mitigate the role of cash flow, a potential reason is that my sample includes a significant number of observations with negative cash flow. This could be because of firms expensing out research and development (R&D) expenditure from cash flow. Given that my sample contains a large proportion of firms in the service sectors, for whom R&D investment is vital (especially in recent years), the ISCF value of 0.0157 could be under-estimated. To account for this, I add back the R&D expenses to both investment and cash flow. After repeating the estimation process, the fitted ISCF equals 0.2 when volatility, EBP, and size are valued at their sample medians and year dummies are included. This value falls into the range suggested by the existing literature. Moreover, as suggested by column (1) of Table 6, all the regression coefficients remain the same sign and achieve statistical significance. Table 7 shows the fitted ISCF when volatility, EBP, and size are evaluated at their different percentile values. Like in the baseline estimation, results are significant in economic terms (less so for size); when volatility, EBP, and size increase from the bottom 10th percentile to the top 10th percentile, fitted ISCF changes by approximately 24%, 27%, and 1%, respectively.

My empirical results also survive other robustness checks, as depicted in Table 6. First, as an alternative way of dealing with the overestimation of macroeconomic Q, I drop observations with a measured Q greater than 40. Second, I replace the macroeconomic Q with an alternative measure of macroeconomic uncertainty. One possible explanation for this is that a firm’s size only acts as an indirect proxy for the financial frictions it faces. Based on the structural estimation in Hennessy and Whited (2007), I put forth that smaller firms face more costly external financing; however, this relationship is not causal but rather a consequence of simultaneity. In other words, ceteris paribus, a firm doesn’t face more frictions because it is small, but because it has certain “deep” characteristics that co-exist with the small size, like information asymmetry. Given that the estimates in Table 5 are conditional on a full set of control variables, the role of size as a proxy for the underlying “deep” characteristics could be undermined if those control variables simultaneously correlate with these unobservable characteristics.

I thank Bruce Petersen for highlighting this constructive suggestion.

For this specification, when lagged 3- and 4-year corporate variables are used as instruments, the AR test indicates an order-3 serial correlation (p-value equal to 0.0499) but not an order-4 serial correlation. I therefore use corporate-level variables in the lagged-4 and -5 years as instruments instead. Maintaining the previous choice of instruments, however, does not change the estimate qualitatively.
with the financial Q, another widely-adopted control variable for investment opportunities. Moreover, I remove the period of the Great Recession from my sample to make certain the results are not driven by specific features of crises besides uncertainty and financial frictions. In addition, I include lagged investment into the regression as an additional explanatory variable as per Eberly et al. (2012), estimating a dynamic panel model. In each of the scenarios, my main findings remain unchanged – uncertainty reduces ISCF, and EBP enlarges ISCF. However, the effects of size are less robust; they are preserved only by adding back R&D, dropping the Great Recession, and adopting the dynamic panel specification.

3.4 Further Discussion

Before moving on, I will provide more discussion surrounding the empirical results. First, ISCF’s decline with volatility further shields my estimation process from the omitted-variable bias. In particular, suppose that ISCF is positive because cash flows have been informative about future investment opportunities. Such information should be more relevant to investment decisions upon heightened uncertainty when information from other sources is less reliable. Uncertainty should thus enlarge ISCF, opposing what is observed in the data. See Abel and Eberly (2011) for a similar argument that the information-based ISCF increases with the value of growth options.

Second, Section 3 proves that the real-options channel leads uncertainty to diminish ISCF. However, as the model’s extension demonstrates, the precautionary motives for avoiding bankruptcy might result in the same empirical patterns. Subsample estimates are useful in disentangling these two explanations. Specifically, the precautionary motives do not appear in my theoretical model until the leverage reaches a threshold (that binds collateral constraints). This is in line with a commonly accepted argument in the precautionary savings literature, that agents with weaker balance sheets have stronger precautionary motives. I therefore divide my sample into two subsamples according to the existing net leverage (based on the median value), with higher values positing weaker balance sheets. If the role of uncertainty in reducing ISCF comes primarily from the precautionary motives, the effect should be more prominent for the subsample with high leverage.

Table 8 features the subsample estimation results, with the first column containing the

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65Results are similar (available upon request) if: (1) I only include manufacturing firms in the sample, and (2) I follow Leahy and Whited (1995) in dropping observations with M&A greater than 15% of total assets.
66See Carroll (1997) for discussions of consumer behavior.
67The strength of balance sheets should not be regarded as the degree of financial frictions. It instead measures the needs for external funds rather than the costs to obtain external funds. See Hennessy and Whited (2007) for more discussion.
subsamples with low existing net leverage and the second column those for the high existing net leverage. After comparing the two estimates, I find that cash flow interacting with volatility is significantly negative only for the subsample with low leverage. In other words, uncertainty diminishes ISCF only when firms have stronger, rather than weaker, balance sheets. This rejects the precautionary motives driving the empirical patterns and therefore indirectly justifies the real-options channel. Moreover, the term of cash flow’s interaction with EBP is significantly positive only for the subsample with low leverage; the interaction term between cash flow and size, however, reverses its sign for the subsample with high leverage. This summarizes that the timing channel in which more financial frictions increase ISCF is prominent only for the financially strong firms. For financially weak firms, however, the empirical evidence is in line with the capacity channel that more frictions lead to lower ISCF.

