Monetary Policy, the Skill Premium, and Unemployment Across Skills*

Sherif Khalifa†
Johns Hopkins University
December 13, 2005

Abstract

This paper studies the implications of a monetary policy shock on the skill premium and unemployment across skills. A dataset on the skill premium and the unemployment ratios of the high and low educated is compiled using the Outgoing Rotation Group of the Current Population Survey. A vector autoregressive analysis demonstrates that a contractionary policy induces a decline in the skill premium and a larger and more persistent increase in the unemployment ratio of the low educated relative to that of the high educated. To account for these patterns, a framework characterized by search frictions is developed where the population is divided into those high and low educated. There are two types of firms: wholesalers and retailers. Wholesalers post two types of vacancies: the complex that can be matched with the high educated, and the simple that can be matched with both the high and low educated. On the job search is allowed. Wholesalers produce intermediate goods using labor only, and sell their product to retailers in a perfectly competitive market. Retailers purchase these intermediate goods, differentiate and sell them to households in a monopolistically competitive market. Nominal rigidities are generated as only some of the retailers adjust their price every period. The monetary authority utilizes the nominal interest rate as the instrument for the conduct of monetary policy, and a positive monetary policy shock replicates most of the observations.

Keywords: monetary policy, skill premium, unemployment, search and matching, sticky prices.

JEL Classification: E12, E52, J24, J31,J41.

*I gratefully acknowledge the guidance of Professors Thomas Lubik and Louis Maccini during the preparation of this paper.
†Department of Economics, Mergenthaler Hall, 3400 North Charles Street, Baltimore, MD, 21218, USA. Email: sherif@jhu.edu
1 Introduction

There is a surge in interest in documenting and analyzing the underlying causes of certain significant changes that the labor market in the United States has been witnessing. A sample of these changes include a widening wage disparity amidst those with different observable skill characteristics, attributed by Acemoglu (1998,1999,2002,2003) to a technological change that is biased towards the skilled, increasing their productivity relative to the unskilled, thus generating the observed inequality. The labor market participants, with distinguishable skills, also experience differences in terms of the exposure to and duration of unemployment, mainly due to the decline in the demand for the unskilled as demonstrated by Berman et al. (1995,1998). This decline is ascribed to skill biased technological change and outsourcing, besides the evidence provided by Sicherman (1991) that the skilled occupy temporary jobs in which they are overqualified considering their education level, thus crowding out the unskilled into unemployment. All these studies convey the message that agent heterogeneity across skills is becoming an inevitable ingredient in any thorough analysis of contemporary labor market conditions.

On the other hand, considerable amount of research is devoted to analyze the effects of the conduct of monetary policy on economic activities. Naturally, comprehending the repercussions that labor markets face after the economy is exposed to a monetary policy shock captured the attention of a lot of these studies. Nevertheless, though this market experiences significant changes, a study of the effect of monetary shocks on this changing environment is not sufficiently considered. This paper assumes the task of filling this gap by analyzing the effect of a monetary policy shock on the skill premium and the unemployment ratios of the skilled and the unskilled. To understand the implications of a monetary policy shock, a vector autoregressive analysis is undertaken where the set of endogenous variables include the Fed funds rate, the inflation rate, real gross domestic product, the skill premium, and the unemployment ratios of the high and low educated. A positive shock to Fed funds rate causes a decline in the inflation rate, real gross domestic product and the skill premium, while induces a larger and more persistent increase in the unemployment ratio of the low educated relative to that of the high educated, indicating that the low educated bear the brunt of the increase in unemployment after a contractionary policy.

In order to capture the factors critical in generating these observations, the paper incorporates the theoretical setup developed in Khalifa (2005) into a new Keynesian framework with nominal rigidities. The model relies on search frictions as it became the standard environment in analyzing the cyclical behavior of labor market variables eversince Merz (1995) and Andolfatto (1996) introduced two sided search into an otherwise real business cycle model and succeeded in reproducing some stylized facts that
the Walrasian model either resolved in an unsatisfactory manner or has not been able to address at all. On the other hand, the new Keynesian paradigm that integrates imperfect competition and nominal rigidities into an optimizing general equilibrium setup gained consensus as an appropriate framework to analyze the economy’s response to monetary policy shocks. Nevertheless, a drawback of the new Keynesian model is attributed to its failure to address a set of stylized facts that characterize the labor market due to the lack of frictions that can generate equilibrium unemployment. This stimulated the incorporation of search frictions into new Keynesian models. These extensions focused on the demand channel of monetary policy in which nominal rigidities in the Calvo (1983) sense allows monetary transmission through its influence on aggregate demand, as adopted in Cheron and Langot (2000), Soto (2001), Riascos (2002), Krause and Lubik (2003), Gerke and Rubart (2003), Trigari (2003, 2004), Christoffel and Linzert (2004), Walsh (2003, 2005), Tang (2005) and Moyen and Sahuc (2005).

This paper builds upon the progress established in these studies and extends their framework, that combines labor search and an optimizing based monetary policy model, into one that is characterized by two sided heterogeneity. In this context, the population is divided into those high and low educated. There are two types of firms: wholesalers and retailers. Wholesalers post two types of vacancies: the complex that can be filled by the high educated and the simple that can be filled by both the high and low educated. Though the variables of interest do not feature job heterogeneity, Khalifa (2005) demonstrates that job competition and the possible mismatch between the employees education level and the qualification of jobs they are occupying are essential aspects in understanding the flows in and out of unemployment in a labor market with heterogeneous agents. The wholesalers use labor as their only factor of production, and sell their output to retailers in a perfectly competitive market. The retailers use the wholesale output as their intermediate good, which they differentiate costlessly and sell to households in a monopolistically competitive market. Nominal rigidities occur as only a portion of the retailers are assumed to adjust their prices every period. The monetary authority utilizes the nominal interest rate as the instrument for the conduct of monetary policy. A positive monetary policy shock reduces wage inequality and shows that the low educated bear the brunt of the increase in unemployment, consistently with the VAR results.

This paper can also be perceived as a contribution to a series of studies focusing on the asymmetric implications of monetary policy shocks. This line of research is stimulated by the fact that though monetary policy transmission has always been of interest to policy makers, most of the research in this area concentrates on the aggregate activity ignoring possible asymmetries on more disaggregate levels in the economy, either across sectors or between heterogeneous agents. In this context, some studies evaluated
the hypothesis that different sectors react to changes in monetary policy according to their interest rate sensitivity. For instance, Gertler and Gilchrist (1994) compare the response of small versus large manufacturing firms, and find that a funds rate shock has a greater cumulative impact on small relative to large firms, and that small firms exhibit an asymmetric response to monetary policy but large firms do not. Arnold et al. (2005) employ state level data on industries for the period 1958 to 2001, and find evidence that support cross industry dissimilarities in response to monetary shocks, which can be accounted for by their sensitivity to interest rates. They also find that regional differences in the effectiveness of monetary policy arise from cross region dissimilarities in the relative importance of small businesses. Similarly, Bouakez et al. (2005) estimate a model, for the period 1959 to 2002, with heterogeneous production sectors: agriculture, mining, construction, durables manufacturing, nondurables manufacturing and services. The results imply that, after an unexpected temporary increase in the growth rate of money supply, the output and hours of work in construction and durable manufacturing increase proportionally more than those of the other sectors, and that the relative prices in the services sectors rises the most. Similarly, Thorbecke (1997) provide evidence that a Fed funds rate shock reduces employment in construction and durable goods sectors the most, and that the burden of unemployment fall disproportionately on low income individuals. These studies, however, can be translated to an analysis of the effects of monetary policy across skills, in as much as the sectors analyzed impose a minimum skill requirement for recruitment. However, Carpenter and Rodgers (2004) find that innovations to the Fed funds rate lowers the employment-population ratio of the less skilled, and the larger responses are not due to their higher likelihood of being employed in industries and occupations that are more sensitive to contractionary policy.

Other studies analyze the implications of monetary shocks on heterogeneous agents in the economy. For instance, Romer and Romer (1998) find that unanticipated inflation is associated with a lower Gini coefficient in the short run. Heer and Smith (2003) present evidence that inflation reduces the inequality of the earnings distribution. Finally, Fowler (2005) finds that monetary policy systematically responds over the business cycle to deviations in income inequality, and concludes that as the Gini is a measure of the poor’s share of total income which is derived mainly from wages, then there is evidence that monetary policy responds to wage inequality.

The remainder of the paper is organized as follows: section 2 covers the vector autoregressive analysis, section 3 the model, section 4 the calibration, section 5 the results, section 6 concludes, and section 7 includes the data and derivations appendices. References, tables and figures are included thereafter.
2 Observations

The task of examining the effect of a monetary policy shock on the skill premium and the composition of unemployment across skills is undertaken by compiling a time series from the Outgoing Rotation Group of the Current Population Survey\(^1\). The Current Population Survey is a rotating panel. After the fourth month in the survey, the participants take an eight month hiatus. Afterwards, they are interviewed for another four months, and after the eighth month in sample, they are completely dropped from the survey. The fourth and eighth month-in-sample groups from all 12 months play a special role as they are given additional questions, the answers to which are collected as the Outgoing Rotation Group. These provide us with information about the participants’ employment status, level of education, type of occupation, weekly earnings, and weekly hours of work. The data is monthly and covers the period from January 1979 until December 2004.

To compile a time series out of this survey, the observations in each monthly file are divided into those employed and those unemployed. Each of these groups is further divided into those high and low educated, where the high educated are those who obtained some college education or higher. Then each of the two employed groups is further divided into those working in a complex occupation and those in a simple occupation. This provides four employed and two unemployed types as follows:

1. The high educated employed in a complex occupation.
2. The high educated employed in a simple occupation.
3. The high educated unemployed.
4. The low educated employed in a complex occupation.
5. The low educated employed in a simple occupation.
6. The low educated unemployed.

The low educated in a complex occupation type is dropped as their average proportion in the sample for the period understudy is insignificant. For the remainder of the employed types, weighted average hourly wages are calculated as the ratio of the weighted average weekly earnings to the weighted average weekly hours for each group. For the two unemployed types, levels of unemployment are calculated. Ratios of unemployment of the respective types as a proportion of the total sample are also considered. Using the three wages, the between group wage premium or the skill premium is defined as the ratio of the weighted average wage of the two high educated types to that of the low educated in simple occupations. Therefore, the variables compiled and used in the analysis are as follows:

\(^1\)Detailed data description is included in appendix 7.1.
(1) The between group wage premium, or the skill premium.

(2) The proportion of the high educated unemployed.

(3) The proportion of the low educated unemployed.

