Problem Set 3

Due Date: 10/5/2011 Econ 180.636

1. Sample selection. Assume that u and v are both standard normal with correlation ρ. Assume that each worker in an economy receives a wage offer $W = \alpha + u$. But wage offers are only accepted (and therefore observed) if the worker decides to enter the workplace. In fact, v is a measure of "propensity to work"….a worker enters the workplace only if $v > 0$. Derive an expression for the expected wage of the working population $E(W \mid v > 0)$ as a function of α and ρ.

2. Let X be a random variable with moment generating function $m(t)$ and let a be a positive constant. Prove that

$P(X \geq a) \leq \exp(-at)m(t)$ for $t > 0$

and

$P(X \leq a) \leq \exp(-at)m(t)$ for $t < 0$

3. How many standard normal random numbers do I have to draw to have a 90 percent chance that *at least two* of them are greater than 1.96?

4. X and Y are two random variables such that $E(X) = 1$, $Var(X) = 1$, $E(Y) = -1$, $Var(Y) = 4$ and the correlation between X and Y is 0.5.

(a) What is the expectation of X+2Y+1?

(b) What is the variance of X+2Y+1?

5. Suppose that the distributions of height (X) and weight (Y) of a particular population is bivariate normal with expectations 70 inches and 180 pounds and standard deviations 2 inches and 20 pounds, respectively. The correlation between X and Y is 0.6.

(a) We are told that a particular individual from this population weighs 190 pounds. What is the conditional probability density for his height?

(b) We are told that a particular individual from this population is 68 inches tall. What is the conditional probability density for his weight?

6. X1 and X2 are independent $N(0, \sigma^2)$ random variables.

(a) Find the joint pdf of Y_1 and Y_2 where $Y_1 = X_1^2 + X_2^2$ and $Y_2 = X_1 / \sqrt{Y_1}$.

(b) Prove that Y_1 and Y_2 are independent.

7. Prove the Cauchy-Schwarz inequality.

8. Each day it is either rainy or sunny. On a rainy day, the chance that the next day will be sunny is 50 percent. On a sunny day, the chance that the next day will be rainy is 10 percent.

(a) Write the transition matrix to represent this as a Markov chain.

(b) Today it is sunny. What is the probability that it will be sunny on the day after tomorrow?

(c) What is the steady state distribution of this Markov chain?

9. A spider hunting a fly moves between locations 1 and 2 according to a Markov chain with transition matrix $\begin{pmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{pmatrix}$ starting in location 1. The fly, unaware of the spider, starts in location 2 and moves between these same two locations according to the Markov chain $\begin{pmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{pmatrix}$. The spider catches the fly and the hunt ends whenever they meet at the same location.

Think of the progress of the hunt as a Markov chain in which the three states are as follows.
State A: Fly is in location 1, spider is in location 2.
State B: Fly is in location 2, spider is in location 1.
State C: The hunt is over…they are in the same location. This is an *absorbing* state.

(a) Write down the transition matrix for this Markov chain.
(b) What is the probability that both the spider and fly are in their initial locations after four moves?
(c) What is the cumulative distribution function for the length of the hunt. That is, if T is the length of the hunt, what is $P(T \leq t)$? If on the first move, the fly and spider go to the same place, we say that the hunt lasted one period, T=1. Since they start in different locations, $P(T > 0) = 1$.
(d) What is the probability mass function for the length of the hunt (i.e. the probability that it lasts for *t* periods)?
(e) What is the expected length of time that the hunt lasts? Hint: If $|\alpha| < 1$ then $\Sigma_{j=1}^{\infty} j\alpha^{j-1} = \dfrac{1}{(1-\alpha)^2}$.