Given that both the real-options channel and the timing channel relate to the value of waiting, such subsample evidence supports that waiting is more relevant to firms with stronger balance sheets. Li et al. (2015) find similar results using data from corporations in emerging markets. Despite balance-sheet strength’s affecting the value of waiting being beyond the scope of this paper, such a phenomenon could be understood intuitively. Waiting is valuable because, by delaying investment, firms forgo near-term profits in exchange for either a better (regarding the timing channel) or a less volatile future (regarding the real-options channel). Firms with weak current balance sheets could thus have fewer motives to do so because they value near-term profits more than those with strong current balance sheets do. For instance, investing immediately helps expand their businesses, generating more profits that could service debt or operating expenses. Stated another way, waiting is less relevant for financially weak firms, and so are the two waiting-related channels.

A last point of discussion is centered around the redistribution of resources by the consolidated firms’ CFOs upon uncertainty and financial shocks. This redistribution process is neglected by the theoretical model; it could, however, affect interpretations of the empirical results. The “case-2” scenario at the end of Section 3 is clearly rejected by the data. This is because, if a firm’s CFO always redistributes all resources into the active production units, uncertainty should have no impact on ISCF. With regards to the “case 3” scenario, I postulate that taking money away from active production units upon heightened uncertainty (for precautionary motives) and injecting money into active production units when facing more frictions (to ease financing constraints) could lead to uncertainty diminishing ISCF and

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68 Another piece of evidence disfavoring the precautionary explanation is that the listed firms in my sample are typically far from their default boundaries. I calculate their default probability following Bharath and Shumway (2008). Less than 10% of firms have a default probability that exceeds 1% in the next year.
financial frictions enlarging ISCF, fulfilling the same roles as are relevant to the real-options channel and the timing channel. If the CFOs’ redistribution is indeed the underlying reason for the observed empirical patterns, such phenomena should be more prominent for firms with weaker current balance sheets because they possess stronger precautionary motives and higher needs for funds. Subsample estimates in Table 8 again rule out this possibility.

4 Policy Implications

This section gives an example of the policy implications of ISCF in the real world. Seeing that recessions are typically periods of both heightened uncertainty and tighter financial constraints, how stimulative policy could overcome these obstacles to investment is of great policy importance. Hubbard (1997) documents that a crucial channel through which monetary and fiscal policy could boost investment is strengthening firms’ internal cash flows to ease financial constraints (called the “income effect”). For instance, expansionary monetary policy that lowers interest rates reduces firms’ debt-service burden, thereby adding to cash flow. Fiscal policy, like tax incentives, enhances after-tax cash flow. However, as suggested in the previous sections, the final effectiveness of policy in boosting investment could be dwarfed by heightened uncertainty because of the associated small ISCF. I utilize the Bonus Depreciation policy enacted during and in the aftermath of the Internet Bubble (2001–2004) and Great Recession (2008–2011) to illustrate this point. I show that the investment of firms with higher volatility responds less in the case of a common policy shock.

Bonus depreciation is a temporary tax incentive for equipment investment. Its stimulative role is rooted in allowing firms to depreciate equipment expenditure faster, thereby reducing the present value of tax payments. Conceptually, suppose a firm purchases $1 million worth of computers, a five-year item. Under normal tax policy, right after the purchase, the firm can deduct 20% of the value for depreciation ($200,000). From the first through the fifth year, the corresponding deduction values are: $320,000, $192,000, $115,000, $115,000, and $58,000, respectively. The values of tax shields resulting from such deductions are therefore (under a 35% corporate tax rate): $70,000, $112,000, $67,200, $40,300, $40,300, and $20,200. The total tax benefit is calculated as the sum of present values of these flows of tax shields. Bonus depreciation permits a firm to deduct more at the time of investment and then depreciate the remainder according to the normal schedule. In the context of the

69 Besides the income effect, there exist other channels, for example, the neoclassical channel via affecting the users’ costs of capital, the currency channel through exchange rates, etc.

70 This example comes from Table 1 in Zwick and Mahon (2017).
computer example, a 50% bonus depreciation endows the firm with the privilege to deduct 50% × $1 million right after the purchase beyond the normal schedule, leading to a total immediate deduction of $600,000. As a result, each subsequent deduction falls by half. Even though the total value of deductions does not change, each of them has been accelerated. The total tax benefit, as the sum of present values of tax shields, therefore increases. Even though bonus depreciation only applies to equipment investment, it acts as a cash windfall that eases financing constraints and therefore should promote investment across all categories. Moreover, given that it does not change future investment opportunities, this policy serves as a suitable example that is immune to the omitted-variable bias.\footnote{I thank Matthew D. Shapiro for this constructive suggestion.}

