International Trade, Intellectual Property,
and Patterns of Skill Premia*

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Abstract

Trade theory predicts that the skill premium should increase in the skill abundant developed countries, and decline in the skill scarce developing countries. Empirical evidence shows that though wage inequality declined in some developing countries, others experienced an increase in the skill premium. This paper develops a North-South model in which each country produces two types of final goods: a skilled-intensive and an unskilled-intensive good. The former is produced using skilled workers and a range of skilled complementary intermediates, while the latter is produced using unskilled workers and a range of unskilled complementary intermediates. The labor markets in the two countries feature search frictions. The skill premium in autarky is compared to the premium after trade openness. Conditions ensuring that wage inequality increases after trade liberalization are different than the case when labor markets are assumed Walrasian. In this context, there are plausible outcomes consistent with empirical evidence that openness can increase the skill premium in some developing countries, while the rest can experience a decline in wage inequality.

Keywords: intellectual property rights, trade liberalization, wage inequality.

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1 Introduction

Despite the relative increase in the supply of skilled workers in the developed countries, the skill premium or the wage differential between skilled and unskilled workers has increased. Observers concluded that the demand for skilled workers must have increased by more than the increase in the supply to account for this increase in the skill premium. The literature attributed this increase in demand to two possible causes. The first is a skill biased technological progress, where as technology is designed to complement skilled workers, the demand for this labor type increases. The second explanation pertains to the effects of trade liberalization on labor market outcomes. Traditional trade theory predicts that openness induces countries to export the good that intensively uses the relatively abundant factor of production and import the good that intensively uses the factor of production that is relatively scarce. As developed countries are considered skill abundant, they export the good that intensively uses skilled workers. This contributes to an increase in the relative price of the skilled-intensive good, a rise in the relative demand for skilled workers, and accordingly an increase in the skill premium. The theory also predicts that skill scarce developing countries are expected to experience a decline in the relative price of their skilled-intensive good and consequently a reduction in wage inequality.

Theoretical predictions are, however, not supported by empirical evidence. Some developing countries experienced an increase in wage inequality, while the skill premium declined in others. Evidence as to the asymmetric response of wage inequality in the South is documented by Hanson and Harrison (1995), Robbins (1996), Wood (1997), and Goldberg and Pavcnik (forthcoming). This poses a challenge to trade theorists. Foremost is how to explain the widening wage gap both in the developed and in some developing countries, in addition to the asymmetric response to trade openness amidst developing countries.

Previous studies attempted to address this puzzle, in order to resolve the inconsistencies between the predictions of the theory and the stylized facts. For instance, Acharrya and Marjit (2003) argue that latent complementarity between unskilled-intensive and skilled-intensive exports, segmented labor markets between formal and informal sectors and the existence of a large non traded sector, contribute to a rising wage inequality in the South. Beaulieu et al. (2004) present a model in which intratrade trade in the skilled-intensive high-tech sector is driven by international differences in adoption lags incorporating new technology. A reduction in trade barriers within the high-tech sector leads to an increase in inequality in both the North and the South. Epifani and Gancia (2002) show that an elasticity of consumption greater than one and higher scale economies in the skilled-intensive sector imply that any increase in the volume of trade, even between identical countries, tend to be skill biased. If the skill bias effect is strong enough
to overcome the standard factor proportions effect, international trade will spur inequality even in the skill scarce developing economies. Xu (2003) shows that trade liberalization can cause wage inequality in the South in a framework where there are nontraded goods whose range is endogenously determined by the level of trade barriers. A tariff reduction causes an export expansion both in the North and the South, and can explain rising wage inequality in both regions. Xu (2001) shows that in a model with endogenous technology bias, trade opening induces skill bias technical progress that is biased towards the labor intensive sector, causing wage inequality to rise in the North and the South. This is important for small open Southern economies, as it is the sector rather than the factor bias of technical progress that determines relative wages. Zhu (2004) and Zhu and Treffer (2005) develop models where the creation of skilled-intensive goods induces the North to transfer production of older less skilled-intensive goods to the South. These relocated goods are the most skilled-intensive by Southern standards, and thus raises the relative demand for skilled workers and wage inequality within both regions. Neary (2002a,2002b) presents a model of general oligopolistic equilibrium, in which an increase in foreign competition encourages more strategic investment by incumbent firms in order to give themselves an advantage in competing with their rivals. Assuming that these investments require more skilled workers, trade liberalization increase the demand for this type of labor, causing an increase in the skill premium in the North and the South. Thoenig and Verdier (2000) consider a model in which firms can endogenously bias the direction of technological change. When there is a differential degree of protection of property rights, the optimal outcome by innovating firms is the emergence of endogenous technological bias toward skilled labor technologies, and thus the model generates an increase in wage inequality in the North and the South.

Nevertheless, these studies focus on generating an increase in the skill premium in both the North and the South, but not the asymmetry of the response of the skill premium to trade openness between developing countries. Given the empirical evidence, any explanation must be able to generate symmetric changes in wage inequality in the developed and some of the developing countries, and asymmetric changes within the developing countries. Therefore, this paper attempts to reconcile the empirical evidence with a new role of international trade in explaining wage inequality. It compares the conditions under which the skill premium in the North and in the South increases after trade liberalization if labor markets are Walrasian, and if they feature search frictions. The introduction of search frictions and the explicit formalization of the wage setting and bargaining processes is consistent with the findings in Freeman and Oostendorp (2000) who study the October Inquiry Survey of Wages conducted by the International Labor Organization over the period from 1983 to 1998. Their results suggest that the principal forces that affect the occupational wage
structure around the world are the level of GDP per capita and wage setting institutions.

This paper introduces a two country setup. In each country, households are divided into skilled and unskilled workers. There are two types of firms: producers of final goods and producers of intermediate goods. There are two types of final goods: a skilled-intensive and an unskilled-intensive good. The former is produced utilizing skilled workers and a range of skilled complementary intermediates. The latter is produced utilizing unskilled workers and a range of unskilled complementary intermediates. In this model, the North innovates and the South imitates. Therefore, skilled and unskilled complementary intermediates are produced by technology monopolists in the North, and used by producers of final goods both in the North and in the South. Labor markets in the two countries feature search frictions. Therefore, producers of the skilled-intensive final good post complex vacancies that can be filled by skilled workers only, while producers of the unskilled-intensive final good post simple vacancies that can be filled by unskilled workers only. The conditions under which the skill premium increases after trade liberalization in the North and in the South are considered, when protection of intellectual property is enforced in the South and when it is not. Intellectual property is introduced as Acemoglu (2002, 2003) find that the effects of trade opening on technology bias depends critically on the degree to which the South protects intellectual property rights of Northern innovators. With full protection, trade opening implies an increase in the relative market size of labor complementary technologies which leads to unskilled biased technological change, while if there is no protection, trade openness increases the price of the skill-using intermediate good in the North and leads to skill biased technological progress.

The results in this paper suggest that while the wage gap can readily increase in both the North and the South, there are also plausible conditions where the wage gap increases in one country and decreases in another in the South. The remainder of the paper is organized as follows: section 2 presents the model, section 3 the conclusion, section 4 the derivations appendix. References are included thereafter.
2 Model

There are two countries: the North and the South indexed by \( i \in (N, S) \) respectively. In each country, households are divided ex ante into skilled and unskilled workers. There are two types of firms: producers of final goods and producers of intermediate goods. There are two types of final goods: a skilled-intensive and an unskilled-intensive good. The former is produced utilizing skilled workers and a range of skilled complementary intermediates. The latter is produced utilizing unskilled workers and a range of unskilled complementary intermediates. Skilled and unskilled complementary intermediates are produced by technology monopolists in the North. These intermediates are purchased by producers of final goods in the North, and in the South only if protection of intellectual property is enforced. Otherwise, they are copied by producers of final goods in the South. The labor markets in the two countries feature search frictions. Therefore, producers of the skilled-intensive final good post complex vacancies that can be matched with skilled workers only, while producers of the unskilled-intensive final good post simple vacancies that can be matched with unskilled workers only. Labor is recruited via a matching process that is a function of vacancies posted and effective searchers.

2.1 Autarky

2.1.1 Aggregate Production

Country \( i \) produces two goods: the unskilled-intensive good \( Y^l_{it} \), and the skilled-intensive good \( Y^h_{it} \). The aggregate production is given by a constant elasticity of substitution composite of the two goods as follows

\[
Y_{it} = \left[ \gamma \left( Y^l_{it} \right)^{\frac{1}{\varepsilon}} + (1 - \gamma) \left( Y^h_{it} \right)^{\frac{1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon - 1}}
\]  

(1)

where \( \varepsilon \in [0, \infty) \) is the elasticity of substitution between the two goods, and \( \gamma \in (0, 1) \) is a distribution parameter which determines the importance of the two goods in the aggregate production. The skilled-intensive good is produced using skilled workers \( L^h_{it} \) and skilled complementary intermediates \( x^h_{it}(j) \), whose range is denoted \( A^h_t \), according to the following production function

\[
Y^h_{it} = \frac{1}{1 - \beta} \left[ \int_0^{A^h_t} x^h_{it}(j)^{1-\beta} dj \right] (L^h_{it})^\beta
\]  

(2)

The unskilled-intensive good in country \( i \) is produced using unskilled workers \( L^l_{it} \) and unskilled complementary intermediates \( x^l_{it}(j) \), whose range is denoted \( A^l_t \), according to the following production function
\[ Y_{it}^l = \frac{1}{1 - \beta} \left[ \int_0^{A_i} x_{it}^j (j)^{1 - \beta} \, dj \right] (L_{it}^l)^{\beta} \] (3)

Where \( \beta \in (0,1) \). The prices of final goods in country \( i \) are thus given by

\[ P_{it}^h = (1 - \gamma) \left( \frac{Y_{it}}{Y_{it}^h} \right)^{\frac{1}{\varepsilon}} \] (4)

\[ P_{it}^l = \gamma \left( \frac{Y_{it}}{Y_{it}^l} \right)^{\frac{1}{\varepsilon}} \] (5)

The price of the aggregate good in country \( i \) is normalized to 1 as follows

\[ \left[ \gamma^{\varepsilon} \left( P_{it}^h \right)^{1 - \varepsilon} + (1 - \gamma)^{\varepsilon} \left( P_{it}^l \right)^{1 - \varepsilon} \right]^{\frac{1}{1 - \varepsilon}} = 1 \] (6)

2.1.2 Households

The population in country \( i \) is of measure 1, and is divided ex ante into \( \delta_i \) skilled and \( (1 - \delta_i) \) unskilled. Search frictions in the labor market allow for equilibrium unemployment. Therefore, the skilled and the unskilled household members are divided into those employed and those unemployed as follows

\[ L_{it}^h + U_{it}^h = \delta_i \] (7)

\[ L_{it}^l + U_{it}^l = 1 - \delta_i \] (8)

Where \( U_{it}^h \) and \( U_{it}^l \) denote the number of the skilled and the unskilled unemployed in country \( i \) respectively. Time for the unemployed types is normalized to one. A skilled unemployed in country \( i \) uses a portion \( d_{it} \) of its time to search for a complex occupation, and \( (1 - d_{it}) \) for domestic activities. An unskilled unemployed in country \( i \) uses a portion \( g_{it} \) of its time to search for a simple occupation, and \( (1 - g_{it}) \) for domestic activities. As different employment histories amongst members of a household can lead to heterogeneous wealth positions, we follow the literature in assuming that each household is thought of as an extended family whose members perfectly insure each other against variations in labor income due to employment or unemployment. Remaining within the confines of complete markets allows solving the program of a representative household, who chooses consumption and search intensities to maximize the discounted expected infinite sum of its instantaneous utility of consumption. Assuming the household has
the following value function $\Gamma_{it} = \Gamma (L^h_{it}, L^l_{it})$, the optimization problem can be written in the following recursive form

$$\Gamma_{it} = \max_{\{C_{it}, d_{it}, g_{it}\}} \{\ln(C_{it}) + RE_t [\Gamma_{it+1}]\} \tag{9}$$

where $E_t$ is the expectation operator conditional on the information set available in period $t$, and $R$ is the discount factor. The period $t$ utility of household consumption $C_{it}$ is given by the logarithm of consumption expenditures. This is subject to the following budget constraint

$$C_{it} = L^l_{it} W^l_{it} + L^h_{it} W^h_{it} + (1 - d_{it}) U^h_{it} \phi^h_i + (1 - g_{it}) U^l_{it} \phi^l_i + \Psi_{it} \tag{10}$$

where $W^h_{it}$ and $W^l_{it}$ are the period $t$ bargained wages of the skilled and unskilled workers in country $i$ respectively. $\phi^h_i$ and $\phi^l_i$ denote the unemployment benefits or the return to domestic activities for the skilled and unskilled unemployed respectively. $\Psi_{it}$ denotes the dividends distributed by firms. The households also take into consideration the employment dynamics of the two labor types. The skilled employed in period $t + 1$ are comprised of those of that type who are not exogenously separated from a complex occupation in period $t$ according to the exogenous separation rate $\eta^c_i$, in addition to the new matches from the searchers pool of the skilled unemployed as follows

$$L^c_{it+1} = (1 - \eta^c_i) L^c_{it} + \rho^c_{it} d_{it} U^c_{it} \tag{11}$$

Similarly, the unskilled employed in period $t + 1$ are comprised of those of that type who are not exogenously separated from a simple occupation in period $t$ according to the separation rate $\eta^s_i$, in addition to the new matches from the searchers pool of the unskilled unemployed as follows

$$L^s_{it+1} = (1 - \eta^s_i) L^s_{it} + \rho^s_{it} g_{it} U^s_{it} \tag{12}$$

$\rho^c_{it}$ and $\rho^s_{it}$ are the endogenous probabilities that a searcher is matched with a complex or a simple occupation respectively, and are defined as the ratio of the respective matches to the respective effective searchers as follows

$$\rho^c_{it} = \frac{M^c_i}{d_{it} U^c_{it}} = \frac{M^c_i (V^c_{it})^{\alpha_c} (d_{it} U^h_{it})^{1-\alpha_c}}{d_{it} U^h_{it}} = M^c_i (V^c_{it})^{\alpha_c} (d_{it} U^h_{it})^{-\alpha_c} \tag{13}$$
\[ \rho_{it}^s = \frac{M_{it}^s}{g_{it}U_{it}^l} = \frac{M_{it}^s (V_{it}^s)\alpha^s (g_{it}U_{it}^l)^{1-\alpha^s}}{g_{it}U_{it}^l} = M_{it}^s (V_{it}^s)^{\alpha^s} (g_{it}U_{it}^l)^{-\alpha^s} \] (14)

\( M_{it}^c \) and \( M_{it}^s \) represent the number of complex and simple occupation matches respectively, and they are constant returns to scale homogeneous of degree one functions of the number of corresponding vacancies, \( V_{it}^c \) and \( V_{it}^s \), and effective searchers. \( \alpha^c \) and \( \alpha^s \) are the elasticities of matching with respect to complex and simple vacancies respectively. \( M_{it}^c \) and \( M_{it}^s \) are the efficiency parameters of the complex and simple matching processes respectively. The household chooses their consumption such that the marginal utility of consumption equals the Lagrange multiplier \( \lambda_{it} \)

\[ \frac{1}{C_{it}} = \lambda_{it} \] (15)

The household chooses the optimal proportion of time the skilled unemployed allocate to search for a complex occupation \( d_{it} \) such that the unemployment benefit earned by the skilled unemployed is equal to the discounted expected value of an additional skilled worker to the household as follows

\[ \lambda_{it} \phi_i^h = \Re \rho_{it}^h E_t \left[ \frac{\partial \Gamma_{it+1}}{\partial L_{it+1}^h} \right] \] (16)

The household chooses the optimal proportion of time the unskilled unemployed allocate to search for a simple occupation \( g_{it} \) such that the unemployment benefit earned by the unskilled unemployed is equal to the discounted expected value of an additional unskilled worker to the household as follows

\[ \lambda_{it} \phi_i^l = \Re \rho_{it}^s E_t \left[ \frac{\partial \Gamma_{it+1}}{\partial L_{it+1}^l} \right] \] (17)

