Economics 180.367: Investments and Portfolio Management

Solutions to Problem Set 3

(1a) In this situation, Jason effectively has $20,000 invested in Alaska Oil before he makes any investment decisions. The Markowitz model assumes that our initial wealth is in raw dollars, so we'll have to make an adjustment here. If Jason shorts $20,000 of Alaska stock, then he will gain $20,000 in cash today and have an asset that completely offsets the riskiness of his end-of-year labor income. Now Jason has $120,000 in cash. But Jason's wealth goes even further, as he is guaranteed a labor income of $200,000 at the end of the year. This income is risk-free, so we can create an offsetting asset by borrowing (shorting) $200,000 of the risk-free asset today at a zero interest rate today. Now Jason has $320,000 of cash in hand to invest as he best sees fit.

He should invest all this wealth through the typical 'Markowitz model method': find the expected returns of all risky assets, find the variance-covariance matrix for the returns of these assets, calculate the risky asset portfolio that has the highest Sharpe ratio, form the optimal CAL as the ray from the risk-free asset through the optimal risky asset portfolio, and then find the point on the CAL that is tangent to an indifference curve. This point will represent the utility-maximizing combination of the optimal risky asset portfolio and the risk-free asset, and Jason will invest all of his investable cash in it.
Note that it might be the case that after shorting Alaska Oil, the 'Markowitz model method' might tell Jason to then buy more than $20,000 of Alaska Oil. If this was the case, why did Jason short the stock just to then go long in it? In reality, Jason would calculate his optimal portfolio using the Markowitz method as if he already had all $320,000 on hand, and then he would adjust the optimal portfolio value of Alaska Oil down by $20,000 and purchase the adjusted portfolio.

(1b) The CAPM predicts that the optimal risky-asset portfolio is the market portfolio. Thus Jason's work has been cut down, as he should short $20,000 of Alaska Oil, borrow $200,000 at the risk-free rate and then form the optimal CAL as the ray from the risk-free asset through the market portfolio.

Jason should then invest all of his $320,000 in the indifference curve tangency portfolio on the optimal CAL.

Note: the key difference between part a and part b is that in part a the tangency portfolio may not be the market portfolio. By contrast, in part b the tangency portfolio is the market portfolio. In both parts, the optimal strategy for Jason is to sell his exposure to his firm, and then invest on the optimal CAL.

(1c) If Jason was barred from trading in Alaska Oil, his utility would probably decline. There are limited circumstances under which Jason's utility would be unaffected, but it is not possible for his utility to increase. Jason achieves maximal utility if he can choose his portfolio from among all possible portfolios on the steepest CAL that is feasible. As before, the CAPM says that the market portfolio is the risky-asset portfolio with the highest Sharpe ratio, and thus in (1b) we chose to purchase a combination of it and the risk-free asset because these portfolio choices are on the optimal CAL line, and its slope is the Sharpe ratio of the market portfolio.

If $20,000 of Jason's total wealth is effectively invested in Alaska Oil, then this prevents him from choosing a portfolio that is on the optimal CAL. Thus, his utility will in general be lower than in (1b).

What are the circumstances such that Jason's utility would not be affected? There are two circumstances, and both are very unrealistic. The first is that the return of Alaska Oil is perfectly correlated with the return of a portfolio of other energy firms. In that circumstance, Jason can short the other energy firms, and it is equivalent to shorting Alaska oil. This lets us implement the choices we made in 2a. The first circumstance is unrealistic because Alaska oil probably still has firm-specific risk that is uncorrelated with the returns of other energy firms. This firm-specific risk could be associated with the quality of the management of Alaska oil, or events that affect Alaska oil but not other energy firms. However, even if shorting other energy firms is not the perfect solution, it is probably a good strategy because it eliminates much of the energy risk from Jason's portfolio.

The second circumstance where Jason's happiness would not be affected is if the return of Alaska oil is perfectly correlated with return of the tangency portfolio. If it was, then he would already be on the optimal CAL, and by investing in the tangency portfolio and the risk-free asset, he could choose his portfolio along the CAL.

