Economics 180.367: Investments and Portfolio Management

Solutions to Problem Set 2

1. This is the variance covariance matrix

	NoDur	Durbl	Manuf	Enrgy	Chems	BusEq	Telcm	Utils	Shops	Hlth	Money	Other
NoDur	21.91	27.35	26.68	17.48	22.75	26.45	14.74	18.74	23.77	21.56	27.34	25.46
Durbl	27.35	60.69	46.03	28.40	37.35	46.82	22.88	27.56	36.39	28.61	42.95	41.21
Manuf	26.68	46.03	46.38	29.67	35.51	45.28	21.35	26.58	33.30	28.80	40.97	41.61
Enrgy	17.48	28.40	29.67	35.99	23.81	28.32	14.19	21.09	20.43	19.45	27.84	27.60
Chems	22.75	37.35	35.51	23.81	34.27	35.74	17.87	22.66	27.42	25.54	33.18	31.66
BusEq	26.45	46.82	45.28	28.32	35.74	58.83	24.18	27.56	35.37	31.69	41.98	41.86
Telcm	14.74	22.88	21.35	14.19	17.87	24.18	21.69	16.79	18.42	16.10	22.92	21.04
Utils	18.74	27.56	26.58	21.09	22.66	27.56	16.79	31.90	21.63	20.36	30.51	25.93
Shops	23.77	36.39	33.30	20.43	27.42	35.37	18.42	21.63	34.46	24.91	33.81	31.57
Hlth	21.56	28.61	28.80	19.45	25.54	31.69	16.10	20.36	24.91	32.92	29.98	27.42
Money	27.34	42.95	40.97	27.84	33.18	41.98	22.92	30.51	33.81	29.98	48.02	39.91
Other	25.46	41.21	41.61	27.60	31.66	41.86	21.04	25.93	31.57	27.42	39.91	44.67

and the means, standard deviations and Sharpe ratios

	Mean	StandardDeviation	Sharpe Ratio
NoDur	0.977	4.683	0.1617
Durbl	1.093	7.794	0.1120
Manuf	1.033	6.814	0.1194
Enrgy	1.071	6.002	0.1418
Chems	1.041	5.857	0.1401
BusEq	1.098	7.674	0.1144
Telcm	0.838	4.660	0.1326
Utils	0.879	5.651	0.1167
Shops	0.975	5.873	0.1286
Hlth	1.070	5.740	0.1480
Money	1.000	6.933	0.1126
Other	0.828	6.687	0.0909

2. The highest expected return is from business equipment. The highest Sharpe Ratio (for the CAL) is from energy.

3. You are minimizing $SD[r_P] = \sqrt{W'\Sigma W}$ subject to the constraint that the weights sum to one ($W'i = 1$, where i is a vector of ones). The solution has the following weights

NoDur	0.692872
Durbl	-0.03494
Manuf	-0.24322
Enrgy	0.232171

Chems	0.098076
BusEq	-0.08979
Telcm	0.511886
Utils	0.143792
Shops	0.044747
Hlth	0.057631
Money	-0.42225
Other	0.009028

and has mean of 0.885 and standard deviation of 3.589.

4. The weights are

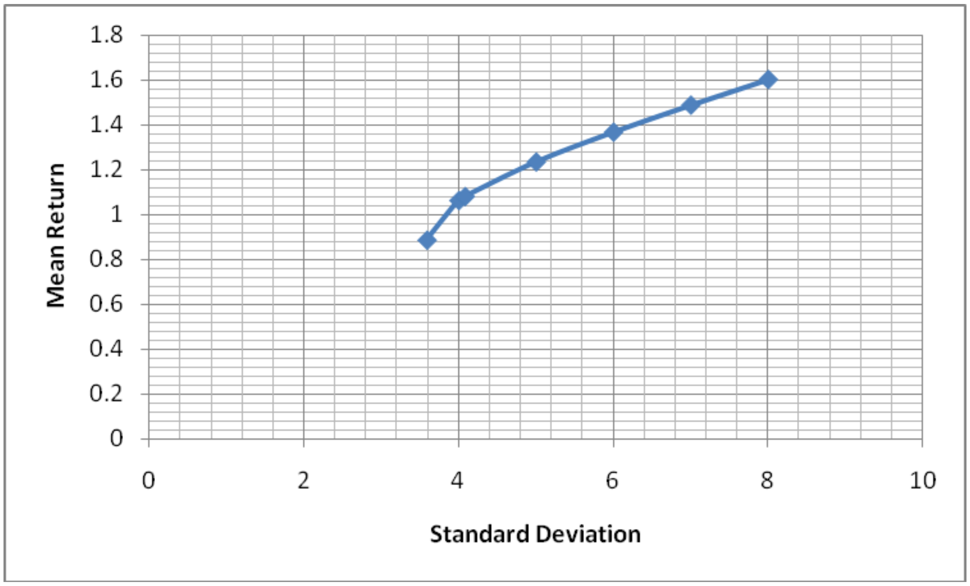
NoDur	0.831727
Durbl	0.054971
Manuf	-0.07348
Enrgy	0.39293
Chems	0.083254
BusEq	0.062205
Telcm	0.316617
Utils	-0.00931
Shops	-0.03438
Hlth	0.198702
Money	-0.27185
Other	-0.55137

