Problem Set 6

Due Date: 12/7/2011 Econ 180.636

1. Let $\{X_1, X_2, ...X_n\}$ and $\{Y_1, Y_2, ...Y_n\}$ be two sets of random variables, each of which is iid standard normal, but where the correlation between X_i and Y_i is ρ. Let $\hat{\rho}$ denote the sample correlation.
(a) What the rejection region for a 5 percent test of the hypothesis that $\rho = 0$ when $n = 64$ (against a two-sided alternative)?
(b) By a Monte-Carlo simulation in Matlab, simulate the power of the test in (a) against the alternatives $\rho = -0.5, -0.48, -0.46, ...0.48, 0.5.$. Use 10,000 replications in each simulation. Plot the power function of the test (don't show any other results: just the graph).

2. Market Research Inc. wants to know if shoppers are sensitive to the prices of items produced in a supermarket. They obtained a random sample of 802 shoppers and found that 378 supermarket shoppers were able to state the correct price of an item immediately after putting it into the cart.
(a) Test at the 5% level the null hypothesis that half of all shoppers are able to state the correct price (against the two sided alternative).
(b) Find the power of this test against the alternative that the true proportion of shoppers that are able to state the correct price is 0.45 (analytically, not using a Monte-Carlo simulation).

3. Let the distribution X have a probability density function

$$f(x) = \frac{1}{2}\lambda \exp(-|x|\lambda)$$

for $-\infty < x < \infty$. Let $X_1, X_2, ...X_n$ be a random sample from this distribution. Suppose that $n = 20$ and $\Sigma_{i=1}^n |X_i| = 100$.
(a) Find the maximum likelihood estimator of λ.
(b) What is the large-sample (asymptotic) distribution of λ?
(c) Test the null hypothesis $\lambda = 0.125$ against the alternative $\lambda \neq 0.125$ by a likelihood ratio test.
(d) Do the same test as in (c) by a Wald test.

4. Let $X_1, X_2, ...X_n$ be a sample from an iid $N(\mu, \sigma^2)$ distribution. Suppose that $n = 20$, $\Sigma_{i=1}^n X_i = 40$ and $\Sigma_{i=1}^n X_i^2 = 180$.
(a) Find the maximum likelihood estimators of μ and σ^2.
(b) Test the hypothesis that $\mu = 1$ and $\sigma^2 = 4$ by a likelihood ratio test. Note carefully that you are testing the hypothesis on the two parameters *jointly*.
(c) Test the same hypothesis as in (b) by a Wald test.
(d) Test the same hypothesis as in (b) by a Lagrange multiplier test.