From the envelope theorem, an additional skilled worker accrue a value to the household that is given by the current labor income earned by that type less the unemployment benefits foregone, in addition to the discounted expected value of the match if this worker is not separated exogenously after being matched. This also includes the value forgone had it been that this member of the household is not matched in the first place, continued to search and got matched with a complex vacancy as follows

\[ \frac{\partial \Gamma_{it}}{\partial L_{it}^h} = \lambda_{it} W_{it}^h - \lambda_{it} \phi_i^h (1 - d_{it}) + \Re (1 - \eta_i^c - \rho_{it}^c d_{it}) E_t \left[ \frac{\partial \Gamma_{it+1}}{\partial L_{it+1}^h} \right] \] (18)

An additional unskilled worker accrue a value to the household that is given by the current labor income earned by this type less the unemployment benefits foregone, in addition to the discounted expected value
of the match if this worker is not separated exogenously after being matched. This also includes the value forgone had it been that this member of the household is not matched in the first place, continued to search and got matched with a simple vacancy as follows

$$\frac{\partial \Gamma_{it}}{\partial L_{it}} = \lambda_{it} W_{it} - \lambda_{it} \phi_{i}^{l} (1 - g_{it}) + R (1 - \eta_{i}^{s} - \rho^{s}_{it} g_{it}) E_{t} \left[ \frac{\partial \Gamma_{it+1}}{\partial L_{it+1}} \right]$$  \hspace{1cm} (19)$$

2.1.3 Producers of Final goods

Producers of final goods take the prices of final goods and intermediates as given, and decide on their demand for the intermediates and the vacancies to post so as to maximize their profits. The discount factor of firms is given such that it effectively evaluate profits in terms of the values attached to them by households, who ultimately own the firms. Thus, the utility based and time varying discount factor used by firms is given by $\left( \frac{\lambda_{it+1}}{\lambda_{it}} \right)$. Assuming the producers of the skilled-intensive final good in country $i$ has the following value function $\Omega_{it}^{c} = \Omega^{c} (L_{it}^{h})$, their optimization problem can be written in the following recursive form

$$\Omega_{it}^{c} = \max \left\{ V_{it}^{c}, x_{it}^{c} \right\} \left[ P_{it}^{h} Y_{it}^{h} - W_{it}^{h} L_{it}^{h} - \omega^{c} V_{it}^{c} - \int_{0}^{A_{h}} \Phi_{it}^{h} (j) x_{it}^{h} (j) \, dj + \mathbb{R} E_{t} \left[ \frac{\lambda_{it+1}}{\lambda_{it}} \Omega_{it+1}^{c} \right] \right\}$$  \hspace{1cm} (20)$$

Where $\Phi_{it}^{h} (j) = \chi_{it}^{h} (j)$, which is the price of purchase of skilled complementary intermediates for the North and the South producers if protection of intellectual property is enforced in the South. Otherwise, $\Phi_{it}^{h} (j) = k \frac{\alpha_{it}^{h}}{\beta_{it}^{h}} < 1$, which is a fixed cost for copying the intermediates by the South producers if intellectual property rights are not enforced. This is subject to the following employment dynamics

$$L_{it+1}^{h} = (1 - \eta_{i}^{c}) L_{it}^{h} + q_{it}^{c} V_{it}^{c}$$  \hspace{1cm} (21)$$

where $q_{it}^{c}$ is the probability of filling a complex vacancy, and is given by the ratio of complex matches to complex vacancies posted, $q_{it}^{c} = \frac{M_{it}}{V_{it}^{c}}$. On the other hand, assuming the producers of the unskilled-intensive good in country $i$ has the following value function $\Omega_{it}^{s} = \Omega^{s} (L_{it}^{l})$, their optimization problem can be written in the following recursive form

$$\Omega_{it}^{s} = \max \left\{ V_{it}^{s}, x_{it}^{s} \right\} \left[ P_{it}^{l} Y_{it}^{l} - W_{it}^{l} L_{it}^{l} - \omega^{s} V_{it}^{s} - \int_{0}^{A_{l}} \Phi_{it}^{l} (j) x_{it}^{l} (j) \, dj + \mathbb{R} E_{t} \left[ \frac{\lambda_{it+1}}{\lambda_{it}} \Omega_{it+1}^{s} \right] \right\}$$  \hspace{1cm} (22)$$

Where $\Phi_{it}^{l} (j) = \chi_{it}^{l} (j)$, which is the price of purchase of unskilled complementary intermediates for the North and the South producers if protection of intellectual property is enforced in the South. Otherwise,
\( \Phi_{lt} (j) = k^{\frac{\beta}{\sigma}} < 1 \), which is a fixed cost for copying the intermediates by South producers if intellectual property rights are not enforced. This is subject to the following employment dynamics

\[
L_{lt+1}^f = (1 - \eta_s^f) L_{lt}^f + q_{lt}^s V_{lt}^s
\]

where \( q_{lt}^s \) is the probability of filling a simple vacancy, and is given by the ratio of simple matches to simple vacancies posted, \( q_{lt}^s = \frac{M_{lt}^s}{V_{lt}^s} \). The demand for intermediates in the North is given by

\[
x_{Nt}^f (j) = \left[ \frac{P_{Nt}^f}{\chi_{lt}^f (j)} \right]^{\frac{1}{r}} (L_{Nt})
\]

\[
x_{Nt}^h (j) = \left[ \frac{P_{Nt}^h}{\chi_{lt}^h (j)} \right]^{\frac{1}{r}} (L_{Nt})
\]

The demand for intermediates in the South, if intellectual property rights are not enforced, is given by

\[
x_{St}^f (j) = \left( P_{St}^f \right)^{\frac{1}{r}} k^{1-\sigma} (L_{St})
\]

\[
x_{St}^h (j) = \left( P_{St}^h \right)^{\frac{1}{r}} k^{1-\sigma} (L_{St})
\]

The demand for intermediates in the South, if intellectual property rights are enforced, is given by

\[
x_{St}^{(IPR)} (j) = \left[ \frac{P_{St}^f}{\chi_{lt}^f (j)} \right]^{\frac{1}{r}} (L_{St})
\]

\[
x_{St}^{(IPR)} (j) = \left[ \frac{P_{St}^h}{\chi_{lt}^h (j)} \right]^{\frac{1}{r}} (L_{St})
\]

Therefore, the demand for intermediates is increasing in the price of the final good, and the employment of factors, but decreasing in the price of the intermediates or the cost of copying them. The producers of the skilled-intensive final good in country \( i \) chooses the optimal level of complex vacancies \( V_{lt}^c \) to post such that the expected marginal cost of posting this type of vacancy equals the discounted expected value to the firm of an additional skilled worker in a complex occupation as follows

\[
\frac{\omega_{lt}^c}{q_{lt}^c} = RE_{lt} \left[ \frac{\lambda_{lt+1} \partial \Omega_{lt+1}^c}{\lambda_{lt} \partial L_{lt+1}} \right]
\]
The producers of the unskilled-intensive final good in country \( i \) chooses the optimal level of simple vacancies \( V_{it}^s \) to post such that the expected marginal cost of posting this type of vacancy equals the discounted expected value to the firm of an additional unskilled worker in a simple occupation as follows

\[
\frac{\omega_i^s}{q_{it}} = \beta E_t \left[ \frac{\lambda_{it+1} \partial \Omega_{it+1}^s}{\lambda_{it} \partial L_{it+1}} \right]
\]  

(31)

From the envelope theorem, the value to the producer of the skilled-intensive final good of an additional skilled worker in a complex occupation is given by the net current value of the match, which is the difference between the value of its marginal productivity and the bargained wage, in addition to the discounted expected value of the match in case the worker is not exogenously separated as follows

\[
\frac{\partial \Omega_{it+1}^c}{\partial L_{it+1}} = P_{it} \frac{\partial Y_{it+1}^c}{\partial L_{it+1}} - W_{it+1} + (1 - \eta_i) \beta E_t \left[ \frac{\lambda_{it+1} \partial \Omega_{it+1}^c}{\lambda_{it} \partial L_{it+1}} \right]
\]  

(32)

Similarly, the value to the producer of the unskilled-intensive final good of an additional unskilled worker in a simple occupation is given by the net current value of the match, which is the difference between the value of its marginal productivity and the bargained wage, in addition to the discounted expected value of the match in case the worker is not exogenously separated as follows

\[
\frac{\partial \Omega_{it+1}^s}{\partial L_{it+1}} = P_{it} \frac{\partial Y_{it+1}^s}{\partial L_{it+1}} - W_{it+1} + (1 - \eta_i) \beta E_t \left[ \frac{\lambda_{it+1} \partial \Omega_{it+1}^s}{\lambda_{it} \partial L_{it+1}} \right]
\]  

(33)

In equilibrium, matched firms and workers obtain from the match a total return that is strictly higher than the expected return of unmatched firms and workers because if they separate each will have to go through an expensive and time consuming process of search before being matched again. We follow the literature in assuming that a realized match share this surplus through a Nash bargaining problem. Therefore, the wage of a skilled worker in country \( i \) is given by

\[
W_{it+1}^h = \left( 1 - \xi_{it}^h \right) \left[ P_{it} \frac{\partial Y_{it+1}^h}{\partial L_{it+1}} + \frac{\rho_{it}^h d_{it} \omega_{it}^c}{q_{it}} \right] + \xi_{it}^h \left[ \varphi_{it}^h (1 - d_{it}) \right]
\]  

(34)

Where \( \xi_{it}^h \) is the firm’s share of the surplus. The wage is a weighted average of two terms: the first indicates that the worker is rewarded for a fraction \( 1 - \xi_{it}^h \) of both the firm’s revenues from the worker’s productivity and the discounted expected value of the match to the firm. The second term indicates that the worker is compensated by a fraction \( \xi_{it}^h \) for the foregone unemployment benefit. The bargained wage

\[1\] Detailed derivations are included in appendix 4.1.
of the unskilled worker is given by\(^2\)

\[
W^l_{it} = \left(1 - \xi^l_i\right) \left[P^l_{it} \frac{\partial Y^l_{it}}{\partial L^l_{it}} + \frac{\rho^l_{it} g^l_{it} \omega^l_{it}}{q^l_{it}}\right] + \xi^l_i \left[\phi^l_i (1 - g^l_{it})\right]
\]

Where \(\xi^l_i\) is the firm’s share of the surplus. The wage is a weighted average of two terms: the first indicates that the worker is rewarded by a fraction \(1 - \xi^l_i\) for the firm’s revenues from the worker’s productivity and the discounted expected value of the match to the firm. The second term indicates that the worker is compensated by a fraction \(\xi^l_i\) for the foregone unemployment benefit. The skill premium in country \(i\) is thus given by the ratio of the wage of the skilled workers to that of the unskilled workers as follows

\[
W^h_{it} = \frac{1 - \xi^h_i}{1 - \xi^l_i}
\left[P^h_{it} \frac{\partial Y^h_{it}}{\partial L^h_{it}} + \frac{\rho^h_{it} d^h_{it} \omega^h_{it}}{q^h_{it}}\right] + \xi^h_i \left[\phi^h_i (1 - d^h_{it})\right]
\]

2.1.4 Producers of Intermediate Goods

The technology monopolists in the North take as given the marginal cost of producing the intermediate good as well as the demand for their products by the producers of final goods, and choose the price of the intermediate good to maximize their profits. The marginal cost of producing intermediates is assumed to be fixed and denoted \(\psi\). If protection of intellectual property is not enforced in the South, technology producers in the North will not consider the demand of the South in their profit maximization as follows

\[
\pi^l_{Nt}(j) = [\chi^l_j (j) - \psi] x^l_{Nt}(j)
\]

\[
\pi^h_{Nt}(j) = [\chi^h_j (j) - \psi] x^h_{Nt}(j)
\]

Substituting the demands for the intermediates, the optimal prices of the intermediate goods are given by

\[
\chi^l_j (j) = \frac{\psi}{1 - \beta}
\]

\[
\chi^h_j (j) = \frac{\psi}{1 - \beta}
\]

Where the profit maximizing price is a constant markup over marginal cost. If we normalize the marginal cost \(\psi\) to \((1 - \beta)\), then the prices of the intermediates \(\chi^l_j (j) = \chi^l_j (j) = 1\). On the otherhand, if intellectual

\(^2\text{Detailed derivations are included in appendix 4.2.}\)
property rights are enforced in the South, then the demand for the intermediates by producers of final goods in the South are considered in the North innovators maximization problem as follows

\[ \pi_{NT}^h(j)^{IPR} = [\chi_j^h(j) - \psi] \left[ x_{NT}^h(j) + x_{St}^h(j) \right] \]  

\[ \pi_{NT}^l(j)^{IPR} = [\chi_j^l(j) - \psi] \left[ x_{NT}^l(j) + x_{St}^l(j) \right] \]  

The optimal prices of the intermediates yields \( \chi_j^l(j) = \chi_j^h(j) = 1 \) as well.

### 2.1.5 Skill Premia

The skill premium in the North is given by\(^3\)

\[ \frac{W_{St}^h}{W_{NT}^h} = \frac{(1 - \xi_N^h) \left[ \frac{\beta}{1 - \rho} (Y_{NT})^{1 - \frac{\beta}{\rho}} (1 - \beta) \frac{1}{\rho} \left( L_{NT}^h \right)^{\frac{1}{\beta}} (A_l^n)^{-\frac{1}{1 - \rho}} + \frac{\rho_{St} S_{NT} \omega_{St}^N}{q_{St}^N} \right] + \xi_N^h \left[ \phi_N^h (1 - d_{NT}) \right]}{(1 - \xi_N) \left[ \frac{\beta}{1 - \rho} (Y_{NT})^{1 - \frac{\beta}{\rho}} (1 - \beta) \frac{1}{\rho} \left( L_{NT}^h \right)^{\frac{1}{\beta}} (A_l^n)^{-\frac{1}{1 - \rho}} + \frac{\rho_{St} S_{NT} \omega_{St}^N}{q_{St}^N} \right] + \xi_N \left[ \phi_N (1 - g_{NT}) \right]} \]  

Similarly, if intellectual property rights are not enforced in the South, the skill premium in the South is given by\(^4\)

\[ \frac{W_{St}^h}{W_{St}^l} = \frac{(1 - \xi_S^h) \left[ \frac{\beta}{1 - \rho} (Y_{St})^{1 - \frac{\beta}{\rho}} (1 - \beta) \frac{1}{\rho} \left( L_{St}^h \right)^{\frac{1}{\beta}} (kA_l^n)^{-\frac{1}{1 - \rho}} + \frac{\rho_{St} S_{St} \omega_{St}^S}{q_{St}^S} \right] + \xi_S^h \left[ \phi_S^h (1 - d_{St}) \right]}{(1 - \xi_S^h) \left[ \frac{\beta}{1 - \rho} (Y_{St})^{1 - \frac{\beta}{\rho}} (1 - \beta) \frac{1}{\rho} \left( L_{St}^h \right)^{\frac{1}{\beta}} (kA_l^n)^{-\frac{1}{1 - \rho}} + \frac{\rho_{St} S_{St} \omega_{St}^S}{q_{St}^S} \right] + \xi_S^h \left[ \phi_S (1 - g_{St}) \right]} \]  

If intellectual property rights are enforced in the South, the skill premium in the South is given by\(^5\)

\[ \frac{W_{St}^{h(IPR)}}{W_{St}^{l(IPR)}} = \frac{(1 - \xi_S^h) \left[ \frac{\beta}{1 - \rho} (Y_{St})^{1 - \frac{\beta}{\rho}} (1 - \beta) \frac{1}{\rho} \left( L_{St}^h \right)^{\frac{1}{\beta}} (A_l^n)^{-\frac{1}{1 - \rho}} + \frac{\rho_{St} S_{St} \omega_{St}^S}{q_{St}^S} \right] + \xi_S^h \left[ \phi_S^h (1 - d_{St}) \right]}{(1 - \xi_S^h) \left[ \frac{\beta}{1 - \rho} (Y_{St})^{1 - \frac{\beta}{\rho}} (1 - \beta) \frac{1}{\rho} \left( L_{St}^h \right)^{\frac{1}{\beta}} (A_l^n)^{-\frac{1}{1 - \rho}} + \frac{\rho_{St} S_{St} \omega_{St}^S}{q_{St}^S} \right] + \xi_S^h \left[ \phi_S (1 - g_{St}) \right]} \]  

\(^3\)Detailed derivations are included in appendix 4.3.1.

\(^4\)Detailed derivations are included in appendix 4.3.1.