To identify the impacts of bonus depreciation on investment, Zwick and Mahon (2017) present a differences-in-differences specification. In particular, because bonus depreciation contributes via modifying the sum of present values of tax shields, its empirical impact grows with the duration of equipment goods that firms invest in. Therefore, those in industries with most of their investment in long-duration equipment are influenced more, acting as the treatment group, while those in short-duration industries serve as the control group. Zwick and Mahon (2017) construct a continuous-treatment variable (called “$z_{N,t}$”) by combining such industry-level variation with the changes of deduction rules imposed by bonus depreciation; $N$ represents the four-digit NAICS industries, and $t$ signifies years. In technical terms, $z_{N,t}$ is the present discounted value of one dollar of deductions for investment eligible for bonus depreciation.\footnote{Eligible investment includes expenditure for all equipment investment put in place during the current year for which the policy incentives apply (Zwick and Mahon (2017)).} During normal periods without policy incentives, they estimate the industry-level $z_N$ utilizing disaggregated-level data from the Internal Revenue Service.\footnote{The estimated $z_N$ is available at the AEA website.} In bonus years, however, $z_N$ is adjusted by the size of the bonus ($B_t$): $z_{N,t} = B_t + (1 - B_t) \times z_N$. $B_t$ is the additional expense allowed per dollar of investment; in the instance of a 50% bonus depreciation, $B_t$ is 0.5. At different points in time, $B_t$ was set to different values: 0, 0.3, 0.5, or 1.\footnote{Specifically, $B_t$ was 0.3 in 2001 and 2002; 0.5 in 2003, 2004, 2008, and 2009; and 1 in 2010 and 2011. In other years, $B_t$ was 0 (non-bonus years).} By regressing firm-level investment on the treatment variable, $z_{N,t}$, Zwick and Mahon (2017) establish that $z_{N,t}$’s temporary increase in bonus years significantly encourages investment.\footnote{The results hold for both eligible equipment investment and total investment across all categories.}

\begin{equation}
\eta_{i,t}/k_{i,t-1} = (\rho_i + \eta_t) + \varsigma (w_{i,t}/k_{i,t-1}) + X'_{i,t-1} \phi + \mu \sigma_{i,t} + \lambda z_{N,t} + \sigma_{i,t} + \epsilon_{i,t} 
\end{equation}

However, the heterogeneous responses conditional on volatility have not been investi-
gated. To do this, I incorporate \( z_{N,t} \)'s interaction with volatility \( \sigma_{i,t} \) into the regression in equation (20). As before, \( i, w, \) and \( k \) represent capital investment, cash flow, and capital stock, respectively. \( X \) is a vector of controls including lagged Tobin’s Q, sales growth, and net leverage. \( \rho \) and \( \eta \) are firm- and year-level fixed effects. The coefficient of treatment variable \( z_{N,t} \)'s interaction with volatility captures the disproportionate impacts of a common policy shock. Seeing the estimated coefficient \( \varrho \) being negative indicates weaker policy effectiveness under high uncertainty, and vice versa. Given the endogeneity issues regarding individual volatility, Tobin’s Q, and control variables, I estimate the regression adopting the same GMM estimator as in Section 3.3, employing the lagged three- and four-year corporate variables as instruments. Table 9 includes the estimation results where standard errors are clustered at the four-digit NAICS level. The estimation results show that the policy stimulus indeed promotes investment, with the coefficient of \( z_{N,t} \) being positive, but less so if volatility is higher. This is suggested by a significantly negative coefficient of the interaction term.

To gauge the economic significance, Table 10 lists the fitted investment responses to a unit policy shock in \( z_{N,t} \), conditional on volatility valued at its corresponding sample percentiles:

\[
\frac{\partial (\frac{\iota_{i,t}}{k_{i,t} - 1})}{\partial z_{N,t}} = \hat{\lambda} + \hat{\varrho} \sigma_{i,t} \tag{21}
\]

As volatility \( \sigma_{i,t} \) moves from its 10th to its 90th percentile value, the fitted investment response in equation (21) drops by more than 44%, and the fitted response is no longer significant in statistical terms.\textsuperscript{76} This indicates that higher uncertainty might attenuate the stimulating effects of the bonus depreciation policy.

Even though the income effect is a crucial channel in which bonus depreciation can boost investment, it is by no means the only channel. The heterogeneous responses I have identified before should only be interpreted as the different total responses by firms with individual-specific volatility. Formally disentangling the effect of easing financing constraints is beyond the scope of this paper; however, additional discussions conceptually might still be of assistance. In particular, Zwick and Mahon (2017) point out that “bonus depreciation provides a temporary reduction in the after-tax price and a temporary increase in the first-year deduction for eligible investment goods.” I therefore roughly group the total effects into these two categories. Having scrutinized the second category, I claim that it should not be the cause of the estimated differences in Table 10. This is because the benefits of increasing the first-year deduction grow with the effective discount rate that firms employ to discount

\textsuperscript{76} Another way to interpret the economic significance is to say a one-standard-deviation increase in volatility decreases the investment response to policy by 23 percent.
tax benefits. Those with high discount rates should value the acceleration more than those with low discount rates. As volatility enlarges discount rates, the effects of this category should be stronger for firms with higher volatility, which is not observed in the data.

