Banks’ Financial Structure and Business Cycles

Fabian Valencia∗†
The Johns Hopkins University
fvalencia@jhu.edu

January 21, 2006

Abstract

A puzzling fact in macroeconomics is that small and transitory shocks seem to be followed by large and persistent fluctuations in macroeconomic variables. Theoretical models explaining this empirical fact focus on the borrower but most of them neglect to model the bank. This paper proposes a framework in which changes in the financial condition of banks can have persistent aggregate consequences. The bank is modeled as a firm in a dynamic framework, in a world of imperfect capital markets. An important feature of the model is the existence of an optimal financial structure defined as the level of net cash position relative to size of the loans portfolio. In the model, an i.i.d shock may generate a persistent effect on output because the shock may cause a significant weakening of the bank, while the latter restores its financial position only through retained earnings. A similar persistent effect arises in the event of a permanent increase in productivity, in such scenario, the bank adjusts lending gradually rather than instantaneously— as in a frictionless model— until the new target is reached. Finally, the bank’s response to shocks is not symmetric. A negative shock has much stronger effects on lending than a positive one.

JEL classification: C61, E32, E44
Keywords: Banking, Financial Structure, Credit Crunch, Dynamic Programming.

∗I am grateful to Laurence Ball and Christopher Carroll for valuable criticism. Comments from Allen Berger, Bradford Case, Eugenio Cerutti, Carl Christ, Katrina Kosec, Andrew Levin, Ricardo Llaudes, Thomas Lubik, Louis Maccini, Fabio Natalucci, Egon Zakajsek, and participants of the seminar series at the Johns Hopkins University and the Board of Governors of the Federal Reserve System are gratefully acknowledged. Remaining errors are my own.
†Department of Economics, 3400 N. Charles Street, Baltimore, MD 21218.
1 Introduction

In a world of perfect capital markets, the Modigliani and Miller (1958) theorem implies that the capital structure of firms is uninformative for real economic decisions. However, this conclusion may be invalid when information about the default risk of borrowers cannot be costlessly acquired by lenders (Blinder and Stiglitz (1983)). Bernanke and Gertler (1995) and others have argued that these credit market imperfections lie at the heart of the explanation of the puzzling fact that small monetary policy innovations seem to be followed by large and persistent fluctuations in macroeconomic variables. They show that allowing for a variable premium on external finance of borrowers leads to a better understanding of several empirical results about the reaction of the real economy to monetary shocks. The mechanism through which monetary shocks get enhanced and propagated through time, due to credit market imperfections, was called the credit channel of monetary policy.\footnote{Bernanke, Gertler, and Gilchrist (1996) provide empirical evidence in support of the existence of the credit channel using a panel of small and large U.S. manufacturing firms. See also Gertler (1988) and Gorton and Winton (2002) for a comprehensive survey of the literature.}

Empirical evidence suggesting the failure of the Modigliani and Miller (1958) theorem was followed by substantial theoretical work with structural models explaining how the dynamic interaction between borrowers’ access to credit and the value of collateral enhance and propagate shocks to the economy. Important contributions along these lines include Bernanke and Gertler (1989, 1990), Kiyotaki and Moore (1997), Gertler (1992), Greenwald and Stiglitz (1993), Fuerst (1995), Carlstrom and Fuerst (1997, 2001), and Bernanke, Gertler, and Gilchrist (1999);\footnote{Cecchetti (1995) provides a detailed differentiation between the different components of the credit channel.} however, Chari, Kehoe, and McGrattan (2005) argue that such frictions are not important once time-variation in factors such as labor taxes and productivity are taken into account. The debate about the specific mechanisms that enhance and propagate economic shocks is still very active; nevertheless, there is empirical evidence in support of the economic importance of credit market frictions. Levin, Natalucci, and Zakrajsek (2004), for instance, estimate the magnitude and cyclical behavior of financial frictions using publicly traded debt of a sample of U.S. firms and find that they are significant and economically important.

Shifting the discussion to banks, it seems plausible to think that the same questions apply to them because banks are firms as well. In that case, if shocks get enhanced and propagated because some firms face credit frictions, then banks may also cause similar effects should they be subject to borrowing constraints as well.\footnote{For an extension of this literature to study the role of borrowing constraints in small open economies and financial crises see Aghion, Bacchetta, and Banerjee (2004), Caballero and Krishnamurthy (2001), Paasche (2001), Burnside, Eichenbaum, and Rebelo (2000), Mendoza (2004) and Gertler, Gilchrist, and Natalucci (2001).}
Furthermore, there is strong empirical evidence in support of the relevance of banks’ financial structure for lending decisions: Bernanke and Lown (1991), Peek and Rosengren (1994, 1995), and Hancock and Wilcox (1994) find a statistically significant relationship between bank capital and lending. Kashyap and Stein (1995) find a differentiated response of small and large banks to monetary policy shocks while Kashyap and Stein (2000) find that the impact of monetary policy on lending is stronger for banks with less liquid balance sheets. Moreover, Bernanke (1983) and others\textsuperscript{4} have stressed the detrimental economic effects of financial distress in the banking industry. Despite the evidence, it is only in the last few years that the interest on modeling the dynamic interactions between banks’ financial structure, their lending decisions, and the aggregate economy started to grow.

This paper contributes to the literature by developing a tractable model to study the macroeconomic implications of variations in banks’ financial conditions. It explores an additional propagation mechanism that has been neglected by the theoretical literature. The goal is to understand bank behavior in a world of imperfect markets by modeling the bank as a firm in a dynamic framework with explicit micro foundations.

Because the objective is to extract macroeconomic-cycle implications, the infinite-horizon feature of this model and the dynamic inferences that can be derived constitute its main advantage over existing bank models, which in most cases are static or have very short time horizons. Bernanke and Gertler (1987) construct a general equilibrium model\textsuperscript{5} and—by characterizing the solutions from the first order conditions—they establish that factors such as the adequacy of bank capital, the riskiness of bank investments, and the costs of bank monitoring have an impact on economic activity. Holmström and Tirole (1997) conclude that both bank capital and entrepreneurial net worth are important for the equilibrium returns on financial markets and bank loans. Stein (1998) develops a model in which information problems make it difficult for banks to raise funds with instruments other than insured deposits; thus, any constraint on the amount of deposits imposes restrictions on lending as well. Diamond and Rajan (2001, 2003a, 2003b) provide a framework to study the role of real liquidity in the transmission of monetary policy, financial crises and banking failure contagion. Other attempts to study the role of banks in the overall economy have included dynamic general equilibrium models such as Chen (2001), Meh and Moran (2004), and Christiano, Motto, and Rostagno (2004). This study does not offer the complete perspective of a general equilibrium model, but it has the advantage of a richer bank structure than the one offered in the previous examples. Moreover, a non-linear solution is easier to obtain which generates in-


\textsuperscript{5}Although Bernanke and Gertler (1987) is set up as a dynamic general equilibrium model, the authors limit the analysis to the first order conditions of the model.
interesting dynamics that would be missed in a linearized solution—the usual solution method to dynamic general equilibrium models.\textsuperscript{6}

The model is solved numerically using dynamic programming techniques. The borrower-bank relationship is modeled with an explicit loan contract subject to information problems. The borrowers’ balance sheet channel, as a propagation mechanism, is shut down in this model. Capital markets are imperfect, the cost of borrowing increases when the bank is in poor health, and raising equity is infinitely costly. The bank does not fund itself entirely with equity because stockholders are impatient (they discount future dividends with a time preference rate that is larger than the average risk-free rate). An implication of the previous assumptions is the existence of an optimal financial structure, defined as the ratio of net cash position\textsuperscript{7} to outstanding loans. In response to a negative shock, it is optimal for the bank to reduce dividends in order to restore its financial condition. If the deterioration is large enough, a credit crunch can arise, which lasts until the bank reaches its optimal financial structure. These persistent effects on lending and output can arise even when the original impulse is a transitory, one-period i.i.d. (interest rate or productivity) shock. In the event of a permanent increase in productivity, the adjustment in lending is gradual, contrary to the instantaneous correction that occurs in a frictionless case. A decline in revenues or an increase in interest rates deteriorates the net cash position and a lower net cash position increases the cost of borrowing. Hence, the target net cash position works as a cushion against unexpected, adverse conditions that would otherwise hinder bank lending. The model also suggests that this cushion increases—relative to the amount of outstanding loans—whenever the degree of uncertainty of loan revenues increases. Finally, in the model, the bank’s response to shocks is not symmetric. A credit crunch can arise when the cash position of the bank deteriorates. However, lending does not increase when the cash position improves in the same magnitude, in absolute value.

The results obtained from numerical simulations of the model suggest that the financial health of the banking system may contribute significantly to the propagation of economic shocks, especially negative ones. These conclusions are important for economic policy because they suggest that its effectiveness may be affected when banks face financial distress. Consequently, government ‘bailouts’ of insolvent banks may be a reasonable response during episodes of a severe weakening of the banking system.

This paper is organized as follows: the next section provides a general description

\textsuperscript{6}A model that resembles the one presented here is Van Den Heuvel (2002), which examines the role of regulatory constraints on bank behavior. This paper however, does not rely on bank capital regulation, instead, financial frictions are market-driven. Furthermore, the bank-borrower relationship is modeled explicitly in this paper.

\textsuperscript{7}In this model, net cash position refers to the difference between revenues and payments to creditors.
of the model, section 3 models the borrower-bank relationship, section 4 elaborates on the bank’s optimization problem and the solutions, sections 5 and 6 provide some quantitative experiments, and section 7 concludes.

2 The model

The model is developed in a partial equilibrium framework, with the bank facing an exogenously determined stochastic and i.i.d. risk-free rate. The model has two main participants: the bank and borrowers. It is assumed that the bank is a monopoly that develops economies of scale in ex-ante evaluating and ex-post monitoring entrepreneurs’ investment projects, thus justifying in this way the existence of financial intermediation. The bank faces borrowing costs given by the risk-free rate plus a wedge that varies with the financial condition of the bank. This separation facilitates intuition. One possible interpretation is that the supply of deposits is infinitely elastic at the risk-free rate because of deposit insurance, in which case the wedge corresponds to deposit insurance premia.

