Contrary to traditional neoclassical growth models, recent decades have seen a number of developing economies running sizable current account surpluses. In response to “new mercantilist” explanations of this phenomenon that relate holdings of foreign assets to higher levels of economic growth, this paper presents a theoretical model of a small open developing economy that permits a welfare analysis of mercantilist policies and importantly answers the question of whether mercantilist motives alone can explain the recent high levels of observed foreign asset holdings. Using a calibration to match the characteristics of China, the model predicts that while such policies may lead to significant welfare gains, consumers’ desires to smooth consumption generally preclude a positive current account balance under most parameterizations. Therefore, deliberate foreign asset accumulation may be welfare reducing or mercantilist motives may provide only one component of a fuller explanation.

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1 Introduction

In discussions of the rationale behind the recent phenomena of some emerging economies exhibiting persistent current account surpluses, particularly China (See Figure 1), and unprecedented stockpiles of foreign reserves, particularly in developing Asia (see Figure 2), a commonly floated explanation is that of “new mercantilism,” i.e. the theory that asset accumulation in emerging markets is a by-product of promoting exports to developed nations in order to facilitate the creation of jobs in industry and accelerate economic growth. However, most of these explanations have been restricted to stylized narratives (Dooley, et al 2003) or suggestive empirics (Rodrik 2008).

This paper fills a gap in the literature by presenting a dynamic model of a developing small open economy that provides a full welfare analysis of mercantilist policy, calibrated to match the growth of China, the largest global holder of foreign reserves. The main insight provided by the model is that despite significant potential welfare gains the mercantilist hoarding of foreign assets by way of capital controls can not explain current account surpluses under realistic calibrations. Overall, mercantilism may provide a component of the rationale behind foreign asset accumulation, but additional motivation is needed to fully justify the levels currently observed in some emerging economies, especially China.

In recent years, the idea of “new mercantilism”\(^1\) has experienced growing popularity in some corners of academia and is often invoked implicitly in popular media. Dooley, Folkerts-Landau, and Garber (2004) offered perhaps the most concise definition when stating, “Exports mean growth.” Alternatively, Dani Rodrik (2013) offers a fuller description:

“It is more accurate to think of mercantilism as a different way to organize the relationship between the state and the economy…in pursuit of common objectives,

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\(^1\)The breadth of meaning implied by the label of “new mercantilism” has often varied by author. This paper follows Aizenman and Lee (2010) in conceptualizing a “mercantilist” accumulation of assets as having the goal of export competitiveness/real economy growth, as opposed to pure insurance purposes.
such as domestic economic growth. Mercantilists view trade as a means of supporting domestic production and employment, and prefer to spur exports rather than imports.”

Thus, at the heart of mercantilism lies the belief that the exportation of goods and services is intrinsically desirable. In other words, the accumulation of gold that was the principal objective of the mercantilism of the 19th century has been supplanted by the accumulation of foreign assets in its modern incarnation, though the means of financing both has remained the same: current account surpluses. However, just as Adam Smith famously argued in “The Wealth of Nations” that ownership of bullion is not fundamentally equivalent to prosperity, the sense in endlessly accumulating foreign assets is not immediately self-evident. After all, how does one gain by continually working day-after-day in return for IOUs ad infinitum? Therefore, any satisfying explanation of the virtues of “new mercantilism” must answer the two important questions of 1) how the economy induces exports and 2) how exports drive economic growth.

Most popular mercantilist stories explain the promotion of exports through exchange rate depreciation. However, in the absence of persistent price-stickiness, such an approach is problematic in that it may only have short-term effects and/or lead to unwanted inflation. Furthermore, as shown in Jeanne (2012), a policy of maintaining an undervalued exchange rate in a growing economy may incur significant welfare costs. Moreover, exchange rate manipulation alone doesn’t provide an explanation for the second and ultimately more important question of how the act of exporting enables healthier economic growth. Thus, exchange rate dynamics are not essential to a mercantilist story and omitted from the model presented in this paper in the interest of simplicity. Instead, I present a real model that is consistent with the concept of “new mercantilism” through the following two important assumptions: the government uses strict capital flow controls to direct resources into the tradable sector, and the production of tradable goods exhibits important externalities to productivity.

I assume that the application of new technologies to production and/or innovations to productivity is driven by two sources: 1) “learning-by-doing” with respect to the share of labor employed in the production of tradable goods, and 2) the relative distance from a global technological frontier. Consumers and firms are too small to individually take into account the impacts of their consumption/employment decisions on the growth rate of technology applied in production. Therefore, the positive spillover effects of higher employment in the tradable sector are only exploitable by a central planner who can internalize the socially-optimal level of tradable-sector employment, or equivalently a government agent, who may direct the level of employment by influencing private consumption decisions. I show that the government can exert such influence by introducing equal import tariffs and export subsidies, taxing consumption of tradable goods, or subsidizing the production of tradable goods. However, all of these options are considered to be infeasible for one or more of the following reasons: 1) special interests may preclude the use of optimal policy due to political opposition or the incursion of overly dear rent-seeking costs, 2) the government may be incapable of appropriately identifying which
industries to subsidize/tax, and/or most importantly 3) such price-distorting policies may be in violation of international treaties, such as WTO membership obligations. The WTO, for example, rules out explicit import tariffs or export subsidies, in addition to production subsidies that may provide an “unfair” advantage in international market competition. These restrictions carry weight because of the WTO’s dispute resolution mechanism, which may authorize aggrieved parties to take countervailing actions.

I therefore assume that the government promotes exports through the use of capital controls, which do not face the same obstacles as price-distorting policies. That is, there exists no universal framework governing the international flows of capital in the same way that the WTO regulates the trade of physical goods among its member countries. Even the IMF, as the world’s largest overseer of the international monetary system, acknowledges in its Articles of Agreement the rights of its member countries to “exercise such controls as are necessary to regulate international capital movements.” While the IMF does have a history of advocating for greater liberalization of international capital markets, its most direct influence has been mostly restricted to a small set of economies in severe financial crises (e.g. Mexico, Thailand, and Argentina), and its own views have become more accommodative of capital controls in certain circumstances since the 2008 global financial crisis.²

The fundamental intuition underpinning the results of the model lies in the balance between the consumer’s desire to increase short-term tradable consumption by borrowing against higher future output growth so as to smooth consumption over time and the desire to decrease short-term tradable consumption – thus increasing net foreign wealth – in order to fuel quicker output growth via faster convergence to the global technological frontier. This latter motivation is the rationale for mercantilist asset accumulation. The main takeaway from the results of the model is that the former desire for consumption smoothing always wins out. This is not too surprising when one realizes that the consequence of the latter behavior – faster output growth – only serves to intensify the motivation of the former behavior – consumption smoothing via borrowing. In other words, so called “mercantilist” asset accumulation is almost entirely self-defeating, even for a wide range of calibrations.

There have been two main bodies of work in the literature seeking to explain observed/optimal levels of foreign assets. First, several papers have proposed that stocks of assets be viewed as “war chests” of precautionary savings against risk, either at the household level in the case of idiosyncratic risk³ or at the national level in the case of “sudden-stop” access to international credit⁴. These papers have demonstrated mixed success in rationalizing the reserve holdings of China.

A second body of work, which I categorize as “mercantilist,” has alternatively proposed that hoarding assets can be indirectly effectual in driving real economic growth. Dooley et al (2004)

²See IMF (2012).
⁴See Durdu et al (2009) and Jeanne and Ranciere (2011)
theorizes that asset accumulation is a byproduct of a growth strategy that involves a developing periphery focusing on producing exports to take advantage of vast external demand from a rich core economy. While successful in popularizing a narrative of growth-promoting mercantilism, others have sought to expand on its premise by offering more rigorous mathematical foundations. A common theme among these models is an assumption that the tradable sector exhibits special characteristics that can be exploited in conjunction with asset accumulation to achieve positive real effects.

This paper is most closely related to the following two models of mercantilism. Rodrik (2008) presents a simple model relating economic growth rates to a special failure of the tradable sector to appropriate all of its profits. The model presented herein improves on this model by offering a full welfare analysis and presenting a more explicit version of the tradable sector’s exceptionalism. Korinek and Serven (2010) also derive a model allowing a full welfare analysis of mercantilistic behavior, although this paper differs in two important ways: 1) I assume the presence of learning-by-doing benefits to production as opposed to Romer-style learning-by-investing, and 2) this paper importantly allows for an analysis of the optimal path of reserves over time, which is absent from the Korinek and Serven model due to the government effectively “throwing away” all foreign assets.

This paper is also related to a few other strains of research. First, it is closely related to the literature noting the disconnect between the predictions of neoclassical models that capital ought to flow to those economies with high marginal products and actual empirical observations. Lucas (1990) first drew attention to the small amount of capital flowing into developing countries, and Prasad, et al (2007) further highlighted the fact the developing countries were actually exporting capital instead of importing. Gourinchas and Jeanne (2011) refer to this as the “allocation puzzle” and note the positive correlation between capital outflows and productivity growth.

Second, this paper is influenced by endogenous growth models that rely upon dynamics in the level of technology/productivity or innovation, rather than capital deepening, to drive growth in the long-run, such as Helpman (1991), Aghion and Howitt (1992), and Eaton and Kortum (1999). Furthermore, the model draws upon the work of Grossman and Helpman (1991) and Melitz (2003), who demonstrate a link between technological adoption and exposure to international trade at the aggregate level in assuming the presence of a positive externality to technological growth stemming from the tradable sector, which is most closely tied to international sources of innovation and inspiration.