In this section, a vector autoregressive VAR analysis is conducted to demonstrate the dynamic response of an identified exogenous monetary policy shock on the premium and the unemployment measures that are compiled, where the short term interest rate is taken to be the instrument of monetary policy. The vector autoregression is given by

\[ Z_t = \eta + \sum_{i=1}^{p} \Phi_i Z_{t-i} + \epsilon_t \]

where \( \epsilon \) is an independently and identically distributed error term with zero mean and constant variance. The vector of endogenous variables is given by

\[ Z_t = [U^h_t, U^l_t, Y_t, Premium_t, r_t, \pi_t] \]

with the letters denoting in order the proportion of the high educated unemployed, the proportion of the low educated unemployed, the logged real gross domestic product, the logged skill premium, the Fed funds rate and the inflation rate. The vector autoregression is estimated using two lags, \( p = 2 \), as suggested by the Akaike information criterion and the Hannan-Quinn information criterion. The estimations are included in Table 2. Figures 1 – 6 display the impulse responses to a one percentage standard deviation innovation to Fed funds rate. The impulses demonstrate that a positive shock to Fed funds rate causes a decline in real gross domestic product, and a sharp increase reflecting the price puzzle followed by a decline in the inflation rate. The shock also induces an increase in the unemployment ratios of the low and high educated, with a larger and more persistent response of that of the former compared to that of the latter. Besides, the shock induces a decline in the skill premium. The decline in the inflation rate and real gross domestic product is a standard result in the literature. As for the responses of the unemployment ratios of the high and low educated and the skill premium, the results emphasize that after a contractionary monetary policy, though the low educated bear the brunt of the increase in unemployment, the gap between their wage and that of the high educated gets closer. The persistence of the unemployment of the low educated can be explained by either longer durations of the spell of unemployment experienced by this type, or a process of skill loss or obsolescence by the high educated if unemployed for an extended period of time.
3 Model

Consider an economy where time is discrete and the horizon is infinite. The population is of measure 1, and there is a constant fraction $\delta$ of households that are ex ante high educated and $(1 - \delta)$ that are low educated. There are two types of firms: wholesalers and retailers. Wholesalers post complex and simple vacancies. Complex vacancies are filled by high educated workers only, while simple vacancies are filled by both high and low educated workers. A high educated worker in a simple occupation is allowed to perform on the job search for a complex occupation, and a low educated in a simple occupation is hired for one period only. Wholesalers produce using labor as their only factor of production, and sell their intermediate output to retailers in a perfectly competitive market. Retailers, on the other hand, purchase these intermediate goods, costlessly transform and sell them to households in a monopolistically competitive market. Prices at the retail level are sticky as only a fraction of retail firms optimally adjust their price every period.

3.1 Households

The household members of each type are divided into those employed and unemployed as follows

\[ N_t^{hc} + N_t^{hs} + U_t^h = \delta \]  

\[ N_t^{ls} + U_t^l = 1 - \delta \]

where $N_t^{ij}$ denotes the number of workers of education type $i$ in occupation type $j$, where $i = (h, l)$ for high and low educated workers respectively, and $j = (c, s)$ for complex and simple occupations respectively. $U_t^i$ denotes the number of unemployed workers of type $i$. Time for the unemployed types is normalized to one. A high educated unemployed uses a portion $s_t$ of its time to search for a complex occupation, and $(1 - s_t)$ to search for a simple occupation. A low educated unemployed uses a portion $g_t$ of its time to search for a simple occupation, and $(1 - g_t)$ for domestic activities. A high educated worker in a simple occupation spends a fraction $o_t$ of its leisure, which is normalized to one, to search for a complex occupation. As different employment histories amongst members of a household can lead to heterogeneous wealth positions, we follow the literature in assuming that each household is thought of as an extended family whose members perfectly insure each other against variations in labor income due to employment or unemployment. Remaining within the confines of complete markets allows solving the program of a representative household, who chooses a contingency plan of its consumption and search intensities to
maximize the expected discounted infinite sum of its instantaneous utility which is separable in consumption and leisure as follows

\[ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \mathcal{U}(C_t) + N_t^{hs} \Omega(1 - o_t) \right] \right\} \]

where \( E_t \) is the expectation operator conditional on the information set available in period \( t \), \( \beta \) is the discount factor, \( \mathcal{U}(C_t) \) is the utility of period \( t \) consumption of the household, and \( \Omega(1 - o_t) \) denotes the utility of period \( t \) leisure of the high educated in simple occupations. Assuming the household has the following value function \( \Gamma^H_t = \Gamma^H \left( N_t^{hc}, N_t^{hs}, N_t^{ls} \right) \), the optimization problem of the household can be written in the following recursive form

\[ \Gamma^H_t = \max_{\{C_t, o_t, s_t, g_t\}} \left\{ \mathcal{U}(C_t) + N_t^{hs} \Omega(1 - o_t) + \beta E_t \left[ \Gamma^H_{t+1} \right] \right\} \]

subject to the following budget constraint

\[ C_t + \frac{B_t}{P_t} = N_t^{ls} W_t^{ls} + N_t^{hs} W_t^{hs} + N_t^{hc} W_t^{hc} + (1 - g_t) U_t^d W_t^d + \frac{B_t - 1 R^n_{t-1}}{P_t} + D_t \]

where \( W_t^{ij} \) is the period \( t \) real wage for type \( ij \), \( W_t^d \) is the return to domestic activities, \( P_t \) is the price of one unit of the final good, \( B_t \) is households holdings of a nominal one period risk free bond, \( R^n_t \) is the gross nominal interest rate on this bond, and \( D_t \) is the total dividends distributed by wholesale and retail firms. The households also take into consideration the employment dynamics of the three types of workers.

The employment of the high educated workers in complex occupations in period \( t + 1 \) is comprised of the workers of that type who are not exogenously separated in period \( t \) according to the separation rate from complex occupations \( \chi^c \), in addition to the new matches from the searchers pool whether they are high educated unemployed or on the job searchers

\[ N_{t+1}^{hc} = (1 - \chi^c) N_t^{hc} + T^c \left[ s_t U^c_t + o_t N_t^{hs} \right] \]

Similarly, the employment of the high educated workers in simple occupations in period \( t + 1 \) is comprised of those of that type who are neither separated from simple occupations exogenously in period \( t \) according to the separation rate \( \chi^s \), nor are matched with complex occupations as a result of their on the job search activities, in addition to the new matches from the searchers pool of the high educated unemployed.
\[ N_{t+1}^{hs} = [1 - \chi_s - o_t T_c^s] N_t^{hs} + T_t^s [(1 - s_t) U_t^h] \]  

The constant separation rates are justified by Hall (forthcoming), who concludes that job separation rates remained almost constant in the United States over the past 50 years, and by Shimer (2005a) who asserts the acyclicity of separations. Finally, the employment of the low educated workers in simple occupations in period \( t+1 \) evolves according to the new matches from the searchers pool of the low educated unemployed

\[ N_{t+1}^{ls} = T_t^s g_t U_t^l \]  

\( T_t^c \) and \( T_t^s \) are the endogenous probabilities of finding a complex or a simple occupation respectively, and defined as the ratio of the respective matches to total effective searchers as follows

\[ T_t^c = \frac{M_t^c}{s_t U_t^h + o_t N_t^{hs}} \]  

\[ T_t^s = \frac{M_t^s}{(1 - s_t) U_t^h + g_t U_t^l} \]  

\( M_t^c \) and \( M_t^s \) represent the number of complex and simple occupation matches respectively, and they are constant returns to scale homogeneous of degree one functions of the number of corresponding vacancies, \( V_t^c \) and \( V_t^s \), and effective searchers

\[ M_t^c = M^c \left( V_t^c, s_t U_t^h + o_t N_t^{hs} \right) \]  

\[ M_t^s = M^s \left( V_t^s, (1 - s_t) U_t^h + g_t U_t^l \right) \]  

Households choose their consumption level such that the marginal utility of consumption equals the Lagrange multiplier \( \lambda_t \)

\[ \frac{\partial U(C_t)}{\partial C_t} = \lambda_t \]  

They choose the level of bonds to hold to satisfy the following condition for the marginal utility of income

\[ \lambda_t = \beta E_t [R_t\lambda_{t+1}] \]
where $R_t$ is the gross real interest rate defined as

$$R_t = E_t \frac{P_t}{P_{t+1}} R_t^a$$

Combining these two first order conditions yields the following Euler equation

$$\frac{\partial \Omega (C_t)}{\partial C_t} = \beta E_t \left[ R_t \frac{\partial \Omega (C_{t+1})}{\partial C_{t+1}} \right]$$

Households choose their on the job search intensity $o_t$ such that the disutility from increasing on the job search intensity by one unit is offset by the difference between the discounted expected value to the household from an additional high educated worker in a complex occupation and that of an additional high educated worker in a simple occupation as follows

$$\frac{\partial \Omega (1-o_t)}{\partial o_t} + T_c^e E_t \left[ \frac{\partial \Gamma_{t+1}^H}{\partial N_{ht}^c} \right] - T_c^e \beta E_t \left[ \frac{\partial \Gamma_{t+1}^H}{\partial N_{ht}^s} \right] = 0$$

They also choose the optimal proportion of time the high educated unemployed allot to search for a complex occupation $s_t$ such that the discounted expected value of an additional high educated in a complex occupation is equal to that of a high educated in a simple occupation as follows

$$T_c^e E_t \left[ \frac{\partial \Gamma_{t+1}^H}{\partial N_{ht}^c} \right] = T_s^e E_t \left[ \frac{\partial \Gamma_{t+1}^H}{\partial N_{ht}^s} \right]$$

Finally, the households choose the optimal proportion of time the low educated unemployed allocate to search for a simple occupation $g_t$ such that the return to domestic activities equals the discounted expected value of an additional low educated worker in a simple occupation

$$\lambda_t W_t^d = \beta T_s^e E_t \left[ \frac{\partial \Gamma_{t+1}^H}{\partial N_{ht}^s} \right]$$

From the envelope theorem, an additional high educated matched with a complex occupation accrue a value for the household that is given by the current wage earned by that type, in addition to the discounted expected value if this worker is not separated exogenously after being matched. This also includes the value foregone, had it been that this member of the household is not matched in the first place, continued to search and got matched with either a simple or a complex vacancy during period $t$ as follows

$$\frac{\partial \Gamma_{t+1}^H}{\partial N_{ht}^c} = \lambda_t W_t^hc + \beta (1 - \chi^c) E_t \left[ \frac{\partial \Gamma_{t+1}^H}{\partial N_{ht}^hc} \right] - \beta T_c^e g_t E_t \left[ \frac{\partial \Gamma_{t+1}^H}{\partial N_{ht}^hc} \right] - \beta T_s^e (1 - s_t) E_t \left[ \frac{\partial \Gamma_{t+1}^H}{\partial N_{ht}^s} \right]$$
Similarly, an additional high educated matched with a simple occupation accrue a value for the household that is given by the current value obtained by the utility of leisure and the current labor income earned by that type, in addition to the discounted expected value if this additional worker is neither separated exogenously nor matched with a complex occupation as a result of on the job search, besides the value accrued if the worker succeeds in on the job search. This also includes the value forgone if this member of the household is not matched in the first place, continued to search, and got matched with either a simple or a complex vacancy during period \( t \) as follows

\[
\frac{\partial \Gamma^H}{\partial N^H_{ls}} = \Omega (1 - o_t) + \lambda_t W^h_{ls}^t + \beta (1 - \chi^s - o_t T^c_t) E_t \left[ \frac{\partial \Gamma^H_{t+1}}{\partial N^h_{ls}} \right] \\
+ \beta T^c_t o_t E_t \left[ \frac{\partial \Gamma^H_{t+1}}{\partial N^c_{ls}} \right] - \beta T^c_t s_t E_t \left[ \frac{\partial \Gamma^H_{t+1}}{\partial N^c_{ls}} \right] - \beta T^s_t (1 - s_t) E_t \left[ \frac{\partial \Gamma^H_{t+1}}{\partial N^s_{ls}} \right]
\]  

(21)