There are no ex-ante information problems. Before decisions are made, the distributions of unrealized shocks are common knowledge. The bank then makes "take-it-or-leave-it" offers to entrepreneurs, who in turn decide to accept or reject the offer. Entrepreneurs who choose to accept the offer sign a two-period loan contract which specifies the amount of resources to be delivered at the moment of signing the agreement and the amount to be repaid at maturity. The contract implies a fixed interest rate during the lifetime of the credit arrangement. The realization of uncertainty takes place after decisions have been made.

As far as entrepreneurs are concerned, the economy is populated with an uncountable number of risk-neutral borrowers who fund projects with their own resources and borrowings from the banking sector. The bank is their only source of external finance. Once an entrepreneur decides to accept the bank’s offer, he invests in a project whose realized output is his private information. If an entrepreneur successfully accomplishes the project, he pays the bank the agreed amount. Otherwise, the bank pays monitoring costs to observe the output of the project and seizes the entire production.
3 The loan contract

The loan agreement takes the form of a standard debt contract with maximum equity participation, as described in Gale and Hellwig (1985), and similar to the one in Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999). The demand side of credit comes from a continuum of entrepreneurs whose individual size is negligible relative to that of the bank. Entrepreneurs live for only two periods and receive at birth a common endowment of resources, which for simplicity is normalized to 1. The bank makes "take-it-or-leave-it" offers to entrepreneurs that include an amount \( l_t \) and an interest rate \( R_t \). If a borrower accepts the offer, he starts a two-period investment project and expects a return given by

\[
E_t \pi(R_t, l_t, \alpha_{t+2}, \Phi_{t+2})
\]

per dollar, with \( \alpha \) and \( \Phi \) denoting i.i.d. stochastic shocks to be defined momentarily-unknown when the bank makes the offer. If a borrower rejects the offer, he can deposit the endowment at the bank and expect a return

\[
(1 + \rho)^2 \cdot (1 + \rho_{t+2})
\]

which corresponds to the risk-free rate compounded for two periods. The latter follows from assuming that the risk-free rate is i.i.d. and unknown at the moment of making decisions. The problem for an entrepreneur is given by (1).

\[
\max \{ \text{accept, reject} \} \left\{ E_t \left[ \pi(R_t, l_t, \alpha_{t+2}, \Phi_{t+2}) \right] , (1 + \rho)^2 \right\}
\]

The next step is to show how \( \pi(R_t, l_t, \alpha_{t+2}, \Phi_{t+2}) \) is determined. It is assumed that entrepreneurs have access to a common production technology which uses only capital as an input. For simplicity it is assumed that entrepreneurs derive utility only from consumption in the second period of their lives. An entrepreneur’s production \( y \) in \( t + 2 \) is given by (2).

\[
y_{t+2} = \alpha_{t+2}\Phi_{t+2}k_{t+2}
\]
ity shocks of idiosyncratic and aggregate nature denoted by $\alpha$ and $\Phi$ respectively.\textsuperscript{11} Productivity shocks are assumed to be i.i.d, lognormally\textsuperscript{12} distributed over a non-negative support, with $E_\Phi[\Phi_t] = 1$ and $E_\alpha[\alpha_t] > (1 + \overline{\rho})^2$ with $\overline{\rho} > 0$ for all $t$. Capital depreciates fully at the end of an entrepreneur’s life. It is assumed that there exists a minimum scale for entrepreneurs’ projects, requiring a level of investment that is strictly larger than the endowment. Hence, borrowing from the financial intermediary to start the project becomes necessary. Entrepreneurs use the endowment to finance the purchase of capital at a unit price, together with loan proceeds $l_t$ obtained from the bank, bringing the total amount of capital acquired to $k_{t+2} = l_t + 1$. Conditionality from the bank requires entrepreneurs to use the entire endowment, should they choose to invest in the project, which implies maximum equity participation. Then, $y_{t+2} = \alpha_{t+2}\Phi_{t+2}(l_t + 1)$. At the moment of signing the contract, none of the productivity shocks are known (neither to the borrower nor to the lender). Their respective distributions and the size of the endowment, however, are common knowledge.

Following Townsend (1979), Gale and Hellwig (1985), and Williamson (1987), with costly state verification, once idiosyncratic productivity $\alpha$ is realized, it is assumed to remain the private information of entrepreneurs. The bank may observe the productivity realization of an entrepreneur only after paying monitoring costs $1 \geq u > 0$,\textsuperscript{13} expressed as a fraction of a borrower’s project value. An entrepreneur’s return is equal to the outcome of the investment minus the amount (principal plus interest) owed to the bank: $R_t l_t$, as indicated in (3).

$$\pi(R_t, l_t, \alpha_{t+2}, \Phi_{t+2}) = (l_t + 1)\alpha_{t+2}\Phi_{t+2} - R_t l_t$$ (3)

Since an entrepreneur’s decision is limited to accepting or rejecting the bank’s offer and $\alpha$ is assumed to be continuously distributed over a non-negative support, there exists a cutoff value $\underline{\alpha} \in [0, \infty)$ such that the return to an entrepreneur is equal to zero. If an entrepreneur’s realization of productivity turns out to be above this cutoff value, he consumes the surplus after honoring the loan contract; otherwise, he goes bankrupt. If he declares bankruptcy, the bank pays monitoring costs to observe the outcome of the failed project and seizes the existing output. The cut-off value of productivity $\underline{\alpha}$ is defined by

\textsuperscript{11}During the lifetime of each entrepreneur, there is only one realization of productivity that is relevant for his project.

\textsuperscript{12}The assumption of lognormally distributed productivity guarantees a non-rationing outcome (See Bernanke, Gertler, and Gilchrist (1999)).

\textsuperscript{13}With the costly state verification setup, the monitoring costs can also be interpreted as bankruptcy costs.
\[(l_t + 1)\alpha_{t+2} \Phi_{t+2} - R_tl_t = 0 \quad (4)\]
\[\frac{\alpha_t(R_t, l_t, \Phi_{t+2})}{(l_t+1)\Phi_{t+2}} = \frac{R_t l_t}{(l_t+1)\Phi_{t+2}} \quad (5)\]

Since idiosyncratic productivity is drawn from the same distribution for all entrepreneurs and there is no difference in the size of the endowment, all entrepreneurs are ex-ante identical and, therefore, all are offered the same loan contract. Now, if \(\alpha_{t+2} < \alpha_t(R_t, l_t, \Phi_{t+2})\), then the project is seized by the bank. However, by imposing limited liability, a borrower’s return cannot be lower than zero. This condition implies

\[
\pi(R_t, l_t, \alpha_{t+2}, \Phi_{t+2}) = \begin{cases} 
(l_t + 1)\alpha_{t+2} \Phi_{t+2} - R_t l_t & \text{if } \alpha_{t+2} \geq \alpha_t(R_t, l_t, \Phi_{t+2}) \\
0 & \text{if } \alpha_{t+2} < \alpha_t(R_t, l_t, \Phi_{t+2}) \end{cases} \quad (6)
\]

At the moment the contract is signed, the expected return to a borrower is denoted by \(E_t[\pi(R_t, l_t, \alpha_{t+2}, \Phi_{t+2})]\). Participation of any entrepreneur is subject to a rationality constraint that requires the rate of return from the project to be at least as good as his opportunity cost. The bank’s decisions take into account this restriction at the moment of making the offer, which formally is given by (7).

\[
E_t[\pi(R_t, l_t, \alpha_{t+2}, \Phi_{t+2})] \geq (1 + \rho)^2 \quad (7)
\]

With the "take-it-or-leave-it" assumption, and the existence of an interior solution, constraint (7) holds with equality at the levels of lending and interest rate that solve the bank’s problem (presented in the next section). If not, the bank could always charge a slightly higher interest rate and would still have a borrower accepting the offer. Thus, the equation \(E_t[\pi(R_t, l_t, \alpha_{t+2}, \Phi_{t+2})] = (1 + \rho)^2\) implicitly defines the interest rate schedule \(R(l)\) that the monopolistic bank charges.

\[\text{Expected returns are then given by}\]
\[
E_t\left[\pi(R_t, l_t, \alpha_{t+2}, \Phi_{t+2})\right] = \int_0^{\infty} \int_0^{\Phi_{t+2}} \alpha_{t+2}(l_t + 1)\Phi_{t+2} dF_t - \int_0^{\Phi_{t+2}} \left[1 - F_t(\alpha_t(R_t, l_t, \Phi_{t+2}))\right] R_t l_t \text{d}F_{\Phi}
\]

\[\text{This is a standard result in principal-agent problems. See Mas-Colell and Green (1995).}\]

\[\text{Under the assumptions of lognormally distributed shocks, the "take-it-or-leave-it" offer and } E_t[\alpha_t] > (1 + \rho)^2, \text{ the lending rate, } R(l) \text{ is convex in leverage and satisfies } \lim_{l_t \to \infty} R(l) = \lim_{l_t \to 0} R(l) = \infty\]
in order to leave any entrepreneur indifferent between investing in a project or not (see Figure 1). It is assumed that in such a case, an entrepreneur prefers to invest. Because the endowment has been normalized to 1, \( l_t \) can be interpreted as leverage (i.e. the amount borrowed per unit of endowment).

