Solutions to Problem Set 4

1. Here is the code:
replics=1000; randn('seed',123);
for imc=1:replics;
e1=randn(100,1);
e2=(0.9*e1)+(sqrt(0.19)*randn(100,1));
r=e1;
x=filter(1,[1;-0.98],e2);
for t=1:80; rh(t,1)=sum(r(t+1:t+20)); end;
Xmat=[ones(80,1) x(1:80)];
[bhat,vcov]=newey(rh,Xmat,30);
Cover(imc)=abs(bhat(2))<1.96*sqrt(vcov(2,2)); %1 if CI covers zero
end;
disp(mean(Cover))

The Newey function is:
function [bhat,vcov]=newey(y,x,l);
%OLS with HAC standard errors
bhat=inv(x'*x)*x'*y;
res=y-(x*bhat);
[n,p]=size(x);
for i=1:p; z(:,i)=x(:,i).*res; end;
g=(z'*z)/n;
if l>0;
   for j=1:l;
      h=(z(1:n-j,:)'*z(j+1:n,:))/n;  h=h*(l+1-j)/(l+1);
      g=g+h+(h');
   end;
end;
vcov=n*inv(x'*x)*g*inv(x'*x);

The answer is 33.7 percent---very low coverage.
The slope coefficient estimate is -0.2172 (standard error: 0.0762). The sup Wald test statistic is 19.97, which is clearly significant. The estimated break date is 2000:08. Here is the plot of the Wald test statistic against the break date:
3. Here is the code:

```matlab
m = xlsread('pset4.xlsx','Sheet1','A2:D718');
dates = m(1:end-12,1) + ((m(1:end-12,2)-1)/12);
y = m(13:end,3)-m(1:end-12,3);
bigt = length(y);
x = [ones(bigt,1) m(1:end-12,4)];
thetaL = prctile(x(:,2),10);
thetaU = prctile(x(:,2),90);
u_grid = linspace(thetaL,thetaU,50);
for i=1:50; ss(i,1)=thresh(y,x,u_grid(i),1); end;
[~,umin]=min(ss);
thetaest=u_grid(umin);
betastar = thresh(y,x,thetaest,0);
disp('Parameter Estimates: Low unemployment/High unemployment regimes');
disp(betastar);
disp('Threshold Estimate');
disp(thetaest);
```

The `thresh` function is:

```matlab
function out = thresh(y,x,theta,isssr);
%isssr=1 if output is sum of squared residuals; otherwise parameters
[n,p] = size(x);
g1 = find(x(:,2)<=theta);
g2 = find(x(:,2)>theta);
n1 = length(g1);
y = [y(g1);y(g2)];
z = zeros(n,2*p);
z(1:n1,1:p) = x(g1,:);
z(n1+1:end,p+1:end) = x(g2,:);
bhat = inv(z'*z)*z'*y;
R = [1 theta -1 -theta]; r = 0;
betastar = bhat-(inv(z'*z)*R'*inv(R*inv(z'*z)*R')*(R*bhat-r));
res = y - (z*betastar);
if isssr == 1;
   out = sum(res.^2);
else;
   out = reshape(betastar,2,p);
end;
```

The parameter estimates are:

<table>
<thead>
<tr>
<th></th>
<th>Low Unemployment</th>
<th>High Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>1.6014</td>
<td>-0.7709</td>
</tr>
<tr>
<td>Slope</td>
<td>-0.2693</td>
<td>0.0165</td>
</tr>
</tbody>
</table>

and the threshold, \( \theta \) is 8.3.

Note that 1.6014-(0.2693*8.3)=-0.7709+(0.0165*8.3)=-0.634, so the two lines “join up.”