Finally, an additional low educated matched with a simple occupation accrue a value for the household that is given by the current wage earned by this type, less the return to domestic activities lost, in addition to the expected value if the worker is hired again after being fired in period \( t \) as follows

\[
\frac{\partial \Gamma^H}{\partial N^L_{ls}} = \lambda_t W^l_{ls} - \lambda_t (1 - g_t) W^d_t - \beta T^s_t g_t E_t \left[ \frac{\partial \Gamma^H_{t+1}}{\partial N^s_{ls}} \right]
\]

(22)

Substituting the envelope into the first order conditions yields the household’s optimal conditions\(^2\)

\[
\frac{\tau T^s_{ls}}{\beta T^c_t (T^s_t - T^c_t)} = E_t \frac{W^h_{ls}}{C_{t+1}} + E_t \left[ 1 - \chi^c - T^c_{t+1} s_{t+1} \right] \frac{\tau T^s_{t+1}}{T^c_{t+1} (T^s_{t+1} - T^c_{t+1})} - E_t \frac{\tau T^s_{t+1} (1 - s_{t+1})}{(T^s_{t+1} - T^c_{t+1})}
\]

(23)

\[
\frac{\tau}{\beta (T^s_t - T^c_t)} = \tau E_t (1 - o_{t+1}) + E_t \frac{W^h_{ls}}{C_{t+1}} + E_t \frac{\tau T^s_{t+1} (o_{t+1} - s_{t+1})}{(T^s_{t+1} - T^c_{t+1})} \\
+ \tau E_t \left[ 1 - \chi^c - o_{t+1} T^c_{t+1} - s_{t+1} (1 - s_{t+1}) \right] (T^s_{t+1} - T^c_{t+1})
\]

(24)

\[
\frac{W^d_t}{\beta C_t T^c_t} = E_t \frac{W^l_{ls} - W^d_t}{C_{t+1}}
\]

(25)

where \( \tau \) is the marginal utility of leisure of the high educated in a simple occupation.

\(^2\)Detailed derivations are included in appendix 7.2.1.
3.2 Wholesalers

The representative wholesaler chooses the number of complex and simple vacancies to post in order to maximize the discounted expected infinite sum of its future profit streams. The profit function of the firm is given by the difference between the value of its production, and the total cost incurred for posting the two types of vacancies, besides the wages given to the three working types as follows

\[ E_0 \sum_{t=0}^{\infty} \beta^t \lambda t \left[ X_t Y_t - \omega s V^{s}_t - \omega c V^{c}_t - N^{hc}_t W^{hc}_t - N^{hs}_t W^{hs}_t - N^{ls}_t W^{ls}_t \right] \] (26)

where \( X_t \) is the wholesale price of one unit of the intermediate good and \( Y_t \) is the intermediate output. \( \omega c \) is the cost of posting a complex vacancy and \( \omega s \) is the cost of posting a simple vacancy. The discount factor of firms is given such that it effectively evaluates profits in terms of the values attached to them by households, who ultimately own the firms. Thus, the utility based and time varying discount factor used by firms is given by \( \left( \beta + \frac{\lambda t+1}{\lambda t} \right) \). Assuming the firm has the following value function \( \Gamma^F_t = \Gamma^F (N^{hc}_t, N^{ls}_t, N^{hs}_t) \), the optimization problem can be written in the following recursive form

\[ \Gamma^F_t = \max_{\{V^{s}_t, V^{c}_t\}} \left\{ X_t Y_t - \omega s V^{s}_t - \omega c V^{c}_t - N^{hc}_t W^{hc}_t - N^{hs}_t W^{hs}_t - N^{ls}_t W^{ls}_t + \beta E_t \left[ \frac{\lambda t+1}{\lambda t} \Gamma^F_{t+1} \right] \right\} \] (27)

The maximization is subject to the production function which is a composite of the complex occupation output \( (A^h_t N^{hc}_t) \) and the simple occupation output \( (A^l_t N^{ls}_t + A^h_t N^{hs}_t) \) given by

\[ Y_t = Y \left[ A_t, (A^h_t N^{hc}_t), (A^l_t N^{ls}_t + A^h_t N^{hs}_t) \right] \] (28)

where \( A_t \) is the aggregate technology, \( A^h_t \) is the technology biased to high educated workers, and \( A^l_t \) is that biased to low educated workers. The problem of the firm is also subject to the employment dynamics

\[ N^{hc}_{t+1} = (1 - \chi c) N^{hc}_t + q^c V^{c}_t \] (29)

\[ N^{hs}_{t+1} = [1 - \chi s - o_t T^{c}_t] N^{hs}_t + q^{hs} V^{s}_t \] (30)

\[ N^{ls}_{t+1} = q^{ls} V^{s}_t \] (31)
where \( q_c^t \) is the probability of filling a complex vacancy, and is given by the ratio of complex matches to complex vacancies posted as follows

\[
q_c^t = \frac{M_c^t}{V_c^t}
\]  

\( q_h^s \) is the probability that a simple vacancy is filled by a high educated, and is given by the ratio of simple matches to simple vacancies posted multiplied by the proportion of the high educated amongst all effective searchers for a simple occupation as follows

\[
q_h^s = \frac{(1 - s_t) U_h^t}{(1 - s_t) U_h^t + g_t U_l^t} \left( \frac{M_s^t}{V_s^t} \right)
\]

and \( q_l^s \) is the probability that a simple vacancy is filled by a low educated, and is given by the ratio of simple matches to simple vacancies posted multiplied by the proportion of the low educated amongst all effective searchers for a simple occupation as follows

\[
q_l^s = \frac{g_t U_l^t}{(1 - s_t) U_h^t + g_t U_l^t} \left( \frac{M_s^t}{V_s^t} \right)
\]

The firm chooses the optimal level of complex vacancies to post such that the expected marginal cost of posting this type of vacancy is just compensated by the discounted expected value of an additional high educated worker in a complex occupation for the firm as follows

\[
\frac{\omega_c^t}{q_c^t} = \beta E_t \left[ \frac{\lambda_{t+1} \partial \Gamma^F_{t+1}}{\lambda_t \partial N_{hs}^{hc}} \right]
\]

The firm chooses the optimal level of simple vacancies to post such that the cost of posting a simple vacancy is equal to the discounted expected value of creating an occupation from this vacancy, whether it is filled by a high or a low educated worker

\[
\omega^s = \beta E_t \left[ \frac{\lambda_{t+1} \partial \Gamma^F_{t+1}}{\lambda_t \partial N_{hs}^{hs}} \right] q_h^s + \beta E_t \left[ \frac{\lambda_{t+1} \partial \Gamma^F_{t+1}}{\lambda_t \partial N_{ls}^{ls}} \right] q_l^s
\]

From the envelope theorem, the value of an additional high educated worker in a complex occupation for the firm is given by the net current value of the match for the firm, which is the difference between its marginal productivity less the bargained wage, in addition to the discounted expected value of the match
in case the worker is not exogenously separated as follows

$$\frac{\partial \Gamma^F}{\partial N^hc_t} = X_t \frac{\partial Y_t}{\partial N^hc_t} - W^hc_t + (1 - \chi^c) \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \Gamma^F_{t+1}}{\partial N^hc_{t+1}} \right] \quad (37)$$

Similarly, the value of an additional high educated worker in a simple occupation for the firm is given by

$$\frac{\partial \Gamma^F}{\partial N^hs_t} = X_t \frac{\partial Y_t}{\partial N^hs_t} - W^hs_t + (1 - \chi^s - \alpha tT^c_t) \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \Gamma^F_{t+1}}{\partial N^hs_{t+1}} \right] \quad (38)$$

Finally, the value of an additional low educated worker in a simple occupation for the firm is given by

$$\frac{\partial \Gamma^F}{\partial N^ls_t} = X_t \frac{\partial Y_t}{\partial N^ls_t} - W^ls_t \quad (39)$$

Substituting the envelope conditions into the first order conditions yields the firm’s optimal conditions:\footnote{Detailed derivations are included in appendix 7.2.2.}

$$\frac{\omega^c_t}{q^c_t} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( X_{t+1} \frac{\partial Y_{t+1}}{\partial N^hc_{t+1}} - W^hc_{t+1} + (1 - \chi^c) \frac{\omega^c_{t+1}}{q^c_{t+1}} \right) \right] \quad (40)$$

$$\varepsilon_t = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( X_{t+1} \frac{\partial Y_{t+1}}{\partial N^hs_{t+1}} - W^hs_{t+1} + (1 - \chi^s - \alpha tT^c_t) \varepsilon_{t+1} \right) \right] \quad (41)$$

Where $\varepsilon_t$ is defined as

$$\varepsilon_t = \frac{\omega^s_t}{q^s_t} - \frac{q^s_t}{q^c_t} \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( X_{t+1} \frac{\partial Y_{t+1}}{\partial N^ls_{t+1}} - W^ls_{t+1} \right) \right] \quad (42)$$

### 3.3 Wages and the Skill Premium

In equilibrium, matched firms and workers obtain from the match a total return that is strictly higher than the expected return of unmatched firms and workers, because if they separate each will have to go through an expensive and time consuming process of search before being matched again. We follow the literature in assuming that a realized match share this surplus through a Nash bargaining problem. Accordingly, the
wage of the high educated worker in a complex occupation is given by

\[ W_{ht} = \left(1 - \xi_{hc}\right) \left[X_t \frac{\partial Y_t}{\partial N_{ht}} + T_s \left(s_t \xi_t/\xi_t\right)\right] + \xi_{hc} \left[C_t \frac{\tau T_s (1 - s_t)}{(T_s - T_t)}\right] \quad (43) \]

Where \( \xi_{hc} \) is the firm’s share of the surplus. The wage is a weighted average of two terms: the first indicates that the worker is rewarded for a fraction \( \left(1 - \xi_{hc}\right) \) of both the firm’s revenues from the worker’s productivity and the discounted expected value to the firm of an additional worker of this type. The second term indicates that the worker is compensated by a fraction \( \xi_{hc} \) for the foregone benefit from the worker’s outside option from being matched with a simple occupation, expressed in terms of the consumption good.

