Heterogeneous Workers and Occupations: Unemployment, Inequality and Crowding Out*

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Abstract

This paper analyzes the cyclical behavior of labor market variables that reflect agent heterogeneity across education levels and educational requirements of jobs. Using the CPS Outgoing Rotation Group, a dataset is compiled including between and within educational group wage premiums, the employment ratios of the high educated in complex and in simple occupations and of the low educated in simple occupations, the unemployment ratios of the high and low educated, as well as a measure of the crowding out of the low educated by the high educated in occupying simple jobs. A boom is accompanied by a rise in the employment in simple occupations, and a decline in the unemployment of the low educated, followed with a lag by a decline in the crowding out effect, an increase in the employment in complex occupations, and a decline in the unemployment of the high educated. Wage disparity across groups widens with a lag, while within group exhibits acyclicity. To account for these observations, a dynamic general equilibrium framework that features search frictions is developed, where the population is divided into those high and low educated, and firms post two types of vacancies: the complex that can be matched with the high educated, and the simple that can be matched with the high and low educated. On the job search is allowed. A benchmark model where all the wages are determined by Nash bargaining is compared to one where the wage of the high educated in simple occupations is indexed to that of those in complex. The latter reproduces most of the observations.

Keywords: heterogeneity, unemployment, inequality, search and matching, business cycle.

JEL Classification: E24, E32, J31, J64.

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1 Introduction

This paper attempts to enhance our understanding of the economic fluctuations in a labor market with heterogeneous agents, and can be perceived as a contribution to a literature that devoted considerable attention to documenting and analyzing the cyclical behavior of labor market variables that reflect agent heterogeneity across observable skill characteristics or educational levels. Previous studies, however, encountered two problems. First, they focus only on a few variables that can be constructed from available surveys, thus abstracting from other aspects of the labor market that cannot be captured by these variables solely. Second, they provide stylized facts without revealing the factors that are critical in generating them, which renders the set of unexplained observations as puzzles remaining to be properly addressed.

In this context, some studies focus on the distinct cyclical behavior of the wages of the skilled and the unskilled, without consideration to the impact of the flows in and out of unemployment on the wage determination process. For instance, Young (2003) and Lindquist (2004) address the behavior of the skill premium at business cycle frequencies, and conclude that the interaction between the wage determination process and frictions capable of producing equilibrium unemployment is essential to resolve the wage premium puzzle. Other studies such as Keane et al. (1988), Hoynes (2000) and Farber (2005) compare the business cycle pattern of the unemployment of the skilled to that of the unskilled, without taking into consideration the channel of job competition between the two types. The evidence for this channel is provided by Devereux (2000, 2003) who concludes that in a downturn skilled workers occupy jobs that would normally be occupied by the unskilled, and claims that job competition has a significant impact on the process of reallocation of labor.

This paper attempts to overcome the problems encountered by these studies, and its contribution is twofold. First, it compiles a dataset that is used to derive stylized facts for several variables that reflect agent heterogeneity in terms of their educational levels and the educational requirements of jobs they are occupying. This allows for a comprehensive analysis that confirms the previous findings and captures new aspects of the market that are absent in this literature. Second, it proposes a framework capable of identifying the market interactions that are critical in generating the observed behavior. This attempt emphasizes that pertinent studies do not reproduce the documented observations due to their focus on agent heterogeneity across educational levels only, thus ignoring a complementary dimension of heterogeneity across the educational requirements of jobs these agents are occupying. This two sided heterogeneity introduces additional dynamics that enables the suggested model to succeed in what other attempts fall short of accomplishing.
Using the Outgoing Rotation Group of the Current Population Survey, the participants are divided into those employed and those unemployed. The two groups are further divided into those high and low educated, where the former are those with at least some college education. The employed types are further divided into those working in complex and simple occupations, where the former are those jobs that require at least some college education. Therefore, a monthly dataset for the period from 1979 to 2004 is compiled including measures of between and within educational group wage premiums, the employment ratios of the high educated in complex occupations, the high educated in simple occupations and the low educated in simple occupations, the unemployment ratios of the high and low educated, as well as a measure of the crowding out of the low educated by the high educated in occupying simple jobs. The stylized facts suggest that an economic expansion is accompanied contemporaneously by a rise in the employment of all types in simple occupations and a decline in the unemployment of the low educated, followed with a lag by a decline in the crowding out effect, a rise in the employment in complex occupations, and a decline in the unemployment of the high educated. Finally, wage disparity across education groups widens with a lag while the within group wage premium exhibits acyclical.

In an endeavour to comprehend the underlying factors that generates this observed cyclical behavior, the paper proposes a theoretical construct with agent heterogeneity across educational levels and the educational requirements of jobs. This framework allows for competition between those distinguished by their educational levels for a job with a particular educational requirement, the crowding out of the unsuccessful by the successfully matched, the possibility of a mismatch between the educational level of the successful and the educational requirement of the job they occupy. This mismatch serves as an endogenous incentive to continue searching for a job that better suits their qualifications, allowing for job to job mobility. These dynamics are captured in a dynamic stochastic general equilibrium model that features search frictions. The paper relies on search frictions as it became the standard environment in analyzing the cyclical behavior of labor market variables ever since Merz (1995) and Andolfatto (1996) introduced two sided search into an otherwise real business cycle model and succeeded in capturing some stylized facts that the Walrasian model either resolved in an unsatisfactory manner or has not been able to address at all. Nevertheless, their adopted framework is characterized by homogeneous workers and jobs, which is not suitable in analyzing the particular variables of interest in this paper.

Therefore, this paper builds upon the subsequent progress in the literature that extends their framework to incorporate agent heterogeneity. Some of these extensions introduced one sided heterogeneity; that of either workers or jobs, while others adopted two sided heterogeneity; that of both workers and jobs. For
instance, some studies as in Acemoglu (2001), Burgess and Turon (2003) and Julien et al. (forthcoming) develop partial equilibrium models with job heterogeneity, while preserving the aspect of workers homogeneity intact, which is extended by Krause and Lubik (2004,2005) into a dynamic general equilibrium framework to study some business cycle aspects of the labor market in the United States. Other issues, however, entailed an environment that features workers heterogeneity with identical jobs, which is adopted by Burdett and Smith (2002), Ellingsen and Rosen (2003) and Mailath et al. (2000), and extended by Esteban-Pretel (2003) into a dynamic general equilibrium framework. Though one sided heterogeneity has proven a rich environment in which a preponderance of issues are addressed, other issues entailed two sided heterogeneity. This stimulated a series of other contributions in an attempt to construct suitable partial equilibrium frameworks featuring heterogeneity of workers and jobs. An extensive, yet nonexhaustive, list includes Ellingsen and Rosen (1994), Mckenna (1996), Acemoglu (1999), Marimon and Zilibotti (1999), Muysken et al. (1999), Delacroix (2000,2003), Davis (2001), Gautier (2002), Albrecht and Vroman (2002), Shi (2002), and finally Collard et al. (2003) who incorporate two sided heterogeneity into a dynamic general equilibrium framework.

This paper builds upon these latest contributions, and develops a model where the population is divided into those high and low educated, and firms post two types of vacancies: the complex that can be matched with the high educated, and the simple that can be matched with both the high and the low educated. High educated workers in simple occupations are allowed to search on the job for a complex occupation. Labor is recruited via two standard matching functions for the simple and complex vacancies. A benchmark model where all the wages are determined by Nash bargaining is compared to a modified one where the wage of the high educated in simple occupations is indexed to that of those in complex. The economy is exposed to two types of shocks: either an aggregate or a skill biased technological shock. The modified model succeeds in reproducing most of the observations. The results imply that any attempt to capture the economic fluctuations in a labor market with heterogeneous agents ought to consider the interaction between a wage determination process in which workers are rewarded over the cycle according to their education level rather than their job qualification, and a labor reallocation process that considers the possible mismatch between the employee education level and the jobs’ educational requirements.

The remainder of the paper is organized as follows: section 2 presents the stylized facts, section 3 develops the theoretical setup, section 4 includes the calibration, section 5 discusses the analysis of the results and some robustness tests, section 6 concludes, section 7 includes the data and derivations appendices. References, figures and tables are included thereafter.
2 Observations

To derive the business cycle patterns of selected labor market variables that reflect agent heterogeneity in their educational levels and the educational requirements of jobs they are occupying, a time series is compiled from the Outgoing Rotation Group of the Current Population Survey\(^1\). This survey provides monthly information from January 1979 until December 2004 on the participants’ employment status, level of education, type of occupation, weekly earnings, and weekly hours of work. To compile a time series out of this survey, the observations in each monthly file are divided into those employed and those unemployed. Each group is further divided into those high and low educated, where the former are those who obtained at least some college education. Then each of the two employed groups is further divided into those working in a complex occupation and those working in a simple occupation, where the former is a job that requires at least some college education. This provides four employed and two unemployed types: the high educated employed in a complex occupation, the high educated employed in a simple occupation, the high educated unemployed, the low educated employed in a complex occupation, the low educated employed in a simple occupation, and the low educated unemployed.

The low educated employed in a complex occupation type is dropped from the sample. For the remainder of the employed types, weighted average hourly wages are calculated as the ratio of the weighted average weekly earnings to the weighted average weekly hours for each group. Levels of employment are calculated for the three working types, and levels of unemployment are calculated for the two unemployed types. Ratios of employment and unemployment of the respective types as a proportion of the total sample are also derived. Using the three hourly wages, two measures of inequality are considered: a between and a within educational group wage premiums. The former is defined as the ratio of the weighted average wage of the two high educated types to that of the low educated in simple occupations. The latter is defined as the ratio of the wage of the high educated in a complex occupation to that of those in a simple occupation. Finally, a crowding out effect is defined as the proportion of the high educated amongst all those employed in simple occupations, such that its increase reflects an increase in the crowding out process of the low educated by the high educated in occupying this type of job. Therefore, the variables compiled and used in the analysis are as follows:

1. The within group wage premium.
2. The between group wage premium.
3. The proportion of the high educated employed in complex occupations.

\(^1\)Detailed data description is included in appendix 7.1.
(4) The proportion of the high educated employed in simple occupations.
(5) The proportion of the low educated employed in simple occupations.
(6) The proportion of the high educated unemployed.
(7) The proportion of the low educated unemployed.
(8) The crowding out effect.