\(^5\)Detailed derivations are included in appendix 4.3.1.
2.2 Trade

Assume the two countries liberalize trade in final goods, with both goods traded costlessly. Trade equalizes the prices of the final goods such that \( P_{St}^l = P_{St}^h = P_{Wt}^l \) and \( P_{St}^h = P_{St}^l = P_{Wt}^h \), where the \( P_{St}^l \) and \( P_{St}^h \) are the after trade liberalization world prices of the unskilled-intensive and the skilled-intensive final goods respectively. If intellectual property rights are not enforced, the skill premium in the North after trade liberalization is thus given by\(^6\)

\[
\frac{W_{Nt}^h}{W_{Nt}^l} = \frac{1 - \xi_N^l}{1 - \xi_N^h} \left[ \frac{\beta}{1 - \beta} \left( Y_{Wt} \right)^{\frac{1}{\gamma} \left( 1 - \gamma \right)^{\frac{1}{\gamma} \left( L_{Nt}^l + k L_{St}^l \right)^{\frac{1}{\gamma} \left( A_l^h \right)^{\frac{1}{\gamma} \left( \phi_N^l (1 - d_{Nt}) \right) + \xi_N^l \left( \phi_N^l (1 - d_{Nt}) \right)} \right)} \right]
\]

While the skill premium in the South after trade liberalization is given by\(^7\)

\[
\frac{W_{St}^h}{W_{St}^l} = \frac{1 - \xi_S^l}{1 - \xi_S^h} \left[ \frac{\beta}{1 - \beta} \left( Y_{Wt} \right)^{\frac{1}{\gamma} \left( 1 - \gamma \right)^{\frac{1}{\gamma} \left( L_{Nt}^l + k L_{St}^l \right)^{\frac{1}{\gamma} \left( A_l^h \right)^{\frac{1}{\gamma} \left( \phi_N^l (1 - d_{Nt}) \right) + \xi_N^l \left( \phi_N^l (1 - d_{Nt}) \right)} \right)} \right]
\]

If intellectual property rights are enforced, the skill premium in the North after trade liberalization is given by\(^8\)

\[
\frac{W_{Nt}^{h.IP}}{W_{Nt}^{l.IP}} = \frac{1 - \xi_N^l}{1 - \xi_N^h} \left[ \frac{\beta}{1 - \beta} \left( Y_{Wt} \right)^{\frac{1}{\gamma} \left( 1 - \gamma \right)^{\frac{1}{\gamma} \left( L_{Nt}^l + k L_{St}^l \right)^{\frac{1}{\gamma} \left( A_l^h \right)^{\frac{1}{\gamma} \left( \phi_N^l (1 - d_{Nt}) \right) + \xi_N^l \left( \phi_N^l (1 - d_{Nt}) \right)} \right)} \right]
\]

While the skill premium in the South after trade liberalization is given by\(^9\)

\[
\frac{W_{St}^{h.IP}}{W_{St}^{l.IP}} = \frac{1 - \xi_S^l}{1 - \xi_S^h} \left[ \frac{\beta}{1 - \beta} \left( Y_{Wt} \right)^{\frac{1}{\gamma} \left( 1 - \gamma \right)^{\frac{1}{\gamma} \left( L_{Nt}^l + k L_{St}^l \right)^{\frac{1}{\gamma} \left( A_l^h \right)^{\frac{1}{\gamma} \left( \phi_N^l (1 - d_{Nt}) \right) + \xi_N^l \left( \phi_N^l (1 - d_{Nt}) \right)} \right)} \right]
\]

\(^6\)Detailed derivations are included in appendix 4.3.2.
\(^7\)Detailed derivations are included in appendix 4.3.2.
\(^8\)Detailed derivations are included in appendix 4.3.2.
\(^9\)Detailed derivations are included in appendix 4.3.2.
2.3 Steady State

Assuming that in a steady state equilibrium, \( L^h_N = \mu L^h_N \), \( L^l_N = vL^l_N \), \( Y_N = \bar{Y} W \), \( Y_S = (1-\bar{\theta}) Y_W \), and finally \( Y_S = \theta Y_N \), where \( \mu > 0 \), \( v > 0 \), \( 0 < \bar{\theta} < 1 \), \( \theta > 0 \), and \( \bar{\theta} = \frac{1}{1+\theta} \). Therefore, we can state the conditions under which the skill premium increases in the North and in the South after trade liberalization only in terms of the steady state variables \( v \), \( \mu \) and \( \theta \) and the parameters \( \sigma = \beta \varepsilon + 1 - \beta \) and \( k \).

**Proposition 1** in a steady state equilibrium, if labor markets in the North and the South are Walrasian, and if protection of intellectual property is not enforced in the South, the skill premium in the North increases after trade liberalization if and only if

\[
\frac{\mu}{v} > \mu
\]

while the skill premium increases in the South after trade liberalization if and only if

\[
\frac{1+\mu}{1+v} > \frac{1+k\mu}{1+kv}
\]

**Proof.** included in appendix 4.5

As \( v > \mu \) can be rearranged as \( \frac{L^h_N}{L^l_N} > \frac{L^h_S}{L^l_S} \), the skill premium increases in the North if the supply of skilled to unskilled workers in the North is higher than that in the South. If the aggregate production function is Cobb-Douglas, the constant elasticity of substitution \( \varepsilon = 1 \), and thus \( \sigma = 1 \). In this case, the skill premium in the South increases after trade if \( v < \mu \), which is satisfied if the supply of skilled to unskilled workers in the South is higher than that in the North. Therefore, if the aggregate production is Cobb-Douglas, and if the North is skill abundant compared to the South, trade liberalization increases the skill premium in the North and reduces it in the South, consistently with the predictions of traditional trade theory.

**Proposition 2** in a steady state equilibrium, if labor markets in the North and the South are Walrasian, and if protection of intellectual property is enforced in the South, the skill premium in the North increases after trade liberalization if and only if

\[
\left( \frac{1+\mu}{1+v} \right)^{\sigma-2} > \left[ \frac{1+\theta^\frac{\mu}{\sigma} \mu^\frac{\sigma-1}{\sigma}}{1+\theta^\frac{\mu}{\sigma} \frac{\sigma-1}{\sigma}} \right]^{\sigma-1}
\]

while the skill premium increases in the South after trade liberalization if and only if

\[
\left( \frac{\mu}{v} \right)^{\frac{\mu}{\sigma}} \left( \frac{1+\mu}{1+v} \right)^{\sigma-2} > \left[ \frac{1+\theta^\frac{\mu}{\sigma} \mu^\frac{\sigma-1}{\sigma}}{1+\theta^\frac{\mu}{\sigma} \frac{\sigma-1}{\sigma}} \right]^{\sigma-1}
\]
Proof. included in appendix 4.6 ■

If the aggregate production is Cobb-Douglas, these conditions collapse to the same conditions in proposition 1.

**Proposition 3** in a steady state equilibrium, if labor markets in the North and the South are non-Walrasian, featuring search frictions, and if protection of intellectual property is enforced in the South, the skill premium in the North increases after trade liberalization if and only if

\[ v > \theta \]

and

\[
\frac{\left( \frac{1}{1+\theta v} \right)^{\frac{\sigma - 1}{\sigma}}}{\left( \frac{1}{1+\mu} \right)^{\frac{\sigma - 2}{\sigma}}} > \frac{1 + \theta \mu}{1 + \theta v}
\]

**Proof.** included in appendix 4.7 ■

If the aggregate production is Cobb-Douglas, then the skill premium in the North increases after trade liberalization if the ratio of the supply of unskilled workers in the South to that in the North \( v \) is higher than the ratio of the production in the South to that in the North \( \theta \). In addition, the second condition stipulates that \( \theta > \mu \) which means that the ratio of the production in the South to that in the North \( \theta \) is higher than the ratio of the supply of skilled workers in the South to that in the North. Combining these conditions, the skill premium increases in the North if and only if \( v > \theta > \mu \).

**Proposition 4** in a steady state equilibrium, if labor markets in the North and the South are non-Walrasian, featuring search frictions, and if protection of intellectual property is not enforced in the South, the skill premium in the North increases after trade liberalization if and only if

\[ v > \frac{\theta}{k} \]

and

\[
\frac{\left( \frac{1}{1+k\theta v} \right)^{\frac{\sigma - 1}{\sigma}}}{\left( \frac{1}{1+k\mu} \right)^{\frac{\sigma - 2}{\sigma}}} > \frac{1 + k\mu}{1 + k\theta v}
\]

**Proof.** included in appendix 4.8 ■
If aggregate production is Cobb-Douglas, the second condition can be rearranged as \( \frac{\theta - k}{\kappa} > \mu \), thus both conditions collapse to \( v > \frac{\theta}{\kappa} > \mu \).

**Proposition 5** in a steady state equilibrium, if labor markets in the North and the South are non-Walrasian, featuring search frictions, and if protection of intellectual property is enforced in the South, the skill premium in the South increases after trade liberalization if and only if

\[
\theta > v
\]

and

\[
\frac{\left( \frac{v}{1+v} \right)^{\frac{1}{\sigma}}} \left( \frac{\theta}{1+\theta} \right)^{\frac{1}{\sigma}} > \frac{\left( \frac{1+\theta + \mu - 1}{\theta + k - 1} \right)^{\frac{1}{\sigma-2}}} \left( \frac{1}{1+\theta} \right)^{\frac{1}{\sigma-2}}
\]

**Proof.** included in appendix 4.9 □

If aggregate production is Cobb-Douglas, then the skill premium increases in the South if the ratio of the supply of unskilled workers in the South to that in the North \( v \) is less than the ratio of the production in the South to that in the North \( \theta \). The second condition collapses to \( \mu > \theta \), which means that the ratio of the production in the South to that in the North \( \theta \) is less than the ratio of the supply of skilled workers in the South to that in the North. These conditions can thus be combined such that the skill premium increases in the South if and only if \( \mu > \theta > v \).

**Proposition 6** in a steady state equilibrium, if labor markets in the North and the South are non-Walrasian, featuring search frictions, and if protection of intellectual property is not enforced in the South, the skill premium in the South increases after trade liberalization if and only if

\[
\frac{\theta}{1+\theta} > \left( \frac{v}{1+kv} \right)
\]

and

\[
\frac{\left( \frac{v}{1+kv} \right)^{\frac{1}{\sigma}}} \left( \frac{\theta}{1+\theta} \right)^{\frac{1}{\sigma}} > \left( \frac{1}{1+\theta} \right)^{\frac{1}{\sigma}}
\]

**Proof.** included in appendix 4.10 □

If aggregate production is Cobb-Douglas, the skill premium in the south increases after trade if \( v + v\theta < \theta + \thetakv \), and if \( \theta + \theta k\mu < \mu + \mu \). If \( v < \mu \), these are combined to \( v + v\theta < \theta + \theta kv < \theta + \theta k\mu < \mu + \mu \).
3 Conclusion

Traditional trade theory predicts that openness induces countries to export the good that intensively uses the relatively abundant factor of production and import the good that intensively uses the factor of production that is relatively scarce. As developed countries are considered skill abundant, they export the good that intensively uses skilled workers. This in return contributes to an increase in the relative price of the skilled-intensive good, to a rise in the relative demand for skilled workers, and accordingly to an increase in the skill premium. The theory also predicts that developing countries are expected to experience a decline in the relative price of their skilled-intensive good and consequently a decline in the skill premium. Nevertheless, empirical evidence shows that though some developing countries witness a declining skill premium, others experience a widening wage gap.

This paper attempts to address the puzzle or the inconsistency between the theoretical predictions and the stylized facts. The models that have been developed in the previous literature feature Walrasian labor markets with no frictions, in which wages are equal to the value of the marginal productivity of labor. Therefore, as the skill abundant country’s exports of the skilled-intensive goods increase, the relative price of the exported good increases and accordingly, the relative wage for the skilled workers increases as well. This paper introduces a setup in which labor markets are non Walrasian, and feature search frictions. In this context, wages are a weighted average of two terms: one includes the value of the marginal productivity of labor in addition to a measure of market tightness, while the other includes outside options. Therefore, the relative increase in the exports of the skilled-intensive good in the skill abundant country does not translate directly to an increase in demand for skilled workers, but rather translates into an increase of vacancies posted specifically to recruit this type of workers. The increase in recruitment depends on market conditions and the efficiency of the matching process. In this framework, the conditions under which the skill premium increases after trade openness either in the North and in the South permit the possibility that some countries experience an increase in wage inequality while others witness a decline. For instance, according to propositions 3 and 5, the skill premium in the South declines if and only if \( \mu < \theta < \upsilon \). However, this condition allows for the possibility that even though all the developing countries are unskilled abundant, or that \( \mu < \upsilon \), some of these countries might satisfy \( \mu < \upsilon < \theta \) or \( \theta < \mu < \upsilon \), and accordingly experience an increase in wage inequality after trade liberalization. This asymmetry in the response of the developing countries is consistent with the empirical observations.

A possible extension is a framework in which the South could undertake research and development to invent intermediate varieties that are appropriate to their specific conditions. So, firms in the South would
have to choose between adopting the North technology, or inventing their own appropriate technology. Also, the producers of final goods in the North can, rather than buy the intermediate goods from North intermediate producers, outsource the production of the intermediate goods to the South to benefit from lower marginal costs. Analyzing the outcomes of these possibilities on wage inequality in the North and the South is worth pursuing.
Appendix

4.1 The wage of skilled workers

The surplus accrued by the household is expressed in terms of goods rather than marginal utility, and reduces to $\frac{1}{\lambda_{it} \partial L_{it}^h}$ in order to guarantee that both the surplus of the producers of skilled-intensive final good and of households are expressed in the same units. The bargained wage is determined by the maximization of the Nash product as follows

$$W_{it}^h = \text{Argmax} \left[ \frac{1}{\lambda_{it} \partial L_{it}^h} \right]$$

Then the sharing rule implies

$$\xi_{it}^h \left[ \frac{\partial \Gamma_{it}}{\partial L_{it}^h} \right] = \left(1 - \xi_{it}^h\right) \lambda_{it} \left[ \frac{\partial \Omega_{it}^c}{\partial L_{it}^h} \right]$$

Substituting the envelope conditions of the households and the firms yields

$$\xi_{it}^h \left[ \lambda_{it} W_{it}^h - \lambda_{it} (1 - d_{it}) \phi_{it}^h + \Re (1 - \eta_{it}^c - \rho_{it}^c d_{it}) E_t \left( \frac{\partial \Gamma_{it+1}}{\partial L_{it+1}^h} \right) \right] = \left(1 - \xi_{it}^h\right) \lambda_{it} \left[ \frac{\partial \Omega_{it+1}^c}{\partial L_{it+1}^h} \right]$$

As we have from the first order conditions

$$\frac{\omega_{it}^c}{q_{it}^c} = \Re E_t \left[ \frac{\lambda_{it+1} \partial \Omega_{it+1}^c}{\lambda_{it} \partial L_{it+1}^h} \right]$$

Therefore

$$\xi_{it}^h \Re E_t \left[ \frac{\partial \Gamma_{it+1}}{\partial L_{it+1}^h} \right] = \left(1 - \xi_{it}^h\right) \Re E_t \left[ \frac{\lambda_{it+1} \partial \Omega_{it+1}^c}{\lambda_{it} \partial L_{it+1}^h} \right] = \left(1 - \xi_{it}^h\right) \frac{\omega_{it}^c}{q_{it}^c}$$

Substituting yields

$$\xi_{it}^h \left[ \lambda_{it} W_{it}^h - \lambda_{it} (1 - d_{it}) \phi_{it}^h + \left(1 - \xi_{it}^h\right) \frac{\lambda_{it} \omega_{it}^c}{q_{it}^c} (1 - \eta_{it}^c - \rho_{it}^c d_{it}) \right] = \left(1 - \xi_{it}^h\right) \lambda_{it} \left[ \frac{\partial \Omega_{it+1}^c}{\partial L_{it+1}^h} \right]$$

Solving for the equilibrium wage rule for the skilled workers yields
\[ W^h_{it} = \left(1 - \xi_i^h\right) \left[ P^h_{it} \frac{\partial Y^h_{it}}{\partial L^h_{it}} + \frac{\rho^e_{it} d_{it} \omega^s_{it}}{q^s_{it}}\right] + \xi_i^h \left[ \phi^h \left(1 - d_{it}\right)\right] \]

### 4.2 The wage of unskilled workers

The surplus accrued by the household is expressed in terms of goods rather than marginal utility, and reduces to \[\frac{1}{\lambda_{it} \partial L^s_{it}}\] in order to guarantee that both the surplus of the producers of the unskilled-intensive final good and of households are expressed in the same units. The bargained wage is determined by the maximization of the Nash product as follows

\[ W^l_{it} = \text{Argmax} \left[ \frac{1}{\lambda_{it} \partial L^l_{it}} \right]
\]