Recall that in the event of bankruptcy of an entrepreneur, the bank seizes the entire project and takes the residual value of output after paying monitoring costs \( u \), which are expressed as a fraction of the total value of production. If an entrepreneur succeeds, the bank collects loan revenues as specified by the loan contract. Using the interest rate function previously derived, revenues for the bank are then given by (8).

\[
G(l_t, \Phi_{t+2}) = \left\{ \begin{array}{ll}
\frac{R(l_t)l_t}{(1-u)(l_t+1)\Phi_{t+2}\alpha_{t+2}} & \text{if } \alpha_{t+2} \geq \frac{\alpha}{\alpha_t} \left( \frac{R(l_t)l_t, \Phi_{t+2}}{} \right) \\
\frac{R(l_t)l_t}{(1-u)(l_t+1)\Phi_{t+2}\alpha_{t+2}} & \text{if } \alpha_{t+2} < \frac{\alpha}{\alpha_t} \left( \frac{R(l_t)l_t, \Phi_{t+2}}{} \right) 
\end{array} \right.
\] (8)

Following Gale and Hellwig (1985), the contract is incentive-compatible because, in the non-default state, the payment to the lender is fixed and strictly larger than the value of the project in the default state.\(^{17}\) The assumption of a continuum of borrowers and the law of large numbers imply that the bank can perfectly diversify the idiosyncratic component of risk. Ex-post aggregate revenues for the bank depend on leverage and the realization of aggregate productivity. Denoting the mean of a variable across borrowers with \( M[\cdot] \), ex-post revenues for the bank are given by (9).\(^{18}\)

\[
G(l_t, \Phi_{t+2}) = M\left[ g(l_t, \alpha_{t+2}, \Phi_{t+2}) \right]
\] (9)

Notice that with a fixed endowment and i.i.d shocks, expected revenues are time-invariant and therefore accelerator effects stemming from the borrower are shut down. The marginal revenue function for the bank, \( G^l(l_t, \Phi_{t+2}) \), is plotted in Figure 2 and the derivation is shown in the appendix.

\(^{17}\) Similar to Bernanke, Gertler, and Gilchrist (1999), an additional assumption is needed to rule out the possibility of having the bank making unbounded profits on a contract where an entrepreneur has a probability of default equal to 1. In this model, a sufficient assumption for that is \( (1-u)E[\alpha] < (1+\rho)^2 \), since \( \lim_{\alpha \to \infty} g(\alpha, l_t, \alpha_{t+2}, \Phi_{t+2}) = (1-u)E[\alpha_{t+2}] \).

\(^{18}\) An equivalent representation for the revenue function is

\[
G(l_t, \Phi_{t+2}) = [1 - F_\alpha(\alpha_t(l_t, l_t, \Phi_{t+2} \Phi_{t+2}))] R(l_t)l_t + (1-u) \left\{ \int_0^{\alpha_t(l_t, l_t, \Phi_{t+2})} (l_t + 1)\alpha_{t+2}\Phi_{t+2}dF_\alpha \right\}
\]
4 The bank’s optimization problem

Bank managers simultaneously choose amounts of lending \((l_t)\), dividends \((d_t)\), and deposits \((c_t)\) in order to maximize the market value of the bank, given by (10).

\[
\max_{\{l_t, d_t, c_t\}} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} d_s
\]  

(10)

Stockholders are assumed to be risk-neutral and impatient in the sense that they discount the future stream of dividends at a rate higher than the average risk-free rate; that is, \(\frac{1}{\beta} > 1 + \rho\).\(^{19}\) Lending amounts are expressed in aggregate levels. As far as transitions between periods are concerned, decisions are made in the middle of each period and uncertainty is revealed between periods. Figure 3 shows the sequence of events for the bank.

In order to understand the transition between periods, it is useful to start by defining the conditions that describe the state of the bank right after decisions for the previous period have been made; that is, \(t - 1\). At that moment, the state is described by three variables: loans originated in period \(t - 2\), \(l_{t-2}\), credit extended in period \(t - 1\), \(l_{t-1}\), and outstanding deposits \(c_{t-1}\).

Between periods \(t - 1\) and \(t\), uncertainty is realized and the values of the stochastic shocks \(\Phi_t\) and \(\rho_t\) become known. The realization of uncertainty determines loan revenues \(G(l_{t-2}, \Phi_t)\) and obligations to depositors \((1 + \rho_t)c_{t-1}\). Where \(G(l_{t-2}, \Phi_t)\) corresponds to the previously derived revenue function. The amount of loans that have not matured yet, \(l_{t-1}\), is kept as outstanding loans’ portfolio. It is assumed that before the bank reaches the middle of the period, it faces frictional costs—to be explained momentarily—denoted by \(f_t\). The net cash position of the bank is the result of loan collections minus payment to depositors and frictional costs\(^{20}\)

\[
h_t = G(l_{t-2}, \Phi_t) - (1 + \rho_t)c_{t-1} - f_t
\]  

(11)

The bank reaches the middle of period \(t\) with two state variables: the net cash position \(h_t\) and outstanding loans \(l_{t-1}\). For given values of the state, the bank

\(^{19}\)The economic motivation for this assumption is that stockholders want to avoid potential agency problems with managers. Once the impatience assumption is introduced, the distinction between shareholders and managers is immaterial. This assumption guarantees that the bank does not fund itself entirely with internal funds. An alternative approach to generate a similar outcome is to introduce taxes.

\(^{20}\)Because of the maturity mismatch between loans and deposits, \(h_t\) can be negative.
chooses lending \((l_t)\), dividends \((d_t)\), and deposits \((c_t)\) with the objective of maximizing \((10)\). Notice that if \(h_t < 0\), then part of the amount of deposits will be used to pay old depositors. Thus the total amount of cash available for lending and dividends is \(c_t + h_t\), which implies that

\[ c_t + h_t \geq d_t + l_t \quad (12) \]

Equity markets in this model are imperfect. For simplicity, it is assumed that issuing equity is infinitely costly. This assumption is modeled by constraining dividends to be nonnegative

\[ d_t \geq 0 \quad (13) \]

After decisions are made, the bank has state variables \(l_t, c_t\) and \(l_{t-1}\).

Equations (14) to (17) summarize the bank’s problem, written in Bellman’s equation form\(^{21}\)

\[
V_t(h_t, l_{t-1}) = \max_{\{l_t, d_t, c_t\}} \{d_t + \beta E_t V_{t+1}(h_{t+1}, l_t)\} \\
\text{s.t. } c_t + h_t \geq d_t + l_t \quad (15) \\
d_t \geq 0 \quad (16) \\
h_{t+1} = G(l_{t-1}, \Phi_{t+1}) - (1 + \rho_{t+1})c_t - f_{t+1} \quad (17)
\]

In this context, the Modigliani and Miller (1958) theorem implies that the value of the bank is unaffected by its capital structure. With information problems in capital markets, this conclusion may not hold (hence the assumption of imperfect equity markets). However, it is also assumed that raising deposits is affected by agency problems between regulators and the bank (because deposits are insured) or between depositors and the bank in the absence of deposit insurance.\(^{22}\) These agency problems are modeled by assuming that the bank faces costs \(f_{t+1} = (1 + \)

\(^{21}\)Extending the maturity of loans to \(n\) periods is trivial as long as the assumption of all payments made at maturity is maintained. In the \(n\)-period case, the only additional complication is to separate the total amount of outstanding loans and the fraction that is maturing in the current period.

\(^{22}\)Berger and Bonaccorsi (2002) find evidence consistent with the hypothesis that increases in the leverage of U.S. banking firms raises agency costs.
\[\rho_{t+1} c_t (1 + \rho_{t+1} c_t) s,\] which are increasing and twice continuously differentiable in deposits. Without loss of generality, possible interpretations of \(f_{t+1}\) include stealing costs, costs associated with financial distress, scrutiny of regulators as the bank’s financial position weakens, etc. The appendix provides a more formal justification for \(f_{t+1}\) by modeling a depositor-bank contract using the same structure as the bank-borrower relationship. In that particular case, \(f\) arises endogenously, embedded in the contractual deposit rate as expected bankruptcy costs. Because the objective of this paper is not to argue in favor of some specific form of imperfection, it suffices to use the reduced-form modeling device introduced through \(f\).

If the left-hand side of equation (15) is strictly larger than the right-hand side, and the resulting cash balance earns the risk-free rate, then the bank can always increase profits by reducing the amount of deposits. That is because the cash balance would not only cost the bank \((1 + \rho_{t+1})\) but would also increase \(f_{t+1}\). Consequently, equation (15) holds with equality and the choice of lending and dividends determines the amount of deposits.

### 4.1 Solution

The model is solved numerically using the logic of endogenous gridpoints developed in Carroll (2005). The appendix provides the assumed values for the parameters of the model and a detailed description of the solution algorithm. For intuition purposes, it is useful to write the problem in terms of middle-of-period and end-of-period state variables. Using the fact that equation (15) holds with equality to substitute out deposits, and denoting end-of-period variables with Gothic letters, the problem can be written as

\[
V_t(h_t, l_{t-1}) = \max \{h_t - q_t + \mathcal{V}_t(q_t, l_t, l_{t-1})\} \quad (18)
\]

\[h_t - q_t \geq 0 \quad (19)\]

Where the net cash position as of the end of the period is defined as \(q_t = h_t - d_t\) and the end-of-period value function, \(\mathcal{V}_t(q_t, l_t, l_{t-1})\), given by

---

\(^{23}\) with \(s > 0\)

\(^{24}\) The endogenous gridpoints method involves using end-of-period values of the state variables, the marginal value functions, and first order conditions to construct middle-of-period levels of the state variables.
\( \mathcal{V}_t(q_t, l_t, l_{t-1}) = \beta E_t V_{t+1}(h_{t+1}, l_t) \) \tag{20} \\
\text{s.t.} \\
h_{t+1} = G(l_{t-1}, \Phi_{t+1}) - (1 + \rho_{t+1}) [l_t - q_t] \tag{21} \\
- (1 + \rho_{t+1}) [l_t - q_t] \frac{(1+\rho_{t+1})(l_t - q_t)}{G(l_{t-1}, \Phi_{t+1})} \tag{22} \\
with first order conditions

\[
1 = \mathcal{W}^0_t(q_t, l_t, l_{t-1}) \tag{23}
\]
\[
0 = \mathcal{W}^1_t(q_t, l_t, l_{t-1}) \tag{24}
\]

where

\[
\mathcal{W}^0_t(q_t, l_t, l_{t-1}) = E_t \beta \left[ 1 + \rho_{t+1} - f_t^a \right] V_{t+1}^h(h_{t+1}, l_t) \tag{25}
\]
\[
\mathcal{W}^1_t(q_t, l_t, l_{t-1}) = E_t \beta \left[ \left[ -1 - \rho_{t+1} - f_t^l \right] V_{t+1}^h(h_{t+1}, l_t) + V_{t+1}^l(h_{t+1}, l_t) \right] \tag{26}
\]
\text{s.t.} \\
h_{t+1} = G(l_{t-1}, \Phi_{t+1}) - (1 + \rho_{t+1}) [l_t - q_t] \\
- (1 + \rho_{t+1}) [l_t - q_t] \frac{(1+\rho_{t+1})(l_t - q_t)}{G(l_{t-1}, \Phi_{t+1})} \tag{27}
\]

Figure 4 shows the infinite-horizon policy rules for the above problem, with the dots indicating the target values for each variable\(^{25}\). The converged solutions are used to derive all numerical results shown in this paper.