The cross correlation coefficients between real gross domestic product in period $t$ and each of these variables in lag and lead periods are displayed in table 5. In addition, figures 1 – 8 depict the cyclical components of each of these variables and that of real gross domestic product. These patterns demonstrate that the within group wage premium is acyclical as the cross correlation coefficients are statistically insignificant. On the otherhand, the between group wage premium has an insignificant correlation coefficient with contemporaneous output, but is procyclical with a lag where the fourth lagged cross correlation coefficient is statistically significant with a p-value of 0.0216. In addition, the proportion of the high educated in complex occupations is procyclical and lags the cycle by 3 quarters, as the cross correlation coefficient with output reaches 0.3891 which is statistically significant with a p-value of 0.0001. The proportions of the high and the low educated employed in simple occupations are positively correlated with contemporaneous output with cross correlation coefficients of 0.3746 and 0.5055 respectively, that are statistically significant with p-values of almost zero. The proportion of the high educated unemployed is countercyclical and lags the cycle by 2 quarters where the cross correlation coefficient with output reaches $-0.4353$ and is statistically significant, while the proportion of the low educated unemployed is countercyclical with a cross correlation coefficient with contemporaneous output of around $-0.595$ and is also statistically significant. The crowding out effect is countercyclical with a lag, where the lagged cross correlation coefficients are statistically significant. These patterns are summarized as follows:

(1) The within group wage premium is acyclical.
(2) The between group wage premium is procyclical with a lag.
(3) The proportion of the high educated in complex occupations is procyclical with a lag.
(4) The proportion of the high educated in simple occupations is procyclical.
(5) The proportion of the low educated in simple occupations is procyclical.
(6) The proportion of the high educated unemployed is countercyclical with a lag.
(7) The proportion of the low educated unemployed is countercyclical.
(8) The crowding out effect is countercyclical with a lag.

Few studies, either in the applied labor realm or in the business cycle literature, attempted an inves-
tigation of the cyclical behavior of some of these variables. Though the variables in these studies are not entirely compatible with the ones in this paper, due to their reliance on different data sources and alternative methodologies, they can serve as a reference to check the validity of the stylized facts presented in this paper.

The variable that received the most attention is the between educational group wage premium, or what is usually referred to as the skill premium. The premium is considered countercyclical originally by Reder (1955) who asserts that an increase in the work force occurs largely amongst the unskilled workers, driving down their incomes relative to those of the skilled. The countercyclical tendency of the wage differentials is further emphasized by Azariadis (1976) and Kydland (1984). The dependence on aggregated data for the measurement of real wages, however, has been criticized as it obscures the effect of compositional changes in the work force over the cycle, as argued by Bils (1985) and Solon et al. (1994). Therefore, some studies resorted to longitudinal data for the analysis of the cyclical pattern of the wage premium. In this context, Raisian (1983) uses the Panel Study of Income Dynamics for the period 1967-1979, and finds that skilled workers accept procyclical wage variability in exchange for more stable employment compared to unskilled workers, which induces a procyclical wage premium. Ziliak et al. (1999) also use the Panel Study of Income Dynamics for the period 1971-1990, and find that the rates of return to schooling are procyclical for whites but tends to be countercyclical for blacks. Keane and Prasad (1993) use the National Longitudinal Survey of Young Men for the period 1966-1981, and find the premium to be uncorrelated with contemporaneous measures of the business cycle.

Recent studies relied on compiled data from the Current Population Survey. In this context, Young (2003) finds that during the period 1979-2001, the premium has significant volatility and a positive weak association with industrial production, used as an indicator for the business cycle. Castro and Coen-Pirani (2005) find that the skill premium remains acyclical and not very volatile relative to gross domestic product throughout the period from 1979 to 2003. Finally, Lindquist (2004) finds that during the period 1979-2002, the mean skill premium is weakly positively correlated, while the median skill premium to be uncorrelated with contemporaneous output, and that both lag output. These findings are consistent with the observation, presented in this paper, that the between educational group wage premium is uncorrelated with contemporaneous output and exhibits lagged procyclicality.

Other studies compared the cyclical patterns of the employment and unemployment of the high and low educated. For instance, Kydland (1984) uses the Panel Study of Income Dynamics for the period 1970-1980, and concludes that "unskilled workers in the United States labor force exhibit greater employment
fluctuations over the business cycle than do skilled workers." Similarly, Hoynes (2000) uses the Current Population Survey Outgoing Rotation Group for the period 1979-1992 and the March Annual Demographic files covering the period 1975-1997, and provides evidence that the employment rates of those with lower education exhibit more cyclical fluctuation and variability than those with higher education. These results are consistent with the higher correlation coefficients with output of the employment proportion of the low educated compared to those of the high educated.

Moreover, Keane et al. (1988) use the National Longitudinal Survey of Young Men for the period 1966-1981, and find that "those with observed characteristics corresponding to lower education are more likely to leave employment in a cyclical downturn." Farber (2005) uses the Displaced Workers Survey over the period 1981-2003, and concludes that "there is a strong cyclical pattern in job loss rates for less educated workers, but the cyclical pattern is weaker for more educated workers." These results are also consistent with the evidence provided in this paper as to the higher correlation coefficient with contemporaneous output for the unemployment ratio of the low educated compared to that of the high educated.

Other studies focused on the crowding out of the low educated by the high educated and the quality adjustment of jobs. For instance, Devereux (2000,2003) uses the Panel Study of Income Dynamics for the period 1976-1992, and shows that the quality of matches vary systematically over the business cycle, indicating that in a recession the skilled occupy jobs that would normally be occupied by the unskilled, and that the probability of making transition to employment is more cyclical for less skilled workers. This is consistent with the observation that the crowding out effect is countercyclical, such that in a recession the high educated occupy simple jobs, thus increasing their proportion amongst all those employed in simple occupations.
3 Model

Consider an economy where time is infinite and discrete. The population is of measure 1, and there is a constant fraction \( \delta \) of households that are ex ante high educated and \( (1 - \delta) \) that are low educated. The representative firm posts complex and simple vacancies. The complex vacancies can be filled by high educated workers only, while the simple vacancies can be filled by both high and low educated workers. A high educated worker in a simple occupation is allowed to perform on the job search for a complex occupation. This is justified as the two types of vacancies differ according to their creation costs, and these costs generate rents which give rise to equilibrium wage differentials between occupation types, and the implied differences in the value of employment motivate workers in simple occupations to search for employment in complex occupations. A low educated worker in a simple occupation is hired for one period only.

3.1 Households

In this context, the high and low educated household members are divided into those employed and those unemployed as follows

\[
N_{h}^{hc} + N_{h}^{hs} + U_{h} = \delta
\]

(1)

\[
N_{l}^{ls} + U_{l}^{l} = 1 - \delta
\]

(2)

where \( N_{i}^{ij} \) denotes the number of workers of education type \( i \) in occupation type \( j \), where \( i = (h, l) \) for high and low educated workers respectively, and \( j = (c, s) \) for complex and simple occupations respectively. \( U_{i} \) denotes the number of the unemployed of type \( i \). Time for the unemployed types is normalized to one. A high educated unemployed uses a portion \( s_{t} \) of its time to search for a complex occupation, and \( (1 - s_{t}) \) to search for a simple occupation. A low educated unemployed uses a portion \( g_{t} \) of its time to search for a simple occupation, and \( (1 - g_{t}) \) for domestic activities. A high educated worker in a simple occupation spends a fraction \( o_{t} \) of its leisure, which is normalized to one, to search for a complex occupation. As different employment histories amongst members of a household can lead to heterogeneous wealth positions, we follow the literature in assuming that each household is thought of as an extended family whose members perfectly insure each other against variations in labor income due to employment or unemployment. Remaining within the confines of complete markets allows solving the program of a representative household, who chooses consumption and search intensities to maximize the expected
discounted infinite sum of its instantaneous utility which is separable in consumption and leisure as follows

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \bar{U}(C_t) + N_t^{hs} \Omega (1 - o_t) \right] \right\}$$  \hspace{1cm} (3)$$

where $E_t$ is the expectation operator conditional on the information set available in period $t$, $\beta$ is the discount factor, $\bar{U}(C_t)$ is the utility of period $t$ consumption of the household $C_t$, and $\Omega (1 - o_t)$ denotes the utility of period $t$ leisure of the high educated workers in a simple occupation. Assuming the household has the following value function $\Gamma^H_t = \Gamma^H \left( N_t^{hc}, N_t^{hs}, N_t^{ls} \right)$, the optimization problem of the household can be written in the following recursive form

$$\Gamma^H_t = \max_{\{C_t, o_t, s_t, g_t\}} \left\{ \bar{U}(C_t) + N_t^{hs} \Omega (1 - o_t) + \beta E_t \left[ \Gamma^H_{t+1} \right] \right\}$$  \hspace{1cm} (4)$$

subject to the following budget constraint

$$C_t = N_t^{ls} W_t^{ls} + N_t^{hs} W_t^{hs} + N_t^{hc} W_t^{hc} + (1 - g_t) U_t^{l} W_d^{l} + D_t$$  \hspace{1cm} (5)$$

where $W_t^{ij}$ is the period $t$ wage for type $ij$, $W_t^{d}$ is the return to domestic activities and is a function of average productivity, and $D_t$ is the dividends distributed by firms. The households also take into consideration the employment dynamics of the three types of workers. The high educated employed in complex occupations in period $t + 1$ are comprised of those of that type who are not exogenously separated in period $t$ according to the separation rate from complex occupations $\chi^c$, in addition to the new matches from the searchers pool whether they are high educated unemployed or on the job searchers as follows

$$N_{t+1}^{hc} = (1 - \chi^c) N_t^{hc} + P_t^c \left( s_t U_t^{hc} + o_t N_t^{hs} \right)$$  \hspace{1cm} (6)$$

Similarly, the high educated employed in simple occupations in period $t + 1$ are comprised of those of that type who are neither separated from simple occupations exogenously in period $t$ according to the separation rate $\chi^s$, nor are matched with complex occupations as a result of on the job search, in addition to the new matches from the searchers pool of the high educated unemployed as follows

$$N_{t+1}^{hs} = [1 - \chi^s - o_t P_t^s] N_t^{hs} + P_t^s \left( (1 - s_t) U_t^{hs} \right)$$  \hspace{1cm} (7)$$

The constant separation rates are justified by Hall (forthcoming), who concludes that over the past fifty years job separation rates remained almost constant in the United States, and by Shimer (2005) who
demonstrates that separation rates exhibit acyclicality. Finally, the employment of the low educated in simple occupations considers the assumption that workers of that type in period \( t \) are hired for one period, and thus are comprised only of the new matches from the searchers pool of the low educated unemployed every period

\[
N_{l,t+1}^{ls} = P_l^s g_t U_l^t
\]  

(8)