and the mean and standard deviation are 1.080 and 4.082, respectively. To get this, we maximize $\frac{W'\mu - 0.22}{\sqrt{W'\Sigma W}}$ subject to the constraint $W'i = 1$.

5. The solutions give the following points on the efficient frontier (including the minimum variance and tangency solutions)

Standard Deviation	Mean
3.589239	0.885168
4	1.062315
4.081597	1.080153
5	1.234418
6	1.367559
7	1.488125
8	1.602485

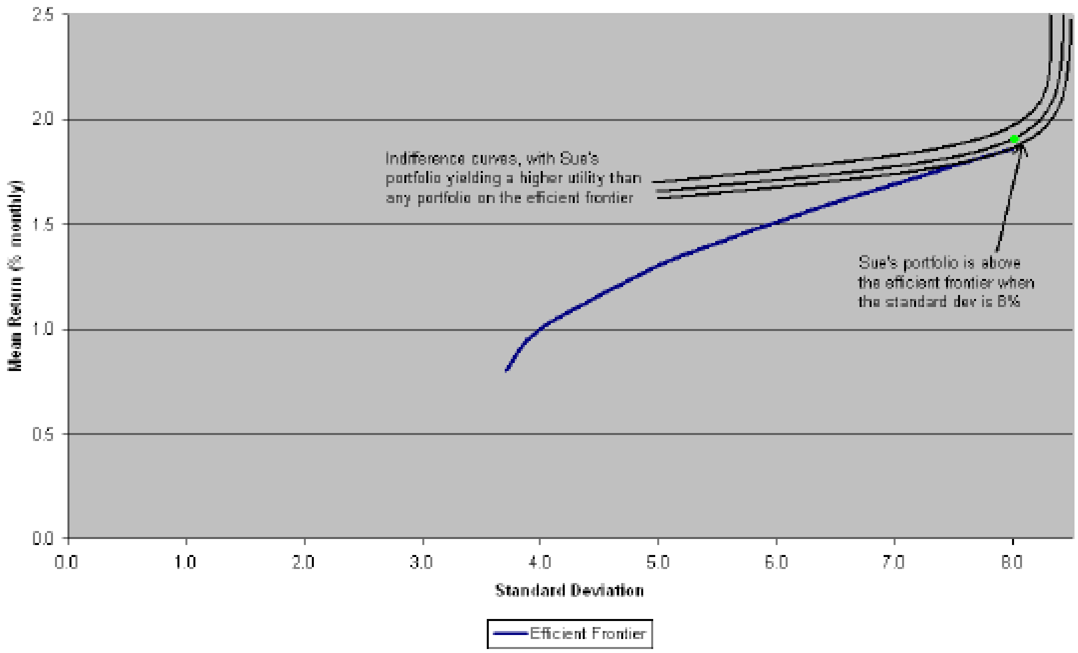
Ploting them gives the following graph



6. The optimal CAL is defined as all portfolios on the ray extending from the risk free asset through the optimal risky portfolio. The optimal risky portfolio is the portfolio that generates the highest Sharpe ratio among feasible portfolios (those whose weights sum to 1). It is clear that the intercept of the CAL will be the risk free net return, in this case 0.22. As described above, the slope will be the Sharpe ratio of the optimal portfolio. In our case, this is 0.211. Thus, the equation will be

