Problem Set 3

Due Date: November 1, 2019

1. Please download data on real GDP (billions of chained 2012 dollars, seasonally adjusted) from the economic database at the Saint Louis Fed from 1947Q1 to 2019Q2:

https://fred.stlouisfed.org/series/GDPC1

Take 400 times the first differences of the log data, which are effectively annualized percentage changes. Call these data \( y_t \). Consider applying a Markov switching model to these data:

\[
y_t = \alpha + \beta S_t + \epsilon_t
\]

where \( \epsilon_t \) is iid \( N(0, \sigma^2) \) and \( S_t \) is a Markov switching process with two states 0 and 1 such that \( P(S_t = 1 | S_{t-1} = 1) = p \) and \( P(S_t = 0 | S_{t-1} = 0) = q \).

(a) Please find the maximum likelihood point estimates of the five parameters.

(b) Plot the filtered probability of being in the low growth state.

2. Suppose that a bond pays coupons \( C \) and has a face value of $1 and that the yield is compounded at the same frequency as the coupons. Let the price of the bond be \( P \), the maturity be \( m \), and the yield be \( y \). Define duration as

\[
D = \frac{1}{P} \left\{ \frac{C}{(1+y)} + \frac{2C}{(1+y)^2} + \frac{3C}{(1+y)^3} + \ldots + \frac{m(1+C)}{(1+y)^m} \right\}
\]

Prove that \( \frac{d \log(P)}{dy} = -\frac{D}{(1+y)} \).

3. Suppose that the yields on zero-coupon bonds are as shown in the Table below.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Yield (Cont. Comp)</th>
<th>Maturity</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 months</td>
<td>5.4</td>
<td>3 years</td>
<td>5.5</td>
</tr>
<tr>
<td>1 year</td>
<td>5.3</td>
<td>3-1/2 years</td>
<td>6</td>
</tr>
<tr>
<td>18 months</td>
<td>5</td>
<td>4 years</td>
<td>6.5</td>
</tr>
<tr>
<td>2 years</td>
<td>4.5</td>
<td>4-1/2 years</td>
<td>7</td>
</tr>
<tr>
<td>2-1/2 years</td>
<td>5</td>
<td>5 years</td>
<td>7.5</td>
</tr>
</tbody>
</table>

(a) What will the price of a bond that pays coupons of 3 percent twice a year (called a 6 percent bond) be? Assume that the first coupon is in exactly 6 months.

(b) Compute the duration of the bond in (a).