\( P_l^c \) and \( P_l^s \) are the endogenous probabilities that a searcher is matched with a complex or a simple occupation respectively, and are defined as the ratio of the respective matches to the respective effective searchers as follows

\[
P_l^c = \frac{M_l^c}{s_t U_l^h + o_t N_{l,t}^{hs}}
\]  

(9)

\[
P_l^s = \frac{M_l^s}{(1 - s_t) U_l^h + g_t U_l^l}
\]  

(10)

\( M_l^c \) and \( M_l^s \) represent the number of complex and simple occupation matches respectively, and they are constant returns to scale homogeneous of degree one functions of the number of corresponding vacancies, \( V_l^c \) and \( V_l^s \), and effective searchers as follows

\[
M_l^c = M^c \left( V_l^c, s_t U_l^h + o_t N_{l,t}^{hs} \right)
\]  

(11)

\[
M_l^s = M^s \left( V_l^s, (1 - s_t) U_l^h + g_t U_l^l \right)
\]  

(12)

The households choose their consumption such that the marginal utility of consumption equals the Lagrange multiplier \( \lambda_t \)

\[
\frac{\partial \Omega(C_t)}{\partial C_t} = \lambda_t
\]  

(13)

They choose their on the job search intensity \( o_t \) such that the disutility from increasing on the job search intensity by one unit is offset by the difference between the discounted expected value to the household from an additional high educated worker in a complex occupation and that of an additional high educated worker in a simple occupation as follows
\[
\frac{\partial \Omega (1 - o_t)}{\partial o_t} + P^c_t \beta E_t \left[ \frac{\partial \Gamma_{t+1}^{hc}}{\partial N_{t+1}^{hc}} \right] - P^c_t \beta E_t \left[ \frac{\partial \Gamma_{t+1}^{hs}}{\partial N_{t+1}^{hs}} \right] = 0
\]

(14)

They also choose the optimal proportion of time the high educated unemployed allot to search for a complex occupation \(s_t\) such that the discounted expected value of an additional high educated in a complex occupation is equal to that of a high educated in a simple occupation as follows

\[
P^c_t E_t \left[ \frac{\partial \Gamma_{t+1}^{hc}}{\partial N_{t+1}^{hc}} \right] = P^s_t E_t \left[ \frac{\partial \Gamma_{t+1}^{hs}}{\partial N_{t+1}^{hs}} \right]
\]

(15)

Finally, the household chooses the optimal proportion of time the low educated unemployed allocate to search for a simple occupation \(g_t\) such that the return to domestic activities equals the discounted expected value of an additional low educated worker in a simple occupation as follows

\[
\lambda_t W^d_t = \beta P^s_t E_t \left[ \frac{\partial \Gamma_{t+1}^{hc}}{\partial N_{t+1}^{hc}} \right]
\]

(16)

From the envelope theorem, an additional high educated matched with a complex occupation accrue a value for the household that is given by the current value or the labor income earned by that type, in addition to the discounted expected value of the match if this worker is not separated exogenously after being matched. This also includes the value forgone, had it been that this member of the household is not matched in the first place, continued to search and got matched with either a simple or a complex vacancy as follows

\[
\frac{\partial \Gamma_{t}^{hc}}{\partial N_{t}^{hc}} = \lambda_t W_t^{hc} + \beta (1 - \chi^c) E_t \left[ \frac{\partial \Gamma_{t+1}^{hc}}{\partial N_{t+1}^{hc}} \right] - \beta P^c_t s_t E_t \left[ \frac{\partial \Gamma_{t+1}^{hc}}{\partial N_{t+1}^{hc}} \right] - \beta P^s_t (1 - s_t) E_t \left[ \frac{\partial \Gamma_{t+1}^{hc}}{\partial N_{t+1}^{hc}} \right]
\]

(17)

Similarly, an additional high educated matched with a simple occupation accrue a value for the household that is given by the current value obtained by the utility of leisure and the labor income earned by that type, in addition to the discounted expected value of the match if this additional worker is neither separated exogenously nor matched with a complex vacancy as a result of on the job search, besides the value accrued if the worker succeeds in on the job search. This also includes the value forgone if this member of the household is not matched in the first place, continued to search and got matched with either a simple or a complex vacancy as follows

\[
\frac{\partial \Gamma_{t}^{hs}}{\partial N_{t}^{hs}} = \Omega (1 - o_t) + \lambda_t W_t^{hs} + \beta (1 - \chi^s - o_t) P^c_t E_t \left[ \frac{\partial \Gamma_{t+1}^{hc}}{\partial N_{t+1}^{hc}} \right] - \beta P^c_t s_t E_t \left[ \frac{\partial \Gamma_{t+1}^{hc}}{\partial N_{t+1}^{hc}} \right] - \beta P^s_t (1 - s_t) E_t \left[ \frac{\partial \Gamma_{t+1}^{hs}}{\partial N_{t+1}^{hs}} \right]
\]

(18)
Finally, an additional low educated matched with a simple occupation accrue a value for the household that is given by the current value obtained by the labor income earned by this type, less the return to domestic activities foregone in addition to the discounted expected value if the worker is hired again after being fired as follows

\[
\frac{\partial \Gamma_H^t}{\partial N_{ls}^t} = \lambda_t W_{ls}^t - \lambda_t (1 - g_t) W_{ls}^d - \beta P_{ls}^s g_t E_t \left[ \frac{\partial \Gamma_H^{t+1}}{\partial N_{ls}^{t+1}} \right] \]  

Substituting the envelope conditions into the first order conditions yields the following household’s optimal conditions

\[
\frac{\tau P_{ls}^s}{\beta P_{ls}^s (P_{ls}^d - P_{ls}^c)} = E_t W_{ls}^{hc} + E_t \left[ 1 - \chi^c - P_{c+1}^{t+1} s_{t+1} \right] \frac{\tau P_{ls}^s}{P_{c+1}^t (P_{ls}^d - P_{ls}^c)} - E_t \frac{\tau P_{ls}^s (1 - s_{t+1})}{(P_{ls}^d - P_{ls}^c)} \]  

\[
\frac{\tau E_t (1 - o_{t+1}) + E_t W_{ls}^{hs} / C_{t+1} + E_t \frac{\tau P_{ls}^s (o_{t+1} - s_{t+1})}{P_{ls}^d - P_{ls}^c}}{\beta (P_{ls}^d - P_{ls}^c)} = E_t W_{ls}^{hc} + E_t \left[ 1 - \chi^c - o_{t+1}^{P_{ls}^d} - P_{ls}^c (1 - s_{t+1}) \right] \]  

\[
\frac{W_{ls}^d}{\beta C_t P_{ls}^d} = E_t \frac{W_{ls}^{hc} + W_{ls}^{hs} - W_{ls}^{hc} + W_{ls}^{hs} - W_{ls}^{hc} + W_{ls}^{hs}}{C_{t+1}} \]  

where \( \tau \) is the marginal utility of leisure of the high educated in simple occupations.

### 3.2 Firms

The representative firm chooses the number of complex and simple vacancies to post in order to maximize the expected discounted infinite sum of its future profit streams. The profit function is given by the difference between the value of its production, where the price of one unit of output is normalized to one, and the total cost incurred for creating the two types of vacancies, as well as the total wages given to the three working types as follows

\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \lambda_t \left[ Y_t - \omega_{c} V_t - \omega_{s} V_t - N_{h}W_{h} - N_{s}W_{s} - N_{ls}W_{ls} \right] \right\} \]  

where \( \omega_{c} \) is the cost of posting a complex vacancy and \( \omega_{s} \) is the cost of posting a simple vacancy. The discount factor of firms is given such that it effectively evaluates profits in terms of the values attached

\[\text{2 Detailed derivations are included in appendix 7.2.1.}\]
to them by households, who ultimately own the firms. Thus, the utility based and time varying discount factor used by firms is given by \( \left( \beta \frac{\lambda_{t+1}}{\lambda_t} \right) \). Assuming the firm has the following value function \( \Gamma^F_t = \Gamma^F (N^{hc}_t, N^{ls}_t, N^{hs}_t) \), the optimization problem can be written in the following recursive form

\[
\Gamma^F_t = \max_{(V^c_t, V^s_t)} \left\{ Y_t - \omega^c V^c_t - \omega^s V^s_t - N^{hc}_t W^{hc}_t - N^{hs}_t W^{hs}_t - N^{ls}_t W^{ls}_t + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \Gamma^F_{t+1} \right] \right\}
\]

(24)

The maximization is subject to the production function which is a composite of the complex occupation output \( A^h_t N^{hc}_t \) and the simple occupation output \( A^l_t N^{ls}_t + A^h_t N^{hs}_t \) given by

\[
Y_t = Y [A_t, (A^h_t N^{hc}_t), (A^l_t N^{ls}_t + A^h_t N^{hs}_t)]
\]

(25)

where \( A_t \) is the aggregate technology, \( A^h_t \) is the technology biased to high educated workers, and \( A^l_t \) is that biased to low educated workers. Bresnahan et al. (2002) provide evidence that a firm that is higher than the mean in some complementary aspects to skill biased technological change, such as information technology and workplace organization, has predicted productivity higher than the mean. In addition, Black and Lynch (2001) show that the higher the average educational level of production workers within a plant, the more likely the plant has performed better than average. This evidence provides justification to introduce a skill biased technological shock that increases the productivity of the high educated relative to that of the low educated, and in the same time increases the productivity of the firm above the mean as well. This also allows us to study the dynamic evolution of the variables of interest along with a deviation of output from its trend as a consequence of a skill biased technological shock. The maximization problem of the firm is also subject to the following employment dynamics

\[
N^{hc}_{t+1} = (1 - \chi^c) N^{hc}_t + q^c_t V^c_t
\]

(26)

\[
N^{hs}_{t+1} = [1 - \chi^s - o_t P^{c}] N^{hs}_t + q^{hs}_t V^s_t
\]

(27)

\[
N^{ls}_{t+1} = q^{ls}_t V^s_t
\]

(28)

where \( q^c_t \) is the probability of filling a complex vacancy, and is given by the ratio of complex matches to complex vacancies posted as follows
\[ q_t^c = \frac{M_t^c}{V_t^c} \]  

(29)

$q_t^{hs}$ is the probability that a simple vacancy is filled by a high educated, and is given by the ratio of simple matches to simple vacancies posted multiplied by the proportion of the high educated amongst all effective searchers for a simple occupation as follows

\[ q_t^{hs} = \frac{(1 - s_t) U_t^h}{(1 - s_t) U_t^h + g_t U_t^l} \left( \frac{M_t^s}{V_t^s} \right) \]  