Then the sharing rule implies

\[ \xi_i^l \left[ \frac{\partial \Gamma_{it}}{\partial L^l_{it}} \right] = \left(1 - \xi_i^l\right) \lambda_{it} \frac{\partial \Omega^s_{it}}{\partial L^l_{it}} \]

Substituting the envelope conditions of the households and the firms yields

\[ \frac{\omega^s_{it}}{q^s_{it}} = \mathcal{R} E_t \frac{\lambda_{it+1} \partial \Omega^s_{it+1}}{\lambda_{it} \partial L^l_{it+1}} \]

As we have from the first order conditions

\[ \xi_i^l \mathcal{R} E_t \left[ \frac{\partial \Gamma_{it+1}}{\partial L^l_{it+1}} \right] = \left(1 - \xi_i^l\right) \mathcal{R} E_t \left[ \frac{\lambda_{it+1} \partial \Omega^s_{it+1}}{\lambda_{it} \partial L^l_{it+1}} \right] = \left(1 - \xi_i^l\right) \frac{\omega^s_{it}}{q^s_{it}} \]

Substituting yields

\[ \xi_i^l \left[ \lambda_{it} W^l_{it} - \lambda_{it} \phi^l_{it} (1 - g_{it}) + \mathcal{R} (1 - \eta^s_{it} - \rho^s_{it} g_{it}) E_t \left( \frac{\partial \Gamma_{it+1}}{\partial L^l_{it+1}} \right) \right] = \left(1 - \xi_i^l\right) \frac{\omega^s_{it}}{q^s_{it}} \]
Solving for the equilibrium wage rule for the unskilled workers yields

\[ W_{lt}^l = (1 - \xi_l^l) \left[ P_{lt}^l \frac{\partial Y_{lt}^l}{\partial L_{lt}^l} + \frac{\rho_{lt}^l \omega_{lt}^l}{q_{lt}^l} \right] + \xi_l^l \left[ \phi_l^l (1 - g_{lt}) \right] \]

### 4.3 Skill Premia

#### 4.3.1 Autarky

**North** Substituting the demand for and the price of unskilled-intensive intermediates into the North production function of the unskilled-intensive final good yields

\[ Y_{Nt}^l = \frac{1}{1 - \beta} \left[ \int_0^{A_t} \left( [P_{Nt}^l]^{\frac{\beta}{\gamma}} (L_{Nt}^l)^{1-\beta} \right) dj \right] (L_{Nt}^l)^{\beta} = \frac{1}{1 - \beta} (P_{Nt}^l)^{\frac{1-\beta}{\sigma}} L_{Nt}^l A_t^l \]

The price of the unskilled-intensive good in the North is thus given by

\[ P_{Nt}^l = (Y_{Nt})^{\frac{\beta}{\gamma}} (Y_{Nt})^{\frac{\beta}{\sigma}} = (Y_{Nt})^{\frac{\beta}{\gamma}} \left[ \frac{1}{1 - \beta} (P_{Nt}^l)^{\frac{1-\beta}{\sigma}} L_{Nt}^l A_t^l \right]^{\frac{\sigma}{1-\beta}} \]

Define \( \sigma = \beta \epsilon + 1 - \beta \), this can be rearranged as

\[ P_{Nt}^l = (Y_{Nt})^{\frac{\beta}{\gamma}} (1 - \beta)^{\frac{\beta}{\sigma}} (L_{Nt}^l A_t^l)^{\frac{\sigma}{1-\beta}} \]

Substitute the demand for intermediate goods and the prices of intermediate and final goods to the marginal productivity of labor in the wage formula. Thus, the value of the marginal productivity in the formula of the wage of the unskilled workers is given by

\[
P_{Nt}^l \frac{\partial Y_{Nt}^l}{\partial L_{Nt}^l} = P_{Nt}^l \left[ \frac{\beta}{1 - \beta} \left[ \int_0^{A_t} \left( [P_{Nt}^l]^{\frac{\beta}{\gamma}} (L_{Nt}^l)^{1-\beta} \right) dj \right] (L_{Nt}^l)^{\beta-1} \right] = P_{Nt}^l \left[ \frac{\beta}{1 - \beta} (P_{Nt}^l)^{\frac{1-\beta}{\sigma}} A_t^l \right] = \frac{\beta}{1 - \beta} \left[ (Y_{Nt})^{\frac{\beta}{\gamma}} (1 - \beta)^{\frac{\beta}{\sigma}} (L_{Nt}^l A_t^l)^{\frac{\sigma}{1-\beta}} \right]^{\frac{1}{\sigma}} A_t^l \]

Substituting the demand for and the price of skilled-intensive intermediates into the North production function of the skilled-intensive final good yields
The price of the skilled-intensive final good in the North is thus given by

\[ \begin{align*}
P_{Nh}^h &= (Y_{Nh})^{\frac{\beta}{\sigma}} \gamma \left( \frac{1}{1 - \gamma} \right) \left( \frac{\beta}{\sigma} \right) \left( L_{Nh}^h \right)^{-\frac{\beta}{\sigma}}.
\end{align*} \]

This can be rearranged as

\[ \begin{align*}
P_{Nh}^h &= (Y_{Nh})^{\frac{\beta}{\sigma}} \gamma \left( \frac{1}{1 - \gamma} \right) \left( \frac{\beta}{\sigma} \right) \left( L_{Nh}^h \right)^{-\frac{\beta}{\sigma}}.
\end{align*} \]

The value of the marginal productivity in the formula of the wage of the skilled workers is given by

\[ \begin{align*}
P_{Nh}^h \frac{\partial Y_{Nh}^h}{\partial L_{Nh}^h} &= P_{Nh}^h \frac{\beta}{1 - \beta} \left[ \int_0^{A_t^h} \left( \frac{1}{1 - \beta} \right)^{-\frac{\beta}{\sigma}} \left( L_{Nh}^h \right)^{-\frac{\beta}{\sigma}} \right] \left( L_{Nh}^h \right)^{-\frac{\beta}{\sigma}} \frac{1}{A_t^h} \\
&= P_{Nh}^h \frac{\beta}{1 - \beta} \left( P_{Nh}^h \right)^{-\frac{\beta}{\sigma}} A_t^h = P_{Nh}^h \frac{\beta}{1 - \beta} \left( P_{Nh}^h \right)^{-\frac{\beta}{\sigma}} A_t^h
\end{align*} \]

Substituting the values of the marginal productivities into the formulas of the wages of the skilled and the unskilled workers, the skill premium in the North is given by (43).

**South** If intellectual property rights are not enforced in the South, substituting the demand for and the price of the unskilled-intensive intermediates into the South production function of the unskilled-intensive final good yields

\[ \begin{align*}
Y_{St}^l &= \frac{1}{1 - \beta} \left[ \int_0^{A_t^l} \left( L_{St}^l \left( P_{St}^l \right)^{\frac{1}{\sigma}} k_{St}^l \right)^{\frac{1}{1 - \beta}} \right] \left( L_{St}^l \right)^{\frac{1}{1 - \beta}} = \frac{1}{1 - \beta} \left( P_{St}^l \right)^{-\frac{\beta}{\sigma}} k_{St}^l A_t^l.
\end{align*} \]

The price of the unskilled-intensive good in the South is thus given by

\[ \begin{align*}
P_{St}^l &= (Y_{St})^{\frac{\beta}{\sigma}} \gamma \left( \frac{1}{1 - \gamma} \right) \left( \frac{\beta}{\sigma} \right) \left( P_{St}^l \right)^{-\frac{\beta}{\sigma}} k_{St}^l A_t^l
\end{align*} \]

This can be rearranged as

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The value of the marginal productivity in the formula of the wage of the skilled workers is given by

$$P_{St} = (Y_{St})^\frac{\beta}{\gamma} (1 - \beta)^\frac{\beta}{\gamma} (kL_{St}A_{t}^h)^{-\frac{\beta}{\gamma}}$$

The price of the skilled-intensive final good yields

$$Y_{St} = \left[ \int_{0}^{A_{t}^h} \left( L_{St}^h \left( P_{St}^{h} \right)^{\frac{\beta}{\gamma}} k \frac{\beta}{\gamma} \right) \left( L_{St}^h \right)^{\beta} \right] = \frac{1}{1 - \beta} \left( P_{St}^{h} \right)^{\frac{1 - \beta}{\beta}} kL_{St}^hA_{t}^h$$

The price of the skilled-intensive final good in the South is thus given by

$$P_{St}^{h} = (Y_{St})^\frac{\beta}{\gamma} (1 - \gamma) (Y_{St}^{h})^{-\frac{1}{\gamma}} = (Y_{St})^\frac{\beta}{\gamma} (1 - \gamma) \left[ \frac{1}{1 - \beta} \left( P_{St}^{h} \right)^{\frac{1 - \beta}{\gamma}} kL_{St}^hA_{t}^h \right]^{-\frac{1}{\gamma}}$$

This can be rearranged as

$$P_{St}^{h} = (Y_{St})^\frac{\beta}{\gamma} (1 - \gamma)^{\frac{\beta}{\gamma}} (1 - \beta)^{\frac{\beta}{\gamma}} (kL_{St}^hA_{t}^h)^{-\frac{\beta}{\gamma}}$$

The value of the marginal productivity in the formula of the wage of the skilled workers is given by

$$P_{St}^{h} \frac{\partial Y_{St}^{h}}{\partial L_{St}^h} = \left[ \int_{0}^{A_{t}^h} \left( L_{St}^h \left( P_{St}^{h} \right)^{\frac{\beta}{\gamma}} k \frac{\beta}{\gamma} \right) \left( L_{St}^h \right)^{\beta} \right] = \frac{1}{1 - \beta} \left( P_{St}^{h} \right)^{\frac{1 - \beta}{\beta}} kL_{St}^hA_{t}^h$$
Substituting the values of the marginal productivities into the formulas of the wages of the skilled and the unskilled workers, the skill premium in the South, if intellectual property rights are not enforced, is given by (44).

If intellectual property rights are enforced in the South, substituting the demand for and the price of the unskilled-intensive intermediates into the South production function of the unskilled-intensive final good yields

\[
Y_{St}^{(IPR)} = \frac{1}{1 - \beta} \left[ \int_0^{A_t^l} \left( L_{St}^l \left( P_{St}^{(IPR)} \right)^{1 - \beta} \right) \, dj \right] (L_{St}^l)^{\beta}
\]

\[
= \frac{1}{1 - \beta} \left( P_{St}^{(IPR)} \right)^{1 - \beta} L_{St}^l A_t^l
\]

The price of the unskilled-intensive good in the South is thus given by

\[
P_{St}^{(IPR)} = (Y_{St})^{\frac{\beta}{\gamma}} \left( Y_{St}^{(IPR)} \right)^{\frac{1 - \beta}{\gamma}} = (Y_{St})^{\frac{\beta}{\gamma}} \left( 1 - \gamma \right) \left[ \frac{1}{1 - \beta} \left( P_{St}^{(IPR)} \right)^{1 - \beta} L_{St}^l A_t^l \right]^{\frac{1 - \beta}{\gamma}}
\]

This can be rearranged as

\[
P_{St}^{(IPR)} = (Y_{St})^{\frac{\beta}{\gamma}} \left( \gamma \left( Y_{St}^{(IPR)} \right)^{\frac{1 - \beta}{\gamma}} \right)^{\frac{\beta}{1 - \beta}} = (Y_{St})^{\frac{\beta}{\gamma}} \left( 1 - \beta \right)^{\frac{\beta}{1 - \beta}} \left( L_{St}^l A_t^l \right)^{\frac{1 - \beta}{\gamma}}
\]

The value of the marginal productivity in the formula of the wage of the unskilled workers is given by

\[
P_{St}^{(IPR)} \frac{\partial Y_{St}^{(IPR)}}{\partial P_{St}^l} = P_{St}^l \frac{\beta}{1 - \beta} \left[ \int_0^{A_t^l} \left( L_{St}^l \left( P_{St}^{(IPR)} \right)^{1 - \beta} \right) \, dj \right] (L_{St}^l)^{\beta - 1}
\]

\[
= P_{St}^l \frac{\beta}{1 - \beta} \left( P_{St}^{(IPR)} \right)^{\frac{1 - \beta}{\gamma}} A_t^l = \frac{\beta}{1 - \beta} \left( P_{St}^{(IPR)} \right)^{\frac{1 - \beta}{\gamma}} A_t^l
\]

\[
= \frac{\beta}{1 - \beta} \left( Y_{St} \right)^{\frac{\beta}{\gamma}} (1 - \beta)^{\frac{\beta}{\gamma}} (L_{St}^l A_t^l)^{\frac{1 - \beta}{\gamma}} A_t^l
\]

\[
= \frac{\beta}{1 - \beta} \left( Y_{St} \right)^{\frac{1 - \beta}{\gamma}} (1 - \beta)^{\frac{1 - \beta}{\gamma}} \gamma \left( L_{St}^l \right)^{\frac{1 - \beta}{\gamma}} (A_t^l)^{\frac{1 - \beta}{\gamma}}
\]

Substituting the demand for and the price of skilled-intensive intermediates into the South production
function of the skilled-intensive final good yields

$$Y^{h(IPR)}_{St} = \frac{1}{1 - \beta} \left[ \int_{0}^{A^h_{St}} \left( P^{h(IPR)}_{St} \right)^{\frac{1}{\beta}} \right] \left( L^{h}_{St} \right)^{\beta} \left( L^{h}_{St} \right)^{1 - \beta} = \frac{1}{1 - \beta} \left( P^{h(IPR)}_{St} \right)^{\frac{1}{\beta}} L^{h}_{St} A^{h}_{St}$$

Similarly, if intellectual property rights are enforced in the South, the price of the skilled-intensive final good in the South is given by

$$P^{h(IPR)}_{St} = (Y_{St})^{\frac{1}{\beta}} (1 - \gamma) \left( Y^{h(IPR)}_{St} \right)^{\frac{1}{\beta}} = (Y_{St})^{\frac{1}{\beta}} (1 - \gamma) \left[ \frac{1}{1 - \beta} \left( P^{h(IPR)}_{St} \right)^{\frac{1}{\beta}} L^{h}_{St} A^{h}_{St} \right]$$

This can be rearranged as

$$P^{h(IPR)}_{St} = (Y_{St})^{\frac{1}{\beta}} (1 - \gamma) \left( Y^{h(IPR)}_{St} \right)^{\frac{1}{\beta}} \left( L^{h}_{St} A^{h}_{St} \right)^{\frac{1}{\beta}}$$

The value of the marginal productivity in the formula of the wage of the skilled workers is given by

$$P^{h(IPR)}_{St} \frac{\partial Y^{h(IPR)}_{St}}{\partial L^{h}_{St}} = P^{h(IPR)}_{St} \frac{\beta}{1 - \beta} \left[ \left( P^{h(IPR)}_{St} \right)^{\frac{1}{\beta}} (L^{h}_{St})^{\beta} \right] = \frac{\beta}{1 - \beta} \left( P^{h(IPR)}_{St} \right)^{\frac{1}{\beta}} A^{h}_{St}$$

$$= \frac{\beta}{1 - \beta} \left[ (Y_{St})^{\frac{1}{\beta}} (1 - \gamma) \left( Y^{h(IPR)}_{St} \right)^{\frac{1}{\beta}} \left( L^{h}_{St} A^{h}_{St} \right)^{\frac{1}{\beta}} \right]$$

$$= \frac{\beta}{1 - \beta} \left( Y_{St} \right)^{\frac{1}{\beta}} (1 - \beta) \left( 1 - \gamma \right) \left( L^{h}_{St} A^{h}_{St} \right)^{\frac{1}{\beta}}$$

Substituting the values of the marginal productivities into the formulas of the wages of the skilled and the unskilled workers, the skill premium in the South, if intellectual property rights are enforced, is given by (45).