Similarly, the bargained wage of the high educated in a simple occupation is given by

\[ W_{hs} = \left(1 - \xi_{hs}\right) \left[X_t \frac{\partial Y_t}{\partial N_{hs}} + T_s (1 - s_t)\right] + \xi_{hs} \left[C_t \frac{\tau T_s (s_t - o_t)}{(T_s - T_t)}\right] \quad (44) \]

Where \( \xi_{hs} \) is the firm’s share of the surplus. The wage is a weighted average of two terms: the first indicates that the worker is rewarded by a fraction \( \left(1 - \xi_{hs}\right) \) for both the firm’s revenues from the worker’s productivity and the discounted expected value of the match for the firm. The second term indicates that the worker is compensated by a fraction \( \xi_{hs} \) for the outside options of foregone leisure and the benefit from being matched with a complex occupation. Finally, the bargained wage of the low educated in a simple occupation is given by

\[ W_{ls} = \left(1 - \xi_{ls}\right) \left[X_t \frac{\partial Y_t}{\partial N_{ls}}\right] + \xi_{ls} \left[W_t^d\right] \quad (45) \]

Where \( \xi_{ls} \) is the firm’s share of the surplus. The wage is a weighted average of two terms: the first indicates that the worker is rewarded by a fraction \( \left(1 - \xi_{ls}\right) \) for the firm’s revenues from this worker’s productivity. The second term indicates that the worker is compensated by a fraction \( \xi_{ls} \) for the foregone return to domestic activities. Using the three wages, the skill premium is defined as the ratio of the weighted average wage of the two high educated types \( W^h_t \), to the wage of the low educated in simple occupations as follows

\[ \text{Premium}_t = \frac{W^h_t}{W^s_t} \quad (46) \]
3.4 Retailers

There is a continuum of monopolistically competitive retailers indexed by \( j \) over the unit interval. Retailers buy the homogeneous intermediate output produced by wholesalers \( Y_t \) at a price \( X_t \) in a competitive market, differentiate them with a technology that transforms a unit of wholesale good into a unit of retail good \( Y_t(j) \) at no cost, and then retailer \( j \) sells them to the households at a nominal price of \( P_{jt} \). Final goods are the CES aggregate

\[
Y_t = \left( \int_0^1 Y_t(j) \frac{\theta - 1}{\theta} dj \right)^{\frac{\theta}{\theta - 1}} \tag{47}
\]

where \( \theta \) is the elasticity of substitution across the differentiated retail goods. Given the aggregate output index, the corresponding price index which is defined as the minimum expenditure required to purchase retail goods resulting in one unit of the final good, is given by

\[
P_t = \left( \int_0^1 P_{jt} \frac{1}{\theta} dj \right)^{-\frac{1}{\theta}} \tag{48}\]

As an optimizing household allocates its consumption spending across alternative differentiated goods at date \( t \) so as to minimize the total expenditure required to achieve a given value of the index \( C_t \), each retailer faces a downward sloping demand curve for his goods given by

\[
C_t(j) = \left( \frac{P_{jt}}{P_t} \right)^{-\theta} C_t \tag{49}\]

Each retailer takes as given the price of the wholesale output besides the demand curve, and chooses its optimal price to maximize its discounted expected stream of future profits under the hypothesis that the price they set at date \( t \) applies at date \( t + s \) with probability \( \sigma \), and is given by

\[
E_t \left\{ \sum_{s=0}^{\infty} \sigma^s \beta^s \frac{\lambda_t}{\lambda_t} \left( \frac{P_{jt}}{P_{t+s}} - \frac{X_{t+s}}{P_{t+s}} \right) C_{t+s}(j) \right\} \tag{50}\]

Substituting the demand curve yields

\[
E_t \left\{ \sum_{s=0}^{\infty} \sigma^s \beta^s \frac{\lambda_t}{\lambda_t} \left( \frac{P_{jt}}{P_{t+s}} - \frac{X_{t+s}}{P_{t+s}} \right) \left( \frac{P_{jt}}{P_{t+s}} \right)^{-\theta} C_{t+s} \right\} \tag{51}\]

Therefore, each retailer chooses the optimal price \( P_{jt} \) such that\(^7\)

\(^7\)Detailed derivations are included in appendix 7.2.6.
\[
\left( \frac{P_{jt}}{P_t} \right) = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t \left\{ \sum_{s=0}^{\infty} \sigma^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[ X_{t+s} \frac{P_{t+s}}{P_t} \right] \theta C_{t+s} \right\}}{E_t \left\{ \sum_{s=0}^{\infty} \sigma^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left( \frac{P_{t+s}}{P_t} \right)^{\theta - 1} C_{t+s} \right\}}
\]

(52)

A forward looking retailer sets its price equal to a markup over a weighted average of expected future marginal costs, where \( \frac{\theta}{\theta - 1} \) is the steady state markup. To introduce price stickiness, assume that each retailer can update its price with a fixed probability \((1 - \sigma)\) that is independent of the time elapsed since the last price adjustment. This can be perceived as a fraction \((1 - \sigma)\) of retailers who get to set a new price, while the remaining \(\sigma\) must continue to sell at their previously posted prices. Therefore, if all firms choose the same price \(P_{jt} = P_t^*\), the aggregate price level evolution is given by

\[
P_t = \left[ \sigma (P_{t-1})^\theta + (1 - \sigma) (P_t^*)^{1-\theta} \right]^{\frac{1}{1-\theta}}
\]

(53)

Combining the log linearized optimal condition and the log linearized aggregate price level yields the following forward looking Phillips curve\(^8\)

\[
\hat{\pi}_t = \frac{1 - \sigma}{\sigma} (1 - \sigma \beta) \left( \hat{X}_t - \hat{P}_t \right) + \beta E_t \pi_{t+1}
\]

(54)

where \(\pi_t\) is the period \(t\) inflation rate.

### 3.5 Monetary Authority

The monetary authority conducts monetary policy using the short term nominal interest rate as the policy instrument and lets the nominal amount of money adjusts accordingly. The Taylor rule that is considered is forward looking as follows

\[
R_t^n = \left[ R^n_{t-1} \right]^{\alpha_r} \left[ (E_t \pi_{t+1})^{\alpha_\pi} (Y_t)^{\alpha_Y} \right]^{1-\alpha_r} e_t^m
\]

(55)

where the parameter \(\alpha_r\) measures the degree of interest rate smoothing. The parameters \(\alpha_\pi\) and \(\alpha_Y\) are the response coefficients of inflation and output respectively. Finally, \(e_t^m\) is an independently and identically distributed monetary policy shock with a standard deviation \(\sigma_m\).

\(^8\)Detailed derivations are included in appendix 7.2.7.
4 Calibration

The functional forms are determined and the parameters are calibrated in order to solve the model numerically. The instantaneous utility function of consumption is represented by the logarithm of consumption expenditures as follows

\[ \mathcal{U}(C_t) = \ln(C_t) \]  

(56)

The instantaneous utility of leisure of the high educated worker in a simple occupation is defined as

\[ \Omega(1 - o_t) = \tau(1 - o_t) \]  

(57)

The matching function for the complex and simple occupations is represented as a Cobb-Douglas specification with constant returns to scale as follows

\[ M^c_t = m^c(V^c_t)^{\gamma^c} \left( s_t U^h_t + o_t N^{hs}_t \right)^{1 - \gamma^c} \]  

(58)

\[ M^s_t = m^s(V^s_t)^{\gamma^s} \left( (1 - s_t) U^h_t + g_t U^l_t \right)^{1 - \gamma^s} \]  

(59)

Where \( \gamma^c \in (0, 1) \) and \( \gamma^s \in (0, 1) \) are the elasticities of complex and simple vacancies respectively, while \( m^c \) and \( m^s \) are the level parameters of the two functions, and capture all factors that influence the efficiency of matching. The technological constraints faced by the representative firm is also represented by a constant returns to scale Cobb-Douglas function with elasticity of final output with respect to the complex occupation output \( \mu \in (0, 1) \), and is given by

\[ Y_t = A_t \left( A^h_t N^{hc}_t \right)^{\mu} \left( A^h_t N^{hs}_t + A^l_t N^{ls}_t \right)^{1 - \mu} \]  

(60)

The production function features three technological shocks, the aggregate technological shock \( A_t \), besides the high and low educated biased technological shocks \( A^h_t \) and \( A^l_t \) respectively. The logarithm of the technological shocks are assumed to follow an AR(1) process as follows

\[ \log A_{t+1} = \rho \log A_t + \epsilon_{t+1}^A \]  

(61)
\[
\log A_{t+1}^h = \rho^h \log A_t^h + \epsilon_{t+1}^h
\]

\[
\log A_{t+1}^l = \rho^l \log A_t^l + \epsilon_{t+1}^l
\]

The \( \epsilon_{t+1}^a, \epsilon_{t+1}^h, \text{ and } \epsilon_{t+1}^l \) are identically and independently distributed random variables drawn from a normal distribution with mean zero and standard deviations denoted by \( \sigma^A, \sigma^Ah, \text{ and } \sigma^Al \) respectively. The return to domestic activities \( W_t^d \) is a function of the average productivity as follows

\[
W_t^d = \psi \left[ \frac{Y_t}{N_t^s + N_t^{hs} + N_t^{hc}} \right]
\]

Table 1 shows the values chosen for the parameters of the model. The first group includes those related to households such as the fixed proportion of the high educated in the population \( \delta \) which is set at 0.5, close to the data average of the year 2004 given by 0.5067. In addition to the household’s discount factor \( \beta \) which is given by 0.98, and the parameter \( \tau \) in the utility of leisure function given by 1.08. The second set pertains to the matching technology, where the level parameters in the matching functions \( m^c \) and \( m^s \) are given by 0.3 and 0.6 respectively. We follow the literature in setting the elasticity of matches with respect to vacancies \( \gamma^c \) and \( \gamma^s \) to 0.6. The separation rates \( \chi^c \) and \( \chi^s \) from the complex and simple occupations are given by 0.045 and 0.02 respectively. These are selected such that their average of 3.256% is consistent with the weighted average separation rate of 3.23% calculated by Hall (forthcoming). The third set includes the technological parameters, where the elasticity parameter in the production function \( \mu \) is set at 0.5. The autoregressive coefficients in the technological laws of motion \( \rho^A, \rho^Ah \text{ and } \rho^Al \) are given by 0.9, 0.95 and 0.95 respectively, and the standard deviation of the three technologies \( \sigma^A, \sigma^Ah \text{ and } \sigma^Al \) are chosen as 0.0049, 0.002 and 0.002 respectively. The fourth set includes the parameters in the bargained wages and the return to domestic activities equations, where the firm’s share of the surplus \( \xi^{hc}, \xi^{hs} \text{ and } \xi^{ls} \) are set around 0.5, 0.58 and 0.5 respectively. The parameter \( \psi \) is set such that the return to domestic activities is 0.16 times the average productivity. The costs of creating the complex vacancy \( \omega^c \) and the simple vacancy \( \omega^s \) are given by 1.043 and 0.08 respectively. The last group includes the parameters of the new Keynesian aspects of the model. The elasticity of substitution across the differentiated goods \( \theta \) is given by 11, to get a markup of 1.1 as is standard in the literature. The proportion of retailers that do not adjust their price \( \sigma \) is given by 0.75. The interest rate smoothing parameter \( \alpha_r \) is given by 0.85, and the parameters \( \alpha_x \) and \( \alpha_Y \) are given by 1.5 and 0.5 respectively. Finally, \( \sigma_m \) is chosen to be 0.1 as standard in the literature.
5 Analysis

The model is solved by computing the nonstochastic steady state around which the equation system is linearized. The resulting model is solved by the methods developed in Sims (2002). A monetary authority that tightens through increasing the nominal interest rate causes an increase in the real interest rate due to nominal rigidities. Therefore, consumers face a higher tradeoff between present and future consumption due to the increased returns on savings. This modifies the aggregate consumption behavior of households, and reduces current and future aggregate demand. Since monopolistic competitive retailers produce to meet demand, this reduces their current and future demand for intermediate goods, which they use as inputs. This consequently causes a decline in their production, which can only occur at declined marginal costs, and because prices are set based on expected future real marginal cost, inflation declines. The intermediate firms, faced by a decline in the demand for their goods, reduce the number of posted vacancies which in turn reduces employment. Average productivity increases as the decline in employment exceeds the decline in output. Therefore, the return to domestic activities increases inducing the low educated unemployed to reduce their search for a simple occupation. Therefore, the proportion of the low educated in simple occupations decline, while the proportion of the low educated unemployed increases consistently with the VAR results. The wholesalers respond by reducing the simple vacancies posted. Therefore, the high educated unemployed increase their search for the more available complex occupations. The firms respond by posting more complex vacancies. The high educated working in simple occupations increase their on the job search for a complex occupation as well. These lead to an increase in the proportion of the high educated in complex occupations, and a reduction in the proportion of the high educated in simple occupations. As the latter effect overcomes the former, the proportion of the high educated unemployed increases, consistently with the VAR results. From the impulses, the response of the unemployment rate of the low educated is larger compared to that of the high educated, however it does not show the same persistence displayed by the data.