(30)

and $q_t^{ls}$ is the probability that a simple vacancy is filled by a low educated, and is given by the ratio of simple matches to simple vacancies posted multiplied by the proportion of the low educated amongst all effective searchers for a simple occupation as follows

\[ q_t^{ls} = \frac{g_t U_t^l}{(1 - s_t) U_t^h + g_t U_t^l} \left( \frac{M_t^s}{V_t^s} \right) \]  

(31)

The firm chooses the optimal level of complex vacancies to post $V_t^c$, such that the expected marginal cost of posting this type of vacancy equals the discounted expected value to the firm of an additional high educated worker in a complex occupation as follows

\[ \frac{\omega^c}{q_t^c} = \beta E_t \left[ \frac{\lambda_{t+1} \partial \Gamma_{t+1}^c}{\lambda_t \partial N_{t+1}^{hc}} \right] \]  

(32)

The firm chooses the optimal level of simple vacancies to post $V_t^s$, such that the cost of posting a simple vacancy is equal to the discounted expected value of creating an occupation from this vacancy, whether it is filled by a high or a low educated worker

\[ \omega^s = \beta E_t \left[ \frac{\lambda_{t+1} \partial \Gamma_{t+1}^s}{\lambda_t \partial N_{t+1}^{hs}} \right] q_t^{hs} + \beta E_t \left[ \frac{\lambda_{t+1} \partial \Gamma_{t+1}^s}{\lambda_t \partial N_{t+1}^{ls}} \right] q_t^{ls} \]  

(33)

From the envelope theorem, the value to the firm of an additional high educated worker in a complex occupation is given by the net current value of the match, which is the difference between its marginal productivity and the bargained wage, in addition to the discounted expected value of the match in case the worker is not exogenously separated as follows

\[ \frac{\partial \Gamma_t^c}{\partial N_t^{hc}} = \frac{\partial Y_t}{\partial N_t^{hc}} - W_t^{hc} + (1 - \chi^c) \beta E_t \left[ \frac{\lambda_{t+1} \partial \Gamma_{t+1}^c}{\lambda_t \partial N_{t+1}^{hc}} \right] \]  

(34)
Similarly, the value to the firm of an additional high educated worker in a simple occupation is given by the net current value of the match, or the difference between its marginal productivity and the bargained wage, in addition to the discounted expected value of the match in case the worker is neither exogenously separated nor matched with a complex occupation as a result of on the job search as follows

\[
\frac{\partial \Gamma^F_t}{\partial N^h_t} = \frac{\partial Y_t}{\partial N^h_t} - W^h_t + (1 - \chi^t - \phi_t P^c_t) \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \Gamma^F_{t+1}}{\partial N^h_{t+1}} \right] \tag{35}
\]

Finally, the value to the firm of an additional low educated worker in a simple occupation is given by the net current value of the match, or the difference between its marginal productivity and the bargained wage, and as the low educated is separated from the simple occupation at the end of each period, the match has no future value as follows

\[
\frac{\partial \Gamma^F_t}{\partial N^l_t} = \frac{\partial Y_t}{\partial N^l_t} - W^l_t \tag{36}
\]

Substituting the envelope conditions into the first order conditions yields the following firm’s optimal conditions\(^3\)

\[
\frac{\omega^c}{q^c_t} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\partial Y_{t+1}}{\partial N^{hc}_{t+1}} - W^h_{t+1} + (1 - \chi^t) \frac{\omega^c}{q^c_{t+1}} \right) \right] \tag{37}
\]

\[
\varepsilon_t = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\partial Y_{t+1}}{\partial N^{hs}_{t+1}} - W^h_{t+1} + (1 - \chi^t - \phi_t P^c_t) \varepsilon_{t+1} \right) \right] \tag{38}
\]

Where \(\varepsilon_t\) is defined as

\[
\varepsilon_t = \frac{\omega^c}{q^c_t} - \frac{q^l_t}{q^h_t} \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\partial Y_{t+1}}{\partial N^{ls}_{t+1}} - W^l_{t+1} \right) \right] \tag{39}
\]

### 3.3 Wages and Premiums

In equilibrium, matched firms and workers obtain from the match a total return that is strictly higher than the expected return of unmatched firms and workers because if they separate each will have to go through an expensive and time consuming process of search before being matched again. We follow the literature in assuming that a realized match share this surplus through a Nash bargaining problem. Therefore, the wage of a high educated worker in a complex occupation is given by\(^4\)

\[
W^h_t = \left( 1 - \xi^h_t \right) \left[ \frac{\partial Y_t}{\partial N^h_t} + P^c_t \frac{\omega^c}{q^c_t} \right] + \xi^h_t \left[ \frac{\Gamma^F_t}{P^c_t} \left( 1 - s_t \right) \right] \tag{40}
\]

\(^3\)Detailed derivations are included in appendix 7.2.2.

\(^4\)Detailed derivations are included in appendix 7.2.3.
ξ_{hc} is the firm’s share of the surplus. The wage is a weighted average of two terms: the first indicates that the worker is rewarded for a fraction \((1 - \xi_{hc})\) of both the firm’s revenues from the worker’s productivity and the discounted expected value of the match for the firm. The second term indicates that the worker is compensated by a fraction \(\xi_{hc}\) for the foregone benefit from the worker’s outside option of being matched with a simple occupation, expressed in terms of the consumption good. Similarly, the bargained wage of the high educated in a simple occupation is given by:

\[
W_{hs}^t = \left(1 - \xi_{hs}\right) \left[ \frac{\partial Y_t}{\partial N_{hs}^t} + P_t^s \left(1 - s_t\right) \varepsilon_t \right] + \xi_{hs} \left[ C_t \left( \tau P_t^h \left( s_t - o_t \right) \right) / \left( P_t^s - P_t^c \right) - C_t \tau \left(1 - o_t\right) \right]
\] (41)

\(\xi_{hs}\) is the firm’s share of the surplus. The wage is a weighted average of two terms: the first indicates that the worker is rewarded by a fraction \((1 - \xi_{hs})\) for the firm’s revenues from the worker’s productivity. The second term indicates that the worker is compensated by a fraction \(\xi_{hs}\) for the foregone return to domestic activities.

Other variables of interest are defined in order to analyze wage inequality in this economy. The within group wage premium is defined as the ratio of the wage of the high educated in complex occupations to that of those in simple occupations as follows:

\[
Within_t = \frac{W_{hc}^t}{W_{hs}^t}
\] (43)

Furthermore, the between group wage premium is defined as the ratio of the weighted average wage of the two types of the high educated \(W_t^h\), to the wage of the low educated in simple occupations. The weighted average wage of the two types of the high educated is defined as

---

5 Detailed derivations are included in appendix 7.2.4.
6 Detailed derivations are included in appendix 7.2.5.
W_t^h = \frac{N_t^hcW_t^{hc} + N_t^hsW_t^{hs}}{N_t^hc + N_t^hs} \tag{44}

and the between group wage premium is given by

\[\text{Between}_t = \frac{W_t^h}{W_t^{ls}} \tag{45}\]

3.4 Equilibrium

The equilibrium in this economy is an allocation that satisfies equations (1) and (2) that defines the division of household members, the employment dynamics (6),(7),(8), the endogenous probabilities that searchers match with vacancies (9),(10), and the probabilities that vacancies are filled (29),(30),(31), the matching functions (11),(12), the household’s optimal conditions (20),(21),(22), the production function (25) and the technological laws of motion, the firm’s optimal conditions (37),(38),(39), the bargained wages (40),(41),(42), the wage premiums (43),(44),(45), in addition to the return to domestic activities as a function of average productivity given by

\[W_t^d = W^d\left[\frac{Y_t}{N_t^hc + N_t^hs + N_t^{ls}}\right] \tag{46}\]

Besides the crowding out effect given by the proportion of the high educated in simple occupations amongst all workers in this type of occupation given by

\[\text{Crowding}_t = \frac{N_t^hs}{N_t^hs + N_t^{ls}} \tag{47}\]

and finally the goods market clearing condition

\[Y_t = C_t + \omega^cV_t^c + \omega^sV_t^s \tag{48}\]
4 Calibration

The functional forms are determined and the parameters are calibrated in order to solve the model numerically. The instantaneous utility function of consumption is represented by the logarithm of consumption expenditures as follows

\[ \mathcal{U}(C_t) = \ln(C_t) \]  
(49)

The instantaneous utility function of leisure of a high educated worker in a simple occupation is defined as

\[ \Omega(1 - o_t) = \tau (1 - o_t) \]  
(50)

The matching functions for the complex and simple occupations are represented as a Cobb-Douglas specification with constant returns to scale as follows

\[ M_c^t = m_c (V_c^t)\gamma_c \left( s_t U_t^h + o_t N_t^{hs} \right)^{1-\gamma_c} \]  
(51)

\[ M_s^t = m_s (V_s^t)\gamma_s \left( (1 - s_t) U_t^h + g_t U_t^l \right)^{1-\gamma_s} \]  
(52)

Where \( \gamma_c \in (0, 1) \) and \( \gamma_s \in (0, 1) \) are the elasticities of matching with respect to complex and simple vacancies respectively, while \( m_c \) and \( m_s \) are the level parameters of the two functions and capture all factors that influence the efficiency of matching. The technological constraints faced by the representative firm is also represented by a constant returns to scale Cobb-Douglas function with elasticity of output with respect to the complex occupation output \( \mu \in (0, 1) \), as follows

\[ Y_t = A_t \left( A_t^h N_t^{hc} \right)^{\mu} \left( A_t^h N_t^{hs} + A_t^l N_t^{ls} \right)^{1-\mu} \]  
(53)

The production function features three technologies, the aggregate technology \( A_t \), whose logarithm is assumed to follow an AR(1) process as follows

\[ \log A_{t+1} = \rho^A \log A_t + \epsilon^A_{t+1} \]  
(54)

Following Acemoglu (2002), we also assume two other types of technologies: one representing the productivity of the high educated \( A_t^h \), and the other that of the low educated \( A_t^l \). The logarithms of the two types
are also assumed to follow an AR(1) process as follows

$$logA_{t+1}^h = \rho^A h^t + \alpha^h_{t+1}$$

(55)

$$log A^l_{t+1} = \rho^A l^t + \alpha^l_{t+1}$$

(56)

The $\alpha^h_{t+1}$, $\alpha^A_{t+1}$ and $\alpha^l_{t+1}$ are independently and identically distributed random variables drawn from a normal distribution with mean zero and standard deviations denoted by $\sigma_A$, $\sigma_{Ah}$, and $\sigma_{Al}$ respectively. Finally, the return to domestic activities undertaken by the low educated unemployed $W^d_t$ is a function of the average productivity as follows

$$W^d_t = \psi \left[ \frac{Y_t}{N^l_t + N^hs_t + N^hc_t} \right]$$

(57)