This leaves me with the first category that bonus depreciation reduces the after-tax price of capital goods, which can lead to an income effect. In the previous model in Section 2, the price of capital goods is used as the numeraire. Therefore, cash flow, in theory, is valued in terms of capital goods. As a result, a reduction in the after-tax price of capital goods in reality indicates a positive cash-flow shock that eases financing constraints. Therefore, the policy’s diminishing role in boosting investment as volatility increases is in line with the real-options channel reducing ISCF. However, the income effect is only part of the story—firms would respond to bonus depreciation even without financial frictions, because the decrease in capital price increases the NPVs of investment projects. Further extricating this effect requires more detailed disaggregated-level data. For example, if I had access to firms’ investment in eligible capital goods versus ineligible capital goods, estimating the empirical model in equation (20) using only ineligible investment would presumably isolate the income effect. This is because the NPV effect does not apply to ineligible investment. I leave this to future research.\footnote{Another potential channel in which bonus depreciation may boost investment is by making firms more (or less) optimistic about future economic prospects.}

5 Conclusions

In the wake of the Great Recession, financial frictions and uncertainty have received greater attention in the macroeconomic literature as drivers of business cycles. Previous studies have found investment to be low when financial markets tighten and uncertainty soars. In this paper, I instead study how financial frictions and uncertainty affect ISCF. ISCF is important because it characterizes firms’ marginal propensity to invest in response to liquidity provision.

My paper revisits a long-standing discussion in the corporate finance literature of whether financial frictions increase or decrease ISCF. Through a dynamic heterogeneous-agent model with options to delay investment, I provide a unified way to understand the two opposing forces at work. A higher degree of financial frictions shrinks ISCF by reducing firms’ capacity to borrow and invest. This is the capacity channel. However, via reducing the option value of waiting, more frictions also accelerate investment, leading to a rise in ISCF. This is the timing channel. Their relative strength depends on the cross-sectional distribution of production units within a consolidated firm. The model further predicts that
increased uncertainty diminishes ISCF, as it raises the option value of waiting; this is known as the real-options channel.

Studying a group of U.S. listed firms over the past 30 years reveals evidence supporting the timing channel and the real-options channel, with the results significant in both statistical and economic terms. These two results highlight the importance of the option value of waiting in firms’ investment decisions. The former demonstrates that my model offers a theoretical explanation for the empirical patterns in Fazzari et al. (1988), while the latter is useful in predicting the effectiveness of liquidity-provision policy. I also examine the empirical effects of the Bonus Depreciation policy (which enhances after-tax cash flow) on individual firms’ investment. The estimation results find that firms respond less to policy shocks when accompanied by greater uncertainty.

I conclude that ISCF plays a crucial part in investment dynamics. Both financial frictions and uncertainty contribute to ISCF. The latter mechanism lessens the effects of liquidity-provision policy on boosting investment. This is because, when facing heightened uncertainty, firms display lower ISCF. My work therefore suggests the important role of reducing uncertainty in policy considerations.
References


Figures and Tables

Figure 1: Realized Marginal Value Functions

This figure portrays the realized marginal value of capital and savings as functions in \( z_j \), at the beginning of period 2, for production unit \( j \), under the calibration in Table 1. The line with circular markers plots the marginal value of capital, \( MVK_{2j} = \frac{\partial V_{2j}(k_{2j}, b_{Lj}^f, z_j)}{\partial k_{2j}} \). The line without markers represents the marginal value of savings, \((- MVB_{2j})\), where \( MVB_{2j} = \frac{\partial V_{2j}(k_{2j}, b_{Lj}^f, z_j)}{\partial b_{Lj}} \) refers to the marginal value of debt; savings are regarded as negative debt.
This figure portrays the expected marginal value of capital and savings as functions in $\sigma_j$, in period 1, for production unit $j$, under the calibration in Table 1. The line with circular markers plots the expected marginal value of capital, $x_{kj} = \mathbb{E}\left[\frac{\partial V_j^2(k_{2j}, b_{Lj}, z_j)}{\partial k_{2j}}\right]$. The solid line without markers represents the expected marginal value of savings, $(-x_{bj})$, where $x_{bj} = \mathbb{E}\left[\frac{\partial V_j^2(k_{2j}, b_{Lj}, z_j)}{\partial b_{Lj}}\right]$ refers to the expected marginal value of debt; savings are regarded as negative debt. The dash line instead shows the investment determinant in period 1: $(x_{kj} + x_{bj})$. 