It is useful now to give some intuition behind the first order conditions of the problem. Notice first that in the absence of imperfect equity markets, the bank will always arrive at the end of the period with the values of \(q_t\) and \(l_t\) that satisfy the first order conditions (23) and (24). In that case, the cash position of the bank as of the middle of the period plays no role in lending decisions. This is because in the event of an adverse shock that weakens its cash position, the bank instantaneously

\(^{25}\)The targets correspond to the ergodic means.
and costlessly substitutes raising debt for issuing equity. The argument is similar to what Romer and Romer (1994) concluded in reference to how banks can substitute various funding sources with different reserve requirements in the event of a shock to cash reserves.

In the presence of infinitely high costs of equity finance, the amount of dividends distributed is such that the marginal value of dividends equals the marginal value of the net cash position. The right-hand side of equation (23), which is shown in more detail in equation (25), corresponds to the marginal value of cash position. Changes in cash position affect its marginal value through two channels: the impact on the value of cash position of the bank in the next period, given by the interest rate and by \( f_t^q \), and the impact on the future value of the bank, given by \( V_{t+1}^h(h_{t+1}, l_t) \). If the bank reaches the middle of the period with a weak cash position such that it is below the optimal amount with which the bank wishes to end the period, the cost of debt rises because \( f_t^q \leq 0^{26} \) and the marginal value of an extra dollar of cash increases as seen in equation (25).

Notice that in this model, there are no coordination failure problems of the Diamond and Dybvig (1983) type. Unexpected withdrawals of resources are also not an issue here as in Kashyap, Rajan, and Stein (1999). The distinction is relevant to avoid the confusion between the role played by net cash position in this model and the role of cash reserves in liquidity models. From the previous paragraph, one concludes that the role of net cash position is that of a cushion against unexpected adverse conditions that would otherwise inhibit the bank’s lending operations. Notice that its role is very close to the one that the literature assigns to bank capital (See Diamond and Rajan (1999) and Berger, Herring, and Szeg (1995)).

In the case of lending, the amount lent is such that the marginal value of outstanding loans equals the marginal cost of raising funds. This is better appreciated by rewriting equation (26) as

\[
E_t \Phi_t \left[ 1 + \rho_{t+1} + f_t^l \right] V_{t+1}^h(h_{t+1}, l_t) = E_t V_{t+1}^l(h_{t+1}, l_t)
\]

In equation (28), the left-hand side corresponds to the marginal cost of lending. The marginal cost of lending is determined by how much it costs the bank to raise deposits. Increasing deposits today affects the future net cash position of the bank, denoted by \( [1 + \rho_{t+1} + f_t^l] \), but it also affects the future value of the bank through the effect on the future cash position, denoted by \( V_{t+1}^h(h_{t+1}, l_t) \). The right hand-side is the marginal benefit from lending; it represents the expected future profits the

---

26 Since \( f_t^l = (1 + \rho_{t+1}) [l_t - q_t] \left( \frac{1 + \rho_{t+1} - q_t}{1 + \rho_{t+1}} \right) s, f_t^q = -2 \frac{(1 + \rho_{t+1}) (l_t - q_t)}{\gamma_t (l_{t-1} - \Phi_{t+1})} s \leq 0 \), because \( l_t - q_t = c_t \geq 0 \).
bank will obtain from lending operations made in the current period. Again, if the bank reaches the middle of the period with a weak cash position such that reducing dividends does not suffice to restore it to the desired level, the marginal cost of lending increases because $f^{l+1}_{t+1} > 0$. A weak financial condition would then hold back lending. If $f = 0$ the marginal values of cash and outstanding loans are unaffected by the bank's financial structure and lending would not respond to changes in the bank's cash position. This conclusion is similar to the one obtained after relaxing the assumption of imperfect equity markets. Consequently, the assumptions of imperfect capital markets result in the failure of the Modigliani and Miller (1958) theorem; the bank's financial structure does matter for lending decisions.

It is now clear that with the assumption of banks facing market frictions similar to those faced by borrowers, the lender no longer plays a passive role in the economy. This framework thus complements existing work that had previously neglected the role of banks in the exacerbation of business cycles. As Bernanke, Gertler, and Gilchrist (1999) pointed out: "we have not addressed the role of banks in cyclical fluctuations......one possibility is to allow the financial intermediaries which lend to entrepreneurs to face financial frictions in raising funds themselves. In this case, the net worth of the banking sector, as well as the net worth of entrepreneurs, will matter for the models’ dynamics".  

4.2 Uncertainty and the target level of cash

Rewrite equation (25) as

$$E_{\Phi,\rho} [1 + \rho_{t+1} - f^q_{t+1}] V^{h}_{t+1}(h_{t+1}, l_t) = \frac{1}{\beta}$$ (29)

The left-hand side of equation (29) is the marginal value of the cash position of the bank. If internal funds are high enough such that frictions do not matter, the left-hand side of equation (29) would be equal to the average risk-free rate, that is $\lim_{q \to \infty} E_{\Phi,\rho} [1 + \rho_{t+1} - f^q_{t+1}] V^{h}_{t+1}(h_{t+1}, l_t) = 1 + \tilde{p}$. Hence, if the impatience assumption holds (that is, $\frac{1}{\beta} > 1 + \tilde{p}$) for given amounts of outstanding loans, there will be a unique target value of end-of-period cash position $0 < \tilde{q} < \infty$, $\tilde{q} = \tilde{h} - \tilde{d}$ that

27 If the cash position is large enough that the constraint on dividends is non-binding, then from equation (18) one knows that $V^{h}_{t+1}(h_{t+1}, l_{t-1}) = 1$. Rolling forward this equation one period, one gets $V^{h}_{t+2}(h_{t+1}, l_t) = 1$. This is an implication of the Envelope theorem. Keep in mind that $q$ is an endogenous variable. Thus the limit is taken just to illustrate what would happen if there were enough internal funds so that frictions do not matter. When the bank funds itself entirely through internal funds, deposits are zero and therefore agency costs between the bank and depositors dissapear.
satisfies equation (29). For values of cash position above (below) \( \tilde{q} \), the marginal value is lower (higher) than the time preference rate. Hence the bank increases (decreases) dividends until (29) holds with equality.

The target constitutes the optimal financial structure of the bank. At the target, the cash position is low enough to reduce potential agency costs between stockholders and managers–if the reason behind impatience is to avoid those costs–but high enough to reduce agency costs between the bank and depositors. The dots plotted on the policy functions (see Figure 4) denote the corresponding targets for each variable. Figure 5 plots equation (29) as a function of end-of-period cash, normalized by the target value of outstanding loans. The dotted line shows the baseline calibration.

Recall that financial frictions \( f \) are assumed to be twice continuously differentiable. Figure 5 helps to understand the importance of this assumption. Suppose impatience is maintained, but \( f \) is linear; in this case, the marginal value of cash would be a horizontal line above or below the time preference rate, \( (\frac{1}{\beta} - 1) \). In such cases, the bank holds no deposits or it has the incentive to run down the cash position to negative infinity. Suppose now that \( f \) is twice continuously differentiable but \( (\frac{1}{\beta} - 1) = \rho \), that is, impatience is relaxed. Notice first that the degree of impatience is illustrated on the graph by the gap between \( \frac{1}{\beta} - 1 \) and \( \rho \). It is easy to see that, as impatience decreases, the target of cash increases, and the intersection between the marginal value of cash with the time preference rate line \( (\frac{1}{\beta} - 1) \) would move to the right. Therefore, without impatience, the bank would accumulate internal funds up to the point where it would never have to incur frictional costs. This situation occurs when deposits are equal to zero. In the absence of impatience and financial frictions, then the marginal value of cash is a horizontal line that would coincide with the average risk-free rate line. In that case, dividends and internal funds are perfect substitutes and a non-uniqueness problem arises.

The graph is also useful to analyze the role of uncertainty. Recall that the dotted line shows the baseline case, while the solid line illustrates the perfect foresight version of the model. As shown in Figure 5, the bank exhibits a precautionary motive. Additional risk in loan revenues raises the target of cash position relative to the outstanding amount of loans. The key elements that generate this behavior are the assumption that \( f \) is a non-linear function of revenues and the constraint on dividends. The interaction between these two components generate a convex marginal value of cash position as shown in Figure 5. Notice that there is a close analogy between the behavior of the bank and that of a consumer who self-insures against income uncertainty by maintaining a target level of the ratio of cash-on-hand to permanent income.

\[28\] Furthermore, the bank is indifferent between accumulating funds beyond this point or keeping \( q \) equal to loans \( l \).

\[29\] The figure is plotted with lending \( l \) being chosen optimally.

5 Financial distress and persistent real effects

When determining the impact of banks’ behavior on the real economy, there are two research questions. The first is whether the financial structure of banks matters for lending decisions, and the second one is whether banks’ behavior can have exogenous real effects. Bernanke and Lown (1991), Peek and Rosengren (1994, 1995), and Hancock and Wilcox (1994) are some examples studying how bank capital shortfalls affect lending, and the results show statistically significant effects. Kashyap and Stein (1995) uses disaggregate banks’ balance sheet data and find evidence suggesting a different impact of monetary policy on the security and loans portfolio of small and large banks. Kashyap and Stein (2000) find that the impact of monetary policy on lending is stronger for banks with less liquid balance sheets.