Table 4 shows the values chosen for the parameters of the model. The first group includes those related to households such as the fixed proportion of the high educated in the population $\delta$ which is set at 0.5, close to the data average of the year 2004 given by 0.506697. In addition to the household’s discount factor $\beta$ which is given by 0.98, and the parameter $\tau$ in the utility of leisure function given by 1.08. The second set pertains to the matching technology, where the level parameters in the matching functions $m^c$ and $m^s$ are given by 0.3 and 0.6 respectively. We follow the literature in setting the elasticity of matches with respect to vacancies $\gamma^c$ and $\gamma^s$ to 0.6. The separation rates $\chi^c$ and $\chi^s$ from the complex and simple occupations are given by 0.045 and 0.02 respectively. These are selected such that their average given by 3.256% is consistent with the weighted average separation rate of 3.23% calculated by Hall (forthcoming). The third set includes the technological parameters, where the elasticity parameter in the production function $\mu$ is given by 0.5. The autoregressive coefficients in the technological laws of motion $\rho^A$, $\rho^{Ah}$ and $\rho^{Al}$ are given by 0.9, 0.95 and 0.95 respectively, and the standard deviation of the three technologies $\sigma_{A}, \sigma_{Ah}$ and $\sigma_{Al}$ are given by 0.0049, 0.002 and 0.002 respectively. The fourth set includes the parameters in the bargained wages and the return to domestic activities equations, where the firm’s share of the surplus $\xi^{hc}$, $\xi^{hs}$ and $\xi^{ls}$ are set around 0.5, 0.58 and 0.5 respectively. The parameter $\psi$ is set such that the return to domestic activities is 0.16 times the average productivity. Finally, the costs of creating the complex vacancy $\omega^c$ and the simple vacancy $\omega^s$ are given by 1.043 and 0.08 respectively.
5 Analysis

The model exposited is considered as a benchmark. A comparison of the results of the benchmark model to a modified version that assumes the wage of the high educated in a simple occupation is indexed to those in a complex occupation such that $W_{hs}^t = \alpha W_{hc}^t$ is considered. This is undertaken as both the benchmark and the modified models are exposed to two types of exogenous shocks: either an aggregate or a skill biased technological shock. The model is solved by computing the nonstochastic steady state around which the equation system is linearized. The resulting model is solved by the methods developed in Sims (2002).

5.1 Benchmark Model

5.1.1 Aggregate Technological Shock

The impulse responses in figures 9 and 10 show that if the economy is exposed to a positive one percentage aggregate technological shock, the productivity of all workers increases and total output increases. Average productivity rises, inducing the low educated unemployed to reduce their search intensity for simple occupations and increase their domestic activities whose return is proportional to average productivity. Consequently, the proportion of the low educated employed declines while the proportion of those unemployed increases. In addition, the bargained wage of the low educated in simple occupations, which is a weighted average of their productivity and the return to domestic activities, increases due to the increase in both its components. Firms respond by reducing simple vacancies posted, while increasing complex vacancies posted. This induces the high educated unemployed to increase their search for the more available complex vacancies. To avoid competition with the unemployed searchers, the high educated working in simple occupations respond by reducing their on the job search. Despite this, the proportion of the high educated in simple occupations declines as the continuous constant separations more than offset the modest level of new matches. The increase in employment of the high educated in complex occupations is overshadowed by the decline in the proportion of those in simple, thus increasing the proportion of the high educated unemployed. Though the proportion of both types of workers employed in a simple occupation declines, the reduction in the low educated amongst them is higher than that of the high educated, and thus the crowding out effect increases. In this context, total vacancies decline while overall unemployment increases.

As the proportion of the high educated in simple occupations declines, the probability that a simple vacancy is matched with a high educated worker decreases and thus the discounted expected value to the
firm of an additional worker of this type increases. This, in addition to the increase in their productivity, leads to an increase in the wage of the high educated in simple occupations. Furthermore, as the proportion of the high educated workers in complex occupations increases, the probability that a complex vacancy is matched with a high educated increases, and thus the discounted expected value to the firm of an additional worker of this type declines. This offsets any increase in the productivity of the high educated in complex occupations, and thus reduces their bargained wage. Nevertheless, the weighted average wage of the two high educated working types increases. This, along with the increase in the wage of the low educated, leads to a decline in the between group wage premium. The within group wage premium declines as the wage of the high educated in simple occupations increases, while that of those in complex declines.

Obviously, these dynamic responses are not compatible with the observations. This is supported by the cross correlations in table 6 that demonstrate the following patterns:

1. The within group wage premium is countercyclical with a lag.
2. The between group wage premium is countercyclical with a lag.
3. The proportion of the high educated in complex occupations is procyclical with a lag.
4. The proportion of the high educated in simple occupations is countercyclical with a lag.
5. The proportion of the low educated in simple occupations is countercyclical with a lag.
6. The proportion of the high educated unemployed is procyclical with a lag.
7. The proportion of the low educated unemployed is procyclical with a lag.
8. The crowding out effect is procyclical with a lag.
9. The overall unemployment ratio is procyclical with a lag.
10. Total vacancies is countercyclical with a lag.

As this experiment encountered problems in accounting for the observations of interest, a different one is attempted where the economy is exposed to another type of technological shock that affects the productivity of the two types of workers disproportionately.

5.1.2 Skill Biased Technological Shock

As in Acemoglu (2002), a skill biased technological shock is defined as one that increases the productivity of high educated workers relative to that of low educated. The impulse responses in figures 11 and 12 demonstrate that this shock improves upon the results obtained from exposing the model to an aggregate technological shock. As the high educated work in both complex and simple occupations, total output increases as a consequence of a skill biased technological shock. This rise in the productivity of the high
educated induces firms to increase their posting of both simple and complex vacancies. Due to on the job search activities, the high educated unemployed, facing competition from the high educated employed searchers, prefer to increase their search intensity for simple occupations rather than for complex. Thus, the proportion of high educated workers in simple occupations increase, while the proportion of those in complex declines. On the otherhand, the high educated employed in simple occupations increase their on the job search intensity, and thus with a lag the proportion of the high educated in complex occupations starts to increase, and as more of those working in simple occupations start being matched with complex occupations, the proportion of the high educated in simple occupations declines. At any point in time, the proportion of the high educated unemployed declines. As the simple vacancies increase, and the return to domestic activities declines, the low educated increase their search intensity for employment and thus the probability of being matched. The proportion of the low educated in a simple occupation increases, thus reducing the proportion of the low educated unemployed. Though the proportions of both the high and low educated working in a simple occupation rise, the increase in the former is less than in the latter. Consequently, the crowding out effect declines. The results produce a Beveridge curve as the overall unemployment ratio declines while total vacancies rise.

Average productivity declines due to a disproportionate increase in the low educated workers, whose productivity declines. As the productivity of the low educated employed declines due to the skill bias of the technological shock, and as the return to domestic activities declines being proportional to decreasing average productivity, the bargained wage of the low educated in a simple occupation declines. Similarly, as the probability that a simple vacancy is matched with a high educated increases, especially as the high educated unemployed increase their search for simple occupations, the value of an additional worker of this type for the firm declines. This offsets the increase in their productivity and thus reduces their wage. On the otherhand, due to the increase in the productivity of the high educated in a complex occupation and the increase in market tightness, their wage increases. From the impulse responses, as the weighted average wages of the two types of the high educated rises, and as the wage of the low educated in simple occupations declines, the between group wage premium increases. Similarly, as the gap between the wages of the two high educated types widens, the within group wage premium increases as well. The cross correlations of an artificial time series that is exposed to a skill biased technological shock are included in table 7, and can be summarized as follows:

(1) The within group wage premium is procyclical.
(2) The between group wage premium is procyclical.
(3) The proportion of the high educated in complex occupations is procyclical with a lag.

(4) The proportion of the high educated in simple occupations is procyclical.

(5) The proportion of the low educated in simple occupations is procyclical.

(6) The proportion of the high educated unemployed is countercyclical.

(7) The proportion of the low educated unemployed is countercyclical.

(8) The crowding out effect is countercyclical.

(9) The overall unemployment ratio is countercyclical.

(10) Total vacancies is procyclical.

Though introducing a skill biased technological shock improves upon the results obtained from an aggregate productivity shock, there is one discrepancy with the observations that the within group wage premium increases as a response to this shock. This is inconsistent with the acyclicality of the premium observed in the data. Besides, the cross correlations are magnified in the model compared to the data even though they show consistent signs.

5.2 Modified Model

Besides the indexation assumed between the wage of the high educated in simple occupations and that of those in complex, the modified model sets the return to domestic activities as a constant proportion $\psi$ of the steady state level of average productivity. This serves to reduce the incentive for the low educated unemployed to resort to domestic activities, and thus leads the employment and unemployment ratios of this type to be compatible with the observations. The modified model with either an aggregate or a skill biased technological shock proves to be a success compared to the benchmark model. Not only is the behavior of its variables more compatible with the cyclical patterns of their counterparts in the data, but the magnitudes of the correlations are closer to those displayed by the data.

5.2.1 Aggregate Technological Shock

The impulse response functions to a one percentage aggregate technological shock are depicted in figures 13 and 14. In this context, an aggregate technological shock increases average productivity. However, as the return to domestic activities is assumed constant, the low educated unemployed increase their search intensity for simple occupations as long as the discounted expected value of an additional worker of this type to households is higher than this constant alternative. The firms respond by increasing simple vacancies. Therefore, the proportion of the low educated employed increases, while the proportion of the unemployed
declines. The high educated unemployed increase their search intensity for simple occupations, and firms respond by reducing complex occupations. The high educated in a simple occupation reduce their on the job search for a complex occupation. The two factors lead to a decline in the proportion of the high educated in complex occupations, and an increase in the proportion of those in simple occupations. The increase in the latter offset the decline in the former, leading to a decline in the proportion of the high educated unemployed. The crowding out effect exhibits countercyclicality as the increase in the proportion of the high educated in simple occupations is less than that of the low educated. The results produce a Beveridge curve as the overall unemployment ratio declines along with an increase in the total vacancies. The wages of the two high educated types change in the same direction due to the indexation assumed leading to an acyclical pattern of the within group wage premium. On the otherhand, the increase in the weighted average wages of the high educated employed is larger compared to the increase in the wage of the low educated, leading to a procyclical between group wage premium. The cyclical behavior reflected by the relevant cross correlations in table 8 is summarized as follows:

1. The within group wage premium is acyclical.
2. The between group wage premium is procyclical.
3. The proportion of the high educated in complex occupations is countercyclical with a lag.
4. The proportion of the high educated in simple occupations is procyclical with a lag.
5. The proportion of the low educated in simple occupations is procyclical.
6. The proportion of the high educated unemployed is countercyclical with a lag.
7. The proportion of the low educated unemployed is countercyclical.
8. The crowding out effect is countercyclical.
9. The overall unemployment ratio is countercyclical.
10. Total vacancies is procyclical.

