Optimal Transition Dynamics in the Leontief Two-Sector Growth Model

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The two-sector optimal growth model due to Uzawa-Srinivasan originating in the mid-sixties has seen a revival in macroeconomics and other applied fields; see Benhabib (1992) for selected papers that deal with endogenously-arising cyclic and chaotic dynamics in equilibrium. In a particularly simple case, the so called Leontief two-sector model, one in which there is a single technique in each sector, Nishimura-Yano (1995) show the possibility of optimal chaos even when future utility is “not so strongly” discounted. However, even in such a simple setting, the transition dynamics of optimal programs have not been fully analyzed and therefore not been understood. In this dissertation we present a complete analysis of optimal growth of this model, by using as our point of departure the general theory of intertemporal resource allocation developed by Brock (1970), Gale (1967), McKenzie (1968) and others; see Majumdar-Mitra-Nishimura (2000) for the current state of the art.

The primary contribution of the dissertation is the identification of a particular parameter $\zeta$, which governs the transition dynamics of optimal programs in the Leontief two-sector model. This parameter $\zeta$ represents a marginal rate of transformation of capital between today and tomorrow under a full utilization of factors. Its role goes beyond the well known capital intensity assumptions, not only in delineating the optimal policy, but also as a threshold in connecting the undiscounted case to the discounted case. When the discount factor is higher than $1/\zeta$, the optimal policy is identical to that derived when future utility is not discounted. Therefore, for this range of the discount factor, there is no possibility of chaos. This result is exactly in line with the McKenzie’s (1983) emphasis on the importance of the analysis in the undiscounted case for understanding of the discounted case, and what has been obtained by Khan-Mitra (2005) for a simpler setting of their RSS model.

Even in the undiscounted case, the transition dynamics are intricate enough to require four separate chapters. As detailed in the c.v., one is forthcoming, another is conditionally accepted and the other two are recently submitted. When the consumption good sector is more capital-intensive than the investment good sector, it is surprising that monotonic convergence emerges as only one special case out of many richer possibilities of transition dynamics. Depending on $\zeta$, an optimal program may converge to a period-two cycle, and even when it converges to the golden-rule stock, it can do so with a ‘jump’ or in a cyclical trajectory that is damped. All this goes against the intuition developed in the substantial literatures in continuous time. Only when capital is durable enough, an optimal program monotonically converges to the golden-rule stock utilizing all available factors within an infinite number of periods. However, here too, the monotonic property holds in conjunction with the requirement that some factors are kept idle when capital is fully circulating as considered by Nishimura-Yano. When the consumption good sector is more labor-intensive, the monotonicity always holds irrespective of capital durability. However, a full utilization of factors is never a possibility along an optimal program. Analysis of the discounted case, especially for those discount factors below $1/\zeta$, is now being conducted.

Even though our analysis so far concerns itself with a pure theory, we believe that the model can be applied to many fields: issues of overlapping generations, market imperfection, learning, R&D and productivity growth as well as Austrian capital-theoretical issues pertaining to forestry and sustainable growth, see Mitra-Wan (1985). Because it illustrates richer possibility of economic dynamics, the model may be particularly useful to explain an economic phenomena currently being addressed by the one-sector model. We also hope that the analytical method developed in the dissertation originating in Khan-Mitra for the RSS model can be extended to more general cases in future work.