2. We use the equation $E(r_i) - r_f = 1.5\lambda_1 - 2\lambda_2$

$0.25 = 1.5\lambda_1 + 2\lambda_2$

$0.21 = 2.2\lambda_1 - 0.2\lambda_2$

Solving these equations gives $\lambda_1 = 0.1$ and $\lambda_2 = 0.05$

Thus, the expected return-beta relationship is:

$E(r_i) - r_f = 0.1\beta_1 + 0.05\beta_2$

3. Since $\beta_F = 0$, the expected return for portfolio F equals the risk-free rate.

For portfolio A, according to APT, we have $0.12 = 0.06 + 1.2\lambda$

For portfolio E, we have $0.08 = 0.06 + 0.6\lambda$

Solving the first equation, gives $\lambda = 5\%$. But solving the second gives $\lambda = 3.33\%$.

The two are inconsistent.

So, it appears as if an arbitrage opportunity exists. For example, let's create portfolio X by combining portfolios A and F with equal weights. Then the expected return for portfolio X is:

$0.5 * 12\% + 0.5 * 6\% = 9\%$

And the beta for portfolio X is

$0.5 * 1.2 + 0.5 + 0 = 0.6$

So, we have created a portfolio with the same beta as portfolio E but a higher expected return. Thus, we can buy portfolio X and sell an equal amount of portfolio E, yielding a profit of

$r_X - r_E = 0.09 + 0.6F - 0.08 - 0.6F = 0.01$

as a sure thing.

4.a. The expected return profit in dollars will be $20,000 on the stocks that the analyst goes long and another $20,000 on the shorts, for a total of $40,000.

The market return will have no effect on the variance of the portfolio because it cancels out in the stocks that the analyst goes long and the stocks that (s)he goes short. So only the idiosyncratic firm returns matter. The variance of the dollar profits from idiosyncratic firm returns is $100000^2 * 0.09$ because $100,000 is invested in each firm (long or short). So the volatility of portfolio returns is

$$100000^2 * 0.09 * 20$$

and the standard deviation of portfolio returns is

$$100,000 * 0.3 * \sqrt{20} = 134,164$$

b. In the same way, the standard deviation of portfolio returns will be

$$\frac{2,000,000}{N} * 0.3 * \sqrt{N}$$

5. (a) The variance of returns on A is $0.8^2 * 0.2^2 + 0.25^2 = 0.0881$
The variance of returns on B is $1^2 * 0.2^2 + 0.1^2 = 0.05$
The variance of returns on C is $1.2^2 * 0.2^2 + 0.2^2 = 0.0976$
(b) The well-diversified portfolio will eliminate idiosyncratic variation. As a result, the mean and variance of the portfolios' excess return will be 0.08 and $0.8^2 * 0.2^2 = 0.0256$, respectively.

For the well diversified portfolio of type B securities, the mean and variance of excess returns are 0.10 and $0.2^2 = 0.04$, respectively.,

For the well diversified portfolio of type C securities, the mean and variance of excess returns are 0.12 percent and $1.2^2 * 0.2^2 = 0.0576$, respectively.

(c) An absence of arbitrage must mean that returns satisfy the equation
$E(r_i) - r_f = \lambda \beta_i$
where β_i is the beta of the i th portfolio. So
$0.08 = 0.8\lambda$
$0.1 = 1\lambda$
$0.12 = 1.2\lambda$
This is consistent with $\lambda = 0.1$

Note on Question 5(b) and (c)
You might have interpreted the column of the table labeled "Expected Return" as referring to "Expected Excess Return". If so, the mean excess returns in (b) are 0.10, 0.12 and 0,14. And then there IS an arbitrage opportunity because there is no way of simultaneously solving the equations
$0.1 = 0.8\lambda$
$0.12 = 1\lambda$
$0.14 = 1.2\lambda$