The wage of the low educated in simple occupations increases due to the increase in the return to domestic activities which is one of its components. As the proportion of the high educated in simple occupations decline, the probability that a simple vacancy is filled by one declines as well, and the discounted expected value of an additional worker of that type to the firm increases, thus increasing their wage. On the otherhand, as the proportion of the high educated in complex occupations increases, the probability that a complex vacancy is matched with one increases, and thus the discounted expected value of an additional worker of that type to the firm declines, causing a decline in their wage. As the weighted average wage of
the high educated declines, and the wage of the low educated increases, the between group wage premium declines consistently with the observations displayed by the data as demonstrated in the VAR analysis.

Table 3 presents the cross correlation coefficients between the variables of interest in the model. These are consistent with the VAR analysis and the empirical literature. For instance, the skill premium is negatively correlated with the nominal interest rate with a coefficient of $-0.8904$. Previous studies provide evidence as to the decline in income inequality in response to an increase in inflation, while in this paper wage inequality increases along with a surge in inflation. These results allow us to conclude that the decline in income inequality is mainly influenced by capital rather than labor income. The unemployment ratios of the high and low educated are positively correlated with the nominal interest rate, but the coefficient of the latter, 0.502, is higher than the former, 0.0963, confirming that the low educated bear the brunt of the increase in unemployment after a contractionary policy. The proportion of those employed in complex occupations are positively correlated, while the proportions of both types employed in simple occupations are negatively correlated with the nominal interest rate. This can be interpreted as consistent with the evidence provided by the empirical literature that shows that the employment in the construction and durable manufacturing sectors respond differently than that of the services sector. The results are consistent if the former sectors are assumed to recruit mainly low educated workers, while the latter sector has a high minimum level of education requirement for recruitment. Finally, as is standard in the literature, the real gross domestic product and the inflation rate are negatively correlated with nominal interest rate with correlation coefficients of $-0.5341$ and $-0.8577$ respectively.

6 Conclusion

The repercussions of the conduct of monetary policy on economic activities, and in particular on labor market conditions, is a topic that captured the attention of a lot of studies. Nevertheless, as this market experiences significant changes, the analysis of the effect of monetary policy shocks on labor market variables that reflect this changing environment, especially agent heterogeneity across skills, is not adequately addressed.

This paper assumes the task of filling this gap through analyzing the asymmetric impact of monetary policy on the wages and unemployment ratios across skills. To this purpose, a dataset on the skill premium and the unemployment ratios of the skilled and the unskilled is compiled from the Current Population Survey Outgoing Rotation Group. To analyze the effect of a monetary policy shock, a vector autoregressive analysis is undertaken, where an innovation to Fed funds rate causes a decline in the inflation rate, real
gross domestic product and the skill premium, while induces a larger and more persistent increase in the unemployment of the low educated relative to that of the high educated. To account for these patterns, a framework characterized by search frictions is considered where workers are heterogeneous in terms of their education level. There are two types of firms: wholesalers and retailers. Wholesalers post two types of vacancies: the complex that can be filled by the high educated, and the simple that can be filled by the high and low educated. High educated workers in a simple occupation can continue searching on the job for a complex occupation. The wholesalers produce intermediate goods using labor only, and sell their product to retailers in a perfectly competitive market. Retailers purchase these intermediate goods, differentiate and sell them to households in a monopolistically competitive market. Nominal rigidities are generated as only some of the retailers adjust their price every period. Monetary authority utilizes the nominal interest rate as the instrument for the conduct of monetary policy, and relies on a forward looking Taylor rule. A contractionary monetary policy replicates most of the observations. The success of the model suggests that a combination of agent heterogeneity along education lines and market frictions capable of producing equilibrium unemployment, which also considers the possible mismatch between the employees education level and the qualification of their jobs, are essential aspects in understanding the implications of a monetary shock on a labor market with heterogeneous agents.

The model, however, can not account for the higher persistence of the unemployment of the low educated compared to that of the high educated. The introduction of the aspect of skill obsolescence by the high educated unemployed can increase the persistence of the unemployment of the low educated in the model to match that of the data. In this context, Esteban-Pretel and Faraglia (2005) develop a new Keynesian model with two kinds of rigidities: Calvo price setting for retail firms and search frictions in the labor market. They include the aspect of skill loss by the high educated if unemployed for an extended period of time, and show that this mechanism is essential in explaining the persistence of the overall unemployment ratio in reaction to a monetary policy shock. Finally, the model reveals two effects on the low educated: they bear the brunt of the increase in unemployment after a contractionary policy, however the gap between their wage and that of the high educated gets closer. A possibility for future research is to consider the final welfare effect on the low educated as a result of these two opposing effects.
7 Appendix

7.1 Data

The data set used is the Outgoing Rotation Group of the Current Population Survey. The Current Population Survey is a rotating panel. After the fourth month in the survey, the participants take an eight month hiatus. Afterwards, they are interviewed for another four months, and after the eighth month in sample, they are completely dropped from the survey. The Merged Outgoing Rotation series is a collection of the 4th and 8th month-in-sample groups from all 12 months. These two groups play a special role as they are given additional questions, the answers to which are collected in the Outgoing Rotation Group files. The data is monthly and covers the period from January 1979 until December 2004. At the end of each year, the 12 monthly files for January through December are concatenated into a single annual file. The variables extracted are as follows

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>MONTH</td>
<td>Month of interview</td>
</tr>
<tr>
<td>BEMP2</td>
<td>Employed persons excluding farm and private household workers</td>
</tr>
<tr>
<td>GRDHI</td>
<td>Highest grade attended</td>
</tr>
<tr>
<td>GRDATN</td>
<td>Educational attainment</td>
</tr>
<tr>
<td>OCC</td>
<td>Occupation of job last week</td>
</tr>
<tr>
<td>ERNWKC</td>
<td>Weekly earnings before deductions (1979-1988)</td>
</tr>
<tr>
<td>HOURS</td>
<td>Total hours worked last week</td>
</tr>
<tr>
<td>ERNWGT</td>
<td>Earnings weight</td>
</tr>
</tbody>
</table>

Each annual file is divided into monthly files according to the variable MONTH. For each monthly file, observations are split into those employed and those unemployed according to BEMP2. Both the employed and the unemployed are further split into those high and low educated, where the high educated are those who obtained some college education or higher. The following table shows the variables' ranges defining the high and low educated:
<table>
<thead>
<tr>
<th>Period</th>
<th>High Educated</th>
<th>Low Educated</th>
</tr>
</thead>
<tbody>
<tr>
<td>1979-1988</td>
<td>14 ≤ GRDHI ≤ 19</td>
<td>1 ≤ GRDHI ≤ 13</td>
</tr>
<tr>
<td>1989-1991</td>
<td>13 ≤ GRDHI ≤ 18</td>
<td>1 ≤ GRDHI ≤ 12</td>
</tr>
<tr>
<td>1992-2004</td>
<td>40 ≤ GRDATN ≤ 46</td>
<td>31 ≤ GRDATN ≤ 39</td>
</tr>
</tbody>
</table>

Each worker group, the high or low educated, is further divided into two groups: those employed in complex occupations, and those employed in simple occupations. The complex and simple occupations are defined by the ranges of the variable OCC specified in the following table:

<table>
<thead>
<tr>
<th>Period</th>
<th>Complex Occupation</th>
<th>Simple Occupation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1979-1982</td>
<td>1 – 85</td>
<td>86 – 90</td>
</tr>
<tr>
<td></td>
<td>91 – 96</td>
<td>100 – 101</td>
</tr>
<tr>
<td></td>
<td>102 – 246</td>
<td>260 – 995</td>
</tr>
<tr>
<td></td>
<td>178 – 242</td>
<td>243 – 991</td>
</tr>
<tr>
<td>1992-2002</td>
<td>0 – 163</td>
<td>164 – 165</td>
</tr>
<tr>
<td></td>
<td>166 – 173</td>
<td>174 – 177</td>
</tr>
<tr>
<td></td>
<td>178 – 242</td>
<td>243 – 999</td>
</tr>
<tr>
<td></td>
<td>2100 – 3650</td>
<td>3700 – 9830</td>
</tr>
</tbody>
</table>

Therefore, we have four employed and two unemployed types as follows:

(1) The high educated employed in a complex occupation.
(2) The high educated employed in a simple occupation.
(3) The high educated unemployed.
(4) The low educated employed in a complex occupation.
(5) The low educated employed in a simple occupation.
(6) The low educated unemployed.

The weighted average of the variables on weekly earnings and hours worked last week for each of the working groups are calculated using the proper weights ERNWGT. These weights are created for each month such that, when applied, the resulting counts are representative of the national counts. Thus the proper application of weights enables the results to be presented in terms of the population of the United States as a whole, instead of just the participants in the survey.
The weighted average hourly wage of each worker type is calculated as the ratio of the weighted average weekly earnings to the weighted average hours worked last week for each group. These derived wages are used to calculate the skill premium, or the between group wage premium, defined as the ratio of the weighted average wage of the two high educated types to that of the low educated in simple occupations. To calculate measures of unemployment, the binary variable BEMP2 is used to distinguish those employed and those unemployed. The unemployed are divided into high and low educated in the same manner as explained earlier. The ratios of the two unemployed types to the overall sample are calculated by summing over the weights in each type, and dividing by the sum of the weights of the total sample. Finally, the variables compiled and used in the analysis are as follows:

1. The weighted average wage of the high educated in complex occupations.
2. The weighted average wage of the high educated in simple occupations.
3. The weighted average wage of the low educated in simple occupations.
4. The between group wage premium.
5. The proportion of the high educated unemployed.
6. The proportion of the low educated unemployed.

Finally, the quarterly real gross domestic product (Chained Dollars, seasonally adjusted at annual rates) is extracted from the National Income and Product Accounts NIPA. The inflation rate is the percentage change of the consumer price index available from the Bureau of Labor Statistics. The interest rates are the monthly Federal funds rates from the Board of Governors of the Federal Reserve System. As the Gross Domestic Product data is quarterly, all monthly time series are transformed into quarterly ones by taking three months averages.
7.2 Derivations

7.2.1 Household’s Optimal Conditions

From (18) we have

$$E_t \left[ \frac{\partial \Gamma_{H_{t+1}}}{\partial N_{hc_{t+1}}} \right] = \frac{T_s}{T_f} E_t \left[ \frac{\partial \Gamma_{H_{t+1}}}{\partial N_{hc_{t+1}}} \right]$$

(65)

Substituting (65) in (17) yields

$$E_t \left[ \frac{\partial \Gamma_{H_{t+1}}}{\partial N_{hc_{t+1}}} \right] = \frac{\tau}{\beta (T_s - T_f)}$$

(66)

Substituting (66) in (65) yields

$$E_t \left[ \frac{\partial \Gamma_{H_{t+1}}}{\partial N_{hc_{t+1}}} \right] = \frac{\tau T_s}{\beta T_f (T_s - T_f)}$$

(67)

Substituting the envelope condition (20) in (67) yields the first optimal condition (23), while substituting the envelope condition (21) in (66) yields the second optimal condition (24). Finally, from (19) we have

$$E_t \left[ \frac{\partial \Gamma_{H_{t+1}}}{\partial N_{ls_{t+1}}} \right] = \frac{\lambda_t W^d_t}{\beta P^s_t}$$

(68)

Substituting the envelope condition (22) in (68) yields the third optimal condition (25).