Figure 2: Expected Marginal Value Functions
This figure portrays the negative relationship between $\theta$ and $\sigma^*$ under the calibration in Table 1 with $\theta$ capturing the tightness of collateral constraint and $\sigma^*$ being the root of $x_{kj} + x_{bj} = 0$. As financial shocks reduce $\theta$, the corresponding $\sigma^*$ rises, representing the timing channel.
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<th>Parameters</th>
<th>Symbols</th>
<th>Values</th>
<th>Sources</th>
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<td>COMPUSTAT</td>
</tr>
<tr>
<td>Risk-free rate (gross, annual)</td>
<td>( R_f )</td>
<td>1.04</td>
<td>FRED</td>
</tr>
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<td>Time preference (annual)</td>
<td>( \beta )</td>
<td>0.96</td>
<td>Bloom et al. (2014)</td>
</tr>
<tr>
<td>Depreciation (annual)</td>
<td>( \delta )</td>
<td>0.1</td>
<td>Bloom et al. (2014)</td>
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<tr>
<td>Liquidity price of capital</td>
<td>( q )</td>
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<td>Cui (2014)</td>
</tr>
<tr>
<td>Collateral constraint</td>
<td>( \theta )</td>
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<td>-</td>
</tr>
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</table>

This table summarizes the model calibration that generates Figure 1, Figure 2, and Figure 3. Expected profitability (per unit of capital goods) is set to 0.3 matching the median value of cash flow (as a ratio of capital stock) of COMPUSTAT U.S. listed firms from 1984 to 2015. Cash flow rate is income before extraordinary items plus depreciation and amortization, divided by the year-start value of property, plant and equipment. Gross interest rate \( R_f \) is set to 1.04 per year, equal to the average 2-year Treasury bond yield from 1984 to 2015. The choice of maturity is to match the modelling setup that firms borrow in a two-period bond. Time preference parameter \( \beta \) and annual depreciation \( \delta \) are taken from Bloom et al. (2014). Liquidation price of capital is set to 85% of the purchasing price, as in Cui (2014). In Figure 1 and Figure 2, the collateral constraint parameter \( \theta \) is set to 0.65; together with \( q \), it generates a fire-sale value equal to 55% of the purchasing price of capital goods \((0.85 \times 0.65)\), belonging to the range 40%-60% suggested by the existing literature. In Figure 3, the value of \( \theta \) instead moves all the way from 0 to 1.
Table 2: Summary Statistics

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<th>St.Dev.</th>
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<td>0.403</td>
<td>0.226</td>
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<tr>
<td>Cash-flow Ratio ($w_{j,t}/k_{j,t-1}$)</td>
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<td>0.102</td>
<td>2.690</td>
<td>0.305</td>
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<tr>
<td>Macroeconomic Q</td>
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<td>18.797</td>
<td>39.297</td>
<td>5.885</td>
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<tr>
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<td>0.064</td>
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<td>0.594</td>
<td>1.071</td>
<td>0.257</td>
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<tr>
<td>$\sigma_{i,t}$ by RVOL</td>
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<tr>
<td>$\sigma_{i,t}$ by EGARCH</td>
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Investment ratio is capital expenditure normalized by year-start property, plant and equipment. Cash-flow ratio is income before extraordinary items plus depreciation and amortization, normalized by year-start property, plant and equipment. Macroeconomic Q is the sum of debt and market capitalization less total inventories, normalized by property, plant and equipment. Financial Q is the market-to-book ratio of total assets. Real sales growth is the growth rate of sales (net) adjusted to the 1982 U.S. dollar using the Producer Price Index. Net leverage is the sum of debt and market capitalization less cash and cash equivalents over market capitalization. $\sigma_{i,t}$ by RVOL is the realized equity volatility normalized by (1+Net Leverage). $\sigma_{i,t}$ by EGARCH is the EGARCH-inferred equity volatility normalized by (1+Net Leverage). Size is market value of asset, calculated by book value of assets minus total common equity plus market capitalization (in millions U.S. dollar).
Table 3: Measurement-Errors-Correcting Estimation

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<td>0.0176***</td>
<td>0.0178***</td>
<td>0.0201***</td>
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<tr>
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<td>(0.00283)</td>
<td>(0.00275)</td>
<td>(0.00284)</td>
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<td><strong>Sigma × CF</strong></td>
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<td>-0.0659***</td>
<td>-0.0640***</td>
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<td>(0.0219)</td>
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<td>0.00863***</td>
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Observations | Number of Firms | R-squared |
--------------|-----------------|-----------|
106,355       | 12,278          | 0.305     |

Uncertainty Type | Firm Fixed Effects | Year Dummies | Method |
Realized       | YES             | YES        | Cumulants 5 |
Realized       | YES             | YES        | Cumulants 6 |
Realized       | YES             | YES        | Cumulants 7 |
EGARCH         | YES             | YES        | Cumulants 7 |

Robust standard error in parentheses; *** p<0.01, ** p<0.05, * p<0.1
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Robust standard error in parentheses; *** p<0.01, ** p<0.05, * p<0.1
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<td>0.0093</td>
<td>0.0196</td>
<td>0.0149</td>
<td></td>
</tr>
<tr>
<td>Pct90</td>
<td>0.0034</td>
<td>0.0237</td>
<td>0.0140</td>
<td></td>
</tr>
</tbody>
</table>

\[
\frac{\text{Pct}_{90}}{\text{Pct}_{10}} - 1 = -81\% +109\% -8\%
\]