5.2.2 Skill Biased Technological Shock

The impulse response functions to a one percentage skill biased technological shock are depicted in figures 15 and 16. In this context, the low educated unemployed, faced by a constant return to domestic activities, increase their search for a simple occupation as long as the wage earned from this type of job is higher than the constant alternative. The firms respond by increasing simple vacancies. The high educated unemployed increase their search intensity for simple occupations, and firms respond by reducing complex occupations. The high educated in a simple occupation reduce their on the job search for a complex occupation. The
proportion of the high educated in complex occupations decline, while that of the two employed types in simple occupations increases. As the increase in the low educated amongst them is larger than that of the high educated, the crowding out effect declines. The unemployment proportions of the high and low educated decline. A Beveridge curve is generated as the overall unemployment ratio declines along with a rise in total vacancies.

The wage of the high educated in complex occupations increases as their productivity increases, and as market tightness increases. The wage of the other high educated type follow suit as a consequence of the indexation assumed. Therefore, the within group wage premium does not respond to the shock. The wage of the low educated in simple occupations decline due to their low productivity. As the weighted average wage of the high educated employed increases, the between group wage premium increases. The cyclical behavior reflected by the relevant cross correlations in table 9 is summarized as follows:

(1) The within group wage premium is acyclical.
(2) The between group wage premium is procyclical.
(3) The proportion of the high educated in complex occupations is countercyclical with a lag.
(4) The proportion of the high educated in simple occupations is procyclical with a lag.
(5) The proportion of the low educated in simple occupations is procyclical.
(6) The proportion of the high educated unemployed is countercyclical with a lag.
(7) The proportion of the low educated unemployed is countercyclical.
(8) The crowding out effect is countercyclical.
(9) The overall unemployment ratio is countercyclical.
(10) Total vacancies is procyclical.

5.3 Robustness

The robustness of the results of the model is examined to check whether the dynamic evolution of the variables of interest are sensitive to the chosen calibration of the parameters. In the benchmark model, the response to an aggregate technological shock is influenced to a large extent by the increase in average productivity, which raises the return to domestic activities inducing the low educated unemployed to reduce their search for employment, and consequently increasing their unemployment. The first robustness test targets the parameter $\psi$. Reducing the parameter allows the return to domestic activities to be less sensitive to average productivity. The results, however, are not altered as though the outside options for the low educated unemployed with a lower $\psi$ are not as tempting after an increase in average productivity, the
alternative given by the bargained wage remains proportional to the return to domestic activities, and thus does not serve as a sufficient incentive for the low educated unemployed to increase their search intensity for employment.

The other parameter that influences the results is the separation rate from simple occupations $\chi^s$. As shown in the analysis, though the high educated in simple occupations reduce their on the job search for fear of competition from unemployed searchers, their proportion declines as the constant separations offset the modest level of matches. This in return contributes to the increase in the unemployment of the high educated, which contradicts the observations. Therefore, a reduction of $\chi^s$ is considered to examine the robustness of these results. The results are not affected even after a considerable reduction of this parameter.

Considering the skill biased technological shock impulses, the factor that influences the results is on the job search, as the decision by the high educated unemployed to channel their search activities towards the simple occupations is influenced by the fear from competition with on the job searchers. This reduces the proportion of workers in complex occupations, which contradicts the observations. Therefore, constricting job to job mobility is attempted by assuming that only a proportion $\gamma$ of the successful on the job searchers are allowed to be matched. Thus, the employment dynamics of the two types of the high educated workers are changed such that

$$N_{t+1}^hc = (1 - \chi^c) N_{t}^hc + P^c_t \left[s_t U_t^h + \gamma_o t N_{t}^{hs}\right]$$

$$N_{t+1}^{hs} = [1 - \chi^s - \gamma_o t P^c_t] N_{t}^{hs} + P^s_t \left[(1 - s_t) U_t^h\right]$$

where $\gamma$ is set at 0.7, which means that only 70% of those who succeed in on the job search are actually permitted to reallocate to a complex occupation. Though this assumption reduces the competition from on the job searchers, it does not change the results.
6 Conclusion

This paper analyzes the cyclical behavior of selected labor market variables that reflect agent heterogeneity in educational levels and the educational requirements of jobs they are occupying. A set of stylized facts imply that an economic expansion is accompanied contemporaneously by a rise in the employment of all types in simple occupations, and a decline in the unemployment of the low educated, followed with a lag by a decline in the crowding out effect, an increase in the employment in complex occupations, and a decline in the unemployment of the high educated. Finally, wage disparity across education groups widens with a lag, while the within group wage premium exhibit acyclicality. In order to comprehend the factors behind the evolution of these patterns, a model is developed where workers of heterogeneous education levels search for two types of vacancies: complex and simple, and on the job search is allowed. This environment where wages are determined through a Nash bargaining process is compared to a modified one in which the wage of the high educated in complex occupations is indexed to that of those in simple. The modified version accounts for most of the observations.

The results suggest that any comparison between the experiences of the high and the low educated over the cycle in terms of their wages, employment probabilities and exposure to unemployment, ought to consider the interaction between two processes. The first is a wage determination process in which the workers are rewarded according to their education level rather than their job qualification, and the second is a labor reallocation process that takes into consideration the possible mismatch between the education level of the employees and their job qualification.

An agenda for future work includes examining the implications of monetary policy shocks on labor market variables that reflect the prevalent agent heterogeneity. In addition, the introduction of skill acquisition and obsolescence in a model of human capital investment allows to address the effects of the relative increase in the supply of the skilled, which makes it profitable to create jobs designed specifically for this type. Consequently, the composition of jobs undergoes a qualitative change altering the structure of wages and unemployment. In addition, investment in physical capital can be introduced with the feature of capital skill complementarity. This is expected to have interesting implications as it provides the households with an alternative channel for intertemporal consumption smoothing other than creating vacancies, and thus can have an effect on the creation and composition of vacancies in the economy.
7 Appendix

7.1 Data

The data set used is the Outgoing Rotation Group of the Current Population Survey. The Current Population Survey is a rotating panel. After the fourth month in the survey, the participants take an eight month hiatus. Afterwards, they are interviewed for another four months, and after the eighth month in sample, they are completely dropped from the survey. The Merged Outgoing Rotation series is a collection of the 4th and 8th month-in-sample groups from all 12 months. These two groups play a special role as they are given additional questions, the answers to which are collected in the Outgoing Rotation Group files. The data is monthly and covers the period from January 1979 until December 2004. At the end of each year, the 12 monthly files for January through December are concatenated into a single annual file. The variables extracted are as follows

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>MONTH</td>
<td>Month of interview</td>
</tr>
<tr>
<td>BEMP2</td>
<td>Employed persons excluding farm and private household workers</td>
</tr>
<tr>
<td>GRDHI</td>
<td>Highest grade attended</td>
</tr>
<tr>
<td>GRDATN</td>
<td>Educational attainment</td>
</tr>
<tr>
<td>OCC</td>
<td>Occupation of job last week</td>
</tr>
<tr>
<td>ERNWKC</td>
<td>Weekly earnings before deductions (1979-1988)</td>
</tr>
<tr>
<td>HOURS</td>
<td>Total hours worked last week</td>
</tr>
<tr>
<td>ERNWGT</td>
<td>Earnings weight</td>
</tr>
</tbody>
</table>

Table 1: Extracted variables

Each annual file is divided into monthly files according to the variable MONTH. For each monthly file, observations are split into those employed and those unemployed according to BEMP2. Both the employed and the unemployed are further split into high educated and low educated households, where the high educated are the workers who obtained some college education or higher. The following table shows the variables’ ranges defining the high and low educated
Period | High Educated | Low Educated
---|---|---
1979-1988 | 14≤GRDHI≤19 | 1≤GRDHI≤13
1989-1991 | 13≤GRDHI≤18 | 1≤GRDHI≤12
1992-2004 | 40≤GRDATN≤46 | 31≤GRDATN≤39

Table 2: Ranges for the high and low education levels

Each worker group, the high or the low educated, is further divided into two groups: those employed in complex occupations and those employed in simple occupations. The complex and simple occupations are defined by the ranges of the variable OCC specified as follows:

<table>
<thead>
<tr>
<th>Period</th>
<th>Complex Occupation</th>
<th>Simple Occupation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1979-1982</td>
<td>1 – 85</td>
<td>86 – 90</td>
</tr>
<tr>
<td></td>
<td>91 – 96</td>
<td>100 – 101</td>
</tr>
<tr>
<td></td>
<td>102 – 246</td>
<td>260 – 995</td>
</tr>
<tr>
<td></td>
<td>178 – 242</td>
<td>243 – 991</td>
</tr>
<tr>
<td>1992-2002</td>
<td>0 – 163</td>
<td>164 – 165</td>
</tr>
<tr>
<td></td>
<td>166 – 173</td>
<td>174 – 177</td>
</tr>
<tr>
<td></td>
<td>178 – 242</td>
<td>243 – 999</td>
</tr>
<tr>
<td></td>
<td>2100 – 3650</td>
<td>3700 – 9830</td>
</tr>
</tbody>
</table>

Table 3: Ranges for the complex and simple occupation types

Therefore, we have four employed and two unemployed types as follows:

(1) The high educated employed in a complex occupation.
(2) The high educated employed in a simple occupation.
(3) The high educated unemployed.
(4) The low educated employed in a complex occupation.
(5) The low educated employed in a simple occupation.
(6) The low educated unemployed.

The weighted average of the variables on weekly earnings and hours worked last week for each of the working groups are calculated using the proper weights ERNWGT. These weights are created for each month such that, when applied, the resulting counts are representative of the national counts. Thus the
proper application of weights enables the results to be presented in terms of the population of the United States as a whole, instead of just the participants in the survey.