Third, this paper takes cues from the extensive work on technological convergence, or the “Veblen-Gerschenkron effect.” Among the first to propose dynamic mathematical models of

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5 Aizenman and Lee (2007) discuss how different types of externalities can lead to significantly different policy recommendations.
6 There also exists a large empirical literature examining the relationship between exporting and productivity at the firm level, although the findings are very mixed. Wagner (2007) provides a survey of this work.
7 Veblen (1915) and Gerschenkron (1952) were among the first economists to comment on the advantages of “relative backwardness” in catching-up to innovators.
catch-up to a “global frontier” were Nelson and Phelps (1966) and Findlay (1978), with Barro and Sala-i-Martin (1997), Howitt (2000), and Acemoglu et al (2006) providing more modern examples.

Each paper proposes a different variable that affects the speed of convergence, including “educational attainment,” R&D expenditures, and managerial skill. I assume that technology converges to a global frontier at a rate that is determined by the level of employment in the tradable-goods sector, which can be interpreted as a “learning-by-doing” style of technological progress. This assumption provides a conceptual link between the economy’s dynamic growth and the consumption/saving decisions of the consumer.

Fourth, this paper is also marginally related to the literature on “Dutch disease,” which posits a relationship between technological growth and the allocation of resources in sectors of tradable production, as outlined in Corden and Neary (1982). Since increasing the rate of technological adoption involves channeling productive resources into the tradable sector, then an exogenous addition of tradable goods to domestic output would counteract the gains from doing so. Van Wijnbergen (1984) demonstrates the need for government intervention in the context of an economy with learning-by-doing effects in order to limit the dynamic loss associated with the resource discovery. However, Torvik (2001) points out that the danger of Dutch disease depends on the degree to which the nontradable sector also exhibits learning-by-doing effects, as I assume is the case, and the composition of tradable and nontradable sectors, which may drastically differ across countries.

The paper is structured as follows. Section 2 presents the basic setup of the model and alternative options for government policy. Section 3 presents an empirical calibration of the model, and Section 4 discusses results of a benchmark model along with various tests of robustness. Section 5 provides a brief conclusion.

2 Model

2.1 Consumer Demand

I consider a small open economy comprising many identical, infinitely-lived consumers. The economy permits a representative consumer who seeks to maximize his lifetime discounted utility, given by

\[ U_0 = \int_0^\infty u(C_t)e^{-\rho t} dt, \]  

(1)

where \( \rho \) is the consumer’s temporal discount rate and \( u(\bullet) \) is a CRRA felicity function of the form \( C_t^{1-\theta}/(1 - \theta) \), such that \( 1/\theta \) is the elasticity of intertemporal substitution. Real consumption \( C_t \) is a composite of two goods: an internationally tradable good and a domestically-consumed good.
nontradable good (denoted by $T$ and $N$, respectively). The real consumption index exhibits the following CES form

$$C(C_{Tt}, C_{Nt}) = \left[ \phi C_{Tt}^{\sigma - 1} + (1 - \phi) C_{Nt}^{\sigma - 1} \right]^{\frac{\sigma}{\sigma - 1}},$$

such that $\sigma$ measures the elasticity of substitution between tradable and nontradable goods.

The representative consumer earns income by owning and renting out units of capital to firms as well as inelastically providing labor to firms in exchange for wages. Consumers make decisions on how best to allocate their income between consumption, investment in additional physical capital, and the purchase of interest-bearing foreign assets. New capital is created from tradable goods according to the following

$$\dot{K}_t = I_t - \delta K_t,$$

where $K_t$ is the aggregate stock of physical capital in the economy, $I_t$ is aggregate investment and $\delta$ is the rate of capital depreciation. For simplicity, I assume that only tradable goods can be converted into physical capital and that the economy’s physical capital stock is entirely owned by domestic agents, although these assumptions could easily be relaxed.

In aggregate, the representative consumer’s dynamic budget constraint can therefore be expressed as

$$\dot{B}_t = r^* B_t + W_t \bar{L} + R_t K_t - \frac{1}{q_t} C_t - I_t,$$

where $B_t$ represents net claims on future foreign income, $r^*$ is the exogenously determined fixed rate of interest on foreign assets, $W_t$ is the wage rate, $R_t$ is the rental rate of physical capital, and $I_t$ is investment in new physical capital, all of which are measured in tradable goods. Supposing no transportation costs and perfect competition among firms, I assume the law of one price holds such that the price of tradables is held constant over time by the global market and normalized to unity, such that tradable goods serve as numeraire for the economy. $\bar{L}$ is the fixed level of aggregate labor (hereafter normalized to unity), and $q_t$ represents the price of tradables in terms of composite real consumption, which serves as the real exchange rate for the economy (an increase in $q_t$ reflecting a real depreciation).

Additionally, the consumer is restricted in his dynamic consumption choices by the following condition

$$\lim_{t \to \infty} B_t e^{-r^* t} \geq 0,$$

which rules out Ponzi-type borrowing schemes. If consumers are non-satiated, then this condition will hold with equality as the consumer finds it optimal not to leave assets “on the table”.

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9In particular, the assumption of domestic capital ownership is made in order to easily delineate the unique paths of financial assets and physical capital. However, allowing for foreign direct investment would not influence the equilibrium path of consumption since assets are risk-free, such that the individual consumers’ dynamic utility maximization only depends on their net level of wealth and not its exact composition.
Korinek and Serven (2010), by contrast, requires consumers to abandon their claims on tradable assets in order to achieve the economy’s optimal dynamic path.

The representative consumer seeks to maximize his real consumption at every point in time by optimizing the mix of tradable and nontradable consumption subject to some allocated level of expenditures, resulting in the following intratemporal first-order conditions

\[
\frac{p_{Ct}}{p_{Ct} \frac{\partial C_t}{\partial N_t}} = p_{Nt} \\
\frac{p_{Ct}}{p_{Ct} \frac{\partial C_t}{\partial T_t}} = p_T \Rightarrow p_t = \frac{1-\phi}{\phi} \left( \frac{C_{Tt}}{C_{Nt}} \right)^{1/\sigma},
\]

where \( p_t = p_{Nt}/p_T \) is the price of nontradable goods in terms of tradable goods and \( p_{Ct} \) is the price of composite real consumption. Using the conditions in (6), the real exchange rate can be expressed as

\[
q_t = \phi \left[ \phi + (1-\phi) \left( \frac{\phi}{1-\phi} p_t \right)^{1-s} \right]^{1/(s-1)}.
\]

Therefore, the real exchange rate for the economy is fundamentally tied to the relative price of nontradables.

2.2 Production

The supply side of the economy comprises numerous firms in two sectors producing tradable and nontradable goods. In aggregate, these sectoral outputs can be respectively expressed as

\[
Y_{Tt} = K_{Tt}^{\alpha} (A_t L_{Tt})^{1-\alpha}, \\
Y_{Nt} = K_{Nt}^{\alpha} (A_t L_{Nt})^{1-\alpha},
\]

where \( A_t \) represents the economy-wide level of labor-augmenting technology or general productivity. The assumption of homogeneous technologies across sectors is made mainly for mathematical simplicity, and could easily be relaxed. Firms choose their optimal allocations of productive resources, which are subject to the following constraints in aggregate

\[
K_{Tt} + K_{Nt} = K_t \\
L_{Tt} + L_{Nt} = 1,
\]

where capital and labor are both freely mobile between sectors.

Similar to Rodrik (2008), the final key assumption of the model is the presence of a learning-by-doing externality in the production of tradable goods, such that the evolution of technology over time is related to the endogenously-determined amount of labor employed in the tradable sector. More specifically, I assume the aggregate measure of technology/efficiency in production
A_t exhibits the following rate of growth similar to Nelson and Phelps (1966),

\[ \frac{\dot{A}_t}{A_t} = g \left( L_{Tt}, \frac{A^*_t}{A_t} \right), \tag{12} \]

where \( A^*_t \) denotes the world “frontier” level of technology, which grows at the exogenous rate of \( g^* \). The intuition for the transition equation of technology is that developing countries catch-up to the frontier \( A^*_t \) by absorbing previously pioneered expertise/know-how and instituting established best practices. The speed of convergence to the frontier is related to the size of the labor share in the tradable sector, since tradable production is by its nature intrinsically more directly connected to the wealth of international knowledge and thereby able to foster quicker advancement. The function \( g(\bullet) \) satisfies the following conditions:

\[ \frac{\partial g}{\partial L_{Tt}} > 0, \tag{13} \]
\[ \frac{\partial g}{\partial \left( \frac{A^*_t}{A_t} \right)} > 0, \tag{14} \]
\[ g(L_{Tt}, f) = g^* \forall L_{Tt} \in [0,1], \tag{15} \]

where \( f \) represents the long-run level of technology to which the economy converges relative to the frontier. Full convergence to the frontier would be represented by \( f = 1 \). Assuming the economy begins with a lower level of technology such that \( A_0 < A^*_0 \), then the growth rate of domestic technology will converge to the frontier growth rate over time, by construction, and the speed of this convergence can be improved by employing a larger share of labor in tradable production. However, institutional weaknesses that inhibit the absorption and implementation of new technologies may prohibit the economy from ever fully reaching the level of the frontier at any point in time, as in the case of \( f > 1 \), though the economy may still converge to the same rate of technological growth \( g^* \).

