Problem Set 5

Due Date: 11/9/2011 Econ 180.636

1. Suppose that $X_1, X_2, ...X_n$ are iid normal with mean zero and variance σ^2.

(a) Find a sufficient statistic for σ^2.

(b) Find the Cramer-Rao lower bound for the variance of any unbiased estimator of σ^2.

2. Supose that $X_1, X_2, ...X_n$ are iid, where each random variable takes on the values 0, 1 and 2 with probabilities $1-3p$, p and $2p$, respectively, with $0 < p < 1/3$. Let $n_0 = \Sigma_{i=1}^n 1(X_i = 0)$ be the number of observations that are equal to zero, and n_1 and n_2 be similarly defined as the number of observations that are equal to 1 and 2, respectively.

(a) Find the MLE of p.

(b) Find the Cramer-Rao lower bound for the variance of any unbiased estimator of p.

3. Consider the bivariate random vector (X,Y) where the marginal distribution of X is uniform with support on the interval [1/2,1] and the conditional distribution of Y given that $X = x$ is $N(0, \lambda x^2)$.

(a) Find the joint probability density of X and Y.

(b) Show that $Z = Y / X$ is $N(0, \lambda)$.

(c) Derive the log-likelihood function for λ when observing $\{X_1, Y_1, X_2, Y_2, ...X_n, Y_n\}$, which are n iid draws with distribution (X,Y). Hint: The log-likelihood where there are multiple variables is just the sum of the log of their joint densities: $l(\lambda) = \Sigma_{i=1}^n \log f(X_i, Y_i)$.

(d) Find the MLE of λ.

4. Use Matlab to conduct a Monte-Carlo simulation in which you draw 10,000 samples each with $n = 30$ Poisson random variables with parameter $\lambda = 2$. In each sample compute the maximum likelihood estimator (analytically). Show the histogram over all 10,000 samples of the maximum likelihood estimates. Superimpose on this the theoretical asymptotic distribution. Repeat all this for $n = 100$. You can use the poissrnd function in Matlab to create Poisson random variables.