4.3.2 Trade

North After trade openness, if intellectual property rights are enforced, the world supply of the skilled-intensive good is given by

$$Y^{h(IPR)}_{Wt} = \frac{1}{1 - \beta} \left[ \int_{0}^{A^h_{Nt}} \left( P^{h(IPR)}_{Nt} \right)^{\frac{1}{\beta}} (L^{h}_{Nt})^{\beta} \right] \left( L^{h}_{Nt} \right)^{1 - \beta} + \int_{0}^{A^h_{St}} \left( P^{h(IPR)}_{St} \right)^{\frac{1}{\beta}} (L^{h}_{St})^{\beta}$$
As the prices of the skilled-intensive goods in the North and South are equalized after trade, this can be rearranged as

\[ Y_{Wt}^h(IPR) = \frac{1}{1-\beta} \left( P_{Wt}^h(IPR) \right)^{1-\beta} \left( L_{Nt}^h + L_{St}^h \right) A_t^h \]

The world price of the skilled-intensive good is thus given by

\[ P_{Wt}^h(IPR) = (Y_{Wt})^{\frac{1}{\beta}} (1-\gamma) \left( Y_{Wt}^h(IPR) \right)^{-\frac{1}{\beta}} = (Y_{Wt})^{\frac{1}{\beta}} (1-\gamma) \left[ \frac{1}{1-\beta} \left( P_{Wt}^h(IPR) \right)^{1-\beta} \left( L_{Nt}^h + L_{St}^h \right) A_t^h \right]^\frac{1}{\beta} \]

This can be rearranged as

\[ P_{Wt}^h(IPR) = (Y_{Wt})^{\frac{1}{\beta}} (1-\gamma) \left( Y_{Wt}^h(IPR) \right)^{-\frac{1}{\beta}} (1-\beta)^{\frac{1}{\beta}} \left( L_{Nt}^h + L_{St}^h \right) \frac{\sigma}{\beta} \left( A_t^h \right)^{-\frac{1}{\beta}} \]

The value of the marginal productivity in the formula of the wage of the skilled workers is given by

\[ P_{Wt}^h(IPR) \left( \frac{\partial Y_{Wt}^h}{\partial L_{Nt}^h} \right) = \frac{P_{Wt}^h(IPR)}{1-\beta} \left[ \int_0^{A_t^h} \left( \frac{P_{Wt}^h(IPR)}{1-\beta} \left( L_{Nt}^h \right)^{1-\beta} dj \right) \left( L_{Nt}^h \right)^{\beta - 1} \right] = \frac{P_{Wt}^h(IPR)}{1-\beta} \left( A_t^h \right)^{1-\beta} \left( L_{Nt}^h \right)^{\beta - 1} \left( A_t^h \right)^{\beta - 1} \]

Similarly, the world supply of the unskilled-intensive good is given by

\[ Y_{Wt}^i(IPR) = \frac{1}{1-\beta} \left[ \int_0^{A_t^i} \left( P_{Nt}^i \right)^{1-\beta} \left( L_{Nt}^i \right)^{1-\beta} dj \left( L_{Nt}^i \right)^{\beta} + \int_0^{A_t^i} \left( P_{St}^i(IPR) \right)^{1-\beta} \left( L_{St}^i \right)^{1-\beta} \left( L_{St}^i \right)^{\beta} \right] \]

As the prices of the skilled-intensive goods in the North and South are equalized after trade, this can be rearranged as

\[ Y_{Wt}^i(IPR) = \frac{1}{1-\beta} \left( P_{Wt}^i(IPR) \right)^{1-\beta} \left( L_{Nt}^i + L_{St}^i \right) A_t^i \]

The world price of the unskilled-intensive good is thus given by
\[ P_{Wt}^{(IPR)} = (Y_{Wt})^\frac{\gamma}{\beta} \left( Y_{Wt}^{(IPR)} \right)^{1/\gamma} = (Y_{Wt})^\frac{\gamma}{\beta} \left[ \frac{1}{1-\beta} \left( P_{Wt}^{(IPR)} \right)^{1-\beta} (L_{Nt} + L_{St}) A_i \right]^{1/\gamma} \]

This can be rearranged as

\[ P_{Wt}^{(IPR)} = (Y_{Wt})^{\frac{\beta}{\gamma}} (1-\beta)^{\frac{\beta}{\gamma}} (L_{Nt} + L_{St})^{-\frac{\beta}{\gamma}} (A_i)^{-\frac{\beta}{\gamma}} \]

The value of the marginal productivity in the formula of the wage of the unskilled workers is given by

\[ P_{Wt}^{(IPR)} \frac{\partial Y_{Nt}^{(IPR)}}{\partial L_{Nt}} = P_{Wt}^{(IPR)} \frac{\beta}{1-\beta} \left[ \int_0^{A_i} \left( \left[ P_{Wt}^{(IPR)} \right]^{1-\beta} (L_{Nt}) \right) \frac{1-\beta}{\beta} A_i \right] \left( L_{Nt} \right)^{\beta-1} \]

\[ = P_{Wt}^{(IPR)} \frac{\beta}{1-\beta} \left( P_{Wt}^{(IPR)} \right)^{1-\beta} = \frac{\beta}{1-\beta} \left( P_{Wt}^{(IPR)} \right)^{1-\beta} A_i \]

Substituting the values of the marginal productivities into the formulas of the wages of the skilled and the unskilled workers, the skill premium in the North, if intellectual property rights are enforced, is given by (46).

If intellectual property rights are not enforced, the world supply of the skilled-intensive good is given by

\[ Y_{Wt}^h = \frac{1}{1-\beta} \left( P_{Wt}^h \right)^{\frac{1-\beta}{\gamma}} (L_{Nt} + kL_{St}) A_i^h \]

The world price of the skilled-intensive good is thus given by

\[ P_{Wt}^h = (Y_{Wt})^{\frac{\beta}{\gamma}} (1-\gamma)^{\frac{\beta}{\gamma}} (1-\beta)^{\frac{\beta}{\gamma}} (L_{Nt} + kL_{St})^{-\frac{\beta}{\gamma}} (A_i^h)^{-\frac{\beta}{\gamma}} \]

The value of the marginal productivity in the formula of the wage of the skilled workers is given by
Similarly, the world supply of the unskilled-intensive good is given by

$$Y_{Wt}^l = \frac{1}{1-\beta} \left( P_{Wt}^l \right)^{1-\beta} \left( L_{Nt}^l + kL_{St}^l \right) A_i^l$$

The world price of the unskilled-intensive good is thus given by

$$P_{Wt}^l = (Y_{Wt}^l)^{\frac{\sigma}{\gamma}} (1-\beta)^{\frac{\gamma}{\sigma}} (L_{Nt}^l + kL_{St}^l) \frac{-\sigma}{\alpha} (A_i^l)^{-\frac{\sigma}{\alpha}}$$

The value of the marginal productivity in the formula of the wage of the unskilled workers is given by

$$P_{Wt}^l \frac{\partial Y_{Nt}^l}{\partial L_{Nt}^l} = P_{Wt}^l \frac{\beta}{1-\beta} \left[ \int_0^{A_i^l} \left( [P_{Wt}^l]^{\frac{1}{\alpha}} (L_{Nt}^l)^{1-\beta} \right) \left( L_{Nt}^l \right)^{\beta-1} \right]$$

$$= P_{Wt}^l \frac{\beta}{1-\beta} \left( P_{Wt}^l \right)^{\frac{1-\beta}{\alpha}} A_i^l = \frac{\beta}{1-\beta} \left( P_{Wt}^l \right)^{\frac{1-\beta}{\alpha}} A_i^l$$

$$= \frac{\beta}{1-\beta} \left( Y_{Wt}^l \right)^{\frac{1}{\gamma}} (1-\beta)^{\frac{1}{\gamma}} (L_{Nt}^l + kL_{St}^l)^{-\frac{\gamma}{\alpha}} (A_i^l)^{-\frac{\gamma}{\alpha}}$$

Substituting the values of the marginal productivities into the formulas of the wages of the skilled and the unskilled workers, the skill premium in the North, if intellectual property rights are not enforced, is given by (48).

**South**  In the South, if intellectual property rights are not enforced, the value of the marginal productivity of workers in the formula of the wage of the unskilled workers is given by
\[ P^h_{Wt} \frac{\partial Y^h_{St}}{\partial L^h_{St}} = P^h_{Wt} \frac{\beta}{1 - \beta} \left[ \int_0^{A^h_t} \left( \left[ P^{(IPR)}_{Wt} \right]^\frac{\beta}{\gamma} \frac{k}{1 - \sigma} \left( L^h_{St} \right)^{-\frac{\gamma}{\sigma}} \right) dj \right] \left( L^h_{St} \right)^{\beta - 1} \]

\[ = P^h_{Wt} \frac{\beta}{1 - \beta} \left( P^{(IPR)}_{Wt} \right)^{\frac{1 - \beta}{\gamma}} k A^h_t = \frac{\beta}{1 - \beta} \left( P^{(IPR)}_{Wt} \right)^{\frac{1 - \beta}{\gamma}} k A^h_t \]

\[ = \frac{\beta}{1 - \beta} \left( Y_{Wt} \right)^{\frac{\beta}{\gamma}} (1 - \gamma) \gamma \frac{\beta}{\gamma} \left( L^h_{St} + k L^h_{St} \right) \frac{\gamma}{\gamma} \left( A^h_t \right)^{\frac{-\beta}{\gamma}} \frac{1}{\gamma} k A^h_t \]

\[ = \frac{\beta}{1 - \beta} \left( Y_{Wt} \right)^{\frac{\beta}{\gamma}} (1 - \beta) \frac{\beta}{\gamma} \left( L^h_{St} + k L^h_{St} \right) \frac{\gamma}{\gamma} \left( A^h_t \right)^{\frac{-\beta}{\gamma}} \frac{1}{\gamma} k A^h_t \]

The value of the marginal productivity in the formula of the wage of the skilled workers is given by

\[ \frac{P^h_{Wt} \partial Y^h_{St}}{\partial L^h_{St}} = \frac{P^h_{Wt} \beta}{1 - \beta} \left[ \int_0^{A^h_t} \left( \left[ P^{(IPR)}_{Wt} \right]^\frac{\beta}{\gamma} \frac{k}{1 - \sigma} \left( L^h_{St} \right)^{-\frac{\gamma}{\sigma}} \right) dj \right] \left( L^h_{St} \right)^{\beta - 1} \]

Substituting the values of the marginal productivities into the formulas of the wages of the skilled and the unskilled workers, the skill premium in the South, if intellectual property rights are not enforced, is given by (47). If intellectual property rights are enforced, the value of the marginal productivity in the formula of the wage of the unskilled workers is given by

\[ \frac{P^l_{Wt} \partial Y^l_{St}}{\partial L^l_{St}} = \frac{P^l_{Wt} \beta}{1 - \beta} \left[ \int_0^{A^l_t} \left( \left[ P^{(IPR)}_{Wt} \right]^\frac{\beta}{\gamma} \frac{k}{1 - \sigma} \left( L^l_{St} \right)^{-\frac{\gamma}{\sigma}} \right) dj \right] \left( L^l_{St} \right)^{\beta - 1} \]

\[ = \frac{P^l_{Wt} \beta}{1 - \beta} \left( P^{(IPR)}_{Wt} \right)^{\frac{1 - \beta}{\gamma}} A^l_t = \frac{\beta}{1 - \beta} \left( P^{(IPR)}_{Wt} \right)^{\frac{1 - \beta}{\gamma}} A^l_t \]

\[ = \frac{\beta}{1 - \beta} \left( Y_{Wt} \right)^{\frac{\beta}{\gamma}} (1 - \beta) \frac{\beta}{\gamma} \left( L^l_{St} + L^l_{St} \right) \frac{\gamma}{\gamma} \left( A^l_t \right)^{\frac{-\beta}{\gamma}} \frac{1}{\gamma} k A^l_t \]

The value of the marginal productivity in the formula of the wage of the skilled workers is given by
\[
\frac{P^h(\text{IPR})}{W_t} \frac{\partial Y^h(\text{IPR})}{\partial L^h_{St}} = P^h(\text{IPR}) \frac{\beta}{1 - \beta} \left[ \int_0^{A^h_l} \left( \left[ P^h(\text{IPR}) \right] \frac{\beta}{\beta} \left( \frac{L^h_{St}}{L^h_{N}} \right) \right) \right] \left( \frac{L^h_{St}}{L^h_{N}} \right)^{\beta - 1} \\
= P^h(\text{IPR}) \frac{\beta}{1 - \beta} \left( \frac{P^h(\text{IPR})}{P^h(\text{IPR})} \right) A^h_l = \frac{\beta}{1 - \beta} \left( \frac{P^h(\text{IPR})}{P^h(\text{IPR})} \right) A^h_l \\
= \frac{\beta}{1 - \beta} \left[ (Y_{Wt})^\beta (1 - \gamma)^{\beta \gamma} (1 - \beta)^{\beta \gamma} (L^h_{Nl} + L^h_{St}) \frac{\beta}{\beta} \left( A^h_l \right)^\beta \right] A^h_l \\
= \frac{\beta}{1 - \beta} (Y_{Wt})^\beta (1 - \beta)^{\beta \gamma} (1 - \gamma)^{\beta \gamma} (L^h_{Nl} + L^h_{St}) \frac{\beta}{\beta} \left( A^h_l \right)^\beta
\]

Substituting the values of the marginal productivities into the formulas of the wages of the skilled and the unskilled workers, the skill premium in the South, if intellectual property rights are enforced, is given by (49).

### 4.4 Skill Bias

To solve endogenously for the steady state skill bias before and after trade, we utilize the "lab equipment specification" which involves using only the final good in generating new innovations. Therefore, we consider the following production for the intermediate varieties in the North \( A^l = \varphi^l R^l \), and \( A^h = \varphi^h R^h \), where \( R^l \) and \( R^h \) are spending on research for the unskilled and skilled intensive complementaries respectively in terms of the final good. This implies that one unit of the final good spent on research will generate \( \varphi^l \) new varieties of the unskilled complementary intermediate, and \( \varphi^h \) of the skilled complementary intermediate. A firm that discovers a new intermediate receives a perfectly enforced patent and becomes its sole supplier.

If intellectual property rights are not enforced in the South, the profits are given by

\[
\pi^l (j) = [\chi^l (j) - (1 - \beta)] x^l_N (j) = [1 - (1 - \beta)] (P^l_N)^\beta L^l_N = \beta (P^l_N)^\beta L^l_N \\
\pi^h (j) = [\chi^h (j) - (1 - \beta)] x^h_N (j) = [1 - (1 - \beta)] (P^h_N)^\beta L^h_N = \beta (P^h_N)^\beta L^h_N
\]

However, firms consider the net present discounted value of profits which, considering the steady state where \( V^l_N = V^h_N = 0 \), can be expressed as follows

\[
rV^l_N = \pi^l + \hat{V}^l_N \pi^l = \beta (P^l_N)^\beta L^l_N \\
rV^h_N = \pi^h + \hat{V}^h_N \pi^h = \beta (P^h_N)^\beta L^h_N
\]
We also have \( \frac{V_h}{N} = \frac{\varphi}{\varphi^h} \) such that technology monopolists are willing to innovate for both sectors. Therefore, we have from the steady state \( \frac{V_h}{N} = \frac{\varphi}{\varphi^h} = \frac{\pi^h}{\pi} \), which yields \( \varphi^h \pi^h = \varphi^h \pi^l \). This states that it is equally profitable to invest money to invent either type of intermediates. To solve for the skill bias, we have

\[
\frac{\varphi^l}{\varphi^h} = \frac{\pi^h}{\pi^l} = \left( \frac{P^h_N}{P^l_N} \right)^{\frac{\beta}{\sigma}} \frac{L^h_N}{L^l_N}
\]

Substituting for the relative price, and assuming \( \frac{1}{\varphi} = \frac{\varphi^l}{\varphi^h} \), we have

\[
\frac{1}{\varphi} = \left[ \frac{1 - \gamma}{\gamma} \right] \frac{\varphi^h}{\varphi^l} \left[ \frac{L^h_N A^h}{L^l_N A^l} \right] \frac{\pi^h}{\pi^l} \frac{L^h_N}{L^l_N}
\]

Rearranging and solving for the skill bias yields

\[
\frac{A^h}{A^l} = \varphi^h \left( \frac{1 - \gamma}{\gamma} \right)^\varepsilon \left( \frac{L^h_N}{L^l_N} \right)^{\sigma - 1}
\]