More specifically, I assume that the growth rate of technology takes on the following functional form

\[ \frac{\dot{A}_t}{A_t} = g^* + \left( \gamma_0 + \gamma_1 L_{Tt} \right) \left( \frac{A^*_t}{A_t} - f \right), \tag{16} \]

which satisfies the conditions above so long as \( \gamma_1 > 0 \). Therefore, the assumption of homogeneous technologies between sectors can be thought of as the limiting case in allowing for the greatest possible impact of the externality from tradable labor. Alternatively restricting the effect of the externality solely to the level of technology employed in the tradable sector would mainly weaken the magnitude of the results without substantially changing their substance, so I restrict my analysis to the use of the specification in \ref{16}.

Defining \( \kappa_t \equiv K_t/L_t \) as the relative capital intensities employed in each sector, then factor...
mobility and profit maximization now impose the following conditions for wages

\[(1 - \alpha)\kappa_T A_t^{1-\alpha} = W_t \quad (17)\]

\[p_t (1 - \alpha)\kappa_N A_t^{1-\alpha} = W_t \quad (18)\]

and likewise for rental rates

\[\alpha \kappa_T^{1-1} A_t^{1-\alpha} = R_t \quad (19)\]

\[p_t \alpha \kappa_N^{1-1} A_t^{1-\alpha} = R_t \quad (20)\]

For reasons that vary with the structure of the economy (namely, the absence of capital controls or state-control of banks) and will be further clarified later, the domestic economy exhibits interest rate parity with the international rate, such that

\[R_t = r^* + \delta \quad (21)\]

Since the international interest rate is taken as given, the supply-side of the economy intratemporally determines the wage rate, relative factor intensities, and the relative price of nontradables all independently of domestic demand, such that we can express these variables as

\[\kappa_T = \kappa_N = \left(\frac{\alpha}{1+\delta}\right)^{1/(1-\alpha)} A_t \quad (22)\]

\[W_t = (1 - \alpha)\left(\frac{\alpha}{1+\delta}\right)^{\alpha/(1-\alpha)} A_t \equiv \bar{w} A_t \quad (23)\]

\[p_t = 1 \quad (24)\]

A direct consequence of this derivation is that we can easily see by substituting the expression for \(p_t\) in (24) into (7) that both the domestic relative price ratio and the real exchange rate are constant over time. This is consistent with the famous Harrod-Balassa-Samuelson effect, which predicts varying real exchange rates based on intersectoral productivity differences. Since technologies have been assumed to be homogenous across sectors, the economy does not exhibit real exchange rate dynamics.

### 2.3 Asset Accumulation Motive

We are now in a position to further clarify the workings of the model and illustrate the motivation for accumulating foreign assets. This section essentially defines the “mercantilist” aspect of the model, whereby the exports of tradable goods is linked to higher transitional growth rates for the real economy.

First, using the relative price equilibrium in the economy defined by setting the demand-side pricing condition in (6) equal to the supply-side pricing condition in (24) allows us to derive an
expression linking nontradable and tradable consumption

\[ C_{Nt} = \left(1 - \phi\right)^{\sigma} \cdot C_{Tt} \equiv \bar{c}_N \cdot C_{Tt}. \]  

(25)

Therefore, using the nontradability constraint \( Y_{Nt} = C_{Nt} \), the labor resource constraint in (11), the relative factor intensities in (22), and the tradable/nontradable consumption link above, we can derive the following relationship between the tradable labor allocation and tradable consumption

\[
C_{Nt} = K_{Nt}^\alpha (A_t L_{Nt})^{1-\alpha} \\
C_{Nt} = \kappa_{Nt}^\alpha L_{Nt} (A_t L_{Nt})^{1-\alpha} \\
C_{Nt} = \kappa_{Nt}^\alpha A_t^{1-\alpha} (1 - L_{Tt}) \\
L_{Tt} = 1 - \left(\frac{\alpha}{\alpha + \gamma}ight)^{(\alpha-1)\gamma N} \frac{C_{Tt}}{A_t} \equiv 1 - \gamma_2 \frac{C_{Tt}}{A_t}. \]  

(26)

Thus, \( L_{Tt} \) and \( C_{Tt} \) have a linearly inverse relationship.

We can see that the contemporaneous effect of reducing tradable consumption is to drive more labor into the production of tradable goods, thereby increasing tradable output and expanding the country’s trade balance, which may be used to finance the purchase of foreign assets. If we consider the equivalence between the price ratio defined by demand in (6) and that defined by supply in (24), then it’s clear that a reduction in tradable consumption must be matched by a decline in nontradable consumption, and since nontradable output must be fully consumed domestically this can be achieved by redirecting labor from the nontradable sector into the tradable sector. Repressing domestic consumption, thus, leads to higher employment of labor in the tradable sector in equilibrium.

Because of the relationship between tradable and nontradable consumption expressed in (25), we can express the real consumption function in (2) as

\[
C(C_{Tt}, C_{Nt}) = (\phi + (1 - \phi)c_N^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)} \cdot C_{Tt} \equiv \bar{c} \cdot C_{Tt}, \]  

(27)

meaning we can alternatively express the instantaneous utility function in (1) as

\[
u(C_t) = \frac{\bar{c}^{1-\theta}}{1-\theta} \cdot C_{Tt}^{1-\theta}. \]  

(28)

Therefore, a reduction in tradable consumption unambiguously reduces utility at any moment in time, but by virtue of the redirection of labor toward the tradable sector and the externality to tradable labor employment presented in (12), it may also speed the growth in productivity and allow the economy to enjoy a higher level of output in the future than it otherwise would have experienced. Once the domestic technology level attains the frontier, however, the motivation for fettered consumption disappears and the economy may begin to consume the interest income its accumulated assets. The key question at consideration is whether the dynamic gains from resource reallocation offset the early losses from constrained consumption.
2.4 Optimality Conditions

Having presented the intratemporal relationships among the key variables of the model, I now turn to the consideration of the intertemporal optimization of the model. This section shows how dynamic optimization in a laissez-faire economy differs from that of a centrally planned economy, wherein a benevolent social planner seeks to maximize the discounted lifetime utility of the country. The social planner is able to exploit the production externatility in the tradable goods sector by targeting the optimal level of tradable consumption. Later, I show how a government agent can make use of mercantilist policy to replicate the social planner’s choices.

In order to ensure that countries at the technological frontier exhibit a balanced growth path, I assume that the following relationship exists among the model parameters

\[ g^* = \frac{r^* - \rho}{\theta}. \]  

(29)

Therefore, in order to simplify the exposition of the model’s solutions, I henceforth normalize all variables by the frontier level of technology, \( A^*_t \), and denote the transformed variables by lower-case letters, e.g.

\[ c_t \equiv \frac{C_t}{A^*_t}. \]

2.4.1 Laissez-Faire

As stated previously, the representative consumer’s goal is to maximize his lifetime discounted utility presented in (1) subject to the dynamic budget constraint in (4). Since the consumer can freely choose between investing tradable output in additional physical capital or a limitless supply of foreign assets, he will choose to maintain the capital stock at the level which equalizes the domestic net marginal products of capital with the international rate of return, such that the interest rate parity condition in (21) must hold. Thus, the consumer is indifferent between investing in additional physical capital or accumulating additional foreign assets since both will guarantee a constant rate of return \( r^* \) in equilibrium. Therefore, in making his intertemporal consumption choices the consumer only cares about his net level of total assets, which I define as \( m_t \equiv b_t + k_t \). Thus, we can express the representative consumer’s dynamic optimization problem as

\[
\max_{c_t} \int_0^\infty \hat{u}(c_t)e^{-(r^*-g^*)t} \, dt \\
\text{s.t. } \dot{m}_t = (r^* - g^*)m_t + \bar{w} \cdot a_t - \frac{1}{q^t}c_t,
\]

(30)  

(31)

where the consumer takes levels of prices and technology as given, and \( \hat{u}(\bullet) \) is the normalized version of the instantaneous utility function in (1).
Optimizing with respect to consumption yields the following important first-order condition

$$\lambda = \frac{\partial \hat{u}}{\partial c_t} \cdot q_t = \frac{\partial \tilde{u}}{\partial c_T t},$$

(32)

where $\lambda$ is the co-state variable associated with total assets. This condition implies that the marginal utility of normalized tradable consumption must be constant over time. Therefore, (32) dictates that the optimal paths of real consumption, tradable consumption, and nontradable consumption will all be constant over time.

We can utilize the relationship between tradable labor and consumption in (26) to express the transition of technology from (16) in reduced form as

$$\dot{a_t} = \tilde{g}(a_t, c_T t).$$

(33)

Therefore, assuming the consumer has rational expectations, the dynamic equilibrium of the economy can be found by solving for the value of $\lambda$ that defines the level of tradable consumption in (32) and the path of technology in (33) such that the following intertemporal budget constraint is satisfied:

$$\int_0^\infty c_T t e^{-(r^* - g)^t} dt = \int_0^\infty \left( \tilde{y}_T (a_t, c_T t) - (r^* + \delta) \tilde{k}(a_t) \right) e^{-(r^* - g)^t} dt + b_0 + k_0,$$  

(34)

where initial values of capital $k_0$ and net foreign assets $b_0$ are given. See the appendix for a full derivation of this constraint, as well as a full description of the numerical solution methodology.

