Problem Set 2

Due Date: 3/4/2011 Econ 608

1. What is identification?

2. Consider the model where there are data $\{y_i, x_i, z_i\}_{i=1}^n$ which are iid and $y_i = \beta x_i + e_i$ such that $E(e_i x_i) = 0$ and $E(e_i z_i) = 0$. What is the efficient GMM estimator of β?

3. Consider the moment condition $E(h(Y_i, \theta_0))$ where θ is a p x1 parameter vector of which θ_0 is the true value, $h(.,.)$ is a k x1 vector of moment conditions and $k > p$. Consider the GMM estimator

$$\hat{\theta} = \arg\min S(\theta)$$

where

$$S(\theta) = n^{-1/2}\Sigma_{i=}^n h(Y_i,\theta)' W n^{-1/2}\Sigma_{i=}^n h(Y_i,\theta)$$

where the data are iid and we assume that W is the inverse of the variance-covariance matrix of $h(Y_i, \theta_0)$. Derive the limiting distribution of $S(\hat{\theta})$.

4. Consider the same problem as in Q2 but also define the GMM estimator

$$\tilde{\theta} = \arg\min G(\theta)$$

where

$$G(\theta) = n^{-1/2}\Sigma_{i=}^n h(Y_i,\theta)' I n^{-1/2}\Sigma_{i=}^n h(Y_i,\theta)$$

Let $V(.)$ denote the asymptotic variance of the argument. Prove that $V(\tilde{\theta}) - V(\hat{\theta})$ is positive semidefinite.

5. Conduct a Monte-Carlo simulation in which the data are generated according to an AR(1) with an autoregressive parameter 0.95, a sample size of 100, normal shocks and no intercept. In each of 1,000 replications, construct bootstrap confidence intervals for the autoegressive parameter via (a) the other percentile, (b) the percentile (c) the percentile-t and (d) the asymptotic confidence intervals. Compare the effective coverage rates of the different confidence intervals (95 percent nominal coverage).

6. Data on real stock returns, real gross bond returns and real gross per capita consumption are available on the website, constructed from the data on Robert Shiller's website
http://www.econ.yale.edu/~shiller/data.htm

Consider the GMM estimation of the consumption CAPM Euler equation discussed in class.
(a) Compute the one-step, two-step and continuous-updating GMM estimators of δ and γ using a constant, two lags of stock returns bond returns and real consumption as instruments. Report standard errors (these should use analytical, not numerical, derivatives).
(b) Conduct a Hansen J-test of overidentifying restrictions corresponding to the two-step GMM estimator.

(c) Construct a confidence set for δ and γ using the S-set methodology of Stock and Wright (2000).
(d) Log-linearize the Euler equation and estimate the parameters by two-stage least squares.
(e) Include in (d) the results from the first-stage F statistic.

7. Use the data on real stock returns and real gross bond returns to form the Hansen-Jagannathan bound for the mean and standard deviation of the pricing kernel. Report your results by plotting the bound with $E(M_{t+1})$ on the horizontal axis and $\sigma(M_{t+1})$ on the vertical axis.