1. Consider the univariate autoregression \( y_t = \phi y_{t-1} + u_t \) where \( u_t \) is iid with mean zero and variance \( \sigma^2 \) and \( |\phi| < 1 \). If \( \hat{\phi} \) is the OLS estimator of \( \phi \) and \( T \) is the sample size, then

\[
T^{1/2} (\hat{\phi} - \phi) \rightarrow_d N(0, 1 - \phi^2)
\]

(a) Find the asymptotic distribution of \( \hat{\phi} \).
(b) Prove that \( y_t = \alpha y_{t-2} + \nu_t \) where \( \alpha = \phi^2 \) and \( \nu_t = u_t + \phi u_{t-1} \) is an MA(1) process. (Hint: Don’t overthink: it is two lines of algebra).
(c) Let \( \hat{\alpha} \) be the OLS estimator of \( \alpha \) in (b). Find its asymptotic distribution. (Hint: Think about the serial correlation properties of the process \( y_{t-2} \nu_t \)).
(d) Compare the distributions in (a) and (c). Which has the bigger variance?

2. Conduct a Monte-Carlo simulation. Let \( \varepsilon_t = 0.9 \varepsilon_{t-1} + u_t \) be an AR(1) process where \( u_t \) is iid N(0,1), initialized from \( \varepsilon_0 = 0 \). Define \( y_t = \beta + \varepsilon_t \) where \( \beta = 1 \). The sample size is \( T=400 \). You estimate \( \beta \) by the sample mean of \( \{y_t\} \) (which is the OLS estimate from regression on a constant) and form four confidence intervals of 95 percent nominal coverage:

(i) Using White standard errors and standard normal critical values (i.e. confidence interval is point estimate \( \pm 1.96 \) standard errors),
(ii) Using Newey-West standard errors with lag truncation of 1 and standard normal critical values,
(iii) Using Newey-West standard errors with lag truncation of 26 (following the Stock et al. (2018) advice for lag truncation) and standard normal critical values, and
(iv) Using Newey-West standard errors with lag truncation of 26 and Kiefer-Vogelsang critical values. This is the complete advice of Stock et al.. In this case, the Kiefer-Vogelsang critical values are point estimate \( \pm 2.15 \) standard errors.

Please conduct 1,000 replications and report the effective coverage of each of the four confidence intervals. The effective coverage means the fraction of the 1,000 replications when each type of confidence interval includes the true value, which is 1. Please include your Matlab code.

3. Consider the reduced form VAR \( A(L)y_t = u_t \) where \( u_t \) is iid with mean 0 and variance-covariance matrix \( \Sigma \). Suppose that \( \varepsilon_t \) is a vector of structural shocks such that \( u_t = R\varepsilon_t \) and let \( \varepsilon_t = Su_t \). Prove that \( S = R^\prime \Sigma^{-1} \).

4. Monthly data on the civilian unemployment rate, and CPI inflation and the federal funds rate, 1960:01 to 2007:12 are on the first tab of the spreadsheet for this homework, on the course website. Estimate a VAR of order 12 on the data ordered in this way. Using the standard Cholesky factorization and the non-informative prior, find the posterior mean and 95% Bayesian confidence intervals for the effects of a structural monetary policy shock on the unemployment rate, inflation, and the federal funds rate, at horizons 0-60 months. Please include your Matlab code.
5. Consider a 2x1 variable \( Y_t \) where \( Y_t \) is iid with mean 0 and identity variance-covariance matrix.

(a) Is it possible to write \( Y_t = D_0 \varepsilon_t + D_1 \varepsilon_{t-1} \) where \( \varepsilon_t \) is a time series of orthogonal, zero-mean, unit variance structural shocks, other than \( D_0 = I \) and \( D_1 = 0 \)? If so, give an example of the matrices \( D_0 \) and \( D_1 \).

(b) Is the representation in (a) invertible?