Problem Set 2

Due Date: 9/26/2017 Econ 180.636

1. A dice is rolled twice.
   (a) What is the marginal probability of the sum of the two numbers being 10?
   (b) What is the probability of at least one six conditional on the sum of the two numbers being 10?
   (c) What is the expectation of the first number conditional on the sum of the two numbers being 10?
   (d) What is the variance of the first number conditional on the sum of the two numbers being 9?

2. Let X be uniform from 1 to 2 and $Y = \log(X)$.
What is $P(Y \geq \varepsilon)$ for any $\varepsilon > 0$?
Use Chebychev's inequality to bound $P(Y \geq \varepsilon)$.

3. X has the probability density function $f(x) = \frac{1}{\lambda} e^{-x/\lambda}$ with support $0 < x < \infty$. Compute $E(X \mid X > 1)$.

4. X and Y are independent random variables with probability density functions $f_X(x) = \frac{1}{\lambda} e^{-x/\lambda}$ and $f_Y(y) = \frac{1}{\lambda} e^{-y/\lambda}$ with support $0 < x < \infty$ and $0 < y < \infty$, respectively. Let Z be a random variable with the probability density function $f_Z(z) = \frac{ze^{-z/\lambda}}{\lambda^2}$. Using moment generating functions or otherwise, prove that the distribution of X+Y is the same as the distribution of Z.

5. You are on a game show competing with another rival. There is a suitcase full of money, which is being auctioned off. Let i=1,2 denote the two contestants and b_1 and b_2 denote their bids. The suitcase is awarded to the higher bidder, who must pay her bid.

Let V denote the amount of money (in thousands of dollars) in the suitcase. Note that if the winner bids b, her overall profit is $V - b$. If this amount is negative, she will have to *pay* this difference out of her own pocket.

Assume that V is uniformly distributed from 1 to 2.

Before the show begins, each contestant receives a signal of how much money is in the suitcase. This is her private signal, which her rival does not know. Let X_i be the signal received by contestant i which is uniformly distributed from 0 to $2V$. Note that $E(X_i) = V$. Both contestants are naïve and bid $b_i = E(V \mid X_i)$, i.e. each contestant submits a bid that is equal to her expectation of the value given the private signal. This is a monotone increasing function of X_i.

There is a "tie-breaking" rule that if both contestants submit the same bid, then the one with the higher signal wins.

Focus on contestant 1, who receives a signal $X_1 = 1.4$.

(a) Derive $E(V \mid X_1 = 1.4)$, i.e. contestant 1's posterior expectation of V after observing her signal.

(b) Derive $E(V \mid X_1 = 1.4, X_2 < 1.4)$, i.e. contestant 1's posterior expectation of V conditional on both her signal and the event that her rival's signal is lower than hers.

(c) Based on (a) and (b), derivate an expression for contestant 1's expected profit, were she to win the auction. What is wrong here?

(d) In Matlab, simulate 10 million uniform random variables on [1,2], representing the draws of V, the value in the suitcase. For each, take a draw of the signals received by the two contestants (X_1 and X_2). What is the average value of V over only those draws for which X_1 is between 1.38 and 1.42? What is the average value of V over only those draws for which X_1 is between 1.38 and 1.42 and X_2 is less than 1.4? How does this relate to your answers in (a) and (b)?