### 2.4.2 Social Planner

I now consider the case of a benevolent social planner, who maximizes the lifetime discounted utility in (1) by explicitly choosing the path of tradable consumption $C_T t$ while taking the previously presented intratemporal equilibrium conditions as given. I assume that the social planner targets consumption instead of the level of employment in the tradable sector directly because I show later the consistency of this approach with the use of capital controls. The social planner is assumed to be capable of recognizing the impacts of his choice of consumption on other aggregate variables in the economy, including the effects of consumption on intratemporal equilibrium prices. As such, the social planner chooses the path for tradable consumption so as to indirectly exploit its inverse relationship with tradable-sector labor employment and achieve a more optimal rate of technological advancement.

Using the pricing conditions in (6) and (24) to eliminate $q_t$ from the dynamic budget con-
straint, the social planner’s problem can be stated as

$$\max_{c_{Tt}} \int_0^\infty \tilde{u}(c_{Tt}) e^{-(r^* - g^*)t} dt$$  \hspace{1cm} (35)$$

s.t. \hspace{0.5cm} \dot{m} = (r^* - g^*)m + \bar{w}(a_t) - \bar{q}c_{Tt}$$  \hspace{1cm} (36)$$
\dot{a}_t = \bar{g}(a_t, c_{Tt}),$$  \hspace{1cm} (37)$$

where $\bar{q} \equiv \frac{1}{\phi} e^{(\sigma - 1)/\sigma}$, the evolution of technology is the reduced-form version from (33), and $\tilde{u}(\bullet)$ is the normalized version of the utility function in (28) expressed solely as a function of tradable consumption.

Optimization with respect to $c_{Tt}$ yields the following first-order condition

$$\frac{\partial \tilde{u}}{\partial c_{Tt}} = -\frac{\partial \tilde{g}}{\partial c_{Tt}} \mu_t + \bar{q} \lambda,$$  \hspace{1cm} (38)$$

where $\lambda$ is again the constant co-state variable associated with total assets, and $\mu_t$ is the co-state variable associated with technology. Furthermore, the first-order optimality condition with respect to technology yields an expression for the evolution of $\mu_t$

$$\dot{\mu}_t = \left[(r^* - g^*) - \frac{\partial \tilde{g}}{\partial a_t}\right] \mu_t - \bar{w} \lambda.$$  \hspace{1cm} (39)$$

Because of the inverse relationship between labor employed in the tradable sector and tradable consumption, as well as the assumption regarding the effect of $L_{Tt}$ on technology growth in (13), we know that $\frac{\partial \tilde{g}}{\partial c_{Tt}} < 0$. Therefore, the right-hand side of (38) represents the marginal utility of tradable consumption, which is no longer constant (as in the laissez-faire setting) due to the movement in $\mu_t$. Using the assumed form of the technology growth function in (16), we can also express marginal utility with respect to tradable consumption as

$$\frac{\partial \tilde{u}}{\partial c_{Tt}} = \bar{q} \lambda + \gamma_1 \gamma_2 \left(\frac{1}{a_t} - 1\right) \mu_t.$$  \hspace{1cm} (40)$$

Since $a_t$ converges to one from below by construction and (39) implies that $\mu_t$ is bounded in the long-run steady-state, then it’s easy to see that

$$\lim_{t \to \infty} \frac{\partial \tilde{u}}{\partial c_{Tt}} = \bar{q} \lambda.$$  \hspace{1cm} (41)$$

Then since $\mu_t$ is positive, equation (41) implies that the marginal utility of tradable consumption in the long-run is less than the marginal utility at every previous point in time, or equivalently

$$\lim_{t \to \infty} c_{Tt} > c_{Tt} \forall t,$$  \hspace{1cm} (42)$$

that is, tradable consumption reaches its all time zenith in the long-run steady-state.

How does this compare to the laissez-faire solution? The fact that tradable consumption
must increase over time in the social planner’s solution implies that its level must fall below that of the laissez-faire consumer’s fixed consumption at some point in time, since both versions of the economy face the same intertemporal budget constraint. Intuitively, this lower level of consumption is likely to occur early on when the relative benefits to technological accumulation are the highest. In this sense, the mercantilist approach to optimal growth is characterized by an initial period of repressed consumption, which corresponds to a relatively larger net trade balance that could be used to finance the purchase of foreign assets.

Equation (39) together with (38) and the equations of motion for assets in (36) and technology in (37) define a three-dimensional system of differential equations that can be solved for the welfare maximizing path of tradable consumption in conjunction with the intertemporal budget constraint in (34). A full explanation of the numerical solution methodology employed is presented in the appendix.

2.5 Government Policy

Since the gains to technological innovation from labor employed in the tradable sector only arise at the aggregate level, individual consumers and firms in the economy are unable to identify and exploit the potential dynamic gains from reallocating resources within the economy. Assuming the government is large enough to internalize the importance of the externality and has the maximization of aggregate welfare as its primary objective, then it can play an important role by influencing the relative levels of sectoral activity in the economy in order to achieve more optimal outcomes.

I first consider the use of capital controls by the government, by which it may replicate the social planner’s optimal consumption path. Next, I consider various price-distorting policies that may theoretically lead to first-best outcomes, but are nonetheless assumed to be infeasible for reasons discussed in the introduction.

2.5.1 Capital Controls

The government may direct the path of tradable consumption without violating multinational trade agreements by employing strict controls on the flows of private financial capital. Suppose the economy includes a government agent who interacts with the consumer through lump-sum transfer payments and imposes measures such that individual consumers are no longer able to buy or sell assets in the international market (forcing \( b_t = 0 \) for all \( t \)), such that the consumer’s budget constraint becomes

\[
\frac{1}{w}c_t + i_t = w_t + (r^* + \delta)k_t + z_t,
\]
where \( z_t \) represents tradable-goods-denominated net-transfer payments from the government (a negative value implying a net tax). I assume that the government uses any revenue it collects to accumulate foreign assets on the international market, such that its budget constraint is

\[
\dot{b}_t^G + z_t = (r^* - g^*)b_t^G,
\]

(43)

where \( b_t^G \) represents official government reserves of foreign assets. Additionally allowing the government to collect revenues by issuing bonds solely to domestic savers would not change the analysis. I also assume the government is subject to a no-Ponzi-type borrowing constraint, parallel to that of the consumer’s in (5). Combining these two constraints yields a consolidated dynamic budget constraint for the economy

\[
\dot{b}_t^G = (r^* - g^*)b_t^G + w_t + (r^* + \delta)k_t - \frac{1}{\eta_t}c_t - i_t,
\]

(44)

which is nearly equivalent to the representative consumer’s budget constraint in (4), with the sole difference being that the foreign assets are now strictly under the control of the government. To emphasize why this is important, one can use the pricing conditions in (6) and (24) to rewrite the consolidated constraint as

\[
\dot{b}_t^G - (r^* - g^*)b_t^G = y_{Tt} - c_{Tt} - i_t.
\]

(45)

Notice that the left-hand side is the government’s choice variable, and the right-hand side can be expressed a function of \( c_{Tt} \) (see appendix). Therefore, by virtue of the restrictions on private capital flows and its control over the economy’s stock of foreign assets, the government is able to determine the economy’s trade balance and, by extension, direct the consumer’s chosen consumption path. In other words, the government is able to induce “forced saving” on the part of the consumer (via tax transfers invested in foreign assets) in order to achieve a welfare-maximizing path for private consumption after internalizing the extant productivity externality in the tradable goods sector that consumers and firms are too small to individually recognize. Note that if the economy enters a long-run steady state in which the government’s holdings of private assets are positive, then (43) implies that the government should distribute the flows of interest income back to domestic consumers (a feature not possible in the Korinek and Serven (2010) model).

It’s important to note that the discussion in this section implicitly continues to assume that the marginal product of domestic capital is equal to the exogenous international rate of interest, even though domestic savers/investors are disconnected from the foreign asset market. To justify the continued use of the interest rate parity condition in (21), I additionally assume that the government directs the stock of domestic capital by implicitly subsidizing private investment in order to achieve alignment between the marginal product of capital and international rates. This assumption seems especially innocuous in the case of China, where the so-called “Big Four” banks, which are among some of the largest in the world in terms of capital and interest income, are all state-controlled. Otherwise allowing the domestic rate of return on capital to differ from
the international rate would be dynamically inefficient for the country as a whole.

2.5.2 Wage Subsidy

The government’s first-best policy would be to target the source of the dynamic inefficiency: employment in the tradable sector. Suppose that firms producing tradable output receive an ad valorem subsidy of $s_t > 0$ from the government for wages paid to labor (i.e. the subsidy reduces the labor costs faced by firms in the tradable sector), and this subsidy is financed via transfers from consumers so as to be revenue-neutral. Then the following will be true:

**Proposition 1.** The government may implement any feasible consumption path by choosing the appropriate path for a wage subsidy $s_t$ to employment in the tradable sector.

**Proof.** The effect of introducing the subsidy would be to increase equilibrium wages from (23) to

$$W_t = \frac{1-\alpha}{1-s_t} \left( \frac{\alpha}{\mu + \delta} \right)^{\alpha/(1-\alpha)} A_t$$

and therefore alter the supply-side pricing condition from (24) to

$$p_t = \left( \frac{1}{1-s_t} \right) ^{1-\alpha}.$$  

This will in turn have the domino effect making the previous constants of $\bar{c}_N, \bar{c}, \gamma_2$ into implicit functions of $s_t$, which further implies that the real exchange rate $q_t$ will be a function of $s_t$ in equilibrium, and the share of labor in the tradable sector can be expressed as

$$L_{Tt} = 1 - \gamma_2(s_t) \cdot \frac{c_{Tt}}{A_t}.$$  

Since the representative consumer takes prices as given when choosing consumption, the intertemporal optimality condition becomes

$$\frac{\lambda}{q(s_t)} = \frac{\partial \bar{u}}{\partial c_t}.$$  

Relative to the laissez-faire condition in (32), the government can indirectly control the level of consumption (and tradable sector labor by extension) by choosing the appropriate path for $\{s_t\}_{0}^\infty$ in order to produce the optimal outcome for the economy.