This shows that the endogenized direction of technical change or the relative bias of technology is determined by the relative factor supply and the elasticity of substitution between the two factors, and technology is endogenously biased in favor of the more abundant factor if \( \sigma > 1 \). On the other hand, if intellectual property rights are enforced in the South, the profits can be expressed as

\[
\pi^l(\text{IPR}) (j) = [1 - (1 - \beta)] \left[ x^l_N(j) + x^l_S(\text{IPR}) (j) \right] = \beta \left[ (P^l_N)^{\frac{\beta}{\gamma}} L^l_N + (P^l_S)^{\frac{\beta}{\gamma}} L^l_S \right]
\]

\[
\pi^h(\text{IPR}) (j) = [1 - (1 - \beta)] \left[ x^h_N(j) + x^h_S(\text{IPR}) (j) \right] = \beta \left[ (P^h_N)^{\frac{\beta}{\gamma}} L^h_N + (P^h_S)^{\frac{\beta}{\gamma}} L^h_S \right]
\]

Therefore

\[
\frac{\varphi^l}{\varphi^h} = \frac{\pi^h}{\pi^l} = \frac{(P^h_N)^{\frac{\beta}{\gamma}} L^h_N + (P^h_S)^{\frac{\beta}{\gamma}} L^h_S}{(P^l_N)^{\frac{\beta}{\gamma}} L^l_N + (P^l_S)^{\frac{\beta}{\gamma}} L^l_S}
\]

Substituting for the prices, we have

\[
\frac{1}{\varphi} = \left( \frac{1 - \gamma}{\gamma} \right)^\frac{\varepsilon}{\sigma} \left( \frac{L^h_N A^h}{L^l_N A^l} \right)^\frac{1}{\sigma} L^h_N + \left( \frac{1 - \gamma}{\gamma} \right)^\frac{\varepsilon}{\sigma} \left( \frac{L^h_S A^h}{L^l_S A^l} \right)^\frac{1}{\sigma} L^h_S
\]

Rearranging and solving for the skill bias yields
\[ \left( \frac{A^h}{A^l} \right)^{\frac{\bar{\sigma}}{\bar{\sigma}}} = \varphi \left( \frac{1 - \gamma}{\bar{\gamma}} \right)^{\bar{\epsilon}} \left[ \frac{(Y_N)^{\frac{1}{\bar{\gamma}}} (L_N^h)^{\frac{\bar{\sigma}-1}{\bar{\sigma}}} + (Y_S)^{\frac{1}{\bar{\gamma}}} (L_S^h)^{\frac{\bar{\sigma}-1}{\bar{\sigma}}} }{(Y_N)^{\frac{1}{\bar{\gamma}}} (L_N^l)^{\frac{\bar{\sigma}-1}{\bar{\sigma}}} + (Y_S)^{\frac{1}{\bar{\gamma}}} (L_S^l)^{\frac{\bar{\sigma}-1}{\bar{\sigma}}} } \right] \]

Assuming \( Y_S = \theta Y_N \) in the steady state, where \( \theta > 0 \), we have

\[ \frac{A^h}{A^l} = \varphi^\sigma \left( \frac{1 - \gamma}{\bar{\gamma}} \right)^{\bar{\epsilon}} \left[ \frac{(L_N^h)^{\frac{\bar{\sigma}-1}{\bar{\sigma}}} + \theta^{\frac{1}{\bar{\gamma}}} (L_S^h)^{\frac{\bar{\sigma}-1}{\bar{\sigma}}} }{(L_N^l)^{\frac{\bar{\sigma}-1}{\bar{\sigma}}} + \theta^{\frac{1}{\bar{\gamma}}} (L_S^l)^{\frac{\bar{\sigma}-1}{\bar{\sigma}}} } \right]^\sigma \]

After trade liberalization, to endogenize \( \frac{A^h}{A^l} \) using world relative prices, if intellectual property rights are not enforced, we have

\[ \frac{\varphi^l}{\varphi^h} = \frac{\pi^h}{\pi^l} = \left( \frac{P_{W}^h}{P_{W}^l} \right)^{\frac{1}{\bar{\gamma}}} \left( \frac{L_N^h}{L_N^l} \right)^{\frac{\bar{\sigma}}{\bar{\gamma}}} \left( \frac{A^h}{A^l} \right)^{-\frac{\bar{\sigma}}{\bar{\gamma}}} \]

Solving for the skill bias yields

\[ \frac{A^h}{A^l} = \varphi^\sigma \left( \frac{1 - \gamma}{\bar{\gamma}} \right)^{\bar{\epsilon}} \left( \frac{L_N^h}{L_N^l} + k L_S^h \right)^{-\frac{1}{\bar{\gamma}}} \left( \frac{L_N^l}{L_N^l} \right)^{\frac{\bar{\sigma}}{\bar{\gamma}}} \]

To endogenize \( \frac{A^h}{A^l} \) using world relative prices, if intellectual property rights are enforced, we have

\[ \frac{\varphi^l}{\varphi^h} = \frac{\pi^h}{\pi^l} = \left( \frac{P_{W}^{(IPR)}}{P_{W}^{(IPR)}} \right)^{\frac{1}{\bar{\gamma}}} \left( \frac{L_N^h + L_S^h}{L_N^l + L_S^l} \right)^{\frac{\bar{\sigma}}{\bar{\gamma}}} \left( \frac{A^h}{A^l} \right)^{-\frac{\bar{\sigma}}{\bar{\gamma}}} \]

Solving for the skill bias yields

\[ \frac{A^h}{A^l} = \varphi^\sigma \left( \frac{1 - \gamma}{\bar{\gamma}} \right)^{\bar{\epsilon}} \left( \frac{L_N^h + L_S^h}{L_N^l + L_S^l} \right)^{\frac{\bar{\sigma}}{\bar{\gamma}}} \]

### 4.5 Proof of Proposition 1

If the labor markets are assumed Walrasian, then the producers of final goods choose the amount of labor to use in the production process such that the wage of each type is equal to its marginal productivity. The
demand for the skilled and unskilled labor in country \( i \) is thus given by

\[
W_i^l = P_i^l \frac{\beta}{1 - \beta} \left[ \int_0^{A_i^l} x_i^l (j)^{1 - \beta} dj \right] (L_i^l)^{\beta - 1}
\]

\[
W_i^h = P_i^h \frac{\beta}{1 - \beta} \left[ \int_0^{A_i^h} a_i^h (j)^{1 - \beta} dj \right] (L_i^h)^{\beta - 1}
\]

In autarky, the skill premium in country \( i \), after substituting for the demand and relative prices, is given by

\[
\frac{W_i^h}{W_i^l} = \left( \frac{1 - \gamma}{\gamma} \right)^{\frac{\sigma - 1}{\sigma}} \left( \frac{A_i^h}{A_i^l} \right)^{\frac{(\sigma - 1)^2}{\sigma}} \left( \frac{L_i^h}{L_i^l} \right)^{\frac{1}{\sigma}}
\]

After substituting endogenous skill bias, if intellectual property rights are not enforced, the skill premium in the North is given by

\[
W_N^h = \varphi^{\sigma - 1} \left( \frac{1 - \gamma}{\gamma} \right)^{\varepsilon} \left( \frac{L_N^h}{L_N^l} \right)^{\sigma - 2}
\]

After substituting endogenous skill bias, if intellectual property rights are not enforced, the skill premium in the South is given by

\[
W_S^h = \varphi^{\sigma - 1} \left( \frac{1 - \gamma}{\gamma} \right)^{\varepsilon} \left( \frac{L_S^h}{L_S^l} \right)^{\frac{(\sigma - 1)^2}{\sigma}} \left( \frac{L_N^h + kL_S^h}{L_N^l + kL_S^l} \right)^{\frac{1}{\sigma}}
\]

If intellectual property rights are not enforced, the world skill premium is given by the ratio of marginal productivities of the two types of workers

\[
\frac{W_W^h}{W_W^l} = \left( \frac{1 - \gamma}{\gamma} \right)^{\varepsilon} \left( \frac{A_i^h}{A_i^l} \right)^{\frac{(\sigma - 1)^2}{\sigma}} \left( \frac{L_N^h + kL_S^h}{L_N^l + kL_S^l} \right)^{\frac{1}{\sigma}}
\]

After substituting for the endogenous skill bias, we have

\[
\frac{W_W^h}{W_W^l} = \varphi^{\sigma - 1} \left( \frac{1 - \gamma}{\gamma} \right)^{\varepsilon} \left( \frac{L_N^h + kL_S^h}{L_N^l + kL_S^l} \right)^{-1} \left( \frac{L_N^l}{L_N^h} \right)^{\sigma - 1}
\]

So, the skill premium in the North increases after trade liberalization if and only if \( \frac{W_W^h}{W_W^l} > \frac{W_N^h}{W_N^l} \), or if

\[
\left( \frac{L_N^h}{L_N^l} \right) > \left( \frac{L_N^h + kL_S^h}{L_N^l + kL_S^l} \right)
\]
This can be simplified such that

$$1 > \left( \frac{1 + k\mu}{1 + k\upsilon} \right)$$

Similarly, the skill premium in the South increases after trade if and only if $\frac{W^h_{Ww}}{W^h_{Ws}} > \frac{W^h_{Ww}}{W^h_{Ws}}$, or if

$$\left( \frac{L^h_N}{L^h_N} \right)^{(\sigma - 1)} \left( \frac{L^h_S}{L^h_S} \right)^{\frac{\mu}{\upsilon}} \left( \frac{L^h_N + kL^h_S}{L^h_N} + kL^h_S \right)$$

which can be simplified such that

$$\left( \frac{L^h_N}{L^h_N} \right)^{(\sigma - 1)} \left( \frac{\mu L^h_N}{\upsilon L^h_N} \right)^{\frac{\mu}{\upsilon}} > \left( \frac{L^h_N (1 + k\mu)}{L^h_N (1 + k\upsilon)} \right)$$

which can be simplified further to

$$\left( \frac{\mu}{\upsilon} \right)^{\frac{\mu}{\upsilon}} > \left( \frac{1 + k\mu}{1 + k\upsilon} \right)$$

### 4.6 Proof of Proposition 2

In autarky, the skill premium in country $i$ after substituting for the demand and relative prices, is given by

$$\frac{W^h_i}{W^i_i} = \left( \frac{1 - \gamma}{\gamma} \right)^{\frac{\phi}{\gamma}} \left( \frac{A^h_i}{A^i} \right)^{\frac{\sigma - 1}{\gamma}} \left( \frac{L^h_i}{L^i_i} \right)^{\frac{1}{\gamma}}$$

After substituting endogenous skill bias, if intellectual property rights are enforced, the skill premium in the North is given by

$$\frac{W^h_N}{W^N_N} = \phi^{\sigma - 1} \left( \frac{1 - \gamma}{\gamma} \right)^{\frac{\epsilon}{\gamma}} \left( \frac{L^h_N}{L^N_N} \right)^{\frac{\sigma - 1}{\gamma}} \left[ \left( \frac{L^h_N}{L^h_N} \right)^{\frac{\sigma - 1}{\gamma}} + \theta^{\frac{\phi}{\gamma}} \left( \frac{L^h_S}{L^h_S} \right)^{\frac{\sigma - 1}{\gamma}} \right]^{\frac{\sigma - 1}{\gamma}}$$

After substituting endogenous skill bias, if intellectual property rights are enforced, the skill premium in the South is given by
\[ \frac{W^h_S}{W^l_S} = \varphi^{\sigma-1} \left( \frac{1 - \gamma}{\gamma} \right) \varepsilon \left( \frac{L^h_S}{L^l_S} \right)^{-\frac{\gamma}{\sigma}} \left[ \frac{(L^h_N)^{\frac{\sigma-1}{\sigma}} + \theta \frac{\mu}{L^h_N}}{(L^l_N)^{\frac{\sigma-1}{\sigma}} + \theta \frac{\mu}{L^l_N}} \right]^{\sigma-1} \]

Introducing trade, if intellectual property rights are enforced in the South, the world skill premium is given by

\[ \frac{W^h_W}{W^l_W} = \left( \frac{1 - \gamma}{\gamma} \right) \varepsilon \left( \frac{A^h}{A^l} \right)^{\frac{\sigma-1}{\sigma}} \left( \frac{L^h_N + L^h_S}{L^l_N + L^l_S} \right)^{-\frac{1}{\sigma}} \]

After substituting endogenous skill bias, we have

\[ \frac{W^h_W}{W^l_W} = \varphi^{\sigma-1} \left( \frac{1 - \gamma}{\gamma} \right) \varepsilon \left( \frac{L^h_N + L^h_S}{L^l_N + L^l_S} \right)^{-\frac{1}{\sigma}} \]

The skill premium in the North increases after trade liberalization if and only if \( \frac{W^h_W}{W^l_W} > \frac{W^h^S}{W^l^S} \) or if

\[ \left[ \frac{(L^h_N)^{\frac{\sigma-1}{\sigma}} + \theta \frac{\mu}{L^h_N}}{(L^l_N)^{\frac{\sigma-1}{\sigma}} + \theta \frac{\mu}{L^l_N}} \right]^{\sigma-1} < \left( \frac{L^h_N}{L^l_N} \right)^{\frac{\mu}{1+\mu}} \left( \frac{L^h_N + L^h_S}{L^l_N + L^l_S} \right)^{-\frac{1}{\sigma}} \]

This can be simplified such that

\[ \left[ \frac{1 + \theta \frac{\mu}{\sigma}}{1 + \theta \frac{\mu}{\sigma} + \frac{\mu}{1+\mu}} \right]^{\sigma-1} < \left( \frac{1 + \mu}{1+\mu} \right)^{\sigma-2} \]

Similarly, the skill premium increases in the South after trade liberalization if and only if \( \frac{W^h_W}{W^l_W} > \frac{W^h^S}{W^l^S} \) or if

\[ \left[ \frac{(L^h_N)^{\frac{\sigma-1}{\sigma}} + \theta \frac{\mu}{L^h_N}}{(L^l_N)^{\frac{\sigma-1}{\sigma}} + \theta \frac{\mu}{L^l_N}} \right]^{\sigma-1} < \left( \frac{L^h_N}{L^l_N} \right)^{\frac{\mu}{1+\mu}} \left( \frac{L^h_N + L^h_S}{L^l_N + L^l_S} \right)^{-\frac{1}{\sigma}} \]

This can be simplified such that

\[ \left[ \frac{1 + \theta \frac{\mu}{\sigma}}{1 + \theta \frac{\mu}{\sigma} + \frac{\mu}{1+\mu}} \right]^{\sigma-1} < \left( \frac{1 + \mu}{1+\mu} \right)^{\sigma-2} \]

which is satisfied if

\[ \left[ \frac{1 + \theta \frac{\mu}{\sigma}}{1 + \theta \frac{\mu}{\sigma} + \frac{\mu}{1+\mu}} \right]^{\sigma-1} < \left( \frac{1 + \mu}{1+\mu} \right)^{\sigma-2} \]
4.7 Proof of Proposition 3

To establish the conditions under which wage inequality increases in the North after trade liberalization, if protection of intellectual property is enforced in the South, we compare the skill premium in the North in autarky to that in trade. The steady state skill premium in the North after trade liberalization is given by

\[
\frac{W_N^{h(IPR)}}{W_N^{l(IPR)}} = \left(1 - \xi_N^h \right) \left[ \frac{\beta}{1 \beta} (Y_W)^{\frac{1}{\gamma}} (1 - \beta)^{\frac{1}{\gamma}} (1 - \gamma)^{\frac{1}{\gamma}} (L_N^h + L_S^h)^{\frac{1}{\gamma}} (A^h)^{\frac{1 + \gamma}{1 - \gamma}} + \frac{\rho^h_{dN} \omega_N^h}{q_N^h} \right] + \xi_N^h \left[ \phi^h_N (1 - d_N) \right] \\
\quad (1 - \xi_N^l) \left[ \frac{\beta}{1 \beta} (Y_W)^{\frac{1}{\gamma}} (1 - \beta)^{\frac{1}{\gamma}} (1 - \gamma)^{\frac{1}{\gamma}} (L_N^l + L_S^l)^{\frac{1}{\gamma}} (A^l)^{\frac{1 + \gamma}{1 - \gamma}} + \frac{\rho^l_{dN} \omega_N^l}{q_N^l} \right] + \xi_N^l \left[ \phi^l_N (1 - g_N) \right]
\]

Denote

\[ \mathcal{L}_N^h = \left(1 - \xi_N^h \right) \left[ \frac{\beta}{1 \beta} (Y_W)^{\frac{1}{\gamma}} (1 - \beta)^{\frac{1}{\gamma}} (1 - \gamma)^{\frac{1}{\gamma}} (L_N^h + L_S^h)^{\frac{1}{\gamma}} (A^h)^{\frac{1 + \gamma}{1 - \gamma}} \right] \]

\[ \mathfrak{D} = \left(1 - \xi_N^h \right) \frac{\rho^h_{dN} \omega_N^h}{q_N^h} + \xi_N^h \left[ \phi^h_N (1 - d_N) \right] \]

\[ \mathfrak{G} = \left(1 - \xi_N^l \right) \frac{\rho^l_{dN} \omega_N^h}{q_N^l} + \xi_N^l \left[ \phi^l_N (1 - g_N) \right] \]

