Final Formula Sheet (Including Statistical Tables)

Econ 180-636

This sheet will be available in the final exam.

Discrete Distributions

Poisson Distribution

$$P(X=x)=\frac{e^{-\lambda}\lambda^{x}}{x!}$$

$E(X)=\lambda$

$Var(X)=\lambda$

mgf: $\exp(\lambda(e^{t}-1))$

Binomial Distribution

$$P(X=x)=C_{x}^{n}p^{x}(1-p)^{n-x}=\frac{n!}{x!(n-x)!}p^{x}(1-p)^{n-x}$$

$E(X)=np$

$Var(X)=np(1-p)$

mgf: $[pe^{t}+(1-p)]^{n}$

Negative Binomial

$P(X=x)=C_{r-1}^{x-1}p^{r}(1-p)^{x-r},\ x=r,r+1,r+2,\ldots$

$E(X)=r/p$

$$Var(X)=\frac{r(1-p)}{p^{2}}$$

Geometric

$P(X=x)=p(1-p)^{x-1},\ x=1,2,3,\ldots$

$E(X)=1/p$

$$Var(X)=\frac{1-p}{p^{2}}$$

Continuous Distributions

Beta

$$f(x)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}\text{ where }\Gamma(\alpha)=\int_{0}^{\infty}t^{\alpha-1}e^{-t}dt$$

$$E(X)=\frac{\alpha}{\alpha+\beta}$$

$$Var(X)=\frac{\alpha\beta}{(\alpha+\beta)^{2}(\alpha+\beta+1)}$$

Exponential

$f(x) = \frac{1}{\lambda} e^{-x/\lambda}$

$E(X) = \lambda$

$Var(X) = \lambda^2$

Gamma

$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}$

Mean of a truncated normal

$$E(X \mid a < X < b) = \mu + \frac{\phi(\frac{a-\mu}{\sigma}) - \phi(\frac{b-\mu}{\sigma})}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})} \sigma$$

Uniform on (a,b)

$E(X) = (a+b)/2$

$Var(X) = (b-a)^2 \ /12$

Normal with mean μ and variance σ^2

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})$$

Moment generating function: $m(t) = \exp(\mu t + \frac{\sigma^2 t^2}{2})$

Order statistics

Suppose that an iid sample with pdf f and cdf F is ordered $X_{(1)} \le X_{(2)} ..... \le X_{(n)}$.

The pdf for $X_{(j)}$ (the *j*th smallest statistic) is

$$\frac{n!}{(j-1)!(n-j)!} f(x) F(x)^{j-1} (1-F(x))^{n-j}$$

Selected Inequalities

Hölder's Inequality: $|E(XY)| \le |E(X^p)^{1/p} E(Y^q)^{1/q}$ where $\frac{1}{p} + \frac{1}{q} = 1$

Liapounov Inequality: $\{E(|X|^r)\}^{1/r} \le \{E(|X|^s)\}^{1/s}$, $1 < r < s$

Minkowski Inequality: $E(|X+Y|^p)^{1/p} \le E(|X|^p)^{1/p} + E(|Y|^p)^{1/p}$, $p \ge 1$

Covariance Inequality:

- If g is nondecreasing and h is nonincreasing $E(g(X)h(X)) \le E(g(X))E(h(X))$
- If g and h are <u>both</u> nondecreasing or <u>both</u> nonincreasing $E(g(X)h(X)) \ge E(g(X))E(h(X))$

Standard Brownian Motion

First Hitting Time: $P(T_a \le t) = \sqrt{\frac{2}{\pi}} \int_{|a|/\sqrt{t}}^{\infty} e^{-y^2/2} dy$

P(Goes up A before it goes down B): $\frac{B}{A+B}$

Brownian Motion with Drift

If $\mu > 0$, $P(T_a < \infty) = 1$ if $a > 0$ or it is $e^{2\mu a}$ if $a < 0$

If $\mu < 0$, $P(T_a < \infty) = 1$ if $a < 0$ or it is $e^{2\mu a}$ if $a > 0$

P(Goes up A before going down B): $\frac{e^{2\mu B} - 1}{e^{2\mu B} - e^{2\mu A}}$

Cumulants

$\kappa_1 = E(X) = \mu$

$\kappa_2 = E((X-\mu)^2) = \sigma^2$

$\kappa_3 = E((X-\mu)^3)$

$\kappa_4 = E((X-\mu)^4) - 3\sigma^4$

Edgeworth Expansion

$\Phi(x)$ is the $N(0,1)$ cdf and $\Phi^{(j)}(x) = \frac{d^j \Phi(x)}{dx^j}$ and $F_n(x)$ is the cdf of $\sqrt{n}(\frac{\bar{X}-\mu}{\sigma})$

Let $\rho_i = \kappa_i / \sigma^i$

Two terms: $F_n(x) = \Phi(x) - \frac{\rho_3 \Phi^{(3)}(x)}{6\sqrt{n}} + O(\frac{1}{n})$

Three terms: $F_n(x) = \Phi(x) - \frac{\rho_3 \Phi^{(3)}(x)}{6\sqrt{n}} + \frac{1}{n}[\frac{\rho_4 \Phi^{(4)}(x)}{24} + \frac{\rho_3^2 \Phi^{(6)}(x)}{72}] + O(\frac{1}{n^{3/2}})$

