Problem Set 4

Due Date: 4/27/2011 Econ 608

1. Consider the predictive regression

$$r_{t+h}^{(h)} = \beta x_t + \varepsilon_{t+h}^{(h)}$$

where $r_{t+h}^{(h)} = \Sigma_{s=1}^{h} r_{t+s}$ denote h-period returns, r_{t+1} is the one-period return, x_t is a predictor. Assume that $\beta = 0$ and let $\varepsilon_{t+1} = r_{t+1}$ denote the one-period unpredictable return, where $\varepsilon_{t+h}^{(h)} = \Sigma_{s=1}^{h} \varepsilon_{t+s}$. Assume that $E_t(\varepsilon_{t+1}) = 0$. Let the sample size be T and the OLS estimate be $\hat{\beta}$. Prove that

$$T^{1/2}(\hat{\beta} - \beta) \rightarrow_d N(0, V)$$

where V can be estimated by

$$(T^{-1}\Sigma x_t x_t')^{-1} T^{-1} \Sigma w_{t+1} w_{t+1}' (T^{-1}\Sigma x_t x_t')^{-1}$$

$w_{t+1} = r_{t+1} x_t^{(h)}$ and $x_t^{(h)} = \Sigma_{s=1}^{h} x_{t+1-s}$. These are standard errors 1B from Hodrick (1992). Note that they are only valid under the null of no predictability.

2. Consider the ARCH model with no mean

$$r_t = \sigma_t \varepsilon_t$$
$$\sigma_t^2 = \omega + \alpha r_{t-1}^2$$

where ε_t is iid N(0,1). Derive an expression for the unconditional kurtosis, $\frac{E(r_t^4)}{E(r_t^2)^2}$. Under what conditions is it finite?

3. Download the data for this homework from the class website (these are simulated data just for the purpose of the exercise). Use the Kalman filter to fit a random walk plus noise model

$$y_t = \mu_t + \varepsilon_t$$
$$\mu_t = \mu_{t-1} + \eta_t$$

where ε_t and η_t are iid normal with mean zero and variances σ_ε^2 and σ_η^2, respectively, and are mutually uncorrelated. Specifically,

(a) Estimate the parameters σ_ε^2 and σ_η^2 by maximum likelihood.

(b) Conditioning on the estimates in (a), use the Kalman filter to obtain estimates $\mu_{t|t}$.

(c) Now use the Kalman smoother to obtain estimates $\mu_{t|T}$.

4. Go to the data archive of the Journal of Applied Econometrics and download the data from the paper by Mahieu and Schotman (1998; Journal of Applied Econometrics; 13(4))

http://econ.queensu.ca/jae/

The first column of the data represent the Japanese Yen/$ exchange rate returns (weekly frequency) from 1973 to 1994.

Letting these exchange rate returns be denoted as y_t, consider the simple Gaussian autoregressive stochastic volatility (ARSV) model

$$y_t = \sigma_t \varepsilon_t$$
$$\log(\sigma_t^2) = \omega + \phi \log(\sigma_t^2) + \xi u_t$$

where the errors ε_t and u_t are both standard normal. Set the parameters $\phi = 0.98$, $\omega = -0.17$ and $\xi = 0.2$. Approximating a log-chi squared random variable by a mixture of two normals as discussed in class, use the smoother with Gibbs sampling to obtain estimates of $\sigma_{t|T}^2$

5. Suppose that you are comparing the model $y_t = \beta_1 x_{1t} + \varepsilon_{1t}$ and the model $y_t = \beta_1 x_{1t} + \beta_2 x_{2t} + \varepsilon_{2t}$ where x_{1t} and x_{2t} are both scalars. Starting half-way through the sample, you estimate the two models recursively and collect the forecast errors. Let $\hat{u}_{1t}$ and $\hat{u}_{2t}$ denote the two prediction errors. Consider the Diebold-Mariano test stastistic

$$\frac{T^{*-1}\Sigma_{t=1}^{T}(\hat{u}_{1t}^2 - \hat{u}_{2t}^2)}{\hat{\sigma}/\sqrt{T^*}}$$

where $T^* = T/2$ is the number of time periods for the out-of-sample forecast comparison and $\hat{\sigma}^2$ is the sample variance of $\hat{u}_{1t}^2 - \hat{u}_{2t}^2$.

(a) Find the 5 percent critical value of this test statistic for testing the null hypothesis that $\beta_2 = 0$.

(b) Is this the optimal way of testing the hypothesis that $\beta_2 = 0$?