If bank lending reacts to changes in bank health, the next question is how important that is for the aggregate economy. This has been more challenging to prove since it requires one to successfully identify exogenous changes in the supply of credit and to disentangle them from those that simply follow the economic cycle. Despite the difficulties, there is a significant amount of work showing that a deterioration in banks’ financial health indeed has important real effects. Bernanke (1983) explores the role of non-monetary factors during the Great Depression and concludes that the evidence supports the hypothesis that the weakness of financial institutions had real effects. Although his results were not conclusive, later studies support the findings in Bernanke (1983). Dell’Ariccia, Detragiache, and Rajan (2005) explore whether the decline in credit and growth that follows a banking crisis can be partially explained by the weakening of the banking sector. They find evidence suggesting that sectors more dependent on external finance perform relatively worse after banking crises. Using two different sample periods, Gibson (1995, 1997) finds that Japanese firms whose main bank had financial problems had investment levels substantially lower than firms whose bank was not in trouble or those who were listed on the stock market. The overall impact, however, was found to be small because of the relative size of bank-dependent borrowers. Klein, Peek, and Rosengren (2002), using firm-level and bank-level data, find that financial difficulties at banks were economically and statistically important in reducing the number of FDI projects by Japanese firms into the United States. Peek, Rosengren, and Tootell (2003) identify loan supply shocks using CAMEL ratings of U.S. banks and find that bank health has economically significant effects on the U.S. economy. Moreover, they also find that their loan supply measure is particularly important for understanding movements in inventories. Peek and Rosengren (1997, 1999) investigate whether the sharp stock market decline in Japan was transmitted to the United States via U.S. branches. They find statistically and economically significant results suggesting that binding risk-based capital requirements associated with the fall of stock prices resulted in a reduction in lending by Japanese banks in the United States. Peek and Rosengren
(2000) use the banking crises in Japan as a credit supply shock exogenous to the U.S. They establish that loan supply shocks emanating from Japan and transmitted to U.S. markets, where Japanese banks had significant market penetration, had real effects on construction activity in U.S. commercial real estate markets.

This section describes some quantitative experiments using the converged policy rules and derives results that are consistent with the aforementioned studies.

The first experiment consists of a negative productivity shock of one standard deviation for one period. Figure 6 plots the corresponding results. Two distinct starting points are considered. In the first case (top dotted line), the state variables are equal to their corresponding targets. In the second case, the bank starts with a much weaker cash position while outstanding loans are at the target (bottom dashed and solid lines). The latter scenario shows two adjustment paths. One line corresponds to the adjustment path that would be followed in absence of shocks (dashed line). The second line (solid line) shows the adjustment path that results after the productivity shock under analysis. In both scenarios, the transitory shock to productivity causes a decline in revenues which deteriorates the net cash position of the bank, as illustrated in Figure 6a. The deterioration in the cash position generates an increase in the size of financial frictions, as illustrated in figure 6c, for the case of a weak initial condition. However, for the scenario where the bank is at the target when it experiences the shock, its strong financial condition allows it to restore the optimal level of cash position by simply reducing dividends. Notice how in this case the amount of financial frictions suffers only a slight increase, therefore lending is effectively shielded against the shock. The cash position then works as a cushion against drops in revenues.

When the financial condition of the bank is weak, it cannot avoid facing higher frictional costs. The higher the financial frictions, the higher the borrowing costs and the less profitable it is to lend. Consequently, lending declines, but as lending declines, revenues decline as well, which deteriorates the cash position of the bank even further. This result can be appreciated in the inverted hump-shaped response of lending to the shock–Figure 6b. This follows from the fact that because dividends are already at zero, the bank can restore its financial condition only through the accumulation of retained earnings. The bank then attempts to reduce the size of financial frictions by reducing lending and building up cash. Eventually, the bank has reduced lending to a point where financial frictions reach a peak. Financial frictions then start decreasing and the bank starts increasing lending again, but gradually. Notice however, that there exist a persistent gap between the adjustment path for lending following the shock (bottom solid line) and the line that corresponds to the adjustment without the shock (bottom dashed line). This gap persists beyond

---

31 This weaker initial condition can be generated by a 2.7 standard-deviation productivity shock when the bank is at the target.
the period when the shock was realized. As a result, the one-period i.i.d impulse has a long-lasting effect on the economy due to the weak initial condition of the bank.

By comparing the two scenarios, one can determine the economic cost of the credit crunch triggered by the transitory shock. The reduction in credit causes a decrease in investment and hence a decline in output.32 One could interpret this output loss as the costs associated with financial distress, because the only reason behind the decline in output is the poor health of the banking system.

Consider now how the bank reacts to a one-period, 1% increase in the risk-free rate. The results are plotted in Figure 7. An increase in the interest rate raises funding costs. Because there is no hedging in this model, the mismatch in maturity leaves the bank unprotected against interest rate risk. As before, in the case of a weaker initial financial condition, lending exhibits the same persistent effects and a credit crunch arises with similar consequences, as in the previous scenarios. As before, Figure 7c illustrates how financial frictions rise when the bank is already weak when the shock is realized.33

### 6 Asymmetry of responses

A visual inspection of the policy functions suggests that the response of lending to changes in cash position varies with its initial level, suggesting an asymmetric response to shocks. This section formally explores this implication. The exercise involves simulating the response of the bank to an 2.5 standard-deviation productivity shock and 5% change in the risk—free rate, in each direction. Figure 8 shows the results for dividends and lending as a percentage of their corresponding targets. The first conclusion derived from the figure is that, despite the exposure to interest rate risk, when the bank is at the target, interest rate shocks have a much weaker impact than productivity shocks. The qualitative nature of the response, however, suggests that in both cases, negative shocks have a stronger impact than positive ones. When the cash position deteriorates, the bank reduces dividends. If the shock is large enough, the reduction in dividends is not enough to shield lending. In that case, a credit crunch arises, with the consequences explained earlier. This implication is driven by the non-linear nature of the lending policy function. The propensity to lend is much larger at low levels of cash position and outstanding loans.

32 Since endowment is fixed, any reduction in lending directly affects the amount of capital used in production.

33 It is worth noticing that cycles in this model are triggered by shocks exogenous to the banking industry. Gorton and He (2004) show that credit cycles can also be generated by competition among banking firms. They present empirical evidence showing how banks change credit standards in order to secure a larger market share, generating credit cycles even when there are no developments at the macroeconomic level. The study of both types of cycles can be performed by extending the current model to an oligopoly. This task is left for future research.
than at points where cash and loans are close to or above the target. There is some
evidence in the literature suggesting asymmetry in lending dynamics. Dell’Ariccia
and Garibaldi (2005) estimate gross credit flows for the U.S. banking system be-
tween 1979 and 1999 and find that for any given rate of change of net credit, gross
flows are larger in a recession than in a boom and that credit contractions are more
volatile than credit expansions. The asymmetric response suggested by the model
may provide an explanation of the asymmetric effects of monetary policy on output
that some economists have found. Cover (1992), for instance, examines whether
positive and negative money-supply shocks have symmetric effects on output. He
finds that while positive money-supply shocks do not affect output, negative shocks
do.

7 Extension: adding permanent productivity shocks

In the baseline model, entrepreneurial endowment was fixed. In this section, en-
dowment is allowed to vary over time, but its exogenous nature is maintained. It is
therefore assumed that aggregate endowment in the economy evolves according to

\[ W_{t+1} = W_t \Psi_{t+1} \]  \hspace{1cm} (30)

for a permanent shock \( \Psi_{t+1} \), assumed to be i.i.d. lognormally distributed over
\([0, \infty)\). Productivity shocks will now have two perfectly observable components:
a transitory one and a permanent one. The assumption of a time-varying en-
trepreneurial endowment introduces a new state variable. However, it is possible to
keep the structure of the model simple by normalizing all variables by \( W_t \). Given
the two-period horizon of the entrepreneur and therefore of the loan contract, the
distinction between the two shocks is irrelevant from the perspective of the bor-
rrower. Thus, the derivations that pertain to the loan contract must be modified in
a trivial way. Recall from section 2 that after substituting for capital, production
for an entrepreneur, normalized by the level of endowment, was given by

\[ y_{t+2} = \alpha_{t+2} \Phi_{t+2} (l_t + 1) \]  \hspace{1cm} (31)

Now let \( \Phi = \Psi \tau \), with \( \tau \) denoting transitory shocks to productivity, assumed
to be identically and independently lognormally distributed over \([0, \infty)\). Therefore,
the new normalized aggregate revenue function of the bank, \( G(\cdot) \), is constructed by
modifying equation (9) using $\Phi_{t+2} = \Psi_{t+2} \tau_{t+2}$ and $\frac{h_{t+2}}{\Psi_{t+1}}$ as the main argument.\textsuperscript{34} The normalized end-of-period value function is now given by (32).

$$v_t(h_t, l_t, l_{t-1}) = \beta E_t \Psi_{t+1} v_{t+1}(h_{t+1}, \frac{l_t}{\Psi_{t+1}})$$ \hspace{1cm} (32)

s.t.

$$h_{t+1} = \left[ G(\frac{l_t}{\Psi_t}, \Psi_{t+1}, \tau_{t+1}) - (1 + \rho_{t+1})(l_t - q_t) \right] \left[ (1 + \rho_{t+1})(l_t - q_t) \frac{1}{\Psi_t} G(\frac{l_t}{\Psi_t}, \Psi_{t+1}, \tau_{t+1}) \right]$$ \hspace{1cm} (33)

and, as before, the optimization problem can be written as

$$v_t(h_t, l_t, l_{t-1}) = \text{Max} \{ h_t - q_t + v_t(h_t, l_t, l_{t-1}) \}$$ \hspace{1cm} (34)

s.t. $h_t - q_t \geq 0$ \hspace{1cm} (35)

where $V_t(H_t, L_{t-1}, W_t) = W_t v_t(h_t, \frac{l_t}{\Psi_t})$ for all $t \leq T$ with small letters denoting the normalized version of capital letters and with $T$ corresponding to a hypothetical last period of life of the bank.

