Problem Set 1

Due Date: 09/13/2011 Econ 180.636

1. Consider the following two functions

(a) $f(x) = C(2x - x^3)$ if $0 < x < 5/2$

0 otherwise

(b) $f(x) = C(2x - x^2)$ if $0 < x < 3/2$

0 otherwise

Can (a) be a probability density function? If so, determine C. Can (b) be a pdf? If so, determine C.

2. The inhabitants of an island tell the truth one third of the time, and lie with probability 2/3. On an occasion, after one of them made a statement, another islander stepped forward and declared the statement true. What is the probability that the first statement was indeed true?

3. Let X be a random variable that is uniformly distributed on the interval [-1,1]. Derive the pdf and cdf of (i) $Y = \log(X+1)$, (ii) $Y = |X|$ and (iii) $Y = X^3$. Be sure to give the support of Y in each case. Verify by integration that each of these pdfs actually integrates to 1.

4. Consider two gamblers, A and B. A has \$5 and B has \$10. They play a game in which they flip a fair coin; if it comes up heads then A gives B a dollar. If it comes up tails then B gives A a dollar. They continue like this until one player has no money left--the other player is then considered to be the winner. What is the probability that A wins this game?

Hint: Let $p(i)$ denote the probability that player i wins given that this payer has $\$i$. Use conditional probability to show the recursion $(i+1)p(i) = ip(i+1)$ by induction.

5. Suppose that the pdf of a random variable X is $f(x) = \frac{1}{\sqrt{2}\sigma}\exp(-\frac{|x-\mu|}{\sigma/\sqrt{2}})$. Derive $Var(X)$. You may assume that $E(X) = \mu$ as we showed in class. Hint: Use integration by parts.

6. Let X be a binomial random variable with parameters n and p. Derive its moment generating function. Hence compute the variance of X. Use the fact that $\Sigma_{i=0}^{n} C_i^n u^i v^{n-i} = (u+v)^n$ (without proof).

7. Use Matlab to draw 10,000 uniform random numbers on the unit interval. Take minus the log of each of these 10,000 random numbers. Plot the histogram of these data. What is probability density function of minus the log of a uniform random variable on the unit interval? Plot it on the same chart (to get the scale of the histogram and pdf to be the same, multiply the pdf by 10,000).