Therefore, dividing the numerator and denominator by \( \mathcal{L}_N^h \), and assuming \( \xi_N^h = \xi_N^l \), the skill premium can be written as

\[
\frac{W_N^{h(IPR)}}{W_N^{l(IPR)}} = \frac{\left(1 - \gamma \right)^{\frac{1}{\gamma}} (L_N^h + L_S^h)^{\frac{1}{\gamma}} (A^h)^{\frac{1 + \gamma}{1 - \gamma}} + \frac{\mathfrak{D}}{\mathcal{L}_N}} {1 + \frac{\mathfrak{D}}{\mathcal{L}_N}}
\]

Similarly, the steady state skill premium in the North in autarky is given by

\[
\frac{W_N^h}{W_N^l} = \left(1 - \xi_N^h \right) \left[ \frac{\beta}{1 \beta} (Y_N)^{\frac{1}{\gamma}} (1 - \beta)^{\frac{1}{\gamma}} (1 - \gamma)^{\frac{1}{\gamma}} (L_N^h)^{\frac{1}{\gamma}} (A^h)^{\frac{1 + \gamma}{1 - \gamma}} + \frac{\rho^h_{dN} \omega_N^h}{q_N^h} \right] + \xi_N^h \left[ \phi^h_N (1 - d_N) \right] \\
\quad (1 - \xi_N^l) \left[ \frac{\beta}{1 \beta} (Y_N)^{\frac{1}{\gamma}} (1 - \beta)^{\frac{1}{\gamma}} (1 - \gamma)^{\frac{1}{\gamma}} (L_N^l)^{\frac{1}{\gamma}} (A^l)^{\frac{1 + \gamma}{1 - \gamma}} + \frac{\rho^l_{dN} \omega_N^l}{q_N^l} \right] + \xi_N^l \left[ \phi^l_N (1 - g_N) \right]
\]

Denote

\[ \mathcal{L}_N^h = \left(1 - \xi_N^h \right) \left[ \frac{\beta}{1 \beta} (Y_N)^{\frac{1}{\gamma}} (1 - \beta)^{\frac{1}{\gamma}} (1 - \gamma)^{\frac{1}{\gamma}} (L_N^h)^{\frac{1}{\gamma}} (A^h)^{\frac{1 + \gamma}{1 - \gamma}} \right] \]

Therefore, dividing the numerator and denominator by \( \mathcal{L}_N^h \), and assuming \( \xi_N^h = \xi_N^l \), the skill premium can be written as

\[ 37 \]
Then, remaining to establish the conditions under which wage inequality increases after trade liberalization if

\[
\frac{W^b_N}{W^l_N} = \left[ \frac{\left( \frac{1-\gamma}{\gamma} \right)^\frac{\gamma}{\gamma} \left( \frac{L^b_N}{L^l_N} \right)^{-\frac{1}{\gamma}} \left( \frac{A^b}{A^l} \right)^{\frac{\gamma-1}{\gamma}} }{1 + \frac{\gamma}{\gamma} \frac{L^b_N}{L^l_N}} \right] + \frac{\gamma}{\gamma} \frac{L^b_N}{L^l_N}
\]

This can be simplified as

\[
\left[ \frac{\left( \frac{1-\gamma}{\gamma} \right)^\frac{\gamma}{\gamma} \left( \frac{L^b_N + L^b_S}{L^l_N + L^l_S} \right)^{-\frac{1}{\gamma}} \left( \frac{A^b}{A^l} \right)^{\frac{\gamma-1}{\gamma}} }{1 + \frac{\gamma}{\gamma} \frac{L^b_N + L^b_S}{L^l_N + L^l_S}} \right] \mathcal{L}^N + \frac{\gamma}{\gamma} \frac{L^b_N + L^b_S}{L^l_N + L^l_S} \mathcal{L}^A > \left[ \frac{\left( \frac{1-\gamma}{\gamma} \right)^\frac{\gamma}{\gamma} \left( \frac{L^b_N}{L^l_N} \right)^{-\frac{1}{\gamma}} \left( \frac{A^b}{A^l} \right)^{\frac{\gamma-1}{\gamma}} }{1 + \frac{\gamma}{\gamma} \frac{L^b_N}{L^l_N}} \right] \mathcal{L}^N + \frac{\gamma}{\gamma} \frac{L^b_N}{L^l_N} \mathcal{L}^A
\]

If \( \mathcal{L}^T_N < \mathcal{L}^A_N \), which is satisfied if and only if \( \left( \frac{Y^N}{1+\theta^T} \right) > \left( \frac{L^b_N}{L^l_N + L^l_S} \right) \) or if \( \frac{\theta}{1+\theta} > \frac{\gamma}{1+\theta} \), or if \( \frac{1}{1+\theta} > \frac{1}{1+\theta} \) then

\[
\frac{\gamma}{\gamma} \frac{L^b_N + L^b_S}{L^l_N + L^l_S} > \frac{\gamma}{\gamma} \frac{L^b_N}{L^l_N}
\]

Then, remaining to establish the conditions under which

\[
\left[ \frac{\left( \frac{1-\gamma}{\gamma} \right)^\frac{\gamma}{\gamma} \left( \frac{L^b_N + L^b_S}{L^l_N + L^l_S} \right)^{-\frac{1}{\gamma}} \left( \frac{A^b}{A^l} \right)^{\frac{\gamma-1}{\gamma}} }{1 + \frac{\gamma}{\gamma} \frac{L^b_N + L^b_S}{L^l_N + L^l_S}} \right] \mathcal{L}^N > \left[ \frac{\left( \frac{1-\gamma}{\gamma} \right)^\frac{\gamma}{\gamma} \left( \frac{L^b_N}{L^l_N} \right)^{-\frac{1}{\gamma}} \left( \frac{A^b}{A^l} \right)^{\frac{\gamma-1}{\gamma}} }{1 + \frac{\gamma}{\gamma} \frac{L^b_N}{L^l_N}} \right] \mathcal{L}^N
\]

Substituting the endogenous skill bias yields

\[
\left( \frac{L^b_N + L^b_S}{L^l_N + L^l_S} \right)^{-\frac{1}{\gamma}} \mathcal{L}^N > \left( \frac{L^b_N}{L^l_N} \right)^{-\frac{1}{\gamma}} \left[ \frac{(L^b_N)^{-\frac{1}{\gamma}} + \theta^T (L^b_S)^{-\frac{1}{\gamma}}}{(L^l_N)^{-\frac{1}{\gamma}} + \theta^T (L^l_S)^{-\frac{1}{\gamma}}} \right] \mathcal{L}^N
\]

which can be simplified to

\[
\mathcal{L}^N > \left[ \frac{(L^b_N)^{-\frac{1}{\gamma}} + \theta^T (L^b_S)^{-\frac{1}{\gamma}}}{(L^l_N)^{-\frac{1}{\gamma}} + \theta^T (L^l_S)^{-\frac{1}{\gamma}}} \right]^{-\frac{1}{\gamma}} \mathcal{L}^N
\]

Which can be rearranged to

\[
\left( \frac{L^b_N}{L^l_N + L^l_S} \right)^{\frac{1}{\gamma}} > \left[ \frac{(L^b_N)^{-\frac{1}{\gamma}} + \theta^T (L^b_S)^{-\frac{1}{\gamma}}}{(L^l_N)^{-\frac{1}{\gamma}} + \theta^T (L^l_S)^{-\frac{1}{\gamma}}} \right]^{-\frac{1}{\gamma}} \mathcal{L}^N
\]

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This can be simplified to
\[
\left(\frac{1}{1+\theta}\right)^{\frac{1}{\gamma}} \frac{1+\theta}{1+\theta} \frac{\frac{\sigma-1}{\gamma}}{1+\theta} \left(\frac{1}{1+\theta}\right)^{\frac{1}{\gamma}} \frac{1+\theta}{1+\theta}
\]

4.8 Proof of Proposition 4

To establish the conditions under which wage inequality increases in the North after trade liberalization, if protection of intellectual property is not enforced in the South, we compare the skill premium in the North in autarky to that in trade. The steady state skill premium in the North after trade liberalization is given by

\[
\frac{W_{N}^h}{W_{N}^l} = \frac{(1 - \xi_N^h) \left[ \frac{\beta}{1-\beta} (Y_N)^{\frac{1}{\gamma}} (1-\beta)^{\frac{1}{\gamma}} (L_N^h + kL_S^h)^{\frac{1}{\gamma}} (A^h)^{\frac{1}{\gamma}} + \frac{\rho_N d_N \omega_N}{q_N} \right] + \xi_N^h \left[ \phi_N^h (1 - d_N) \right]}{(1 - \xi_N^l) \left[ \frac{\beta}{1-\beta} (Y_N)^{\frac{1}{\gamma}} (1-\beta)^{\frac{1}{\gamma}} (L_N^l + kL_S^l)^{\frac{1}{\gamma}} (A^l)^{\frac{1}{\gamma}} + \frac{\rho_N \omega_N}{q_N} \right] + \xi_N^l \left[ \phi_N^l (1 - g_N) \right]}
\]

Denote

\[
\mathcal{L}_N^T = \left(1 - \xi_N^l\right) \left[ \frac{\beta}{1-\beta} (Y_N)^{\frac{1}{\gamma}} (1-\beta)^{\frac{1}{\gamma}} (L_N^l + kL_S^l)^{\frac{1}{\gamma}} (A^l)^{\frac{1}{\gamma}} \right]
\]

\[
\Xi = \left(1 - \xi_N^h\right) \frac{\rho_N d_N \omega_N}{q_N} + \xi_N^h \left[ \phi_N^h (1 - d_N) \right]
\]

\[
\Gamma = \left(1 - \xi_N^l\right) \frac{\rho_N \omega_N}{q_N} + \xi_N^l \left[ \phi_N^l (1 - g_N) \right]
\]

Therefore, dividing the numerator and denominator by \(\mathcal{L}_N^T\), and assuming \(\xi_N^h = \xi_N^l\), the skill premium can be written as

\[
\frac{W_{N}^h}{W_{N}^l} = \frac{\left(1 - \gamma\right)^\frac{1}{\gamma} \left(\frac{L_N^h + kL_S^h}{L_N^l + kL_S^l}\right)^{\frac{1}{\gamma}} (A^h)^{\frac{1}{\gamma}} + \Xi}{1 + \frac{\Xi}{\Gamma}}
\]

Similarly, the steady state skill premium in the North in autarky is given by

\[
\frac{W_{N}^h}{W_{N}^l} = \frac{(1 - \xi_N^h) \left[ \frac{\beta}{1-\beta} (Y_N)^{\frac{1}{\gamma}} (1-\beta)^{\frac{1}{\gamma}} (L_N^h)^{\frac{1}{\gamma}} (A^h)^{\frac{1}{\gamma}} + \frac{\rho_N d_N \omega_N}{q_N} \right] + \xi_N^h \left[ \phi_N^h (1 - d_N) \right]}{(1 - \xi_N^l) \left[ \frac{\beta}{1-\beta} (Y_N)^{\frac{1}{\gamma}} (1-\beta)^{\frac{1}{\gamma}} (L_N^l)^{\frac{1}{\gamma}} (A^l)^{\frac{1}{\gamma}} + \frac{\rho_N \omega_N}{q_N} \right] + \xi_N^l \left[ \phi_N^l (1 - g_N) \right]}
\]

Denote
\[ \mathcal{L}_N^A = \left( 1 - \xi_N^l \right) \left[ \frac{\beta}{1 - \beta} (Y_N)^{\frac{1}{\beta}} (1 - \beta) \gamma (L_N^h)^{\frac{1}{1 - \beta}} (A^l)^{\frac{1 - \gamma}{\beta}} \right] \]

Therefore, dividing the numerator and denominator by \( \mathcal{L}_N^A \), and assuming \( \xi_N^h = \xi_N^l \), the skill premium can be written as

\[ \frac{W_N^h}{W_N^l} = \left[ \frac{\left( \frac{1 - \gamma}{\gamma} \right) \left( \frac{L_N^h + kL_S^h}{L_N + k} \right) \frac{1}{(A^l)^{\frac{1 - \gamma}{\beta}}} }{1 + \frac{1}{\mathcal{L}_N^A}} \right] + \frac{2}{\mathcal{L}_N^A} \]

This can be simplified as

\[ \frac{\left[ \left( \frac{1 - \gamma}{\gamma} \right) \left( \frac{L_N^h + kL_S^h}{L_N + k} \right) \frac{1}{(A^l)^{\frac{1 - \gamma}{\beta}}} \right]}{\mathcal{L}_N^T + \frac{1}{\mathcal{L}_N^A}} \left[ \left( \frac{1 - \gamma}{\gamma} \right) \left( \frac{L_N^h}{L_N} \right) \frac{1}{(A^l)^{\frac{1 - \gamma}{\beta}}} \right] \mathcal{L}_N^T + \frac{1}{\mathcal{L}_N^A} \]

If \( \mathcal{L}_N^T < \mathcal{L}_N^A \), which is satisfied if and only if \( \left( \frac{Y_N}{Y_W} \right) > \left( \frac{L_N^h}{L_N^h + kL_S^h} \right) \), or if \( \bar{\omega} > \frac{1}{1 + \kappa^u} \), or if \( \frac{1}{\gamma^u} > \frac{1}{1 + \kappa^u} \) then

\[ \frac{\mathcal{L}_N^T}{\mathcal{L}_N^A + \frac{1}{\mathcal{L}_N^A}} > \frac{\mathcal{L}_N^A}{\mathcal{L}_N^A + \frac{1}{\mathcal{L}_N^A}} \]

Then, remaining to establish the conditions under which

\[ \left[ \left( \frac{1 - \gamma}{\gamma} \right) \left( \frac{L_N^h + kL_S^h}{L_N + k} \right) \frac{1}{(A^l)^{\frac{1 - \gamma}{\beta}}} \right] \mathcal{L}_N^T > \left[ \left( \frac{1 - \gamma}{\gamma} \right) \left( \frac{L_N^h}{L_N} \right) \frac{1}{(A^l)^{\frac{1 - \gamma}{\beta}}} \right] \mathcal{L}_N^A \]

Substituting the endogenous skill bias yields

\[ \left( \frac{L_N^h + kL_S^h}{L_N + k} \right)^{-1} \mathcal{L}_N^T > \left( \frac{L_N^h}{L_N} \right)^{-1} \mathcal{L}_N^A \]

which can be simplified to

\[ \frac{\mathcal{L}_N^T}{\mathcal{L}_N^A} > \frac{\left( \frac{L_N^h + kL_S^h}{L_N + k} \right)^{-1}}{\left( \frac{L_N^h}{L_N} \right)^{-1}} \]

Which can be rearranged to

\[ \mathcal{L}_N^T > \mathcal{L}_N^A \]
This can be simplified to

\[
\left( \frac{1}{1 + k\nu} \right)^{\frac{1}{\gamma}} > \frac{1 + k\mu}{1 + k\nu}
\]

### 4.9 Proof of Proposition 5

To establish the conditions under which wage inequality increases in the South after trade liberalization, if protection of intellectual property is enforced in the South, we compare the skill premium in the South in autarky to that in trade. The steady state skill premium in the South after trade liberalization is given by

\[
\frac{W_{S}^{h(IPR)}}{W_{S}^{a(IPR)}} = \left( 1 - \xi_{s}^{h} \right) \left( 1 - \xi_{s}^{l} \right) \left[ \frac{\beta}{1 - \beta} (Y_{W}^{h})^{\frac{1}{\beta}} (1 - \gamma)^{\frac{1}{\gamma}} (L_{N}^{h} + L_{S}^{h})^{-\frac{1}{\beta}} (A^{h})^{\frac{\beta - 1}{\beta}} + \frac{\rho_{S}^{h} d_{S}^{h} \xi_{S}^{h}}{q_{S}^{h}} \right] + \xi_{s}^{h} \left[ \phi_{S}^{h} (1 - d_{S}) \right]
\]