The problem shown above is solved applying the same solution method described earlier, from which a new set of policy functions is obtained (shown in Figure 9). As before, the dots show the location of the corresponding targets. Unlike the previous case, in which an adverse shock to productivity (now equivalent to a transitory shock) generated persistent effects on lending, it is now a positive permanent change in productivity what induces lending inertia. The bank cannot self-insure against a permanent shock to productivity; thus, a one-to-one response in all variables is expected in the long-run. However, if the increase in productivity is large enough, the bank takes more than one period to adjust the levels of all variables to the new corresponding targets. Figure 10 shows the results for the levels of each variable, where the latter are recovered from their normalized versions by multiplying them by the size of endowment. In Figure 10, one can appreciate the reaction of the bank when there is a 2.5 standard-deviation permanent increase to productivity. The two dotted lines in each diagram illustrate the location of the new and old targets as indicated in the figures. In the absence of capital market frictions, the adjustment is instantaneous; the dashed line shows how lending is at the new target one period after the shock, while in the presence of financial frictions it takes the bank one

\textsuperscript{34}The appendix elaborates on the normalization in detail.
8 Conclusions

In the last two decades, macroeconomists have devoted a great deal of attention to developing structural models explaining how small and transitory shocks generate large and persistent fluctuations in real macroeconomic variables. Nevertheless, these models are focused on frictions affecting borrowers but they neglect the bank. In recent years, there has been a growing attention to explicitly model banks’ decisions and their dynamic interactions with the real economy. This paper contributes in this direction by developing a dynamic framework to study the interactions between macroeconomic shocks, banks’ financial positions, their lending decisions, and the real economy. The bank is modeled as a firm in a world of imperfect capital markets. The key feature of the model is the existence of an optimal financial structure in the form of net cash position relative to outstanding loans, generated by a decreasing marginal value of cash—induced by financial frictions—and impatient stockholders. Because borrowers’ creditworthiness is ex-ante exogenous and time-invariant, it is possible to compute the exact aggregate effects of a credit crunch triggered by a financial weakening of the bank.

In the model, first, an i.i.d productivity or interest rate shock can generate persistent real aggregate effects because of its impact on the bank’s financial condition, especially when the initial state of the bank is weak. Second, the response of the bank to shocks is not symmetric; a negative shock has a much stronger impact on lending than a positive one. Third, the net cash position works as a cushion against unexpected losses that would otherwise hinder bank lending; moreover, in the presence of uncertainty, the bank exhibits a precautionary motive. An increase in the risk of loan revenues strengthens the cushion by increasing the targeted level of net cash position. Finally, the inertial effects on lending and output also occur when a permanent increase in productivity takes place in the economy.

The results presented in this paper are important for economic modeling and policy analysis because they suggest that the financial intermediary, when studied as a firm, plays a non-trivial role in the exacerbation of business cycles and in the effectiveness of economic policy. For instance, if banks face financial difficulties, the lending channel of monetary policy may break because banks will need to restore their financial health before lending returns to normal levels. If the economy is in a recession, then a direct intervention of the government to speed up the recovery of the banking system may be justified if the benefit of restoring credit markets sooner outweighs the cost of the intervention. The model also has implications for banking regulation, especially in relation to the cyclical aspects of regulatory requirements as proposed in Basel II. If requirements tighten during recessions, and
they are effectively binding, then the real effects could be stronger than what the results derived here suggest. The asymmetric response of the bank to shocks may constitute one possible explanation for the asymmetric effects of monetary policy and asymmetry in credit flows that some researchers have found in the data. To conclude, the overall importance and quantitative relevance of all of these implications will increase significantly in economies where bank-intermediated credit is the largest funding source for firms, especially in emerging countries, where financial markets have not reached the degree of development of their industrialized counterparts.

References


Figure 1. Lending Rate

Figure 2. Marginal Revenues
Figure 3. Bank’s Sequence of Events

Middle of Period \( t \)
- State variables: \( l_{t-1}, h_t \)
- \( h_t = G(l_{t-2}, \Phi_t) - (1 + \rho_t) c_{t-1} - f_t \)
- Choice variables: \( c_t, d_t, l_t \)

Uncertainty is realized: \( \Phi_t, \rho_t \)

Period \( t-1 \)
- State variables after decisions were made: \( l_{t-2}, c_{t-1}, h_{t-1} \)

Period \( t \)
- State variables after decisions: \( l_{t-1}, c_t, h_t \)

Uncertainty is realized: \( \Phi_{t+1}, \rho_{t+1} \)
Figure 4. Infinite Horizon Policy Functions

4a. Dividends Policy Function

4b. Lending Policy Function
Figure 5. Cash Position Target

Marginal value of cash

Time Preference Rate: $1/\beta - 1$

Mean risk-free rate: $\mathcal{P}$
Figure 6. Response to a Negative Transitory Productivity Shock \((1\sigma_\phi)\)

6a. Cash Position

6b. New Lending

6c. Financial Frictions

6d. Dividends
Figure 7. Response to a Monetary Contraction ($\rho_t = 1\%$)

7a. Cash Position

7b. New Lending

7c. Financial Frictions

7d. Dividends
Figure 8. Asymmetry of Responses

8a. New Lending

8b. Dividends

8c. New Lending

8d. Dividends
Figure 9. Policy Functions with Permanent Shocks

9a. Dividends Policy Function

9b. Lending Policy Function
Figure 10. Permanent Shocks to Productivity ($2.5\sigma_\psi$)

### 10a. Cash Position

- **Level vs. Time**
- **No frictions**
- **With frictions**

### 10b. New Lending

- **Level vs. Time**
- **No frictions**
- **With frictions**

### 10c. Dividends

- **Level vs. Time**
- **No frictions**
- **With frictions**

**Figure Notes**
- The graphs illustrate the impact of permanent shocks to productivity on different economic indicators.
- The green dashed line represents the scenario with frictions, while the solid blue line represents the no-frictions case.
- The graphs are labeled with appropriate axes and data points.

**Data Points**
- Table 36 provides specific values for comparison.
9 Appendix

9.1 Derivation of the marginal revenue function

The aggregate revenue function is given by

\[ G(l_t, \Phi_{t+2}) = [1 - F_\alpha(\alpha(l_t, \Phi_{t+2}))] R(l_t)l_t + (1-u) \int_0^{\Phi_{t+2}} (l_t+1)\alpha_{t+2} \Phi_{t+2} dF_\alpha \]  

(36)

Using \( \alpha(l_t, \Phi_{t+2}) = \frac{R(l_t)l_t}{(l_t+1)\Phi_{t+2}} \) and dropping the arguments of \( \alpha \), (36) can be rewritten as

\[ G(\alpha, l_t, \Phi_{t+2}) = [1 - F_\alpha(\alpha)] (l_t + 1)\alpha_{t+2} + (1-u) \int_0^{\alpha} (l_t + 1)\alpha_{t+2} \Phi_{t+2} dF_\alpha \]  

(37)

\[ G(\alpha, l_t, \Phi_{t+2}) = (l_t + 1)\Phi_{t+2} \left\{ [1 - F_\alpha(\alpha)] \alpha + (1-u) \int_0^{\alpha} \alpha_{t+2} dF_\alpha \right\} \]  

(38)

where

\[ \Gamma(\alpha) = [1 - F_\alpha(\alpha)] \alpha + (1-u) \int_0^{\alpha} \alpha_{t+2} dF_\alpha \]  

(39)

\[ \Gamma'(\alpha) = [1 - F_\alpha(\alpha)] - u\alpha f(\alpha) \]  

(40)

Totally differentiate (38) with respect to \( l \) to obtain

\[ G'(\alpha, l_t, \Phi_{t+2}) = \Phi_{t+2} \left\{ (l_t + 1)\Gamma'(\alpha)\alpha' + \Gamma(\alpha) \right\} \]  

(41)

since \( \alpha(l_t, \Phi_{t+2}) = \frac{R(l_t)l_t}{(l_t+1)\Phi_{t+2}} \), then
$$\alpha' = \frac{1}{(l+1)\Phi_{t+2}} \left[R'(l_t)l_t + \frac{R(l_t)}{l_t+1} \right]$$ \hspace{1cm} (42)$$

The next step is to compute $R'(l_t)$ from equation (7) whose full specification is written below.

$$\int_0^{\infty} \left[ \int_{\alpha}^{\infty} (l_t + 1)\Phi_{t+2} dF_\alpha - [1 - F_\alpha(\alpha)] R(l_t)l_t \right] dF_\Phi = (1 + \rho)^2$$ \hspace{1cm} (43)$$

Using again $\alpha(l_t, \Phi_{t+2}) = \frac{R(l_t)l_t}{(l_t+1)\Phi_{t+2}}$ rewrite (43) as

$$\int_0^{\infty} \left[ \int_{\alpha}^{\infty} (l_t + 1)\Phi_{t+2} dF_\alpha - [1 - F_\alpha(\alpha)] (l_t + 1)\alpha \Phi_{t+2} \right] dF_\Phi = (1 + \rho)^2 \hspace{1cm} (44)$$

where

$$\Omega(\alpha) = \int_{\alpha}^{\infty} dF_\alpha - [1 - F_\alpha(\alpha)] \alpha \hspace{1cm} (47)$$

$$\Omega'(\alpha) = -[1 - F_\alpha(\alpha)] \hspace{1cm} (48)$$

Totally differentiate (46) with respect to $l$ to obtain

$$\int_0^{\infty} \left[ \Phi_{t+2}\Omega(\alpha) + (l_t + 1)\Phi_{t+2}\Omega'(\alpha)\alpha' \right] dF_\Phi = 0 \hspace{1cm} (49)$$

Using (42) one gets
\[
\int_{0}^{\infty} \Phi_{t+2} \Omega(\alpha) dF_{\Phi} = - \int_{0}^{\infty} (l_{t} + 1) \Phi_{t+2} \Omega'(\alpha) (l_{t+1}) \frac{1}{\Phi_{t+2}} \left[ R'(l_{t}) l_{t} + \frac{R(l_{t})}{l_{t+1}} \right] dF_{\Phi} \quad (50)
\]
\[
\int_{0}^{\infty} \Phi_{t+2} \Omega(\alpha) dF_{\Phi} = - \int_{0}^{\infty} \Omega'(\alpha) \left[ R'(l_{t}) l_{t} + \frac{R(l_{t})}{l_{t+1}} \right] dF_{\Phi} \quad (51)
\]
\[
R'(l_{t}) = \frac{\int_{0}^{\infty} \Phi_{t+2} \Omega(\alpha) + \Omega'(\alpha) \frac{R(l_{t})}{l_{t+1}} dF_{\Phi}}{-\int_{0}^{\infty} \Omega'(\alpha) l_{t} dF_{\Phi}} \quad (52)
\]

With (52) on hand, \( G^{l}(\alpha, l_{t}, \Phi_{t+2}) \) is easy to compute.