This table shows the estimated ISCF, i.e. equation (19), employing the regression results in column (1) of Table 4. Pct10, Pct20, ... and Pct90 refer to the sample percentile values of the corresponding Variable, which is the realized volatility of enterprise values, excess bond premium, and size, respectively, in column (1)-(3). Each ISCF is calculated using (19) when the corresponding Variable is valued at the corresponding sample percentile value, while the other two determinants are valued at their sample median values. The last row shows the change of ISCF when Variable moves from its bottom 10th percentile to the top 10th percentile.
Table 6: Robustness

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inv Ratio</td>
<td>Inv Ratio</td>
<td>Inv Ratio</td>
<td>Inv Ratio</td>
<td>Inv Ratio</td>
</tr>
<tr>
<td>Uncertainty (Sigma)</td>
<td>-0.238***</td>
<td>0.0239</td>
<td>-0.104***</td>
<td>-0.0953***</td>
<td>-0.0847***</td>
</tr>
<tr>
<td></td>
<td>(0.115)</td>
<td>(0.0364)</td>
<td>(0.0335)</td>
<td>(0.0322)</td>
<td>(0.0292)</td>
</tr>
<tr>
<td>Cash-flow Ratio (CF)</td>
<td>0.214***</td>
<td>0.0683***</td>
<td>0.0328**</td>
<td>0.0321**</td>
<td>0.0158*</td>
</tr>
<tr>
<td></td>
<td>(0.0267)</td>
<td>(0.0162)</td>
<td>(0.0152)</td>
<td>(0.0134)</td>
<td>(0.00872)</td>
</tr>
<tr>
<td>Sigma × CF</td>
<td>-0.0631*</td>
<td>-0.0535**</td>
<td>-0.0259***</td>
<td>-0.0225**</td>
<td>-0.0173*</td>
</tr>
<tr>
<td></td>
<td>(0.0361)</td>
<td>(0.0222)</td>
<td>(0.00841)</td>
<td>(0.0111)</td>
<td>(0.00891)</td>
</tr>
<tr>
<td>EBP × CF</td>
<td>0.0414***</td>
<td>0.0367***</td>
<td>0.0112**</td>
<td>0.0176***</td>
<td>0.0105***</td>
</tr>
<tr>
<td></td>
<td>(0.0130)</td>
<td>(0.0127)</td>
<td>(0.00533)</td>
<td>(0.00504)</td>
<td>(0.00388)</td>
</tr>
<tr>
<td>lag Q</td>
<td>0.0123***</td>
<td>0.0164***</td>
<td>0.0998***</td>
<td>0.00595***</td>
<td>0.00488***</td>
</tr>
<tr>
<td></td>
<td>(0.000791)</td>
<td>(0.00154)</td>
<td>(0.00619)</td>
<td>(0.000355)</td>
<td>(0.000245)</td>
</tr>
<tr>
<td>lag Real Sales Gr</td>
<td>0.209***</td>
<td>0.118***</td>
<td>0.103***</td>
<td>0.0954***</td>
<td>0.0364</td>
</tr>
<tr>
<td></td>
<td>(0.0526)</td>
<td>(0.0279)</td>
<td>(0.0235)</td>
<td>(0.0269)</td>
<td>(0.0251)</td>
</tr>
<tr>
<td>lag Net Leverage</td>
<td>-0.0850***</td>
<td>-0.0722***</td>
<td>-0.0959***</td>
<td>-0.112***</td>
<td>-0.0927***</td>
</tr>
<tr>
<td></td>
<td>(0.0134)</td>
<td>(0.00830)</td>
<td>(0.00925)</td>
<td>(0.00937)</td>
<td>(0.00817)</td>
</tr>
<tr>
<td>Size</td>
<td>4.16e-06***</td>
<td>-1.79e-07</td>
<td>-1.35e-06</td>
<td>2.39e-06***</td>
<td>3.71e-07</td>
</tr>
<tr>
<td></td>
<td>(1.25e-06)</td>
<td>(6.32e-07)</td>
<td>(9.42e-07)</td>
<td>(7.49e-07)</td>
<td>(7.00e-07)</td>
</tr>
<tr>
<td>Size × CF</td>
<td>-1.45e-06***</td>
<td>3.14e-09</td>
<td>2.10e-07</td>
<td>-1.30e-06***</td>
<td>-4.87e-07*</td>
</tr>
<tr>
<td></td>
<td>(6.02e-07)</td>
<td>(9.01e-07)</td>
<td>(3.32e-07)</td>
<td>(2.31e-07)</td>
<td>(2.64e-07)</td>
</tr>
<tr>
<td>lag Inv Ratio</td>
<td>0.127***</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.0240)</td>
</tr>
<tr>
<td>Observations</td>
<td>81,519</td>
<td>68,633</td>
<td>78,371</td>
<td>69,143</td>
<td>81,121</td>
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<tr>
<td>Number of Firms</td>
<td>9,261</td>
<td>8,068</td>
<td>9,204</td>
<td>9,028</td>
<td>9,232</td>
</tr>
<tr>
<td>Hansen J Test</td>
<td>0.998</td>
<td>0.998</td>
<td>0.999</td>
<td>0.235</td>
<td>1.000</td>
</tr>
<tr>
<td>AR(2) Test</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.358</td>
</tr>
<tr>
<td>AR(3) Test</td>
<td>0.236</td>
<td>0.0823</td>
<td>0.0964</td>
<td>0.177</td>
<td>0.716</td>
</tr>
<tr>
<td>AR(4) Test</td>
<td>0.529</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Robustness</td>
<td>Add R&amp;D</td>
<td>Drop Big Q</td>
<td>Fin Q</td>
<td>Drop GR</td>
<td>Dynamic Panel</td>
</tr>
<tr>
<td>Uncertainty Type</td>
<td>Realized</td>
<td>Realized</td>
<td>Realized</td>
<td>Realized</td>
<td>Realized</td>
</tr>
<tr>
<td>Firm Fixed Effects</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Year Dummies</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Method</td>
<td>A-Bond 3-4</td>
<td>A-Bond 4-5</td>
<td>A-Bond 3-4</td>
<td>A-Bond 3-4</td>
<td>A-Bond 3-4</td>
</tr>
</tbody>
</table>