Such a wage subsidy may be considered the first-best policy option because it directly targets the source of the inefficiency without introducing any additional intertemporal distortions. However, I assume that such a policy is not an option because it would violate WTO provisions that forbid direct subsidization of export-competing industries.
2.5.3 Consumption Tax or Production Subsidy

Alternatively, suppose the government levies a ad valorem tax $\tau_t > 0$ on the consumption of tradable goods by domestic consumers (or equivalently subsidizes every unit of tradable output produced by firms), which is revenue-neutral via transfer payments. Then the following is true:

**Proposition 2.** The government may implement any feasible path for $c_{Tt}$ by choosing the appropriate path for the consumption tax $\tau_t$.

**Proof.** Let $p_T' = p_T(1 + \tau_t)$ denote the tax-inclusive price faced by consumers. Then the real exchange rate may be expressed as $q_t' = \frac{p_T'}{p_{Ct}}$. Since the representative consumer takes prices as given when making his intertemporal consumption decisions, the first-order condition in (32) may now be expressed as $\lambda = \frac{\partial \hat{u}}{\partial c_t} \cdot q_t$, or

$$\lambda/(1 + \tau_t) = \frac{\partial \hat{u}}{\partial c_t} \cdot q_t$$

(50)

Since the government controls the left-hand side of this expression and the right-hand side can be expressed solely as a function of $c_{Tt}$, the government may effectively set the path of tradable consumption.

As discussed in the introduction and similar to the previous section, this approach may also be infeasible because of international agreements, such as the Agreement on Implementation of Article VI of the GATT 1994, which states

“A product is to be considered as being dumped...if the export price of the product exported...is less than the comparable price...for the like product when [consumed] in the exporting country.”

2.5.4 Export Subsidy and Import Tariffs

Alternatively, the government may influence imports and exports according to the following:

**Proposition 3.** The government may implement any feasible path for $c_{Tt}$ by introducing the appropriate path for subsidies $\epsilon_t^X$ on exports and tariffs $\epsilon_t^M$ on imports, where $\epsilon_t^X = \epsilon_t^M$.

**Proof.** A consumer wanting to purchase a unit of tradable consumption could either import it at a tariff-inclusive price of $p_T(1 + \epsilon_t^M)$ or purchase it domestically. However, no domestic seller would be willing to accept a price of less than $p_T(1 + \epsilon_t^X)$ since he could earn that by selling the unit to foreigners. Therefore, the combination of tariffs and subsidies is functionally
equivalent to levying a domestic tradable-consumption tax of $\tau_t = \epsilon_t^X = \epsilon_t^M$, such that the previous proposition applies.

Equivalently, the government could impose a tax on foreign borrowing and/or lending. However, all of these price-distorting approaches are considered to be in direct violation of WTO obligations, such that the government’s best viable option is the imposition of capital flow controls. Therefore, for the remainder of the paper I assume that the government implements a strict ban on private international borrowing and lending.

3 Calibration

In this section, I use cross-country panel data to empirically estimate a plausible range of values for the parameters in the technological growth equation in (16), which drives the possibility of welfare gains via mercantilist policy.

Due to the assumption of homogeneous technologies and factor income shares across the two sectors of production in the economy, we can combine the production functions in (8) and (9) to derive the following consolidated production function for aggregate output

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha},$$  \hspace{1cm} (51)

where the aggregate supply of labor $L_t$ is no longer fixed at unity. Then by using observations of an economy’s total aggregate output, capital stock, and labor force, one can calculate residual estimates of the economy’s overall technology level $A_t$.

I use data from the World Bank’s *World Development Indicators* (WDI) for the size of the economy’s labor force (adult population, ages 15-64) and the *Penn-World Tables 8.0* (PWT\(^{10}\)) for PPP-converted real GDP measures. I follow the development accounting literature’s standard of assuming a constant capital share of income of $\alpha = 1/3$. I also use country-level data on capital transformation shares of GDP from the PWT to estimate national aggregate capital stocks using a perpetual inventory method, assuming a depreciation rate of 6%. I then back out residual values for technology over the period 1960-2009 for 125 countries\(^{11}\) the distributions of which are presented in Figure 3. Observations for each time period are averages over five-year intervals in order to reduce transitory noise and focus on fundamentals-based long-term trends.

I define the technology frontier $A_t^*$ as the highest value of technology out of all countries in the sample in each period, which happens to correspond to the United States in every 5-year average. Individual countries’ (indexed by $i$) technology growth rates are calculated as the log

\(^{10}\)Feenstra, Inklaar, and Timmer (2013).

\(^{11}\)I follow Bernanke and Gurkaynak (2001) and Mankiw, Romer, and Weil (1992) in excluding economies whose GDPs rely predominately on oil production and economies with populations of less than one million.
changes from period $t - 1$ to $t$ and then subtracted from the growth rate of the frontier $g^*_t$ to create a measurement of a country’s “growth premium.” I use data on the share of the labor force engaged in industrial production (also from the WDI) as a proxy for tradable-sector employment, and the ratio of $A_{t-1}^r/A_{t-1}$ as a measure of distance from the frontier. Summary statistics for the data over the period 1980-2009\(^{12}\) are presented in Table 1. Using observations for these variables in the following model,

$$
\Delta \log A_{t,i} - g^*_t = \left( \gamma_0 + \gamma_1 \cdot L_{Tt-1,i} \right) \left( \frac{A_{t-1}^r}{A_{t-1}} - (1 + e^{h_t}) \right),
$$

I estimate parameter values for $\gamma_0$, $\gamma_1$, and $h$ using nonlinear least squares and “cluster-robust” standard errors allowing for correlation in the errors of each country. Results are presented in column (1) of Table 2. By construction, estimates of $h$ restricted to provide values for $f$ from the technology equation of motion in (16) that are greater than or equal to one, and the estimated value of $-33.84$ implies a common cross-country value of $f = 1$, implying perfect convergence to the technological frontier.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \log A_{t,i} - g^*_t$</td>
<td>-.002</td>
<td>.048</td>
<td>-.275</td>
<td>.291</td>
</tr>
<tr>
<td>$L_{Tt-1,i}$</td>
<td>.245</td>
<td>.088</td>
<td>.023</td>
<td>.471</td>
</tr>
<tr>
<td>$A_{t-1}^r/A_{t-1}$</td>
<td>7.08</td>
<td>8.36</td>
<td>1</td>
<td>56</td>
</tr>
<tr>
<td>$law_i$</td>
<td>56.9</td>
<td>27.5</td>
<td>3.5</td>
<td>100</td>
</tr>
<tr>
<td>$political_i$</td>
<td>67.7</td>
<td>22.0</td>
<td>10</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 1: Summary statistics.

Alternatively, each country may actually converge to an individual long-run ratio of the global frontier, similar to Howitt (2000). In considering cross-country differences in productivity, Hall

\(^{12}\)WDI data on shares of labor in industrial output begin in 1980.
and Jones (1999) suggest that these idiosyncrasies may be best explained by “social infrastructure,” such as institutional quality and government policy. In the current context, we must suppose that countries exhibit certain individual characteristics that are influential in the advancement of economic efficiency yet are extremely resistant to dramatic change over time. I therefore attempt to find reasonable country-specific values of $f$ using the following model

$$
\Delta \log A_{t,i} - g_t = \left( \gamma_0 + \gamma_1 \cdot L_{T_{t-1,i}} \right) \left( \frac{A_{t-1,i}}{A_{t-1}} - \min_t \left( \frac{A_t}{A_{it}} \right) + \left( e^{\psi_0 + \psi_1 \text{law}_i + \psi_2 \text{politic}_i} \frac{1}{1 + e^{\psi_0 + \psi_1 \text{law}_i + \psi_2 \text{politic}_i}} \right) \left( 1 - \min_t \left( \frac{A_t}{A_{it}} \right) \right) \right) \left( 1 + e^{\psi_0 + \psi_1 \text{law}_i + \psi_2 \text{politic}_i} \right) \right),
$$

(53)

I estimate the values of the parameters $\gamma_0, \gamma_1, \psi_0, \psi_1, \psi_2$ again using nonlinear least squares and cluster-robust standard errors. The additional covariates represent indices of a country’s “rule of law” and “political stability” from the Global Innovation Index (Cornell, INSEAD, and WIPO 2013). The functional form ensures that estimates of $f$ will fall between 1 and the minimum distance from the frontier actually observed for each country. Estimation results are presented in column (2) of Table 2. Implied estimates of the value of $\hat{f}_i$ for a small sample of countries is presented in Table 3. The correlation between the average of $\text{law}_i$ and $\text{politic}_i$ for each country and the associated estimate of $\hat{f}_i$ is $-0.6$, suggesting that stronger governance enables countries to more closely approach the global technological frontier in the long-run.