7.2.2 Wholesaler’s Optimal Conditions

Substituting the envelope condition (37) in the first order condition (35) yields

$$\frac{\omega^c}{q^c_t} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( X_{t+1} \frac{\partial Y_{t+1}}{\partial N_{hc_{t+1}}} - W_{hc_{t+1}} + (1 - \chi) \frac{\omega^c}{q^c_{t+1}} \right) \right]$$

Substituting the envelope condition (39) in the first order condition (36) yields

$$\omega^s = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \Gamma_{F_{t+1}}}{\partial N_{hs_{t+1}}} q^h_{t+1} + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( X_{t+1} \frac{\partial Y_{t+1}}{\partial N_{ls_{t+1}}} - W_{ls_{t+1}} \right) \right] q^l_{t+1} \right]$$

Which can be rearranged as

$$\varepsilon_t = \frac{\omega^s}{q^h_{t+1}} - \frac{q^l_{t+1}}{q^h_{t+1}} \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( X_{t+1} \frac{\partial Y_{t+1}}{\partial N_{ls_{t+1}}} - W_{ls_{t+1}} \right) \right]$$

$$= \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \Gamma_{F_{t+1}}}{\partial N_{hs_{t+1}}} \right]$$

26
Substituting the envelope condition (38) yields

$$
\varepsilon_t = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( X_{t+1} \frac{\partial Y_{t+1}}{\partial N_{t+1}^{hs}} - W_{t+1}^{hs} + (1 - \chi^s - a_t P_t^c) \varepsilon_{t+1} \right) \right]
$$

7.2.3 The wage of high educated workers in complex occupations

The surplus accrued by the household is expressed in terms of goods rather than marginal utility, and reduces to \([1 / \lambda_t \partial N_t^{hc}]\) in order to guarantee that both the firm’s surplus and the household’s surplus are expressed in the same units. The bargained wage is determined by the maximization of the Nash product as follows

$$
W_t^{hc} = \text{Argmax} \left[ \frac{1}{\lambda_t} \frac{\partial \Gamma_t^H}{\partial N_t^{hc}} \right]^{-\xi^{hc}} \frac{\partial \Gamma_t^F}{\partial N_t^{hc}}^{\xi^{hc}}
$$

Then the sharing rule implies

$$
\xi^{hc} \left[ \frac{\partial \Gamma_t^H}{\partial N_t^{hc}} \right] = (1 - \xi^{hc}) \lambda_t \left[ \frac{\partial \Gamma_t^F}{\partial N_t^{hc}} \right]
$$

Substituting the envelope conditions of the households and the firms yields

$$
\xi^{hc} \left[ \lambda_t W_t^{hc} + \beta E_t \frac{\partial \Gamma_t^H}{\partial N_t^{hc}}^{T_t^s} (1 - \chi^c - T_t^s s_t) - \beta E_t \frac{\partial \Gamma_t^H}{\partial N_t^{hc}}^{T_t^s} (1 - s_t) \right] = (1 - \xi^{hc}) \lambda_t \left[ X_t \frac{\partial Y_t}{\partial N_t^{hc}} - W_t^{hc} + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \Gamma_{t+1}^F}{\partial N_{t+1}^{hc}} \right] (1 - \chi^c) \right]
$$

As we have from the first order conditions

$$
\frac{\omega^c}{q_t^c} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \Gamma_{t+1}^F}{\partial N_{t+1}^{hc}} \right]
$$

Therefore

$$
\xi^{hc} \frac{\beta}{\lambda_t} E_t \left[ \frac{\partial \Gamma_t^H}{\partial N_t^{hc}} \right] = (1 - \xi^{hc}) \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \Gamma_{t+1}^F}{\partial N_{t+1}^{hc}} \right] = (1 - \xi^{hc}) \frac{\omega^c}{q_t^c}
$$

Substituting yields
\[
\xi^{hc} \left[ \lambda_t W^{hc}_t + \left( 1 - \xi^{hc} \right) \frac{1}{\xi^{hc}} \lambda_t \frac{\omega^e}{q^e_t} (1 - \chi^c - P^c_s t) - \frac{\tau T^e_s (1 - s_t)}{(T^e_s - T^e_t)} \right] \\
= \left( 1 - \xi^{hc} \right) \lambda_t \left[ X^t \left( \frac{\partial Y^t}{\partial N^{hc}_t} + \frac{1}{\xi^{hc}} \frac{\omega^e}{q^e_t} (1 - \chi^c) \right) \right]
\]

Solving for the equilibrium wage rule for the high educated workers in complex occupations yields

\[
W^{hc}_t = \left( 1 - \xi^{hc} \right) \left[ X^t \left( \frac{\partial Y^t}{\partial N^{hc}_t} + \frac{1}{\xi^{hc}} \frac{\omega^e}{q^e_t} (1 - \chi^c) \right) \right] + \xi^{hc} \left[ C_t \frac{\tau T^e_s (1 - s_t)}{(T^e_s - T^e_t)} \right]
\]

### 7.2.4 The wage of high educated workers in simple occupations

The bargained wage is determined by the maximization of the Nash product as follows

\[
W^{hs}_t = \text{Argmax} \left[ \frac{1}{\lambda_t} \frac{\partial \Gamma^H_t}{\partial N^{hs}_t} \right]^{1 - \xi^{hs}} \left[ \frac{\partial \Gamma^F_t}{\partial N^{hs}_t} \right]^{\xi^{hs}}
\]

Then the sharing rule implies

\[
\xi^{hs} \frac{\partial \Gamma^H_t}{\partial N^{hs}_t} = \left( 1 - \xi^{hs} \right) \lambda_t \left[ \frac{\partial \Gamma^F_t}{\partial N^{hs}_t} \right]
\]

Substituting the envelope conditions of the households and firms yields

\[
\xi^{hs} \left[ \tau (1 - o_t) + \lambda_t W^{hs}_t + \beta E_t \left( \frac{1}{\lambda_t} \frac{\partial \Gamma^H_t}{\partial N^{hs}_t} \right) (T^e_s o_t - T^e_s s_t) + \beta E_t \left( \frac{1}{\lambda_t} \frac{\partial \Gamma^H_{t+1}}{\partial N^{hs}_{t+1}} \right) [1 - \chi^s - o_t T^e_s - T^e_s (1 - s_t)] \right] \\
= \left( 1 - \xi^{hs} \right) \lambda_t \left[ X^t \frac{\partial Y^t}{\partial N^{hs}_t} - W^{hs}_t + \beta E_t \left( \frac{1}{\lambda_t} \frac{\partial \Gamma^F_{t+1}}{\partial N^{hs}_{t+1}} \right) [1 - \chi^s - o_t T^e_s] \right]
\]

We can also derive from the first order conditions

\[
\xi^{hs} \frac{\beta}{\lambda_t} E_t \left[ \frac{\partial \Gamma^H_{t+1}}{\partial N^{hs}_{t+1}} \right] = \left( 1 - \xi^{hs} \right) \beta E_t \left( \frac{1}{\lambda_t} \frac{\partial \Gamma^F_{t+1}}{\partial N^{hs}_{t+1}} \right) = \left( 1 - \xi^{hs} \right) \varepsilon_t
\]

Substituting both in the sharing rule yields
\[
\begin{align*}
\xi^{hs} & \left[ \tau (1 - o_t) + \lambda_t W^{hs}_t + \frac{\tau T^s_t}{T^s_t - T^c_t} (T^s_t - T^c_t s_t) + [1 - \chi^s - o_t T^c_t - T^s_t (1 - s_t)] \left( \frac{1 - \xi^{hs}}{\xi^{hs}} \right) \lambda_t \right] \\
= & \left( 1 - \xi^{hs} \right) \lambda_t \left[ X_t \frac{\partial Y_t}{\partial N^{hs}_t} - W^{hs}_t + [1 - \chi^s - o_t T^c_t] \xi_t \right]
\end{align*}
\]

Solving for the equilibrium wage rule for the high educated workers in simple occupations gives

\[
W^{hs}_t = \left( 1 - \xi^{hs} \right) \left[ X_t \frac{\partial Y_t}{\partial N^{hs}_t} + T^s_t (1 - s_t) \xi_t \right] + \xi^{hs} \left[ C_t \frac{\tau T^s_t (s_t - o_t)}{(T^s_t - T^c_t)} - C_t \tau (1 - o_t) \right]
\]

### 7.2.5 The wage of low educated workers in simple occupations

The bargained wage is determined by the maximization of the Nash product as follows

\[
W^{ls}_t = \text{Argmax} \left[ \frac{1}{\lambda_t} \frac{\partial \Gamma^H_t}{\partial N^{ls}_t} \right]^{1 - \xi^{ls}} \left[ \frac{\partial \Gamma^F_t}{\partial N^{ls}_t} \right]^{\xi^{ls}}
\]

Then the sharing rule implies

\[
\xi^{ls} \left[ \frac{\partial \Gamma^H_t}{\partial N^{ls}_t} \right] = \left( 1 - \xi^{ls} \right) \lambda_t \left[ \frac{\partial \Gamma^F_t}{\partial N^{ls}_t} \right]
\]

Substituting in the sharing rule yields

\[
\xi^{ls} \left[ \lambda_t W^{ls}_t - \lambda_t (1 - g_t) W^d_t - \beta E_t \left[ \frac{\partial \Gamma^H_{t+1}}{\partial N^{ls}_{t+1}} \right] T^s_t g_t \right] = \left( 1 - \xi^{ls} \right) \lambda_t \left[ X_t \frac{\partial Y_t}{\partial N^{ls}_t} - W^{ls}_t \right]
\]

Substituting from the first order conditions yields

\[
\xi^{ls} \left[ \lambda_t W^{ls}_t - \lambda_t (1 - g_t) W^d_t - T^s_t g_t \lambda_t \frac{W^d_t}{p^d_t} \right] = \left( 1 - \xi^{ls} \right) \lambda_t \left[ X_t \frac{\partial Y_t}{\partial N^{ls}_t} - W^{ls}_t \right]
\]

Solving for the equilibrium wage for low educated workers in simple occupations gives

\[
W^{ls}_t = \left( 1 - \xi^{ls} \right) \left[ X_t \frac{\partial Y_t}{\partial N^{ls}_t} \right] + \xi^{ls} \left[ W^d_t \right]
\]

### 7.2.6 Retailers Optimal Condition

Nominal profits of the retailers are given by the difference between their total revenue and total cost as follows
The real profits are thus given by

\[
[P_{jt} - X_t] C_t (j)
\]

The retailers choose their optimal price to maximize the discounted expected future profits given the demand for their output and under the hypothesis that the price they set at date \( t \) applies at date \( t + s \) with probability \( \sigma \)

\[
E_t \left\{ \sum_{s=0}^{\infty} \sigma^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[ \frac{P_{jt}}{P_{t+s}} - \frac{X_{t+s}}{P_{t+s}} \right] C_{t+s} (j) \right\}
\]

subject to the demand curve

\[
C_{t+s} (j) = \left( \frac{P_{jt}}{P_{t+s}} \right)^{-\theta} C_{t+s}
\]

Substituting the demand curve yields

\[
E_t \left\{ \sum_{s=0}^{\infty} \sigma^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[ \frac{P_{jt}}{P_{t+s}} - \frac{X_{t+s}}{P_{t+s}} \right] \left( \frac{P_{jt}}{P_{t+s}} \right)^{-\theta} C_{t+s} \right\}
\]