The hourly wage of each worker type is calculated as the ratio of the weighted average weekly earnings to the weighted average hours worked last week for each group. These derived wages are used to calculate selected measures of wage disarary. The within group wage premium defined as the ratio of the weighted average hourly wage of the high educated in complex occupations to that of those in simple occupations. Also, the between group wage premium is defined as the ratio of the weighted average hourly wage of the two types that are high educated to that of the low educated in simple occupations. To calculate measures of employment and unemployment ratios, the binary variable BEMP2 is used to distinguish the two groups. The employed are divided into four types as explained earlier. The ratios of the employed and the unemployed types to the total sample are calculated by summing over the weights in each type, and dividing by the sum of the weights of the total sample. The crowding out effect is calculated as the proportion of the high educated amongst all those employed in simple occupations. Finally, the variables compiled and used in the analysis are as follows:

1. The weighted average hourly wage of the high educated in complex occupations.
2. The weighted average hourly wage of the high educated in simple occupations.
3. The weighted average hourly wage of the low educated in simple occupations.
4. The within group wage premium.
5. The between group wage premium.
6. The proportion of the high educated employed in complex occupations.
7. The proportion of the high educated employed in simple occupations.
8. The proportion of the low educated employed in simple occupations.
9. The proportion of the high educated unemployed.
10. The proportion of the low educated unemployed.
11. The crowding out effect.

Finally, the Real Gross Domestic Product data (Chained Dollars, seasonally adjusted at annual rates) is extracted from the National Income and Product Accounts NIPA. As the Gross Domestic Product data is quarterly, these monthly time series are transformed into quarterly ones by taking three months averages. All variables except employment and unemployment ratios are logged, while all variables are detrended using the Hodrick Prescott filter with a smoothing parameter of 1600.
7.2 Derivations

7.2.1 Household’s Optimal Conditions

From (15) we have

\[ E_t \left[ \frac{\partial \Gamma^H_{t+1}}{\partial N_{hc}^{t+1}} \right] = \frac{P_s^t}{P_c^t} E_t \left[ \frac{\partial \Gamma^H_{t+1}}{\partial N_{hs}^{t+1}} \right] \]  

(58)

Substituting (58) in (14) yields

\[ E_t \left[ \frac{\partial \Gamma^H_{t+1}}{\partial N_{hc}^{t+1}} \right] = \frac{\tau}{\beta (P_s^t - P_c^t)} \]  

(59)

Substituting (59) in (58) yields

\[ E_t \left[ \frac{\partial \Gamma^H_{t+1}}{\partial N_{hc}^{t+1}} \right] = \frac{\tau P_s^t}{\beta P_c^t (P_s^t - P_c^t)} \]  

(60)

Substituting the envelope condition (17) in (60) yields the first optimal condition (20), while substituting the envelope condition (18) in (59) yields the second optimal condition (21) of households. Finally, from (16) we have

\[ E_t \left[ \frac{\partial \Gamma^H_{t+1}}{\partial N_{ls}^{t+1}} \right] = \frac{\lambda_t W_d^t}{\beta P_s^t} \]  

(61)

Substituting the envelope condition (19) in (61) yields the third optimal condition of households (22).

7.2.2 Firm’s Optimal Conditions

Substituting the envelope condition (34) in the first order condition (32) yields the first optimal condition of firms

\[ \frac{\omega^c}{q_t^c} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\partial Y_{t+1}}{\partial N_{hc}^{t+1}} - W_{t+1}^{hc} + (1 - \chi^c) \frac{\omega^c}{q_t^{t+1}} \right) \right] \]

Substituting the envelope condition (36) in the first order condition (33) yields

\[ \omega_s = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial Y_{t+1}}{\partial N_{ls}^{t+1}} q_t^{ls} + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\partial Y_{t+1}}{\partial N_{ls}^{t+1}} - W_{t+1}^{ls} \right) \right] q_t^{ls} \right] \]

Which can be rearranged as
\[ \varepsilon_t = \omega^s - q_{hs}^t \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\partial Y_{t+1}}{\partial N_{t+1}^{hs}} - \psi_{ls}^{t+1} \right) \right] = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial F_{t+1}^s}{\partial N_{t+1}^{hs}} \right] \]

Substituting the envelope condition (35) yields the second optimal condition of firms

\[ \varepsilon_t = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\partial Y_{t+1}}{\partial N_{t+1}^{hs}} - \psi_{hs}^{t+1} + (1 - \chi^s - \psi_{s}^{t+1}) \varepsilon_{t+1} \right) \right] \]

7.2.3 The wage of high educated workers in complex occupations

The surplus accrued by the household is expressed in terms of goods rather than marginal utility, and reduces to \( \left[ \frac{1}{\lambda_t} \frac{\partial \Gamma_H}{\partial N_{t}^{hc}} \right] \) in order to guarantee that both the firm’s surplus and the household’s surplus are expressed in the same units. The bargained wage is determined by the maximization of the Nash product as follows

\[ W_{hc}^t = \operatorname{Argmax} \left[ \frac{1}{\lambda_t} \frac{\partial \Gamma_H}{\partial N_{t}^{hc}} \right]^{1-\xi^{hc}} \left[ \frac{\partial F_{t+1}^s}{\partial N_{t+1}^{hc}} \right]^{\xi^{hc}} \]

Then the sharing rule implies

\[ \xi^{hc} \left[ \frac{\partial \Gamma_H}{\partial N_{t}^{hc}} \right] = \left( 1 - \xi^{hc} \right) \lambda_t \left[ \frac{\partial F_{t+1}^s}{\partial N_{t+1}^{hc}} \right] \]

Substituting the envelope conditions of the households and the firms yields

\[ \xi^{hc} \left[ \lambda_t W_{hc}^t + (1 - \chi^c - \psi_{s}^{t+1}) \beta E_t \frac{\partial \Gamma_H}{\partial N_{t+1}^{hc}} - \psi_{hc}^s (1 - \psi_{t+1}^c) \beta E_t \frac{\partial \Gamma_{t+1}^F}{\partial N_{t+1}^{hc}} \right] = \left( 1 - \xi^{hc} \right) \lambda_t \left[ \frac{\partial Y_{t}}{\partial N_{t}^{hc}} - W_{hc}^t + (1 - \chi^c) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial F_{t+1}^s}{\partial N_{t+1}^{hc}} \right] \]

As we have from the first order conditions

\[ \frac{\omega^c}{q_{hs}^t} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial F_{t+1}^s}{\partial N_{t+1}^{hc}} \right] \]

therefore

\[ \xi^{hc} \beta E_t \left[ \frac{\partial \Gamma_{t+1}^H}{\partial N_{t+1}^{hc}} \right] = \left( 1 - \xi^{hc} \right) \frac{\omega^c}{q_{hs}^t} \]

Substituting yields
\[ \xi_{hc} \left[ \lambda_t W_{hc}^t + \frac{1 - \xi_{hc}}{\xi_{hc}} \lambda_t \frac{\omega^c}{q_t^c} (1 - \chi^c - P_t^c s_t) - \tau P_t^s (1 - s_t) \right] \]
\[ = \left( 1 - \xi_{hc} \right) \lambda_t \left[ \frac{\partial Y_t}{\partial N_{hc}^t} - W_{hc}^t + \frac{\omega^c}{q_t^c} (1 - \chi^c) \right] \]

Solving for the equilibrium wage rule for the high educated workers in complex occupations yields

\[ W_{hc}^t = \left( 1 - \xi_{hc} \right) \left[ \frac{\partial Y_t}{\partial N_{hc}^t} + P_t^c s_t \frac{\omega^c}{q_t^c} \right] + \xi_{hc} \left[ C_t \tau P_t^s (1 - s_t) \right] \]

7.2.4 The wage of high educated workers in simple occupations

The bargained wage is determined by the maximization of the Nash product as follows

\[ W_{hs}^t = \text{Argmax} \left[ \frac{1}{\lambda_t} \frac{\partial \Gamma_H^t}{\partial N_{hs}^t} \right]^{1-\xi_{hs}} \left[ \frac{\partial \Gamma_F^t}{\partial N_{hs}^t} \right]^\xi_{hs} \]

Then the sharing rule implies

\[ \xi_{hs} \left[ \frac{\partial \Gamma_H^t}{\partial N_{hs}^t} \right] = \left( 1 - \xi_{hs} \right) \lambda_t \left[ \frac{\partial \Gamma_F^t}{\partial N_{hs}^t} \right] \]

Substituting the envelope conditions of the households and firms yields

\[ \xi_{hs} \left[ \tau (1 - o_t) + \lambda_t W_{hs}^t + (P_t^c o_t - P_t^c s_t) \beta E_t \frac{\partial \Gamma_H^t}{\partial N_{hs}^t} + [1 - \chi^s - o_t P_t^c - P_t^s (1 - s_t)] \beta E_t \frac{\partial \Gamma_H^{t+1}}{\partial N_{hs}^{t+1}} \right] \]
\[ = \left( 1 - \xi_{hs} \right) \lambda_t \left[ \frac{\partial Y_t}{\partial N_{hs}^t} - W_{hs}^t + (1 - \chi^s - o_t P_t^c) \beta E_t \frac{\lambda_{t+1} \partial \Gamma_{F_{t+1}}}{\lambda_t \partial N_{hs_{t+1}}^{t+1}} \right] \]

We can also derive from the first order conditions

\[ \xi_{hs} \beta \lambda_t E_t \left[ \frac{\partial \Gamma_{H_{t+1}}}{\partial N_{hs_{t+1}}^{t+1}} \right] = \left( 1 - \xi_{hs} \right) \beta E_t \left[ \frac{\lambda_{t+1} \partial \Gamma_{F_{t+1}}}{\lambda_t \partial N_{hs_{t+1}}^{t+1}} \right] + \left( 1 - \xi_{hs} \right) \varepsilon_t \]

Substituting yields

34
\[ \xi_{hs} \left[ \tau (1 - o_t) + \lambda_t W_{hs}^t + \frac{\tau P_i^s}{P_i^s (P_i^s - P_i^c)} (P_i^c o_t - P_i^c s_t) + [1 - \chi^s - o_t P_i^c - P_i^c (1 - s_t)] \left( \frac{1 - \xi_{hs}}{\xi_{hs}} \right) \lambda_t \varepsilon_t \right] = \left( 1 - \xi_{hs} \right) \lambda_t \left[ \frac{\partial Y_t}{\partial N_{hs}^t} - W_{hs}^t + (1 - \chi^s - o_t P_i^c) \varepsilon_t \right] \]

Solving for the equilibrium wage rule for the high educated workers in simple occupations gives

\[ W_{hs}^t = \left( 1 - \xi_{hs} \right) \left[ \frac{\partial Y_t}{\partial N_{hs}^t} + P_i^s (1 - s_t) \varepsilon_t \right] + \xi_{hs} \left[ C_t \frac{\tau P_i^s (s_t - o_t)}{(P_i^c - P_i^s)} - C_t (1 - o_t) \right] \]