Denote

\[
\mathcal{L}_{S}^{h} = \left( 1 - \xi_{s}^{h} \right) \left[ \frac{\beta}{1 - \beta} (Y_{W}^{h})^{\frac{1}{\beta}} (1 - \beta)^{\frac{1}{\beta}} (L_{N}^{h} + L_{S}^{h})^{-\frac{1}{\beta}} (A^{h})^{\frac{\beta - 1}{\beta}} \right] - \xi_{s}^{l} \left[ \phi_{S}^{l} (1 - d_{S}) \right] + \xi_{s}^{l} \left[ \phi_{S}^{l} (1 - g_{S}) \right]
\]

Therefore, dividing the numerator and denominator by \( \mathcal{L}_{S}^{h} \), and assuming \( \xi_{s}^{h} = \xi_{s}^{l} \), the skill premium can be written as

\[
\frac{W_{S}^{h(IPR)}}{W_{S}^{a(IPR)}} = \left( 1 - \gamma \right)^{\frac{1}{\gamma}} \left( \frac{L_{N}^{h} + L_{S}^{h}}{L_{N}^{h} + L_{S}^{h}} \right)^{\frac{1}{\beta}} \left( \frac{A^{h}}{A^{h}} \right)^{\frac{\beta - 1}{\beta}} + \frac{\rho_{S}^{h} d_{S}^{h} \xi_{S}^{h}}{q_{S}^{h}}
\]

Similarly, the steady state skill premium in the South in autarky is given by

\[
\frac{W_{S}^{h(IPR)}}{W_{S}^{a(IPR)}} = \left( 1 - \xi_{s}^{l} \right) \left[ \frac{\beta}{1 - \beta} (Y_{S}^{h})^{\frac{1}{\beta}} (1 - \gamma)^{\frac{1}{\gamma}} (L_{S}^{h})^{-\frac{1}{\beta}} (A^{h})^{\frac{\beta - 1}{\beta}} + \frac{\rho_{S}^{l} g_{S}^{l} \xi_{S}^{l}}{q_{S}^{l}} \right] + \xi_{s}^{l} \left[ \phi_{S}^{l} (1 - g_{S}) \right]
\]
Denote
\[ \mathcal{L}_S^A = \left(1 - \zeta_S^l\right) \left[ \frac{\beta}{1 - \beta} (Y_S)^{\frac{1}{\gamma}} (1 - \beta)^{\frac{1}{\gamma}} (L_S^h)^{\frac{\gamma+2}{\gamma+1}} (A^l)^{-\frac{\gamma+1}{\gamma}} \right] \]

Therefore, dividing the numerator and denominator by \( \mathcal{L}_S^A \), and assuming \( \xi_S^h = \xi_S^l \), the skill premium can be written as
\[ \frac{W_S^A(IPR)}{W_S^A(IPR)} = \left[ \frac{\left(\frac{1-\gamma}{\gamma}\right)^{\frac{1}{\gamma}} \left(\frac{L_S^h}{L_S^l}\right)^{\frac{\gamma+1}{\gamma}} \left(\frac{A^h}{A^l}\right)^{\frac{\gamma+1}{\gamma}}}{1 + \frac{1}{\mathcal{L}_S^A}} \right] \left[ \frac{\left(\frac{1-\gamma}{\gamma}\right)^{\frac{1}{\gamma}} \left(\frac{L_S^h}{L_S^l}\right)^{\frac{\gamma+1}{\gamma}} \left(\frac{A^h}{A^l}\right)^{\frac{\gamma+1}{\gamma}}}{1 + \frac{1}{\mathcal{L}_S^A}} \right] \]

Therefore, wage inequality increases after trade liberalization if
\[ \frac{\left(\frac{1-\gamma}{\gamma}\right)^{\frac{1}{\gamma}} \left(\frac{L_S^h}{L_S^l}\right)^{\frac{\gamma+1}{\gamma}} \left(\frac{A^h}{A^l}\right)^{\frac{\gamma+1}{\gamma}}}{\mathcal{L}_S^T + \gamma} > \frac{\left(\frac{1-\gamma}{\gamma}\right)^{\frac{1}{\gamma}} \left(\frac{L_S^h}{L_S^l}\right)^{\frac{\gamma+1}{\gamma}} \left(\frac{A^h}{A^l}\right)^{\frac{\gamma+1}{\gamma}}}{\mathcal{L}_S^A + \gamma} \]

This can be simplified as
\[ \frac{\mathcal{L}_S^T + \gamma}{\mathcal{L}_S^T + \gamma} > \frac{\mathcal{L}_S^A + \gamma}{\mathcal{L}_S^A + \gamma} \]

If \( \mathcal{L}_S^T < \mathcal{L}_S^A \), which is satisfied if and only if \( \left(\frac{Y_S}{\gamma}\right) > \left(\frac{L_S^h}{L_N^h + L_S^l}\right) \), or if \( 1 - \beta > \frac{\psi}{\frac{\theta}{1+\theta}} \) or if \( \frac{\theta}{1+\theta} > \frac{\psi}{1+\psi} \) then
\[ \frac{\mathcal{L}_S^T + \gamma}{\mathcal{L}_S^A + \gamma} > \frac{\mathcal{L}_S^A + \gamma}{\mathcal{L}_S^A + \gamma} \]

Then, remaining to establish the conditions under which
\[ \left[ \frac{\left(\frac{1-\gamma}{\gamma}\right)^{\frac{1}{\gamma}} \left(\frac{L_S^h}{L_N^h + L_S^l}\right)^{\frac{\gamma+1}{\gamma}} \left(\frac{A^h}{A^l}\right)^{\frac{\gamma+1}{\gamma}}}{\mathcal{L}_S^T} \right] > \left[ \frac{\left(\frac{1-\gamma}{\gamma}\right)^{\frac{1}{\gamma}} \left(\frac{L_S^h}{L_S^l}\right)^{\frac{\gamma+1}{\gamma}} \left(\frac{A^h}{A^l}\right)^{\frac{\gamma+1}{\gamma}}}{\mathcal{L}_S^A} \right] \]

Substituting the endogenous skill bias yields
\[ \left(\frac{L_S^h}{L_N^h + L_S^l}\right)^{\sigma^2} \mathcal{L}_S^T + \left(\frac{L_S^h}{L_S^l}\right)^{\frac{1}{\sigma}} \left[ \frac{\left(\frac{1-\gamma}{\gamma}\right)^{\frac{1}{\gamma}} \left(\frac{L_S^h}{L_S^l}\right)^{\frac{\gamma+1}{\gamma}} \left(\frac{A^h}{A^l}\right)^{\frac{\gamma+1}{\gamma}}}{\mathcal{L}_S^A} \right]^{\sigma^{-1}} \]

which can be simplified to
\[
\mathcal{L}_S^T \mathcal{L}_S^T > \left[ \frac{\left( L_N h \right)^{\frac{\sigma - 1}{\sigma} + \theta \frac{\sigma}{v} \beta h \frac{\sigma - 1}{\sigma}} \left( L_S h \right)^{\frac{\sigma - 1}{\sigma} + \theta \frac{\sigma}{v} \beta S h \frac{\sigma - 1}{\sigma}} \left( L_N + L_S \right) \frac{1}{v_S h \gamma h} + \sigma_{d_s S h} \omega_{S h} \right]}{\left( L_N + L_S \right)^{\frac{1}{\sigma} + \theta \frac{\sigma}{v} \beta S h \frac{1}{\sigma} + \sigma_{d_s S h} \omega_{S h}} \left( L_N + L_S \right)^{\frac{1}{\sigma} + \theta \frac{\sigma}{v} \beta S h \frac{1}{\sigma} + \sigma_{d_s S h} \omega_{S h}}} \right]^{\sigma - 1}\]

Which can be rearranged to

\[
\left( \frac{L_N h}{L_N + L_S h} \right)^{\frac{1}{v}} \left( \frac{L_N + L_S h}{L_N + L_S h} \right)^{\frac{1}{\sigma} + \theta \frac{\sigma}{v} \beta S h \frac{1}{\sigma} + \sigma_{d_s S h} \omega_{S h}} \left( L_N + L_S h \right)^{\frac{1}{\sigma} + \theta \frac{\sigma}{v} \beta S h \frac{1}{\sigma} + \sigma_{d_s S h} \omega_{S h}} \right]^{\sigma - 1}\]

which can be simplified as

\[
\left( \frac{1}{L_N h} \right)^{\frac{1}{v}} \left( \frac{1}{L_N + L_S h} \right)^{\frac{1}{\sigma} + \theta \frac{\sigma}{v} \beta S h \frac{1}{\sigma} + \sigma_{d_s S h} \omega_{S h}} \left( L_N + L_S h \right)^{\frac{1}{\sigma} + \theta \frac{\sigma}{v} \beta S h \frac{1}{\sigma} + \sigma_{d_s S h} \omega_{S h}} \right]^{\sigma - 1}\]

\[\frac{W^h_S}{W^l_S} = \left( 1 - \xi_S h \right) \left[ \frac{\beta}{1 - \beta} \left( Y_W \right) \frac{1 - \beta}{\frac{1}{v} + \beta \left( L_N h + k L_S h \right)^{\frac{1}{\sigma} + \theta \frac{\sigma}{v} \beta h \frac{1}{\sigma} + \sigma_{d_s S h} \omega_{S h} \gamma h} + \xi_S h \phi_S \left( 1 - d_s \right) \right] \left( 1 - \xi_S l \right) \left[ \frac{\beta}{1 - \beta} \left( Y_W \right) \frac{1 - \beta}{\frac{1}{v} + \beta \left( L_N + k L_S \right)^{\frac{1}{\sigma} + \theta \frac{\sigma}{v} \beta S h \frac{1}{\sigma} + \sigma_{d_s S h} \omega_{S h}} + \xi_S \phi_S \left( 1 - g_s \right) \right] \right]
\]

Denote

\[
\mathcal{L}_S^T = \left( 1 - \xi_S l \right) \left[ \frac{\beta}{1 - \beta} \left( Y_W \right) \frac{1 - \beta}{\frac{1}{v} + \beta \left( L_N + k L_S \right)^{\frac{1}{\sigma} + \theta \frac{\sigma}{v} \beta S h \frac{1}{\sigma} + \sigma_{d_s S h} \omega_{S h}} + \xi_S \phi_S \left( 1 - g_s \right) \right] \right]
\]

\[
\mathcal{L}_S = \left( 1 - \xi_S l \right) \left[ \frac{\beta}{1 - \beta} \left( Y_W \right) \frac{1 - \beta}{\frac{1}{v} + \beta \left( L_N + k L_S \right)^{\frac{1}{\sigma} + \theta \frac{\sigma}{v} \beta S h \frac{1}{\sigma} + \sigma_{d_s S h} \omega_{S h}} + \xi_S \phi_S \left( 1 - g_s \right) \right] \right]
\]

Therefore, dividing the numerator and denominator by \( \mathcal{L}_S^T \), and assuming \( \xi_S h = \xi_S l \), the skill premium can be written as

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\[
\frac{W^h_S}{W^l_S} = \left[ \left( \frac{1 - \gamma}{\gamma} \right)^{\frac{k}{b}} \left( \frac{L^h_t}{L^h_N + kL^h_T} \right)^{\frac{1}{\tau}} \left( A^h \right)^{\frac{\sigma - 1}{\sigma}} \right] + \frac{\zeta}{\zeta^*}
\]

Similarly, the steady state skill premium in the South in autarky is given by

\[
\frac{W^h_S}{W^l_S} = \left( 1 - \xi^l_S \right) \left[ \frac{\beta}{1 - \beta} (Y_S)^{\frac{k}{b}} (1 - \beta)^{\frac{1}{\tau}} (L_S)^{\frac{k}{b}} (kA^h)^{\frac{\sigma - 1}{\sigma}} + \frac{\phi^l_S (1 - d_S)}{q_S} \right] + \xi^l_S \left[ \phi^l_S (1 - g_S) \right]
\]

Denote

\[
\mathcal{L}^A_S = \left( 1 - \xi^l_S \right) \left[ \frac{\beta}{1 - \beta} (Y_S)^{\frac{k}{b}} (1 - \beta)^{\frac{1}{\tau}} (L_S)^{\frac{k}{b}} (kA^h)^{\frac{\sigma - 1}{\sigma}} \right]
\]

Therefore, dividing the numerator and denominator by \( \mathcal{L}^A_S \), and assuming \( \xi^h_S = \xi^l_S \), the skill premium can be written as

\[
\frac{W^h_S}{W^l_S} = \left[ \left( \frac{1 - \gamma}{\gamma} \right)^{\frac{k}{b}} \left( \frac{L^h_t}{L^h_N + kL^h_T} \right)^{\frac{1}{\tau}} \left( A^h \right)^{\frac{\sigma - 1}{\sigma}} \right] + \frac{\zeta}{\zeta^*}
\]

Therefore, wage inequality increases after trade liberalization if

\[
\mathcal{L}^A_T + \frac{\bar{\xi}}{\tau} > \mathcal{L}^A_S + \frac{\bar{\xi}}{\tau}
\]

This can be simplified as

\[
\left[ \left( \frac{1 - \gamma}{\gamma} \right)^{\frac{k}{b}} \left( \frac{L^h_t}{L^h_N + kL^h_T} \right)^{\frac{1}{\tau}} \left( A^h \right)^{\frac{\sigma - 1}{\sigma}} \right] \frac{\mathcal{L}^T_S}{L^T_S} + \frac{\bar{\xi}}{\tau} > \left[ \left( \frac{1 - \gamma}{\gamma} \right)^{\frac{k}{b}} \left( \frac{L^h_t}{L^h_N + kL^h_T} \right)^{\frac{1}{\tau}} \left( A^h \right)^{\frac{\sigma - 1}{\sigma}} \right] \frac{\mathcal{L}^A_S}{L^A_S} + \frac{\bar{\xi}}{\tau}
\]

If \( \mathcal{L}^T_S < \mathcal{L}^A_S \), which is satisfied if and only if \( \left( \frac{Y_N}{Y_W} \right) > \left( \frac{L^h_t}{L^h_N + kL^h_T} \right) \), or if \( 1 - \bar{\phi} > \frac{\bar{\xi}}{1 + k\bar{\phi}} \), or if \( \frac{\theta}{1 + \phi} > \frac{\bar{\xi}}{1 + k\bar{\phi}} \) then

\[
\frac{\bar{\xi}}{\tau} > \frac{\bar{\xi}}{\tau}
\]

Then, remaining to establish the conditions under which

\[
\left[ \left( \frac{1 - \gamma}{\gamma} \right)^{\frac{k}{b}} \left( \frac{L^h_t}{L^h_N + kL^h_T} \right)^{\frac{1}{\tau}} \left( A^h \right)^{\frac{\sigma - 1}{\sigma}} \right] \frac{\mathcal{L}^T_S}{L^T_S} > \left[ \left( \frac{1 - \gamma}{\gamma} \right)^{\frac{k}{b}} \left( \frac{L^h_t}{L^h_N + kL^h_T} \right)^{\frac{1}{\tau}} \left( A^h \right)^{\frac{\sigma - 1}{\sigma}} \right] \mathcal{L}^A_S
\]
Substituting the endogenous skill bias yields

\[
\left( \frac{L^b_N + kL^b_S}{L^T_N + kL^T_S} \right)^{\sigma - 1} \left( \frac{L^b_N}{L^T_N} \right)^{\frac{(\sigma - 1)}{\sigma}} L^T_S > \left( \frac{L^b_S}{L^T_S} \right)^{\frac{1}{\sigma}} L^T_S
\]

which can be simplified to

\[
\frac{L^T_N}{L^T_S} > \left( \frac{L^b_N + kL^b_S}{L^T_N + kL^T_S} \right)^{\frac{(\sigma - 1)}{\sigma}} \left( \frac{L^b_N}{L^T_N} \right)^{\frac{1}{\sigma}}
\]

Which can be rearranged to

\[
\left( \frac{L^T_N}{L^T_S} \right)^{\frac{1}{\sigma}} > \left( \frac{L^b_N + kL^b_S}{L^T_N + kL^T_S} \right)^{\frac{(\sigma - 1)}{\sigma}} \left( \frac{L^b_N}{L^T_N} \right)^{\frac{1}{\sigma}}
\]

which can be simplified as

\[
\left( \frac{1}{1 + k\theta} \right)^{\frac{1}{\sigma}} > \left( \frac{1 + k\mu}{1 + k\theta} \right)^{\frac{1}{\sigma}} \left( \frac{\mu}{\theta} \right)^{\frac{1}{\sigma}}
\]
References


