Problem Set 4

Due Date: 11/2/2011 Econ 180.636

1. Suppose that $X_1, X_2, ...X_n$ are iid with probability density function

$$f(x) = \theta x^{\theta-1}, \ \ 0 \le x \le 1.$$

(a) Derive the log likelihood and show that it depends on the data only through $\Sigma_{i=1}^{n} \log X_i$.

(b) Derive the maximum likelihood estimator for θ.

(c) Derive the method of moments estimator for θ.

(d) Now suppose that we observe data such that $n = 30$, $\Sigma_{i=1}^{n} X_i = 20$ and $\Sigma_{i=1}^{n} \log X_i = -13.67$. Calculate the MLE and method of moments estimates.

(e) In Matlab write a function that takes θ as input and calculates the log-likelihood, based in the data values in (d). The function should have only one input, θ. Plot the log-likelihood against θ for θ from 0.01 to 10 and discuss the shape of the log-likelihood function.

(f) Use the fminsearch function as demonstrated in class to numerically maximize the log-likelihood function.

2. Let $X_1, X_2, ...X_n$ be iid with one of two pdfs: if $\theta = 0$, then $f(x) = 1(0 < x < 1)$ while if $\theta = 1$ then $f(x) = \frac{1}{2\sqrt{x}} 1(0 < x < 1)$. Find the MLE of θ.

3. An *unfair* coin is tossed 80 times. It comes up heads 49 times and tails 31 times. Let p denote the probability of heads on each toss. What is the likelihood function at p=1/2? At p=3/4? What is the maximum likelihood estimate?

4. Suppose that $X_1, X_2, ...X_n$ are iid uniform on $[\theta, \theta+1]$. What is the likelihood function. Is the MLE unique?

5. Based on past data and other information, your beliefs about the mean nominal annual stock return excluding dividends is normal with mean 8 percent and standard deviation 4 percent. Also, you know that the standard deviation of stock returns is 16 percent. Then you are told the nominal annual stock returns in percentage points for the last ten years, which are as follows:

1998	21.24	2003	18.38
1999	18.28	2004	10.41
2000	-7.09	2005	5.11
2001	-15.05	2006	11.54
2002	-24.16	2007	4.34

Given your prior beliefs and these data what is your posterior estimate of the mean of annual stock returns?

6. Suppose that $\{x_i\}_{i=1}^{n}$ is iid $N(\mu, \sigma^2)$. Let x denote the sample and $\bar{x} = n^{-1}\Sigma_{i=1}^{n} x_i$. Assume that the prior for μ and σ^2 is

$$p(\mu) \sim N(m, s^2)$$

$$p(\sigma^2) \sim IG(a, b)$$

where IG(.,.) denotes the inverse Gamma distribution, meaning that σ^{-2} is gamma distributed with parameters a and b. It turns out that there is no closed form expression for the posterior distribution of the parameters in this case. However, there are expressions for the following conditional distributions:

$$\mu \mid \sigma^2, x \sim N(m^*, s^{*2}) \text{ where } m^* = \frac{\frac{m}{s^2} + \frac{n}{\sigma^2}\bar{x}}{\frac{1}{s^2} + \frac{n}{\sigma^2}} \text{ and } s^{*2} = \frac{1}{\frac{1}{s^2} + \frac{n}{\sigma^2}}$$

$$\sigma^2 \sim IG(a + \frac{n}{2}, b + \frac{1}{2}\Sigma_{i=1}^{n}(x_i - \mu)^2)$$

Set the following priors: $m = 0$, $s^2 = 10$, $a = 2$, $b = 3$. Create an artificial dataset of size 50 that is N(5,2) using the lines:
randn('seed',123);
x=5+(sqrt(2)*randn(50,1))

Then use the Gibbs sampler to simulate the joint posterior of μ and σ^2.