1. (20 pts) Suppose that the price of a barrel of oil in December 2019 is sure to be some integer ($1, $2, …). Consider an options strategy where you buy 1 call option at a strike of $54, buy 1 call option at a strike of $56, and short 2 call options at a strike of $55. (This is called a \textit{butterfly} strategy). All options are European and you can assume that risk-free interest rates are zero.

(a) What will be the total value of this position if the oil price at expiration is $55?
(b) What will be the total value of this position if the oil price at expiration is anything other than $55 (remember it must be an integer, by assumption)?
(c) If you were to have bought this position on Friday November 8, 2019 at the closing price for that day, what would the net cost have been?
(d) Based on this information, what is the probability of oil being $55 at expiration, if investors are risk neutral?

2. (30 pts) The price of XYZ corp is currently $100. There are four possible paths for its price over the next two years:

(i) It could go up to $110 in one year and up again to $121 in two years.
(ii) It could go up to $110 in one year and back down to $100 in two years.
(iii) It could go down to $90 in one year and back up to $100 in two years.
(iv) It could go down to $90 in one year and down again to $81 in two years.

The riskfree rate is 5 percent per year. What is the price of a European call option on the stock of XYZ corp at a strike price of $100 maturing in two years’ time?

3. (30 pts) Suppose that the price of a non-dividend paying stock is $50 and the volatility of this stock is 0.4 (annualized). The risk-free rate is zero. There are 1,000,000 shares outstanding. Warrants are issued for 100,000 additional shares with an expiration of 2 years hence with a strike price of $60. According to the Black-Scholes formula, what should the price of these warrants be?

4. (20 pts) Consider each of the following possible scenarios for the prices of options. Say if it is possible or impossible. In all cases assume that riskfree rates are zero, the options are European, and the underlying assets pay no dividends. If impossible, please say why not.

(a) A call option with a strike of $20 costs $8. The underlying asset is worth $40.
(b) A call option with a strike of $20 costs $30. The underlying asset is worth $40.
(c) A call option with a strike of $20 costs $45. The underlying asset is worth $40.