That can be rearranged as

\[
E_t \left\{ \sum_{s=0}^{\infty} \sigma^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[ (P_{jt})^{1-\theta} \left( \frac{1}{P_{t+s}} \right)^{1-\theta} - \frac{X_{t+s}}{P_{t+s}} \left( \frac{P_{jt}}{P_{t+s}} \right)^{-\theta} \right] C_{t+s} \right\}
\]

The first order condition with respect to \( P_{jt} \) is given by

\[
E_t \left\{ \sum_{s=0}^{\infty} \sigma^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[ (1-\theta) (P_{jt})^{-\theta} \left( \frac{1}{P_{t+s}} \right)^{1-\theta} + \theta \frac{X_{t+s}}{P_{t+s}} (P_{jt})^{-\theta-1} \left( \frac{1}{P_{t+s}} \right)^{-\theta} \right] C_{t+s} \right\} = 0
\]

Which can be arranged as

\[
E_t \left\{ \sum_{s=0}^{\infty} \sigma^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[ (1-\theta) \left( \frac{1}{P_{t+s}} \right)^{1-\theta} + \theta \frac{X_{t+s}}{P_{t+s}} \left( \frac{P_{jt}}{P_{t+s}} \right)^{-\theta} \right] C_{t+s} \right\} = 0
\]
This can be written as

\[ E_t \left\{ \sum_{s=0}^{\infty} \sigma^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[ (1 - \theta) \left( \frac{1}{P_{t+s}} \right) + \theta \frac{X_{t+s}}{P_{j,t}} \left( \frac{P_{j,t}}{P_{t+s}} \right)^{-\theta} C_{t+s} \right] \right\} = 0 \]

Multiply and divide by \( P_t \)

\[ E_t \left\{ \sum_{s=0}^{\infty} \sigma^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[ (1 - \theta) \left( \frac{1}{P_{t+s}} \right) + \theta \frac{X_{t+s}}{P_{j,t}} \left( \frac{P_{j,t}}{P_{t+s}} \right)^{-\theta} C_{t+s} \right] \right\} = 0 \]

Divide both sides by \( \left( \frac{P_{j,t}}{P_t} \right)^{-\theta} \) and rearrange as

\[ E_t \left\{ \sum_{s=0}^{\infty} \sigma^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[ (1 - \theta) \left( \frac{1}{P_{t+s}} \right) \left( \frac{P_{j,t}}{P_{t+s}} \right)^{-\theta} \right] + \theta \frac{X_{t+s}}{P_{j,t}} \left( \frac{P_{j,t}}{P_{t+s}} \right)^{-\theta} C_{t+s} \right\} = 0 \]

This can be rearranged as

\[ E_t \left\{ \sum_{s=0}^{\infty} \sigma^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[ \left( \frac{1}{P_{t+s}} \right) \left( \frac{P_{j,t}}{P_{t+s}} \right)^{-\theta} \right] \right\} = \frac{\theta}{\theta - 1} E_t \left\{ \sum_{s=0}^{\infty} \sigma^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[ \frac{X_{t+s}}{P_{j,t}} \left( \frac{P_{j,t}}{P_{t+s}} \right)^{-\theta} \right] \right\} \]

Multiply both sides by \( P_{j,t} \) to get

\[ E_t \left\{ \sum_{s=0}^{\infty} \sigma^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[ \left( \frac{P_{j,t}}{P_{t+s}} \right) \left( \frac{P_{j,t}}{P_{t+s}} \right)^{-\theta} \right] \right\} = \frac{\theta}{\theta - 1} E_t \left\{ \sum_{s=0}^{\infty} \sigma^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[ \frac{X_{t+s}}{P_{j,t}} \left( \frac{P_{j,t}}{P_{t+s}} \right)^{-\theta} C_{t+s} \right] \right\} \]

Finally, we have
\[
\left( \frac{P_{jt}}{P_t} \right) = \left( \frac{\theta}{\theta - 1} \right) E_t \left\{ \sum_{s=0}^{\infty} \sigma^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[ \frac{X_{t+s}}{P_{t+s}} \left( \frac{P_t}{P_{t+s}} \right)^{\theta - 1} C_{t+s} \right] \right\}
\]

Which can be arranged as

\[
\left( \frac{P_{jt}}{P_t} \right) = \left( \frac{\theta}{\theta - 1} \right) E_t \left\{ \sum_{s=0}^{\infty} \sigma^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[ \left( \frac{P_{t+s}}{P_t} \right)^{\theta - 1} C_{t+s} \right] \right\}
\]

7.2.7 Phillips Curve

To derive the Phillips curve, substitute the log linearized version of the price index into that of the optimal retail price equation. The price index can be written as

\[
P_t = \left[ (1 - \sigma) (P_t^*)^{1-\theta} + \sigma P_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}
\]

To log linearize the price index, divide both sides by \( (P_t)^{1-\theta} \) to get

\[
1 = (1 - \sigma) \left( \frac{P_t^*}{P_t} \right)^{1-\theta} + \sigma \left( \frac{P_{t-1}}{P_t} \right)^{1-\theta}
\]

Log linearization yields

\[
0 = (1 - \sigma) (1 - \theta) (P^*)^{-\theta} (P)^{\theta-1} P_t^* \hat{P}_t + (1 - \sigma) (\theta - 1) (P^*)^{1-\theta} (P)^{\theta-1} \hat{P}_t + \sigma (1 - \theta) (P)^{-\theta} (P^*)^{\theta-1} \hat{P}_{t-1} + \sigma (\theta - 1) (P) \hat{P}_t + \hat{P}_t
\]

Substituting the steady state condition \( P = P^* \), and rearranging yields

\[
\sigma \hat{P}_t + (1 - \sigma) \hat{P}_t = (1 - \sigma) \hat{P}_t^* + \sigma \hat{P}_{t-1}
\]

This can be rewritten as

\[
\hat{P}_t = (1 - \sigma) \hat{P}_t^* + \sigma \hat{P}_{t-1}
\]

The steady state condition for the optimal retail price equation is given by
\[ P^* = \frac{\theta}{\theta - 1} X \]

The log linearization of the optimal retail price equation yields after substituting the steady state

\[ \sum_{s=0}^{\infty} \sigma^s \beta^s C \hat{P}_t^* = E_t \sum_{s=0}^{\infty} \sigma^s \beta^s C \frac{\theta}{\theta - 1} X \hat{X}_{t+s} - E_t \sum_{s=0}^{\infty} \sigma^s \beta^s C \frac{\theta}{\theta - 1} \hat{P}_{t+s} + E_t \sum_{s=0}^{\infty} \sigma^s \beta^s C \hat{P}_{t+s} \]

This can be rearranged as

\[ \frac{C}{1 - \sigma \beta} \hat{P}_t^* = E_t \sum_{s=0}^{\infty} \sigma^s \beta^s C \frac{\theta}{\theta - 1} X \hat{X}_{t+s} + E_t \sum_{s=0}^{\infty} \sigma^s \beta^s C \left( 1 - \frac{\theta}{\theta - 1} \right) \hat{P}_{t+s} \]

Which, after substituting the steady state and rearranging, can be written as

\[ \hat{P}_t^* = (1 - \sigma \beta) E_t \sum_{s=0}^{\infty} \sigma^s \beta^s \hat{X}_{t+s} \]

This can be expanded to give

\[ \hat{P}_t^* = (1 - \sigma \beta) \left[ \hat{X}_t + (\sigma \beta) E_t \hat{X}_{t+1} + (\sigma \beta)^2 E_t \hat{X}_{t+1} + \ldots \right] \]

This can be written as

\[ \hat{P}_t^* = (1 - \sigma \beta) \hat{X}_t + \sigma \beta (1 - \sigma \beta) \left[ E_t \hat{X}_{t+1} + \sigma \beta E_t \hat{X}_{t+1} + \ldots \right] \]

Thus can be arranged as

\[ \hat{P}_t^* = (1 - \sigma \beta) \hat{X}_t + \sigma \beta E_t \hat{P}_{t+1} \]

From the log linearized version of the price index, we have

\[ \hat{\pi}_t + (1 - \sigma) \hat{P}_{t-1} = (1 - \sigma) \hat{P}_t^* \]

and can be written as

\[ \hat{P}_t^* = \frac{1}{1 - \sigma} \hat{\pi}_t + \hat{P}_{t-1} \]
Leading by one period, and taking the expectation as per period $t$ yields

$$E_t \hat{P}_{t+1}^* = \frac{1}{1-\sigma} E_t \hat{\pi}_{t+1} + \hat{P}_t$$

Substituting the latest two expressions in both sides of the log linearized version of the optimal retail price equation yields

$$\frac{1}{1-\sigma} \hat{\pi}_t + \hat{P}_{t-1} = (1-\sigma\beta) \hat{X}_t + \sigma\beta \left( \frac{1}{1-\sigma} E_t \hat{\pi}_{t+1} + \hat{P}_t \right)$$

This can be rearranged, after adding and subtracting $\hat{P}_t$ from the right hand side, as

$$\frac{1}{1-\sigma} \hat{\pi}_t = (1-\sigma\beta) \hat{X}_t - \hat{P}_{t-1} + \frac{\sigma}{1-\sigma} \beta E_t \hat{\pi}_{t+1} + \sigma\beta \hat{P}_t + \hat{P}_t - \hat{P}_t$$

Which can be written as

$$\frac{1}{1-\sigma} \hat{\pi}_t = (1-\sigma\beta) \hat{X}_t + \hat{\pi}_t + \frac{\sigma}{1-\sigma} \beta E_t \hat{\pi}_{t+1} + (\sigma\beta - 1) \hat{P}_t$$

This can finally be arranged as

$$\left( \frac{1}{1-\sigma} - 1 \right) \hat{\pi}_t = (1-\sigma\beta) \left( \hat{X}_t - \hat{P}_t \right) + \frac{\sigma}{1-\sigma} \beta E_t \hat{\pi}_{t+1}$$

Which gives us the following Phillips curve

$$\hat{\pi}_t = \frac{1-\sigma}{\sigma} (1-\sigma\beta) \left( \hat{X}_t - \hat{P}_t \right) + \beta E_t \hat{\pi}_{t+1}$$
References