<table>
<thead>
<tr>
<th>Specification</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \log A_{t,i} - g_t$</td>
<td>$-0.0005$</td>
<td>$-0.0007$</td>
</tr>
<tr>
<td>$\hat{\gamma}_0$</td>
<td>$(0.005)$</td>
<td>$(0.007)$</td>
</tr>
<tr>
<td>$[-0.0014, 0.0004]$</td>
<td>$[-0.0021, 0.0007]$</td>
<td></td>
</tr>
<tr>
<td>$\hat{\gamma}_1$</td>
<td>$0.0050$</td>
<td>$0.0111^{**}$</td>
</tr>
<tr>
<td>$(0.0031)$</td>
<td>$(0.0045)$</td>
<td></td>
</tr>
<tr>
<td>$[-0.0011, 0.0111]$</td>
<td>$[0.0021, 0.0200]$</td>
<td></td>
</tr>
<tr>
<td>$\hat{h}$</td>
<td>$-33.84$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\hat{\psi}_0$</td>
<td>$-1015$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\hat{\psi}_1$</td>
<td>$3.265$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\hat{\psi}_2$</td>
<td>$11.41$</td>
<td>$-$</td>
</tr>
<tr>
<td>$R^2$</td>
<td>$0.0129$</td>
<td>$0.0352$</td>
</tr>
<tr>
<td>$n$</td>
<td>$363$</td>
<td>$352$</td>
</tr>
</tbody>
</table>

Table 2: Nonlinear least squares estimation results of regression of $A_t$ growth premium on distance from the technological frontier, and labor share in tradables production. Standard errors are reported in parenthesis, and 95% confidence intervals in brackets. ** signifies statistical significance at the 5% level.

Unfortunately, finding accurate estimates of the country-specific $f$ parameters is very difficult, since there is effectively only one observation per country, and using even the most recent values of the “rule of law” and “political stability” covariates may be a poor estimate of what the long-run characteristics of the country may be some 100 years in the future. Moreover, the estimation presented here ignores some obvious issues with endogeneity, as higher future incomes may enable countries to improve their ability to govern over time. At the very least, the estimates in Tables 2 and 3 give ballpark estimates of the reasonable values for the relevant parameters, and I consider the effects of varying these estimates over wide ranges in the next section.
As the largest individual holder of foreign reserves and a commonly cited example of “mercantilist” behavior, it is illustrative to look at the specific case of China. I calibrate the model to match the characteristics of China in the year 2000, shortly before its accession to the WTO and its subsequent decade of high growth and massive foreign asset accumulation. In this specific case of China in the year 2000, the estimates from Specification (2) in Table 2 suggest that a 10 percentage point increase in the share of labor employed in the tradable sector would increase the growth in technology by approximately 0.4 percentage points using the estimate of $f$ in Table 3 and up to 1.1 percentage points assuming full technological convergence, such that $f = 1$. Though a substantial effect, this is actually substantially less than the one-to-one estimated relationship between industrial labor shares and GDP growth in Rodrik (2008), which makes use of a number of additional covariates in a two-stage least squares regression.

<table>
<thead>
<tr>
<th>Country</th>
<th>$f_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>1</td>
</tr>
<tr>
<td>China</td>
<td>7.4</td>
</tr>
<tr>
<td>Germany</td>
<td>1</td>
</tr>
<tr>
<td>Brazil</td>
<td>3.6</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1</td>
</tr>
<tr>
<td>Greece</td>
<td>1.8</td>
</tr>
<tr>
<td>South Korea</td>
<td>1</td>
</tr>
<tr>
<td>Mali</td>
<td>17</td>
</tr>
<tr>
<td>Thailand</td>
<td>5.5</td>
</tr>
<tr>
<td>El Salvador</td>
<td>12.6</td>
</tr>
</tbody>
</table>

Table 3: Select estimated values of $f$ using equation (53).

These results provide a range of reasonable values for the calibration of the parameters in the model’s technology growth equation. For the benchmark results presented in the next section, I choose values of $\gamma_0$ and $\gamma_1$ that lie within their estimated 95% intervals and correspond to an initial rate of economic growth that matches the average rate experienced by China of 9% over the 2000-2007 period.

With regards to the other model parameters, I choose values for $r^*$, $\rho$, and $\theta$ from standard value ranges so as to equate the technological growth rate, $g^*$, to the average growth rate of the technology frontier over the 1980-2009 period of about 1.8%. Tradable and nontradable goods are assumed to be complementary, such that their elasticity of substitution is less than one.\textsuperscript{13} The initial level of technology at the frontier $A_0^*$ is set to the value of the frontier normalized by the value of $A_0$, which is the estimated level of technology in China in 2000. I also assume that the level of initial holdings of tradable assets has been normalized to zero and that the initial level of the physical capital stock equalizes the marginal product of capital with the international rate of interest based on the initial level of domestic technology. Since the standard errors for the parameters involved in the estimation of $f$ are extremely large, I start by assuming the most intuitive case of full convergence to the technological frontier ($f = 1$) for the benchmark model, before considering alternative values. The chosen values for all parameters are shown in

\textsuperscript{13}See Kravis and Lipsey (1987) and Mendoza (1995) for empirical evidence.
Table 4: Chosen parameter values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g^*$</td>
<td>0.018</td>
<td>Estimated</td>
</tr>
<tr>
<td>$r^*$</td>
<td>0.05</td>
<td>Calibrated to $g^*$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.02</td>
<td>Calibrated to $g^*$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>1.67</td>
<td>Calibrated to $g^*$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.3</td>
<td>Mendoza (1995)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.5</td>
<td>Mendoza (1995)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.33</td>
<td>Standard</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.06</td>
<td>Standard</td>
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<tr>
<td>$B_0$</td>
<td>0</td>
<td>Normalized</td>
</tr>
<tr>
<td>$A_0$</td>
<td>1</td>
<td>Normalized</td>
</tr>
<tr>
<td>$A^*_1$</td>
<td>10</td>
<td>US relative to China in 2000</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>0</td>
<td>China 9% growth</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.0197</td>
<td>China 9% growth</td>
</tr>
<tr>
<td>$f$</td>
<td>1</td>
<td>Estimated</td>
</tr>
</tbody>
</table>

4 Results

I now present the optimal paths of key variables for the economy both in a laissez-faire setting and in the presence of government intervention, which presumably takes the form of capital controls coupled with taxes/transfer payments. I first develop the intuition behind the results of a “benchmark” model using the chosen parameter values in the previous section and then consider the robustness of the results to alternative parameter values.

4.1 Benchmark

The solutions to the utility-maximizing time paths of the economies’ aggregate variables using the parameter values in Table 4 is presented in Figures 4 – 11, where solid lines represent the paths of variables in a laissez-faire economy and dashed lines represent an economy under capital controls. In order to fully understand the impact of government intervention, it’s informative to first consider the case of the evolution of the laissez-faire economy.

The motivations of the representative consumer under laissez-faire are very different from those of the government, since the laissez-faire consumer only anticipates the growth in technology, because of the assumption of rational expectations, but does not internalize the effect of his consumption decisions on technology growth. In other words, because the laissez-faire consumer expects income to continually grow over the foreseeable future, his singular desire is to borrow in the short-run against that future income stream in order to smooth his consumption profile over time. However, the economy can only import tradable goods, by definition, in
Figure 4: Paths of consumption under laissez-faire (solid) and capital controls (dashed) as % of total output.

Figure 5: Paths of net assets under laissez-faire (solid) and capital controls (dashed) as % of total output.

Figure 6: Paths of normalized technology under laissez-faire (solid) and capital controls (dashed).

Figure 7: Paths of investment under laissez-faire (solid) and capital controls (dashed) as % of total output.

Figure 8: Paths of trade balance under laissez-faire (solid) and capital controls (dashed) as % of total output.

Figure 9: Paths of current account under laissez-faire (solid) and capital controls (dashed) as % of total output.
order to achieve levels of consumption above its current productive capabilities. Therefore, the laissez-faire economy funnels all of its productive resources into the nontradable sector, since tradable and nontradable goods are complements and the desired level of tradable consumption can all be imported.

Thus, the laissez-faire economy initially exhibits very low growth, since little or no labor has been allocated to the tradable sector, which is the key stimulus for the advancement of technology. Eventually, however, the economy reaches the point at which it will no longer be able to continue borrowing all of its tradable consumption and still have sufficient resources available to avoid defaulting on its mounting debt. At such a time, the economy goes through a radical transformation as the allocations of labor between sectors are nearly fully reversed, such that production is almost entirely focused on tradable output. As seen in Figures 8 and 9, the economy switches from being a net importer to a net exporter, with all of the exports going to pay off the interest on its accumulated debt. This transformation also causes a large spike in growth as the labor being funneled into the tradable sector to provide the output for interest payments also contributes to a take-off in the absorption of new technology. The higher levels of technology originating from the tradable sector also spillover into the nontradable sector, such that the economy is still able to produce an adequate level of nontradable consumption despite a much lower allocation of labor to that sector. Overall, the laissez-faire consumer has no motivation to run a positive current account balance.

In contrast to the laissez-faire consumer, the government takes into account the impact of allocating labor to the tradable sector, as defined by (12), and accordingly leads the economy through a very different evolution. The central story is demonstrated by the paths of consumption in Figure 11. Initially, the levels of technology, and therefore total output, are equivalent between the two versions of the economy, but consumption is lower under capital controls than it is without. This is by virtue of the fact that the government is able to see the long-run dynamic effects of consumption on the implementation of advanced technologies and therefore effectively intervenes in order to initially allocate a higher level of labor to the production of tradable goods, as seen in Figure 10. The benefit of this lower consumption is apparent in the faster rate of convergence to the technological frontier relative to the laissez-faire economy.
shown in Figure 6. Even though the levels of technology both converge to the same level in the long-run, faster convergence to the frontier means the production of additional consumable output over the short-run that would not otherwise be available. Furthermore, the higher levels of technology require higher levels of capital to maintain international interest rate parity, such that investment is higher under capital controls than laissez-faire.