Robust standard error in parentheses; *** p<0.01, ** p<0.05, * p<0.1
Table 7: Estimated ISCF – R&D included

<table>
<thead>
<tr>
<th>Pct</th>
<th>Variable</th>
<th>(1) ISCF</th>
<th>(2) ISCF</th>
<th>(3) ISCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pct10</td>
<td>0.2092</td>
<td>0.1846</td>
<td>0.2004</td>
<td></td>
</tr>
<tr>
<td>Pct20</td>
<td>0.2076</td>
<td>0.1907</td>
<td>0.2003</td>
<td></td>
</tr>
<tr>
<td>Pct30</td>
<td>0.2058</td>
<td>0.1921</td>
<td>0.2003</td>
<td></td>
</tr>
<tr>
<td>Pct40</td>
<td>0.2034</td>
<td>0.1950</td>
<td>0.2003</td>
<td></td>
</tr>
<tr>
<td>Pct50</td>
<td>0.2002</td>
<td>0.2002</td>
<td>0.2002</td>
<td></td>
</tr>
<tr>
<td>Pct60</td>
<td>0.1959</td>
<td>0.2055</td>
<td>0.2001</td>
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</tr>
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<td>Pct70</td>
<td>0.1895</td>
<td>0.2069</td>
<td>0.1999</td>
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<tr>
<td>Pct80</td>
<td>0.1795</td>
<td>0.2174</td>
<td>0.1993</td>
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<tr>
<td>Pct90</td>
<td>0.1592</td>
<td>0.2336</td>
<td>0.1975</td>
<td></td>
</tr>
</tbody>
</table>

\[
\frac{\text{Pct90}}{\text{Pct10}} - 1 \quad -24\% \quad +27\% \quad -1\%
\]

This table shows the estimated ISCF, i.e. equation (19), employing the regression results in column (1) of Table 6. Pct10, Pct20, ... and Pct90 refer to the sample percentile values of the corresponding **Variable**, which is the realized volatility of enterprise values, excess bond premium, and size, respectively, in column (1)-(3). Each ISCF is calculated using (19) when the corresponding **Variable** is valued at the corresponding sample percentile value, while the other two determinants are valued at their sample median values. The last row shows the change of ISCF when **Variable** moves from its bottom 10th percentile to the top 10th percentile.
<table>
<thead>
<tr>
<th></th>
<th>(1) Inv Ratio</th>
<th>(2) Inv Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty (Sigma)</td>
<td>-0.142***</td>
<td>-0.0884</td>
</tr>
<tr>
<td></td>
<td>(0.0424)</td>
<td>(0.0671)</td>
</tr>
<tr>
<td>Cash-flow Ratio (CF)</td>
<td>0.00236</td>
<td>-0.00489</td>
</tr>
<tr>
<td></td>
<td>(0.00948)</td>
<td>(0.0111)</td>
</tr>
<tr>
<td>Sigma × CF</td>
<td>-0.0260**</td>
<td>0.00183</td>
</tr>
<tr>
<td></td>
<td>(0.0102)</td>
<td>(0.0191)</td>
</tr>
<tr>
<td>EBP × CF</td>
<td>0.00773**</td>
<td>0.0147</td>
</tr>
<tr>
<td></td>
<td>(0.00388)</td>
<td>(0.0117)</td>
</tr>
<tr>
<td>lag Q</td>
<td>0.00506***</td>
<td>0.00825***</td>
</tr>
<tr>
<td></td>
<td>(0.000263)</td>
<td>(0.000647)</td>
</tr>
<tr>
<td>lag Real Sales Gr</td>
<td>0.0314</td>
<td>0.0275</td>
</tr>
<tr>
<td></td>
<td>(0.0278)</td>
<td>(0.0215)</td>
</tr>
<tr>
<td>lag Net Leverage</td>
<td>-0.177***</td>
<td>-0.1000***</td>
</tr>
<tr>
<td></td>
<td>(0.0231)</td>
<td>(0.00624)</td>
</tr>
<tr>
<td>Size</td>
<td>-5.92e-07</td>
<td>9.77e-07</td>
</tr>
<tr>
<td></td>
<td>(8.82e-07)</td>
<td>(9.11e-07)</td>
</tr>
<tr>
<td>Size × CF</td>
<td>-4.40e-07</td>
<td>2.03e-06***</td>
</tr>
<tr>
<td></td>
<td>(2.74e-07)</td>
<td>(2.12e-07)</td>
</tr>
<tr>
<td>Observations</td>
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<td>40,820</td>
</tr>
<tr>
<td>Number of Firms</td>
<td>6,541</td>
<td>6,274</td>
</tr>
<tr>
<td>Hansen J Test</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>AR(2) Test</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>AR(3) Test</td>
<td>0.227</td>
<td>0.269</td>
</tr>
<tr>
<td>Sub-samples</td>
<td>Low Leverage</td>
<td>High Leverage</td>
</tr>
<tr>
<td>Uncertainty Type</td>
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<td>Realized</td>
</tr>
<tr>
<td>Firm Fixed Effects</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Year Dummies</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Method</td>
<td>A-Bond 3-4</td>
<td>A-Bond 3-4</td>
</tr>
</tbody>
</table>