Berry-Esseen Bound $|F_n(x) - \Phi(x)| \le \frac{CE(|X_i|^3)}{\sigma^3} n^{-1/2}$

Bayes estimators

Normal-Normal Case. $X_1, X_2, \ldots X_n$ are iid $N(\mu, \sigma^2)$ where σ^2 is known

Prior: $\mu \sim N(\theta, \tau^2)$

Posterior distribution is:

$\mu | X_1, \ldots X_n \sim N(\frac{n\tau^2}{n\tau^2 + \sigma^2}\bar{X} + \frac{\sigma^2}{n\tau^2 + \sigma^2}\theta, \frac{\sigma^2\tau^2}{n\tau^2 + \sigma^2})$

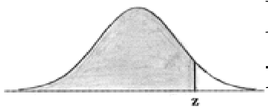
Beta-Binomial Case. X is binomial with parameters n and p. We observe s successes

n is known

The prior for p is beta with parameters α and β.

The posterior for p is beta with parameters $\alpha + s$ and $n - s + \beta$.

Tables of the Standard Normal Distribution



Probability Content from $-\infty$ to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

UPPER PERCENTILES of the T Distribution

DF	0.10	0.05	0.025	0.01	0.005	0.001
1	3.078	6.314	12.71	31.82	63.66	318.3
2	1.886	2.92	4.303	6.965	9.925	22.33
3	1.638	2.353	3.182	4.541	5.841	10.21
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.893
6	1.44	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.86	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.25	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.93
13	1.35	1.771	2.16	2.65	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.12	2.583	2.921	3.686
17	1.333	1.74	2.11	2.567	2.898	3.646
18	1.33	1.734	2.101	2.552	2.878	3.61
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.08	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.5	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.06	2.485	2.787	3.45
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.31	1.697	2.042	2.457	2.75	3.385
40	1.303	1.684	2.021	2.423	2.704	3.307
50	1.299	1.676	2.009	2.403	2.678	3.261
60	1.296	1.671	2	2.39	2.66	3.232
70	1.294	1.667	1.994	2.381	2.648	3.211
80	1.292	1.664	1.99	2.374	2.639	3.195
100	1.29	1.66	1.984	2.364	2.626	3.174
150	1.287	1.655	1.976	2.351	2.609	3.145

Chi-Squared Distribution Cumulative Distribution Function

DF	0.01	0.05	0.1	0.25	0.5	0.75	0.9	0.95	0.99
1	0.00016	0.0039	0.0158	0.102	0.455	1.32	2.71	3.84	6.63
2	0.0201	0.103	0.211	0.575	1.39	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.21	2.37	4.11	6.25	7.81	11.3
4	0.297	0.711	1.06	1.92	3.36	5.39	7.78	9.49	13.3
5	0.554	1.15	1.61	2.67	4.35	6.63	9.24	11.1	15.1
6	0.872	1.64	2.2	3.45	5.35	7.84	10.6	12.6	16.8
7	1.24	2.17	2.83	4.25	6.35	9.04	12	14.1	18.5
8	1.65	2.73	3.49	5.07	7.34	10.2	13.4	15.5	20.1
9	2.09	3.33	4.17	5.9	8.34	11.4	14.7	16.9	21.7
10	2.56	3.94	4.87	6.74	9.34	12.5	16	18.3	23.2
11	3.05	4.57	5.58	7.58	10.3	13.7	17.3	19.7	24.7
12	3.57	5.23	6.3	8.44	11.3	14.8	18.5	21	26.2
13	4.11	5.89	7.04	9.3	12.3	16	19.8	22.4	27.7
14	4.66	6.57	7.79	10.2	13.3	17.1	21.1	23.7	29.1
15	5.23	7.26	8.55	11	14.3	18.2	22.3	25	30.6
16	5.81	7.96	9.31	11.9	15.3	19.4	23.5	26.3	32
17	6.41	8.67	10.1	12.8	16.3	20.5	24.8	27.6	33.4
18	7.01	9.39	10.9	13.7	17.3	21.6	26	28.9	34.8
19	7.63	10.1	11.7	14.6	18.3	22.7	27.2	30.1	36.2
20	8.26	10.9	12.4	15.5	19.3	23.8	28.4	31.4	37.6
21	8.9	11.6	13.2	16.3	20.3	24.9	29.6	32.7	38.9
22	9.54	12.3	14	17.2	21.3	26	30.8	33.9	40.3
23	10.2	13.1	14.8	18.1	22.3	27.1	32	35.2	41.6
24	10.9	13.8	15.7	19	23.3	28.2	33.2	36.4	43
25	11.5	14.6	16.5	19.9	24.3	29.3	34.4	37.7	44.3
26	12.2	15.4	17.3	20.8	25.3	30.4	35.6	38.9	45.6
27	12.9	16.2	18.1	21.7	26.3	31.5	36.7	40.1	47
28	13.6	16.9	18.9	22.7	27.3	32.6	37.9	41.3	48.3
29	14.3	17.7	19.8	23.6	28.3	33.7	39.1	42.6	49.6
30	15	18.5	20.6	24.5	29.3	34.8	40.3	43.8	50.9
35	18.5	22.5	24.8	29.1	34.3	40.2	46.1	49.8	57.3
40	22.2	26.5	29.1	33.7	39.3	45.6	51.8	55.8	63.7