

**Problem Set 3 for Economics 180.367:
Investments and Portfolio Management
Due at the beginning of class on October 7.**

Note: Point totals are shown at the beginning of each question. It is important to show your work.

1. (50 pts total) Jason Smith works for Alaska Oil Company. His pay next year is tied to the performance of Alaska Oil. In particular, his salary at the end of the year will be:

$$\text{Salary} = 200,000 + 20,000 \times R_{alaska}$$

where R_{alaska} is the return on Alaska Oil Company's stock during the year.

In addition, Jason has \$100,000 in wealth that he can invest. Let us assume that Jason can borrow and lend freely at the riskfree interest rate, which is 0.

The rest of this question asks you to provide intuitive answers with explanations and drawings. No further math is required.

(a) (20 pts) Assume that Jason is risk-averse, and he has mean-variance utility over his end-of-year wealth, and that there are risky assets as well as Alaska oil and the riskfree asset, and that Jason can buy and sell shares of Alaska oil, and all other assets. The returns of these assets do not however necessarily satisfy the CAPM.

Based on this information, what portfolio strategy should Jason follow using the Markowitz model. In other words, explain whether the risk-profile of his wealth is optimal, and how he needs to adjust his portfolio to maximize his utility?

(b) (10 pts) What portfolio strategy should Jason follow if the returns of all assets satisfy the CAPM? How is your answer to this part different from the answer to part 1a?

(c) (20pts) Suppose that Jason's employer does not allow him to buy and sell shares of Alaska oil? If the returns of all assets satisfy the CAPM, how does this trading prohibition affect Jason's utility, and his ability to reach the optimal CAL. If Jason can trade other energy stocks, is there any possibility that his utility would not be affected by this trading prohibition? Under what circumstances would Jason's utility not be affected?

2. (10 pts) Suppose that there are two independent economic factors, F_1 and F_2 . The risk-free rate is 6% and all stocks have independent firm-specific components with a standard deviation of 45%. The following are well-diversified portfolios:

Portfolio	Beta on F_1	Beta on F_2	Expected Return
A	1.5	2.0	31%
B	2.2	-0.2	27%

What is the expected return-beta relationship in this economy?

3. (10 pts) Consider the following data for a one-factor economy. All portfolios are well diversified.

Portfolio	Expected Return	Beta
A	12%	1.2
F	6%	0

Suppose that another portfolio, portfolio E, is well diversified with a beta of 0.6 and an expected return of 8%. Would an arbitrage opportunity exist? If so, what would be the arbitrage strategy?

4. (20 pts) Assume that stock market returns have the market index as a factor, and that all stocks in the economy have a beta of 1 on the market index. Firm-specific idiosyncratic returns all have a standard deviation of 30%.

Suppose that an analyst studies 20 stocks and finds that one half have an alpha of +2%, and the other half have an alpha of -2%. Suppose that the analyst buys \$1 million of an equally weighted portfolio of the positive alpha stocks and shorts \$1 million of an equally weighted portfolio of the negative alpha stocks.

- What is the expected profit (in dollars) and the standard deviation of the analyst's profit?
- How does your answer change if the number of assets is N , where half have an alpha of +2% and half have an alpha of -2%?

5. (10 pts) Assume that security returns are generated by the single index model,

$$R_i = \alpha_i + \beta_i R_m + e_i$$

where R_i is the excess return for security i and R_m is the market excess return. The riskfree rate is 2%. Suppose also that there are three securities: A, B and C, characterized by the following data.

Security	Beta	Expected Return	Idiosyncratic Standard Deviation
A	0.8	10%	25%
B	1	12%	10%
C	1.2	14%	20%

- If the standard deviation of the market excess return is 20%, calculate the variance of returns of securities A, B and C.
- Now assume that there are an infinite number of assets with return characteristics identical to those of A, B and C, respectively. If one forms a well-diversified portfolio of type A securities, what will be the mean and variance of the portfolio's excess returns? What about portfolios composed only of type B or type C stocks?
- Is there an arbitrage opportunity in this market? If so, what is it?