Robust standard error in parentheses; *** p<0.01, ** p<0.05, * p<0.1
Table 9: Bonus Depreciation Estimation

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inv Ratio</td>
<td></td>
</tr>
<tr>
<td>Uncertainty (Sigma)</td>
<td>0.735</td>
</tr>
<tr>
<td></td>
<td>(0.503)</td>
</tr>
<tr>
<td>$z_{N,t}$</td>
<td>1.734**</td>
</tr>
<tr>
<td></td>
<td>(0.870)</td>
</tr>
<tr>
<td>Sigma $\times z_{N,t}$</td>
<td>-0.918*</td>
</tr>
<tr>
<td></td>
<td>(0.554)</td>
</tr>
<tr>
<td>Cash-flow Ratio</td>
<td>0.00297</td>
</tr>
<tr>
<td></td>
<td>(0.00673)</td>
</tr>
<tr>
<td>lag Q</td>
<td>0.00530***</td>
</tr>
<tr>
<td></td>
<td>(0.000267)</td>
</tr>
<tr>
<td>lag Real Sales Gr</td>
<td>0.0996***</td>
</tr>
<tr>
<td></td>
<td>(0.0215)</td>
</tr>
<tr>
<td>lag Net Leverage</td>
<td>-0.105***</td>
</tr>
<tr>
<td></td>
<td>(0.00975)</td>
</tr>
<tr>
<td>Size</td>
<td>-7.36e-06</td>
</tr>
<tr>
<td></td>
<td>(5.93e-06)</td>
</tr>
</tbody>
</table>

Observations: 73,621
Number of Firms: 8,150
Hansen’s J Test: 1.000
AR(2) Test: 0.000
AR(3) Test: 0.250

Uncertainty Type: Realized
Firm Fixed Effects: YES
Year Dummies: YES
Method: A-Bond 3-4

Robust standard error in parentheses; *** p<0.01, ** p<0.05, * p<0.1
Table 10: Bonus Depreciation: Estimated Investment Responses

<table>
<thead>
<tr>
<th></th>
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<th></th>
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</thead>
<tbody>
<tr>
<td>Pct10</td>
<td>1.694</td>
<td>0.881</td>
</tr>
<tr>
<td>Pct20</td>
<td>1.670</td>
<td>0.888</td>
</tr>
<tr>
<td>Pct30</td>
<td>1.641</td>
<td>0.896</td>
</tr>
<tr>
<td>Pct40</td>
<td>1.605</td>
<td>0.907</td>
</tr>
<tr>
<td>Pct50</td>
<td>1.556</td>
<td>0.923</td>
</tr>
<tr>
<td>Pct60</td>
<td>1.490</td>
<td>0.944</td>
</tr>
<tr>
<td>Pct70</td>
<td>1.395</td>
<td>0.978</td>
</tr>
<tr>
<td>Pct80</td>
<td>1.246</td>
<td>1.034</td>
</tr>
<tr>
<td>Pct90</td>
<td>0.946</td>
<td>1.161</td>
</tr>
</tbody>
</table>

\[ \frac{\text{Pct90}}{\text{Pct10}} - 1 = -44\% \]

*** p<0.01, ** p<0.05, * p<0.1

This table shows the estimated investment responses to a unit shock in \( z_{N,t} \), i.e. equation (21), employing the regression results in Table 9. Pct10, Pct20, ... and Pct90 refer to the sample percentile values of the realized volatility of enterprise values. Each response is calculated using (21) when the volatility is valued at the corresponding sample percentile value. The column Std Err instead lists the standard errors of the estimated responses.