Midterm Formula Sheet (Including Statistical Tables)

Econ 180-636

This sheet will be available in the midterm exam.

Discrete Distributions

Poisson Distribution

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$E(X) = \lambda$$

$$Var(X) = \lambda$$

mgf: $\exp(\lambda(e^t - 1))$

Binomial Distribution

$$P(X = x) = C_x^n p^x (1-p)^{n-x} = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$E(X) = np$$

$$Var(X) = np(1-p)$$

mgf: $[pe^t + (1-p)]^n$

Negative Binomial

$$P(X = x) = C_{r-1}^{x-1} p^r (1-p)^{x-r}, \quad x = r, r+1, r+2, \ldots$$

$$E(X) = r / p$$

$$Var(X) = \frac{r(1-p)}{p^2}$$

Geometric

$$P(X = x) = p(1-p)^{x-1}, \quad x = 1, 2, 3, \ldots$$

$$E(X) = 1 / p$$

$$Var(X) = \frac{1-p}{p^2}$$

Continuous Distributions

Beta

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} \text{ where } \Gamma(\alpha) = \int_0^\infty t^{\alpha-1}e^{-t}dt$$

$$E(X) = \frac{\alpha}{\alpha+\beta}$$

$$Var(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

Exponential

$$f(x) = \frac{1}{\lambda} e^{-x/\lambda}$$

$$E(X) = \lambda$$

$$Var(X) = \lambda^2$$

Gamma

$$f(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta}$$

Mean of a truncated normal

$$E(X \mid a < X < b) = \mu + \frac{\phi(\frac{a-\mu}{\sigma}) - \phi(\frac{b-\mu}{\sigma})}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}\sigma$$

Uniform on (a,b)

$$E(X) = (a+b)/2$$

$$Var(X) = (b-a)^2 / 12$$

Normal with mean μ and variance σ^2

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})$$

Moment generating function: $m(t) = \exp(\mu t + \frac{\sigma^2 t^2}{2})$

Bivariate and Multivariate Normal

Bivariate Joint density: $f(x) = (2\pi)^{-1} |\Sigma|^{-1/2} \exp(-\frac{1}{2}(x-\mu)'\Sigma^{-1}(x-\mu))$

Bivariate conditional:

$X_2 \mid X_1 = x_1 \sim N(\mu_2 + \frac{\rho\sigma_2}{\sigma_1}(x_1 - \mu_1), \sigma_2^2(1-\rho^2))$ where $\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$

Multivariate (nx1) Joint density: $f(x) = (2\pi)^{-n/2} |\Sigma|^{-1/2} \exp(-\frac{1}{2}(x-\mu)'\Sigma^{-1}(x-\mu))$

Multivariate conditional: $X_2 \mid X_1 = x_1 \sim N(\mu_2 + \Sigma_{21}\Sigma_{11}^{-1}(x_1 - \mu_1), \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12})$

Order statistics

Suppose that an iid sample with pdf f and cdf F is ordered $X_{(1)} \le X_{(2)} ..... \le X_{(n)}$.

The pdf for $X_{(j)}$ (the jth smallest statistic) is

$$\frac{n!}{(j-1)!(n-j)!} f(x) F(x)^{j-1} (1-F(x))^{n-j}$$

Selected Inequalities

Hölder's Inequality: $|E(XY)| \leq |E(X^p)^{1/p} E(Y^q)^{1/q}$ where $\frac{1}{p} + \frac{1}{q} = 1$

Liapounov Inequality: $\{E(|X|^r)\}^{1/r} \leq \{E(|X|^s)\}^{1/s}$, $1 < r < s$

Minkowski Inequality: $E(|X+Y|^p)^{1/p} \leq E(|X|^p)^{1/p} + E(|Y|^p)^{1/p}$, $p \geq 1$

Covariance Inequality:

- If g is nondecreasing and h is nonincreasing $E(g(X)h(X)) \leq E(g(X))E(h(X))$
- If g and h are <u>both</u> nondecreasing or <u>both</u> nonincreasing $E(g(X)h(X)) \geq E(g(X))E(h(X))$

Standard Brownian Motion

First Hitting Time: $P(T_a \leq t) = \sqrt{\frac{2}{\pi}} \int_{|a|/\sqrt{t}}^{\infty} e^{-y^2/2} dy$

P(Goes up A before it goes down B): $\frac{B}{A+B}$

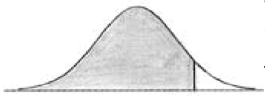
Brownian Motion with Drift

If $\mu > 0$, $P(T_a < \infty) = 1$ if $a > 0$ or it is $e^{2\mu a}$ if $a < 0$

If $\mu < 0$, $P(T_a < \infty) = 1$ if $a < 0$ or it is $e^{2\mu a}$ if $a > 0$

P(Goes up A before going down B): $\frac{e^{2\mu B} - 1}{e^{2\mu B} - e^{2\mu A}}$

Tables of the Standard Normal Distribution



Probability Content from -∞ to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990