### 7.2.5 The wage of low educated workers in simple occupations

The bargained wage is determined by the maximization of the Nash product as follows

\[ W_{ls}^t = \text{Argmax} \left[ 1 \frac{\partial Y_t}{\lambda_t \partial N_{ls}^t} \right] ^{1 - \xi_{ls}} \left[ \frac{\partial Y_t}{\partial N_{ls}^t} \right] ^{\xi_{ls}} \]

Then the sharing rule implies

\[ \xi_{ls} \left[ \frac{\partial Y_t}{\partial N_{ls}^t} \right] = \left( 1 - \xi_{ls} \right) \lambda_t \left[ \frac{\partial Y_t}{\partial N_{ls}^t} \right] \]

Substituting in the sharing rule yields

\[ \xi_{ls} \left[ \lambda_t W_{ls}^t - \lambda_t (1 - g_t) W_d^t - P_t^s g_t \beta E_t \frac{\partial \Gamma_{t+1}^H}{\partial N_{ls}^{t+1}} \right] = \left( 1 - \xi_{ls} \right) \lambda_t \left[ \frac{\partial Y_t}{\partial N_{ls}^t} - W_{ls}^t \right] \]

Substituting from the first order conditions yields

\[ \xi_{ls} \left[ \lambda_t W_{ls}^t - \lambda_t (1 - g_t) W_d^t - P_t^s g_t \lambda_t \frac{W_d^t}{\beta P_t^s} \right] = \left( 1 - \xi_{ls} \right) \lambda_t \left[ \frac{\partial Y_t}{\partial N_{ls}^t} - W_{ls}^t \right] \]

Solving for the equilibrium wage for low educated workers in simple occupations gives

\[ W_{ls}^t = \left( 1 - \xi_{ls} \right) \left[ \frac{\partial Y_t}{\partial N_{ls}^t} \right] + \xi_{ls} \left[ W_d^t \right] \]
References


<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.5</td>
<td>proportion of the high educated in the population</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.98</td>
<td>household discount factor</td>
</tr>
<tr>
<td>$\chi^c$</td>
<td>0.044834595218221</td>
<td>separation rate from complex occupations</td>
</tr>
<tr>
<td>$\chi^s$</td>
<td>0.020284628857494</td>
<td>separation rate from simple occupations</td>
</tr>
<tr>
<td>$m^c$</td>
<td>0.3</td>
<td>efficiency parameter in the complex occupation matching function</td>
</tr>
<tr>
<td>$m^s$</td>
<td>0.6</td>
<td>efficiency parameter in the simple occupation matching function</td>
</tr>
<tr>
<td>$\gamma^c$</td>
<td>0.6</td>
<td>elasticity of complex matches with respect to complex vacancies</td>
</tr>
<tr>
<td>$\gamma^s$</td>
<td>0.6</td>
<td>elasticity of simple matches with respect to simple vacancies</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.5</td>
<td>elasticity of output to complex occupation output</td>
</tr>
<tr>
<td>$\omega^c$</td>
<td>1.042925414254980</td>
<td>cost of posting a complex vacancy</td>
</tr>
<tr>
<td>$\omega^s$</td>
<td>0.08</td>
<td>cost of posting a simple vacancy</td>
</tr>
<tr>
<td>$\xi^{hc}$</td>
<td>0.503664248445526</td>
<td>firm share from bargaining with a high educated in a complex occupation</td>
</tr>
<tr>
<td>$\xi^{hs}$</td>
<td>0.579142850180305</td>
<td>firm share from bargaining with a high educated in a simple occupation</td>
</tr>
<tr>
<td>$\xi^{ls}$</td>
<td>0.500052058210287</td>
<td>firm share from bargaining with a low educated in a simple occupation</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.163237784257017</td>
<td>ratio of return to domestic activities to average productivity</td>
</tr>
<tr>
<td>$\tau$</td>
<td>1.08</td>
<td>parameter in the utility of leisure</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.278430461890875</td>
<td>indexation coefficient</td>
</tr>
<tr>
<td>$\rho^A$</td>
<td>0.9</td>
<td>autoregressive coefficient of aggregate technology</td>
</tr>
<tr>
<td>$\rho^{Ah}$</td>
<td>0.95</td>
<td>autoregressive coefficient of high educated biased technology</td>
</tr>
<tr>
<td>$\rho^{Al}$</td>
<td>0.95</td>
<td>autoregressive coefficient of low educated biased technology</td>
</tr>
<tr>
<td>$\sigma_{eA}$</td>
<td>0.0049</td>
<td>standard deviation of the aggregate technology shock</td>
</tr>
<tr>
<td>$\sigma_{eAh}$</td>
<td>0.002</td>
<td>standard deviation of the high educated biased technology shock</td>
</tr>
<tr>
<td>$\sigma_{eAl}$</td>
<td>0.002</td>
<td>standard deviation of the low educated biased technology shock</td>
</tr>
</tbody>
</table>

Table 4: calibration of model parameters
### Cross correlations of output($t$) and $x(t+i)$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$x(t-4)$</th>
<th>$x(t-3)$</th>
<th>$x(t-2)$</th>
<th>$x(t-1)$</th>
<th>$x(t)$</th>
<th>$x(t+1)$</th>
<th>$x(t+2)$</th>
<th>$x(t+3)$</th>
<th>$x(t+4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Within</strong></td>
<td>-0.096</td>
<td>-0.129</td>
<td>-0.0542</td>
<td>-0.0283</td>
<td>-0.0328</td>
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<td>0.0632</td>
<td>0.0590</td>
<td>0.0609</td>
</tr>
<tr>
<td></td>
<td>(0.3401)</td>
<td>(0.1987)</td>
<td>(0.5883)</td>
<td>(0.7763)</td>
<td>(0.7413)</td>
<td>(0.8348)</td>
<td>(0.5281)</td>
<td>(0.5577)</td>
<td>(0.5472)</td>
</tr>
<tr>
<td><strong>Between</strong></td>
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<td>-0.0750</td>
<td>-0.0050</td>
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<td>0.1236</td>
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<td>0.2143</td>
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</tr>
<tr>
<td></td>
<td>(0.1352)</td>
<td>(0.4560)</td>
<td>(0.9605)</td>
<td>(0.9050)</td>
<td>(0.2115)</td>
<td>(0.1701)</td>
<td>(0.0882)</td>
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<td>(0.0216)</td>
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<td>$N^{hc}$</td>
<td>-0.1154</td>
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<td>-0.0132</td>
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<td>0.3891</td>
<td>0.3802</td>
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<tr>
<td></td>
<td>(0.2530)</td>
<td>(0.5287)</td>
<td>(0.8953)</td>
<td>(0.5414)</td>
<td>(0.0606)</td>
<td>(0.0074)</td>
<td>(0.0024)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
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<tr>
<td>$N^{hs}$</td>
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Table 5: compiled data

p-values in ()

**Within**: within group wage premium

**Between**: between group wage premium

$N^{hc}$: proportion of the high educated in complex occupations

$N^{hs}$: proportion of the high educated in simple occupations

$N^{ls}$: proportion of the low educated in simple occupations

$U^h$: proportion of the high educated unemployed

$U^l$: proportion of the low educated unemployed

**Crowding**: crowding out effect
### Cross correlations of output($t$) and $x(t + i)$

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Table 6: benchmark model with aggregate technological shock. p-values in ()

*Within*: within group wage premium

*Between*: between group wage premium

*N$^{hc}$*: proportion of the high educated in complex occupations

*N$^{hs}$*: proportion of the high educated in simple occupations

*N$^{ls}$*: proportion of the low educated in simple occupations

*$U^h$*: proportion of the high educated unemployed

*$U^l$*: proportion of the low educated unemployed

*Crowding*: crowding out effect

*U*: overall unemployment ratio, *V*: total vacancies
Cross correlations of output($t$) and $x(t+i)$

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Table 7: benchmark model with skill biased technological shock. p-values in ()

**Within**: within group wage premium

**Between**: between group wage premium

$N^{hc}$: proportion of the high educated in complex occupations

$N^{hs}$: proportion of the high educated in simple occupations

$N^{ls}$: proportion of the low educated in simple occupations

$U^h$: proportion of the high educated unemployed

$U^l$: proportion of the low educated unemployed

**Crowding**: crowding out effect

$U$: overall unemployment ratio, $V$: total vacancies
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Table 8: modified model with aggregate technological shock. p-values in ()

$Between$: between group wage premium

$N^{hc}$: proportion of the high educated in complex occupations

$N^{hs}$: proportion of the high educated in simple occupations

$N^{ls}$: proportion of the low educated in simple occupations

$U^h$: proportion of the high educated unemployed

$U^l$: proportion of the low educated unemployed

Crowding: crowding out effect

$U$: overall unemployment ratio

$V$: total vacancies
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Table 9: modified model with skill biased technological shock. p-values in ()

$Between$: between group wage premium

$N^{hc}$: proportion of the high educated in complex occupations

$N^{hs}$: proportion of the high educated in simple occupations

$N^{ls}$: proportion of the low educated in simple occupations

$U^{h}$: proportion of the high educated unemployed

$U^{l}$: proportion of the low educated unemployed

$Crowding$: crowding out effect

$U$: overall unemployment ratio

$V$: total vacancies
Figure 1: Y: cyclical component of real gross domestic product. Within: cyclical component of the within group wage premium.

Figure 2: Y: cyclical component of real gross domestic product. Between: cyclical component of the between group wage premium.
Figure 3: Y: cyclical component of real gross domestic product. NHC: cyclical component of the proportion of high educated workers in complex occupation.

Figure 4: Y: cyclical component of real gross domestic Product. NHS: cyclical component of the proportion of high educated workers in simple occupations.
Figure 5: Y: cyclical component of real gross domestic product. UH: cyclical component of the proportion of the high educated unemployed.

Figure 6: Y: cyclical component of real gross domestic product. NLS: cyclical component of the proportion of low educated workers in simple occupations.
Figure 7: Y: cyclical component of real gross domestic product. UL: cyclical component of the proportion of the low educated unemployed.

Figure 8: Y: cyclical component of real gross domestic product. CROWDING: cyclical component of the crowding out effect.
Figure 9: Benchmark model impulse response functions to a 1% aggregate technological shock.
Figure 10: Benchmark model impulse response functions to a 1% aggregate technological shock.
Figure 11: Benchmark model impulse response functions to a 1% skill biased technological shock.
Figure 12: Benchmark model impulse response functions to a 1% skill biased technological shock.
Figure 13: Modified model impulse response functions to a 1% aggregate technological shock.
Figure 14: Modified model impulse response functions to a 1% aggregate technological shock.
Figure 15: Modified model impulse response functions to a 1% skill biased technological shock.
Figure 16: Modified model impulse response functions to a 1% skill biased technological shock.