The most interesting insight provided by the model is the level of net foreign assets presented in Figure 5. As discussed above, it’s no surprise that the laissez-faire consumer runs up a massive level of debt in order to finance a higher level of initial consumption. However, the intuition behind the outcome under government intervention is more complicated. There are essentially two opposing forces at work in determining the optimal level of assets. First, there is the desire to smooth consumption over time, as given by the elasticity of intertemporal substitution (EIS) and represented in the model by $1/\theta$. Second, there is the desire to boost the rate of technological growth, as represented by $\gamma_1$. The reason these two motives work in opposition is because of the inverse relationship between tradable consumption and tradable-sector labor demonstrated in (26). On the one hand, the curvature of the instantaneous utility function implies that consumers maximize their welfare by borrowing from abroad in the short-term in order to finance higher initial consumption. On the other hand, the fact that allocating more labor to the tradable sector enables higher levels of tradable consumption in the future because of higher productivity implies that consumers can increase their welfare by enduring lower consumption in the short-run.

Therefore, the mercantilist motivation to accumulate assets in order to achieve faster growth is self-defeating, in the sense that the more the government is able to intervene to reallocate more labor to the tradable sector the more consumers want to borrow against the resultant higher future output in order to smooth out their consumption. For the chosen calibration of parameters presented in Table 4, it turns out that the consumption smoothing motive dominates the asset accumulation motive, such that the optimal level of net foreign assets under government intervention is negative. However, the quantity of assets borrowed under government intervention is less than under laissez-faire, meaning the economy does not need to export as much of its tradable output to cover interest payments. Ultimately, the short-term suppression of consumption pays off in terms of higher output levels during the period of convergence to the technological frontier, such that the welfare gain of government intervention is about 20% of permanent consumption.\[14\]

If we suppose that China began pursuing a mercantilist agenda through the use of capital controls and government intervention following its accession to the WTO, then how well does the model fit the Chinese empirical experience over the past decade? For the benchmark calibration of the model, the values of $\gamma_0$ and $\gamma_1$ were chosen so as to set the initial growth in output of the economy under capital controls in Figure 11 near 9% to mimic the empirical experience of China.

\[14\] The welfare premium is measured in terms of the percentage that the constant level of normalized real consumption under laissez-faire would need to be increased to equalize the lifetime discounted utility with that resulting from government intervention.
in the 2000s. This level of growth is fairly sensitive to the chosen value of $\gamma_1$, as shown in Figure 12. The average value of industrial labor share in China during 2000-2008 was about 24%. The model predicts a level closer to 55%, although this discrepancy may simply be a matter of the definitions, since the “tradable” sector includes more than just industry and manufacturing, as parts of the agricultural and even service industries could qualify as well. The average share of investment in GDP in China during the same period was 44%, which is a nearly perfect match to the prediction of the model. However, during this period China ran a current account surplus of nearly 10% of GDP, while the model does not allow for a positive current account during any period of time during the economy’s long transition of over 250 years to steady-state equilibrium.

It should be noted that the model predicts unrealistically high levels of borrowing of over 1000% of GDP under both laissez-faire and capital controls. This should not be surprising, however, given the simplifying assumptions of perfect foresight, infinitely lived consumers, and full convergence to a technological frontier that amounts to a guaranteed long-run ten-fold increase in per-capita output. The important result is that regardless of these simplifications, the consumer is not expected to carry a current account surplus. In the next section, therefore, I consider the sensitivity of this result to alternative calibrations, and the reasons why we should still accept the main implication that the mercantilist story does not seem well suited to standard dynamic macroeconomic growth models.
4.2 Robustness

As discussed in the previous section, the result that the economy does not amass a positive stock of foreign assets under government intervention hinges primarily upon the relative strength of two competing forces: the desire for consumption smoothing, as represented by the elasticity of intertemporal substitution \(1/\theta\), and the strength of the technological advance externality, as represented by the parameters in the technology equation of motion, primarily \(\gamma_1\). In this section, I discuss the robustness of the benchmark model’s results to alternative calibrations of these parameters.

4.2.1 Technology Externality

The degree to which the government is enticed to suppress short term tradable consumption depends on the parameterization of the equation of motion for technology presented in (12). It’s clear from (26) that lowering tradable consumption increases the share of the tradable-sector labor share, which increases tradable output, thus increasing the size of the trade balance. However, a positive net asset position would only be expected if gains from exploiting the externality are sufficiently large to overwhelm the consumption smoothing motive, and this depends primarily upon the values of \(\gamma_0\) and \(\gamma_1\).

![Figure 13: Steady-state levels of net foreign assets as a percentage of GDP under capital controls for different calibrations of \(\gamma_0\) and \(\gamma_1\). Dashed lines represented point estimates (PE) and 95% confidence interval upper (UB) and lower (LB) bounds from Section 3.](image-url)
Figure 13 shows the long-term steady-state levels of assets under capital controls for different calibrations of the technology growth equation. If one restricts consideration to only those values which correspond to initial growth rates of around 9% under government intervention, Figure 13 demonstrates that the long-run impact on foreign borrowing doesn’t differ substantially by parameter values. Note that even for extreme values well outside their estimated 95% confidence intervals the economy is not expected to accumulate a positive net level of external assets. Even considering empirically-unjustifiable very large values of $\gamma_1$, which maximize the externality to tradable labor, the main impact is to increase the speed of convergence to the technological frontier but not to overpower the consumption smoothing motive that drives the economy to borrow capital from abroad. In sum, so long as the elasticity of intertemporal substitution remains sufficiently low, alternative parameterizations of the technology growth equation don’t allow for accumulation of positive net levels of foreign assets.

On the other hand, the overall welfare gains from government intervention vary substantially with the value of $\gamma_1$, as shown in 14. This is because higher values of $\gamma_1$ strengthen the externality to the tradable-sector share of labor and cause faster convergence to the technological frontier. Faster convergence means a higher present discounted value of total output which can be allocated to consumption, thereby increasing total welfare.
4.2.2 Elasticity of Intertemporal Substitution

This section considers the ability of the model to match Chinese levels of external lending for alternative specifications of the elasticity of intertemporal substitution (as represented by $1/\theta$), since the equilibrium outcome of negative assets is driven primarily by the desire for consumption smoothing. Figure 15 presents the evolution of net foreign assets as a percentage of GDP when $\theta$ is decreased to 0.435, the temporal discount rate is correspondingly increased to 0.042 so as to maintain the relationship in (29) when $g^* = 0.018$ (the estimated value in Table 4), and the value of $\gamma_1$ is decreased to the point estimate of 0.0111 so as to continue matching the Chinese growth rate of 9%. This implies that the instantaneous utility function has less curvature and therefore consumers are more willing to substitute consumption across time. As a result, the willingness of consumers to suppress consumption in the short-run under government intervention is sufficient to make the trade balance positive and allow the economy to temporarily become a net lender instead of a net borrower. The elasticity of intertemporal substitution was chosen to match the level of the Chinese current account surplus as a percentage of GDP of around 10% during the 2000s, which is demonstrated by the results of the model in Figure 16.

While the positive level of net assets does better reflect the experience of China, the value of the elasticity of intertemporal substitution needed to justify is problematic. For example, a meta analysis by Havranek, et al (2013) of 169 published studies estimating the elasticity finds a mean estimate of 0.5 (or $\theta = 2$). In sum, the model suggests that it’s very difficult to use a dynamic macroeconomic consumption model to justify the observed levels of foreign reserves in China unless one accepts extreme parameter values that have little empirical justification.
4.2.3 Incomplete Technological Convergence

Finally, it must be noted that the model predicts very extreme levels of borrowing that can’t just be realistically justified. This is because a number of simplifications used in the construction of the model, including infinitely-lived consumers, no risk-premiums on debt, and complete convergence to the technological frontier. While the first two could be addressed by implementing additional complications in the forms of overlapping generations and interest rates that depend on the level of net debt, the assumption of full technological convergence to frontier can be softened in a very straightforward way by altering the value of \( f \). Indeed, the assumption that every country in the world will eventually converge to the same long-term level of technological productivity is perhaps too strong, and it may be more realistic to assume that there will always be a “leader” who maintains a lead over the rest of the world by virtue of some special characteristics, e.g. a economic system that encourages risk, an agglomeration of human capital in a specific location, etc.

Figures 17 and 18 present the results of the model using the parameter values in Table 4 in terms of foreign assets and welfare gains for a range of convergence ratios (e.g., \( f = 2 \) implies that the country never surpasses half of the constantly growing frontier’s level of productivity). Note that the spot estimate of \( f \approx 7 \) for China from Section 3 implies much more reasonable levels of foreign borrowing of around 100% or less of GDP. However, this level of convergence implies that the benefits of mercantilist policy are very short lived, such that the welfare gains would be almost negligible.
Figure 17: Paths of foreign asset borrowing as a percentage of GDP for different degrees of convergence to the technological frontier.