<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.5</td>
<td>proportion of the high educated in the population</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.98</td>
<td>household discount factor</td>
</tr>
<tr>
<td>$\chi^c$</td>
<td>0.045</td>
<td>separation rate from complex occupations</td>
</tr>
<tr>
<td>$\chi^s$</td>
<td>0.02</td>
<td>separation rate from simple occupations</td>
</tr>
<tr>
<td>$m^c$</td>
<td>0.3</td>
<td>efficiency parameter in the complex occupation matching function</td>
</tr>
<tr>
<td>$m^s$</td>
<td>0.6</td>
<td>efficiency parameter in the simple occupation matching function</td>
</tr>
<tr>
<td>$\gamma^c$</td>
<td>0.6</td>
<td>elasticity of complex matches with respect to complex vacancies</td>
</tr>
<tr>
<td>$\gamma^s$</td>
<td>0.6</td>
<td>elasticity of simple matches with respect to simple vacancies</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.5</td>
<td>elasticity of final output to complex occupation output</td>
</tr>
<tr>
<td>$\omega^c$</td>
<td>1.043</td>
<td>cost of posting a complex vacancy</td>
</tr>
<tr>
<td>$\omega^s$</td>
<td>0.08</td>
<td>cost of posting a simple vacancy</td>
</tr>
<tr>
<td>$\xi^{hc}$</td>
<td>0.504</td>
<td>firm share from bargaining with a high educated in a complex occupation</td>
</tr>
<tr>
<td>$\xi^{hs}$</td>
<td>0.58</td>
<td>firm share from bargaining with a high educated in a simple occupation</td>
</tr>
<tr>
<td>$\xi^{ls}$</td>
<td>0.5</td>
<td>firm share from bargaining with a low educated in a simple occupation</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.163</td>
<td>ratio of return to domestic activities to average productivity</td>
</tr>
<tr>
<td>$\tau$</td>
<td>1.08</td>
<td>parameter in the utility of leisure</td>
</tr>
<tr>
<td>$\rho^A$</td>
<td>0.9</td>
<td>autoregressive coefficient of aggregate technology</td>
</tr>
<tr>
<td>$\rho^{Ah}$</td>
<td>0.95</td>
<td>autoregressive coefficient of high educated biased technology</td>
</tr>
<tr>
<td>$\rho^{Al}$</td>
<td>0.95</td>
<td>autoregressive coefficient of low educated biased technology</td>
</tr>
<tr>
<td>$\sigma_{\varepsilon A}$</td>
<td>0.0049</td>
<td>standard deviation of the aggregate technology shock</td>
</tr>
<tr>
<td>$\sigma_{\varepsilon Ah}$</td>
<td>0.002</td>
<td>standard deviation of the high educated biased technology shock</td>
</tr>
<tr>
<td>$\sigma_{\varepsilon Al}$</td>
<td>0.002</td>
<td>standard deviation of the low educated biased technology shock</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>0.1</td>
<td>standard deviation of the monetary policy shock</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.75</td>
<td>proportion of retailers that do not adjust</td>
</tr>
<tr>
<td>$\theta$</td>
<td>1.1</td>
<td>elasticity of substitution across differentiated goods</td>
</tr>
<tr>
<td>$\alpha_r$</td>
<td>0.85</td>
<td>interest rate smoothing parameter</td>
</tr>
<tr>
<td>$\alpha_\pi$</td>
<td>1.5</td>
<td>coefficient of inflation in the Taylor rule</td>
</tr>
<tr>
<td>$\alpha_Y$</td>
<td>0.5</td>
<td>coefficient of output in the Taylor rule</td>
</tr>
</tbody>
</table>

Table 1: Calibration of model parameters
<table>
<thead>
<tr>
<th>Equation 1</th>
<th>Equation 2</th>
<th>Equation 3</th>
<th>Equation 4</th>
<th>Equation 5</th>
<th>Equation 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{t-1}^h$</td>
<td>$U_t^h$</td>
<td>$Y_t$</td>
<td>$Premium_t$</td>
<td>$r_t$</td>
<td>$\pi_t$</td>
</tr>
<tr>
<td>0.600244551</td>
<td>0.592390811</td>
<td>-0.392695688</td>
<td>-0.696270907</td>
<td>-43.0232828</td>
<td>0.170424688</td>
</tr>
<tr>
<td>(0.108158795)</td>
<td>(0.16353604)</td>
<td>(0.361014552)</td>
<td>(0.735293947)</td>
<td>(61.9198272)</td>
<td>(0.260540571)</td>
</tr>
<tr>
<td>[5.54966]</td>
<td>[3.62244]</td>
<td>[-1.08776]</td>
<td>[-0.94693]</td>
<td>[-0.69482]</td>
<td>[0.65412]</td>
</tr>
<tr>
<td>$U_{t-2}^h$</td>
<td>-0.416631004</td>
<td>0.610956713</td>
<td>0.521854120</td>
<td>47.0422578</td>
<td>-0.027384152</td>
</tr>
<tr>
<td>(0.117609561)</td>
<td>(0.177829444)</td>
<td>(0.392559503)</td>
<td>(0.799542913)</td>
<td>(67.3309295)</td>
<td>(0.283306245)</td>
</tr>
<tr>
<td>[3.03895]</td>
<td>[-2.34295]</td>
<td>[1.55634]</td>
<td>[0.65269]</td>
<td>[0.69868]</td>
<td>[-0.09666]</td>
</tr>
<tr>
<td>$U_{t-1}^l$</td>
<td>0.156412615</td>
<td>0.863089365</td>
<td>-0.322873554</td>
<td>0.180448647</td>
<td>-138.8024087</td>
</tr>
<tr>
<td>(0.081395227)</td>
<td>(0.123067706)</td>
<td>(0.271682587)</td>
<td>(0.553347672)</td>
<td>(46.5979522)</td>
<td>(0.196070590)</td>
</tr>
<tr>
<td>[1.92164]</td>
<td>[7.01313]</td>
<td>[-1.18842]</td>
<td>[0.32610]</td>
<td>[-2.97872]</td>
<td>[-1.77901]</td>
</tr>
<tr>
<td>$U_{t-2}^l$</td>
<td>-0.101877312</td>
<td>0.035664221</td>
<td>0.077936215</td>
<td>-1.250358661</td>
<td>88.6043702</td>
</tr>
<tr>
<td>(0.082505454)</td>
<td>(0.124746344)</td>
<td>(0.275388326)</td>
<td>(0.560895310)</td>
<td>(47.2335463)</td>
<td>(0.198744985)</td>
</tr>
<tr>
<td>[-1.23479]</td>
<td>[0.28589]</td>
<td>[0.28300]</td>
<td>[-2.22922]</td>
<td>[1.87588]</td>
<td>[0.36105]</td>
</tr>
<tr>
<td>$Y_{t-1}$</td>
<td>-0.025098187</td>
<td>-0.127878077</td>
<td>1.168793817</td>
<td>0.043985503</td>
<td>20.1204056</td>
</tr>
<tr>
<td>(0.033775824)</td>
<td>(0.051068267)</td>
<td>(0.112737610)</td>
<td>(0.229617565)</td>
<td>(19.3363212)</td>
<td>(0.081361599)</td>
</tr>
<tr>
<td>[-0.74308]</td>
<td>[-2.50406]</td>
<td>[10.36738]</td>
<td>[0.19156]</td>
<td>[1.04055]</td>
<td>[-0.82327]</td>
</tr>
<tr>
<td>$Y_{t-2}$</td>
<td>0.033510410</td>
<td>0.100210428</td>
<td>-0.217974261</td>
<td>-1.00767207</td>
<td>-26.6250387</td>
</tr>
<tr>
<td>(0.034294772)</td>
<td>(0.051852905)</td>
<td>(0.114469765)</td>
<td>(0.233145520)</td>
<td>(19.6334138)</td>
<td>(0.082611678)</td>
</tr>
<tr>
<td>[0.97713]</td>
<td>[1.93259]</td>
<td>[-1.90421]</td>
<td>[-0.43221]</td>
<td>[-1.35611]</td>
<td>[0.22516]</td>
</tr>
<tr>
<td>$Premium_{t-1}$</td>
<td>0.031602601</td>
<td>0.001205569</td>
<td>-0.119807417</td>
<td>0.263394832</td>
<td>-22.6100816</td>
</tr>
<tr>
<td>(0.014639397)</td>
<td>(0.022134430)</td>
<td>(0.048863666)</td>
<td>(0.099522741)</td>
<td>(8.3809080)</td>
<td>(0.035264416)</td>
</tr>
<tr>
<td>[2.15870]</td>
<td>[0.05447]</td>
<td>[-2.45187]</td>
<td>[2.64658]</td>
<td>[-2.69781]</td>
<td>[-1.67724]</td>
</tr>
<tr>
<td>$Premium_{t-2}$</td>
<td>-0.004950208</td>
<td>0.005896756</td>
<td>0.013735427</td>
<td>0.171994896</td>
<td>2.8211246</td>
</tr>
<tr>
<td>(0.015533876)</td>
<td>(0.023486862)</td>
<td>(0.051849275)</td>
<td>(0.105603661)</td>
<td>(8.8929882)</td>
<td>(0.037419100)</td>
</tr>
<tr>
<td>[-0.31867]</td>
<td>[0.25107]</td>
<td>[0.26491]</td>
<td>[1.62868]</td>
<td>[0.31723]</td>
<td>[-0.36996]</td>
</tr>
</tbody>
</table>

Table 2: Vector autoregressive estimation

Standard errors in ( ) and t-statistics in [ ]
Table 2 continued: Vector autoregressive estimation

Standard errors in ( ) and t-statistics in [ ]
<table>
<thead>
<tr>
<th>Premium</th>
<th>$U^h$</th>
<th>$U^l$</th>
<th>$N^{hc}$</th>
<th>$N^{hs}$</th>
<th>$N^{ls}$</th>
<th>$Y$</th>
<th>$R^n$</th>
<th>$\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premium</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$U^h$</td>
<td>$-0.3775$</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$U^l$</td>
<td>$-0.5465$</td>
<td>0.6682</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N^{hc}$</td>
<td>$-0.5098$</td>
<td>0.9284</td>
<td>0.8852</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N^{hs}$</td>
<td>0.4481</td>
<td>$-0.9838$</td>
<td>$-0.7848$</td>
<td>$-0.9799$</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N^{ls}$</td>
<td>0.5465</td>
<td>$-0.6682$</td>
<td>$-1$</td>
<td>$-0.8852$</td>
<td>0.7848</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y$</td>
<td>0.5292</td>
<td>$-0.6028$</td>
<td>$-0.9938$</td>
<td>$-0.8346$</td>
<td>0.7253</td>
<td>0.9938</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$R^n$</td>
<td>$-0.8904$</td>
<td>0.0963</td>
<td>0.502</td>
<td>0.2869</td>
<td>$-0.1898$</td>
<td>$-0.502$</td>
<td>$-0.5341$</td>
<td>1</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.9919</td>
<td>$-0.4129$</td>
<td>$-0.488$</td>
<td>$-0.5001$</td>
<td>0.4625</td>
<td>0.488</td>
<td>0.4652</td>
<td>$-0.8577$</td>
</tr>
</tbody>
</table>

Table 3: Model cross correlation coefficients

*Premium*: the skill premium

$U^h$: proportion of the high educated unemployed

$U^l$: proportion of the low educated unemployed

$N^{hc}$: proportion of the high educated in complex occupations

$N^{hs}$: proportion of the high educated in simple occupations

$N^{ls}$: proportion of the low educated in simple occupations

$Y$: real gross domestic product

$R^n$: nominal interest rate

$\pi$: inflation rate
Figure 1: VAR impulse response of the inflation rate to a shock to Fed funds rate.

Figure 2: VAR impulse response of Fed funds rate to a shock to Fed funds rate.
Figure 3: VAR impulse response of the skill premium to a shock to Fed funds rate.

Figure 4: VAR impulse response of gross domestic product to a shock to Fed funds rate.
Figure 5: VAR impulse response of the proportion of the low educated unemployed to a shock to Fed funds rate.

Figure 6: VAR impulse response of the proportion of the high educated unemployed to a shock to Fed funds rate.
Figure 7: Model impulse response functions to a 1% monetary policy shock.
Figure 8: Model impulse response functions to a 1% monetary policy shock.
Figure 9: Model impulse response functions to a 1% monetary policy shock.
Figure 10: Model impulse response functions to a 1% monetary policy shock.
Figure 11: Model impulse response functions to a 1% monetary policy shock.