### 9.2 Calibration

The model is solved using \( \beta = 0.94 \) and \( 1 + E[\rho] = 1.055 \) in order to satisfy the impatience assumption. The risk-free rate is assumed to be discretely distributed with three outcomes: \{1.05, 1.055, 1.06\}, with probability distribution \{0.25, 0.50, 0.25\}. The expected value of \( \alpha \) is equal to 1.15 and \( \sigma_{\alpha} = 0.5 \), \( \sigma_{\Psi} = 0.3 \), and \( \sigma_{\tau} = 0.3 \). Bankruptcy costs are calibrated as in Carlstrom and Fuerst (1997), with \( u = 0.20 \), which is consistent with the estimates obtained in Levin, Natalucci, and Zakrajsek (2004). Finally, it is assumed that \( \sigma = 0.002 \). These values, in steady state, generate a probability of default of borrowers, \( E_{F\alpha}(\alpha) \), of 3.4 %, a spread between the lending rate and the average risk-free rate of 5.5%, and a leverage ratio of the bank (deposits to total assets) equal to 76%.

### 9.3 Solution algorithm

The starting point is to define a hypothetical last period of life of the bank, \( T \), at which point the bank is liquidated and therefore the optimal decision is to distribute the entire stock of cash \( h_{T} \) as dividends, that is, \( d_{T} = h_{T} \), and to make no new loans. The latter implies that the value function as of \( T \) is \( V_{T}(h_{T}, l_{T-1}) = h_{T} \). The next step involves solving the model from the perspective of period \( T - 1 \).
\[
V_{T-1}(h_{T-1}, l_{T-2}) = \max_{\{d_{T-1}\}} \left\{ d_{T-1} + \beta E_{T-1} h_T \right\}
\]

\[\text{s.t. } c_{T-1} + h_{T-1} = d_{T-1} \]  

\[d_{T-1} \geq 0\]  

\[h_T = G(l_{T-2}, \Phi_T) - (1 + \rho_T) c_{T-1} - \frac{c_{T-1}}{f_{T-1}} s\]

Because it is the second-to-last period of life of the bank, lending is equal to 0. For this period only, it is assumed that \(f_{T-1} = c_{T-1} \frac{c_{T-1}}{f_{T-1}} s\). The immediate implication of this assumption is that uncertainty has no effect on the bank’s optimal behavior in period \(T-1\). The first order condition for the problem, after substituting out deposits, is given by

\[
1 = \beta E_{T-1} \left[ 1 + \rho_T - f_{T-1}^d \right]
\]

from which the optimal solution for dividends is obtained \(d_{T-1}(h_{T-1}, l_{T-2})\), as a function of \(h\) and \(l\). The marginal value functions \(V_{T-1}^h(h_{T-1}, l_{T-2})\) and \(V_{T-1}^l(h_{T-1}, l_{T-2})\) are given by

\[
V_{T-1}^h(h_{T-1}, o_{T-1}) = \beta E_{T-1} \left[ 1 + \rho_T - f_{T-1}^h \right]
\]

\[
V_{T-1}^l(h_{T-1}, l_{T-2}) = \beta E_{T-1} \left[ G^l(l_{T-2}, \Phi_T) - f_{T-1}^l \right]
\]

Because from period \(T - 2\) and backwards the bank will lend positive amounts, it is assumed that \(f_t = (1 + \rho_t)c_{t-1} \frac{(1 + \rho_t)c_{t-1}}{G(l_{t-2}, \Phi_t)} s\) for all \(t \leq T - 2\). Now uncertainty begins to play a role because \(G^l(l_{T-2}, \Phi_T)\) is a nonlinear function of \(\Phi\). With the functions on hand, and using equation (15) to substitute out deposits, the problem as of \(T - 2\) is given by

\[35\] where \(f^s = \frac{\partial f}{\partial c} \frac{dc}{dt}\) unless derivatives are taken with respect to the primitive arguments.
\[ V_{T-2}(h_{T-2}, l_{T-2}) = \max_{\{d_{T-2} \in \mathbb{R}\}} \left\{ d_{T-2} + \beta E_{T-1} V_{T-1}(h_{T-1}, l_{T-1}) \right\} \]  

(60)

\[ d_{T-2} \geq 0 \]  

(61)

\[ h_{T-1} = G(l_{T-2}, \Phi_t) - (1 + \rho_{T-1}) [l_{T-2} - h_{T-2} + d_{T-2}] \]

(62)

\[ -(1 + \rho_{T-1}) [l_{T-2} - h_{T-2} + d_{T-2}] \frac{1}{G(l_{T-2}, \Phi_t)} \]

The structure of the problem from period \( T - 2 \) and backwards is identical in every period; hence it can be generalized to period \( t \). The first step before applying backwards induction is to re-write the problem in terms of middle-of-period and end-of-period variables, as shown in the text, which corresponds to

\[ V_t(h_t, l_{t-1}) = \max \{ h_t - q_t + \Psi_t(q_t, l_t, l_{t-1}) \} \]  

(63)

\[ h_t - q_t \geq 0 \]  

(64)

with first order conditions

\[ 1 = \Psi_t^h(q_t, l_t, l_{t-1}) \]  

(65)

\[ 0 = \Psi_t^l(q_t, l_t, l_{t-1}) \]  

(66)

where

\[ \Psi_t^h(q_t, l_t, l_{t-1}) = E_t \beta \left[ 1 + \rho_{t+1} - f_{t+1}^h \right] V_{t+1}^h(h_{t+1}, l_t) \]  

(67)

\[ \Psi_t^l(q_t, l_t, l_{t-1}) = E_t \beta \left[ -1 - \rho_{t+1} - f_{t+1}^l \right] V_{t+1}^h(h_{t+1}, l_t) + V_{t+1}^l(h_{t+1}, l_t) \]  

s.t.

\[ h_{t+1} = G(l_{t-1}, \Phi_t) - (1 + \rho_{t+1}) [l_t - q_t] \]  

(69)

\[ -(1 + \rho_{t+1}) [l_t - q_t] \frac{(1+\rho_{t+1})[l_t-q_t]}{G(l_{t-2}, \Phi_t)} \]  

(71)
The algorithm is implemented by specifying values for \( h_t \) and \( l_{t-1} \) collected in \( \overrightarrow{h} \) and \( \overrightarrow{l} \) respectively. For each value \( l_{t-1} \in \overrightarrow{l} \), a root-finding procedure is used to determine the values of \( q_t \) and \( l_t \) that satisfy the first order conditions (65) and (66). Define these values as \( q_t \) and \( l_t \). For increased numerical accuracy on the region where the constraint on dividends is binding, \( \overrightarrow{h} \) is augmented with \( q_t \). Notice that \( q_t \) is the level of cash position at which the bank wishes to arrive by the end of the period. Recall also that the transition for outstanding loans between the middle and end of the period implies \( l_{t-1} = l_{t-1} \). Now, for every pair \( \{h_t, l_{t-1}\} \) such that \( h_t \in \overrightarrow{h} \) and \( l_{t-1} \in \overrightarrow{l} \) the solutions are obtained in the following way:

1. If \( h_t \geq q_t \) then constraint (64) is not binding, hence the optimal solutions are \( d_t = h_t - q_t \) and \( l_t = l_t \).
2. If \( h_t < q_t \) then constraint (64) is binding. Therefore the solution for dividends is \( d_t = 0 \) and the solution for lending is obtained by applying a root-finding procedure on equation (66), using the fact that the binding constraint implies that \( q_t = h_t \).\(^{36}\)

The previous steps generate sets of triples \( \{l_t, h_t, l_{t-1}\} \) and \( \{d_t, h_t, l_{t-1}\} \). Linear interpolation between these points yields numerical optimal policy rules \( d_t(h_t, l_{t-1}) \) and \( l_t(h_t, l_{t-1}) \). With these functions on hand, \( V_{h_t}^q(q_t, l_t, l_{t-1}) \) and \( V_{l_t}^q(q_t, l_t, l_{t-1}) \) are constructed in a similar way. This step completes the recursion.\(^{37,38}\) The iteration continues until the policy functions converge.

### 9.4 Deriving the normalized problem with permanent productivity shocks

The bank’s optimization problem in levels is presented in equations (72) to (77). Where \( \mathcal{J}(L_{t-1}, W_t, \Psi_{t+1}, \tau_{t+1}) \) denotes the revenue function before normalization.

\(^{36}\)The great advantage of this algorithm over a standard solution method is that it substantially reduces the number of instances where root-finding procedures are needed.

\(^{37}\)The expectations are approximated using a Gaussian quadrature. See Judd (1998).

