Rationalizability and the Theory of Large Games

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Environments with a continuum of agents have been the subject of increasing investigation in several areas of economic theory, going beyond non-cooperative (and cooperative) game theory to more applied formulations arising in industrial organization, macroeconomic dynamics and even political economy. A continuum of potentially heterogeneous agents pervasive in the economic literature is now studied in a context where interdependence is made explicit and rendered analytically tractable. In the "theory of large games," a player's payoff depends (of course) on his or her actions, but rather than those of "each and everyone else," it depends on a statistical summary of their actions: unlike finite games, each agent is strategically-negligible. The *other* is no longer a player or a fully delineated group of players, but rather the society or the collectivity that is the formalized subject of the game. A player's actions then are influenced by how he or she conceives and conceptualizes the society of which he or she is a part, rather than a specific individual. My thesis is a contribution to this theory, and it takes the form of three essays.

The first substantive essay of the dissertation examines the Bernheim-Pearce *rationalizability* notion in the theory of large games. The second goes beyond Schmeidler's formalization of a *societal response* as an average of individual responses, when a linear structure on the action set is available, or as a distribution of individual responses, when it is not; and examines situations where only more general transformations may be available. The third essay examines a formulation that takes an intermediate position between the *anonymous* and *non-anonymous* polar aspects of the theory, and through the introduction of a space of player characteristics, in addition to a space of players' names, broadens its reach and scope.

[The first essay is to be my job-market paper, but by the time of the completion of the thesis, I am hoping to bring together the richer structures of the second and third essays to bear on it, and thereby offer a coherent and uni-directed contribution to the theory of large games.]

1 Point-Rationalizability in Large Games

In this paper, I characterize *point-rationalizability* in large non-anonymous games with three different formulations of societal interdependence. More specifically, societal interdependence is formulated as distributions of individual responses, integrations of actions and averages of the transformed actions. Given the introspection and "mentalizing" that the rationalizability notions presuppose, a large motivation behind the work is to examine their viability in situations where the terms *rationality* and *full information* can be given a more parsimonious, and thereby more analytically viable, expression.

2 Transformed Societal Responses in Large Games

In this paper, I contribute to the existence theory of pure strategy equilibria in large games with transformed summary statistics. I also generalize, for the same class of games, the existence result for undominated pure strategy Nash equilibria despite the fact that the set of pure strategy Nash equilibria may fail to be weakly compact.

3 Multiplicity of Characteristics in Large Games

In this paper, we present a comprehensive analysis of large non-anonymous games in which every agent has a name as well as a type. We show the existence of pure strategy Nash equilibria under alternative cardinality assumptions on the common set of actions and on the space of types. The space of names is formulated as a *saturated* probability space while the space of types is a complete separable metric space. Also, we show by a counterexample that the existence result fails when the Lebesgue interval is used as the space of names.(This is part of a larger ongoing project with Professors Khan, Rath and Sun)