Figure 18: Welfare gains as a percentage of permanent laissez-faire consumption levels for different degrees of convergence to the technological frontier.
5 Conclusion

Overall, the model suggests that the proclivity for consumption-smoothing is the dominant characteristic of developing economies, such that the ability to exploit a productivity externality in the tradable sector does not prove sufficient to motivate a large net positive foreign asset position, such as that observed in China, under most reasonable parameter values. However, the degree of foreign borrowing is always less under government intervention, including the imposition of capital controls, and leads to sizable welfare gains for consumers. Therefore, although the model may not predict foreign reserve holdings at the magnitudes observed in some developing countries in recent years, it may serve as part of the explanation for foreign asset hoarding in conjunction with other motivating factors, such as precautionary saving.

Further work could be done in this area by introducing additional market rigidities so as to allow for an analysis with interesting real exchange rate dynamics and/or introducing an overlapping generations framework so as to more realistically model saving behavior and demonstrate more plausible levels of foreign asset holdings. Additionally, the model presented in this paper could be extended to a multiple-country setting, which might also allow for the possibility of analyzing the outcomes of “competitive” capital accumulation.
References


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Appendix

Intertemporal Budget Constraint

The constant co-state variable associated with total assets results in the following transversality condition

$$\lim_{t \to \infty} m_t e^{-(r^*-g^*)t} = 0,$$

which combined with the no-Ponzi condition in (5) further implies

$$\lim_{t \to \infty} k_t e^{-(r^*-g^*)t} = 0.$$

Normalizing by $A_t^*$ and utilizing the equilibrium pricing conditions in (6) and (7), we can rewrite the consumer’s dynamic budget constraint as

$$\dot{b}_t = (r^*-g^*)b_t + yT_t - cT_t - i_t.$$  (56)

Integrating over time and applying the no-Ponzi condition (5) yields the following intertemporal version of the budget constraint

$$\int_0^\infty (cT_t + i_t)e^{-(r^*-g^*)t}dt = \int_0^\infty yT_t e^{-(r^*-g^*)t}dt + b_0.$$  (57)

In order to fully solve for the paths of the model, we would like to express this constraint in terms of tradable consumption and technology. Normalizing the production of tradable goods from (8), substituting in the relevant capital intensity from (22), and utilizing the relationship
between tradable labor and consumption in (26) yields the following relationship

\[ y_{Tt} = k_{Tt}^{\alpha} (a_t L_{Tt})^{1-\alpha} \]
\[ y_{Tt} = \kappa_{Tt}^{\alpha} a_t^{-\alpha} L_{Tt} \]
\[ y_{Tt} = (\frac{\alpha}{\alpha - 1})^{\alpha/(1-\alpha)} (a_t - \gamma_2 c_{Tt}) \]
\[ y_{Tt} = \tilde{y}_{T}(a_t, c_{Tt}). \quad (58) \]

Next, the transition equation of physical capital in (3) and international interest parity implies that investment can be expressed as

\[ i_t = \dot{k}_t + (\delta + g^*) k_t, \]

such that

\[ \int_0^\infty i_t e^{-(r^*-g^*)t} dt = \int_0^\infty \dot{k}_t e^{-(r^*-g^*)t} dt + \int_0^\infty (\delta + g^*) k_t e^{-(r^*-g^*)t} dt \]
\[ = \int_0^\infty (\dot{k}_t - (r^*-g^*) k_t) e^{-(r^*-g^*)t} dt + \int_0^\infty (r^* + \delta) k_t e^{-(r^*-g^*)t} dt \]
\[ = \int_0^\infty (r^* + \delta) k_t e^{-(r^*-g^*)t} dt - k_0, \quad (59) \]

where the third equality makes use of the transversality condition in (55).

Finally, the capital intensities in (22) when combined with the resource constraints in (10) and (11) pin down the aggregate capital stock as shown:

\[ K_t = K_{Tt} + K_{Nt} \]
\[ K_t = \kappa_{Tt} L_{Tt} + \kappa_{Nt} L_{Nt} \]
\[ K_t = \kappa_{Tt} L_{Tt} + \kappa_{Nt}(1 - L_{Tt}) \]
\[ K_t = (\kappa_{Tt} - \kappa_{Nt}) L_{Tt} - \kappa_{Nt} \]
\[ K_t = \kappa_{Nt} \]
\[ k_t = (\frac{\alpha}{\alpha - 1})^{1/(1-\alpha)} a_t \]
\[ k_t = \tilde{k}(a_t). \quad (60) \]

Substituting the capital function in (60) into (59) and putting the resulting expression, along with the function of tradable output from (58), into (57) results in the version of the intertemporal budget constraint in (34).

**Model Solution - Laissez-Faire**

Using the first-order optimality condition in (32) and the intratemporal pricing conditions, we can implicitly define the fixed level of tradable consumption as a function of the co-state variable \( \lambda \)

\[ c_{Tt} = \left[ \phi A_0^{\alpha - \theta} c^{1/\theta} \right]^{1/\theta} \quad (61) \]
Normalizing the assumed functional form of the equation of motion for technology in (16) yields the following expression

\[ \dot{a}_t = (\gamma_0 + \gamma_1 L T_t)(1 - f \cdot a_t) \quad (62) \]

\[ \dot{a}_t = (\gamma_0 + \gamma_1 (1 - \gamma_2 c T a_t))(1 - f \cdot a_t) \quad (63) \]

Therefore, once the value of \( \lambda \) is known, then one can solve the differential equation in (63) and use the solution for \( a_t \) to ascertain the paths of all remaining variables in the model.

In order to find the correct value of \( \lambda \) that satisfies the intertemporal budget constraint in (34), I utilize the “time-elimination method” (See Mulligan and Sala-i-Martin (1991)) by considering the equation

\[ m'(a) = \frac{\partial m}{\partial t} \frac{\partial a}{\partial t} = \left( r^* - g^* + f(\gamma_0 + \gamma_1) - \gamma_1 \gamma_2 \frac{CT}{a_t^2} \right) m(a) + \bar{w} \lambda, \quad (64) \]

which defines a new differential equation in \( m \) without time. Assuming a specific value for \( \lambda \), all that is needed to solve this differential equation are boundary points for \( m \) and \( a \). By construction, \( a \) must equal \( 1/f \) in the long-run steady state, and the steady state value of \( m \) can be solved for by setting (31) equal to zero and substituting in the steady state value of \( 1/f \) for \( a \), then solving for \( m \). The differential equation in (64) can then be solved backwards from the steady state point at \( a = 1/f \) to the point at which \( a = a_0 \), and note the value of \( m0 \) implied by the solution. Since the initial points of \{\( a_0, m_0 \}\} are given, one can solve for the correct value of \( \lambda \) by finding the unique value that gives a solution to (64) that corresponds with the given value of \( m_0 \).

**Model Solution - Social Planner**

Using the equation of motion for technology defined in (63), the equation of motion for the co-state variable related to technology in (39) can be expressed as

\[ \dot{\mu}_t = \left[ r^* - g^* + f(\gamma_0 + \gamma_1) - \gamma_1 \gamma_2 \frac{CT}{a_t^2} \right] \mu_t - \bar{w} \lambda, \quad (65) \]

and the expression for marginal utility in (38) can be solved for tradable consumption, yielding

\[ c_{T_t} = \left[ (A_0^e \hat{c})^{\theta - 1} (\gamma_1 \gamma_2 (1 - \frac{1}{a_t} + \frac{\bar{q} \lambda}{\mu_t + \bar{q} \lambda}) \right]^{-1/\theta}. \quad (66) \]

Therefore, by substituting (66) into (65), one can derive an expression for the evolution of \( \mu_t \)

\[ \dot{\mu}_t = \left[ r^* - g^* + f(\gamma_0 + \gamma_1) - \gamma_1 \gamma_2 (1 - \frac{1}{a_t} - f) \mu_t + \bar{q} \lambda \right]^{-1/\theta} \mu_t - \bar{w} \lambda, \quad (67) \]

which is now a function of only \( \mu_t, a_t, \) and \( \lambda \). Likewise, one can substitute the expression for tradable consumption in (66) into the equations of motion for assets in (36) and technology in
to express those equations in terms of $\mu_t$.

Therefore, the evolution of the social planner’s solution can be characterized by the system of three differential equations outlined below

\[
\dot{\mu}_t = \tilde{\mu}(a_t, \mu_t; \lambda) \\
\dot{a}_t = \tilde{a}(a_t, \mu_t; \lambda) \\
\dot{m}_t = \tilde{m}(m_t, a_t, \mu_t; \lambda)
\]

Since the first two equations don’t depend on the level of assets $m_t$, one can use the same time-elimination method outlined in the previous section to define a new differential equation in $\mu'(a)$. Then by choosing a specific value for $\lambda$, setting (67) equal to zero, and substituting the steady value of $1/f$ in for $a$, one can solve for the steady state value of $\mu$. Then it’s possible to solve the differential equation of $\mu'(a)$ for the path of $\mu$ as a function of $a$. Once the path of $\mu$ is so defined, the time-elimination process can be repeated, this time by defining a new differential equation in $m'(a)$ utilizing the solution for $\mu(a)$ from the previous step. Then the process is identical to that of the laissez-faire section: solve for the steady state value of $m$, use the point defined by this steady state value and $a = 1/f$ to solve for $m$ as a function of $a$, and identify the implied initial value of assets $m_0$. If this value is not equal to the given value of $m_0$, iterate the process for different values of $\lambda$ until the given value of initial assets is found. Once one knows the proper value of $\lambda$ that satisfies all of the boundary conditions, then it is straightforward to solve for the paths of all other variables in the economy.