\(^{38}\)The marginal value functions \( V_{h_t}^q(h_{t+1}, l_t) \) and \( V_{l_t}^q(h_{t+1}, l_t) \) are updated after each iteration according to:

\[
V_{h_t}^{q+1}(h, l) = V_{h_t}^{q+2}(h - d_{t+2}(h, l), l_{t+2}(h, l), l),
\]

\[
V_{l_t}^{q+1}(h, l) = V_{l_t}^{q+2}(h - d_{t+2}(h, l), l_{t+2}(h, l), l),
\]

\[
V_{h_t}^{q+2}(h - d_{t+2}(h, l), l_{t+2}(h, l), l) = E_{t+2} \left[ \left( G(l, \Phi_{t+3}) - f_{t+2} \right) V_{l_t}^h(h, l) \right]
\]
\[
\begin{align*}
Max_{\{L_t,D_t\}} \sum_{s=t}^{\infty} \beta^{s-t} D_s \\
\text{s.t. } C_t + H_t &= L_t + D_t \\
H_{t+1} &= J(L_{t-1}, W_t, \Psi_{t+1}, \tau_{t+1}) - (1 + \rho_{t+1})C_t \\
&\quad - (1 + \rho_{t+1})C_t \frac{(1+\rho_{t+1})C_t}{J(L_{t-1}, W_t, \Psi_{t+1}, \tau_{t+1})^s} \\
D_t &\geq 0 \\
W_{t+1} &= \Psi_{t+1}W_t
\end{align*}
\]

(72) \hspace{1cm} (73) \hspace{1cm} (74) \hspace{1cm} (75) \hspace{1cm} (76) \hspace{1cm} (77)

Recall that it is assumed that only the realizations of productivity corresponding to the second period of the lifetime of the loan affect entrepreneurs’ production. Hence, for loans granted in period \( t-1 \), revenues are not affected by the realization of \( \Psi_t \) and \( \tau_t \). The latter implies that \( J(L_{t-1}, W_t, \Psi_{t+1}, \tau_{t+1}) = J(L_{t-1}, W_{t-1}, \Psi_{t+1}, \tau_{t+1}) \).

Assume the bank has arrived at the last period of life \( T \) when the bank is liquidated and the entire stock of cash is distributed in dividends; then \( D_T = H_T \). The value function for period \( T \) is then given by \( V_T(H_T, L_{T-1}, W_T) = H_T \). The value function from the perspective of period \( T-1 \) is then

\[
V_{T-1}(H_{T-1}, L_{T-2}, W_{T-1}) = \max_{\{D_{T-1}, L_{T-1}\}} \{ D_{T-1} + \beta E_{T-1}[H_T] \}
\]

(78)

Defining small letters as the normalized version of capital letters by the level of endowment, that is \( \frac{H_T}{W_T} = h_T \), and using (77), the value function can be written as

\[
\begin{align*}
V_{T-1}(H_{T-1}, L_{T-2}, W_{T-1}) &= \max_{\{D_{T-1}, L_{T-1}\}} \left\{ W_{T-1} \frac{D_{T-1}}{W_{T-1}} + \beta E_{T-1}\left[ \frac{H_T}{W_T} \right] \right\} \\
V_{T-1}(H_{T-1}, L_{T-2}, W_{T-1}) &= \max_{\{D_{T-1}, L_{T-1}\}} \left\{ W_{T-1}d_{T-1} + \beta E_{T-1}[W_{T-1}\Psi_{T-1}] \right\} \\
V_{T-1}(H_{T-1}, L_{T-2}, W_{T-1}) &= \max_{\{d_{T-1}, \beta_{T-1}\}} \left\{ W_{T-1}d_{T-1} + W_{T-1}\beta E_{T-1}[\Psi_{T-1}] \right\} \\
V_{T-1}(H_{T-1}, L_{T-2}, W_{T-1}) &= W_{T-1}v_{T-1}(h_{T-1}, \frac{H_{T-2}}{W_{T-1}})
\end{align*}
\]

(79) \hspace{1cm} (80) \hspace{1cm} (81) \hspace{1cm} (82)

As of \( T-2 \) the value function is given by
The problem can now be generalized to period $t$ knowing that the original problem can always be recovered using $V_t(H_t, L_{t-1}, W_t) = W_tv_t(h_t, l_{t-1}/\beta G_t)$.

The transition equation for the bank’s net cash position is given by

\begin{align}
H_{t+1} &= J(L_{t-1}, W_t, \Psi_{t+1}, \tau_{t+1}) - (1 + \rho_{t+1})C_t - (1 + \rho_{t+1})C_t \left(\frac{(1 + \rho_{t+1})C_t}{J(L_{t-1}, W_t, \Psi_{t+1}, \tau_{t+1})}\right)^s \quad (86) \\
\frac{H_{t+1}}{W_{t+1}} &= \frac{J(L_{t-1}, W_t, \Psi_{t+1}, \tau_{t+1}) - (1 + \rho_{t+1})C_t - (1 + \rho_{t+1})C_t \left(\frac{(1 + \rho_{t+1})C_t}{J(L_{t-1}, W_t, \Psi_{t+1}, \tau_{t+1})}\right)^s W_t}{W_{t+1}} \\
h_{t+1} &= \frac{G(l_{t+1}/\beta G_t, \Psi_{t+1}, \tau_{t+1}) - (1 + \rho_{t+1})C_t - (1 + \rho_{t+1})C_t \left(\frac{(1 + \rho_{t+1})C_t}{G(l_{t+1}/\beta G_t, \Psi_{t+1}, \tau_{t+1})}\right)^s}{\Psi_{t+1}} \quad (87)
\end{align}

with $G(l_{t+1}/\beta G_t, \Psi_{t+1}, \tau_{t+1})$ denoting the normalized version of $J(L_{t-1}, W_t, \Psi_{t+1}, \tau_{t+1})$.

Equation (73) is given by $\frac{L_t + D_t}{W_t} = \frac{C_t + H_t}{W_t}$, which implies $l_t + d_t = c_t + h_t$. The constraint on dividends becomes $0 \leq d_t$. The normalized problem is then

\begin{align}
v_t(h_t, l_{t-1}/\beta G_t) &= \text{Max} \left\{ d_t + \beta E_t \Psi_{t+1} v_{t+1}(h_{t+1}, \frac{l_t}{\Psi_{t+1}}) \right\} \quad (88) \\
\text{s.t.} \quad l_t + d_t &= c_t + h_t \quad (89) \\
h_{t+1} &= \left[ G(l_{t+1}/\beta G_t, \Psi_{t+1}, \tau_{t+1}) - (1 + \rho_{t+1})C_t \left(\frac{(1 + \rho_{t+1})C_t}{G(l_{t+1}/\beta G_t, \Psi_{t+1}, \tau_{t+1})}\right)^s \right] \frac{1}{\Psi_{t+1}} \quad (90) \\
d_t &\geq 0 \quad (91)
\end{align}
Using (89) to substitute out deposits the problem can be written as

\[
v_t(h_t, \frac{l_{t-1}}{\Psi_t}) = \max_{\{d_t, l_t\}} \left\{ d_t + \beta E_t \Psi_{t+1} v_{t+1}(h_{t+1}, \frac{l_{t+1}}{\Psi_{t+1}}) \right\} (92)
\]

\[
s.t. \quad d_t \geq 0 (93)
\]

\[
h_{t+1} = \left[ G(l_{t+1}, \Psi_{t+1}, \tau_{t+1}) - (1 + \rho_{t+1})(l_t - (h_t - d_t)) - (1 + \rho_{t+1})(l_t - (h_t - d_t)) \frac{(1 + \rho_{t+1})(l_t - (h_t - d_t))}{G(\frac{l_{t+1}}{\Psi_{t+1}}, \Psi_{t+1}, \tau_{t+1})} \right] \frac{1}{\Psi_{t+1}} (94)
\]

### 9.5 Agency costs in the depositor-bank relationship

This section provides an alternative way to model credit market frictions. Following the costly state verification framework, the realization of \( \Phi \) is private information of the bank. As before, the underlying distributions of shocks are common knowledge and there are no ex-ante information asymmetries. For simplicity, bankruptcy is defined as a situation in which the book value of equity of the bank is negative. The assumption of infinitely high costs of equity finance is maintained. Then, the bank is closed down in period \( t + 1 \) if

\[
G(l_{t-1}, \Phi_{t+1}) + l_t - \tilde{i}_t c_t < 0 (95)
\]

where the left-hand side of (95) denotes the difference in book value of assets and liabilities as of the beginning of period \( t + 1 \). Where \( \tilde{i}_t c_t \) denotes maturing deposits, which include principal and interest. Ruling out states where bankruptcy is imminent,\(^\text{39}\) there will be some realization of \( \Phi \), such that the book value of equity is exactly zero; thus,

\[
G(l_{t-1}, \tilde{\Phi}_{t+1}) + l_t - \tilde{i}_t c_t = 0 (96)
\]

If the realization of \( \Phi \) is above \( \tilde{\Phi} \), the bank pays depositors the agreed amount and remains open. If not, the bank goes bankrupt, depositors seize the bank and keep the liquidation value of its assets, distributed proportionally to all depositors. The return to depositors in period \( t + 1 \) is summarized by

\(^\text{39}\)The leverage levels at which the bank goes bankrupt with probability 1 are greater than 99.5%, that is, when only 0.5% or less of loans is funded with equity.
\begin{equation}
\begin{aligned}
j(l_{t-1}, l_t, i_t, c_t, \Phi_{t+1}) = \\
\left\{
\begin{array}{ll}
i_t c_t & \text{if } \Phi_{t+1} \geq \Phi(l_{t-1}, l_t, i_t, c_t) \\
(1 - \lambda)(G(l_{t-1}, \Phi_{t+1}) + l_t) & \text{if } \Phi_{t+1} < \Phi(l_{t-1}, l_t, i_t, c_t)
\end{array}
\right.
\end{aligned}
\end{equation}

(97)

where \( \lambda \) denotes monitoring costs, with \( \lambda > u \) to emphasize the superior monitoring technology of the bank. Under risk neutrality, depositors leave the money at the bank in period \( t \) if the return satisfies a participation constraint that requires

\begin{equation}
E_t \left[ j(l_{t-1}, l_t, i_t, c_t, \Phi_{t+1}) \right] \geq E \left[ \rho_{t+1} c_t \right]
\end{equation}

(98)

where the expected return from a bank deposit should be at least as good as the opportunity cost of the depositor. Because depositors do not have other investment alternatives, equation (98) holds with equality. Otherwise, the bank can always pay a slightly lower interest rate to depositors and would still have them leaving the money at the bank. Figure A1 plots the resulting interest rate at which (98) holds with equality, assuming \( l_{t-1} = l_t = 0.39 \), which corresponds to the target values obtained in the baseline model, and for \( \lambda = 0.60 \), while all other parameters are kept unchanged relative to the baseline version.

![Figure A1. Deposit Rate](image-url)