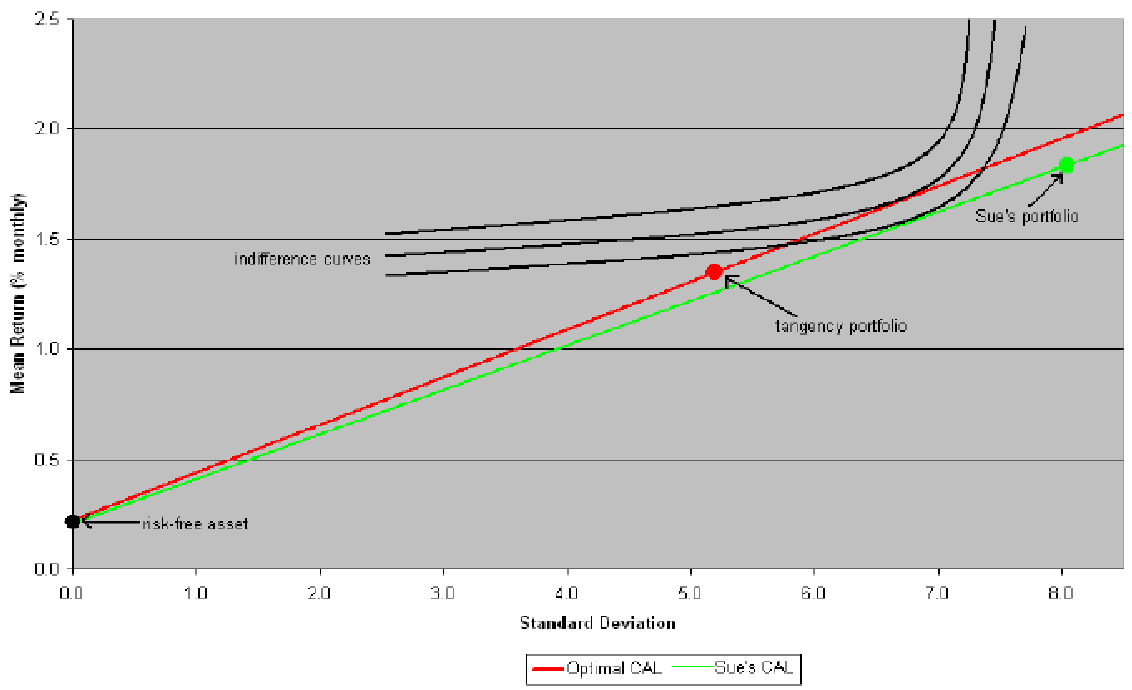
$$\mu = 0.22 + 0.211\sigma$$

7. Yes, there are some investors who should invest in Sue's portfolio rather than a portfolio composed of the 11 risky assets. We can see this because Sue's portfolio has a higher expected mean return than the efficient frontier when the standard deviation of net return is 8% (Sue's 1.8 versus the efficient frontier's 1.602). Without Sue's portfolio available to select, an investor will choose the portfolio on the efficient frontier that is tangent to some indifference curve. It is possible to construct typical indifference curves that yield a higher utility for Sue's portfolio than for the indifference curve -
efficient frontier tangency portfolio. One such example can be seen in the chart below. Note that not all investor's will prefer Sue's portfolio to one that we can construct from the 12 risky assets, but some will.



8. No, there are no investors that will want to purchase a combination of Sue's portfolio and a risk free asset rather than a combination of one of our portfolios and the same risk free asset. Any investor who combines the 12 risky assets with the risk free asset will want to have a portfolio that is on our capital accumulation line (red).

If an investor were to purchase Sue's portfolio and combine it with risk free assets, he would be somewhere on the green line. For any given standard deviation of return, our optimal CAL has a higher mean return than Sue's portfolio's CAL. Thus any investor would prefer to be on the optimal CAL rather than 'Sue's line', assuming he likes higher returns and lower risk. This can also be seen by the fact that Sue's portfolio has a Sharpe ratio of $\frac{1.8-0.22}{8} = 0.1975$. This is less than the Sharpe ratio for our optimal risky portfolio, 0.211, thus the optimal CAL is superior.


2.5
2.0
1.5
1.0
0.5
0.0
Mean Return (% monthly)
indifference curves
tangency portfolio
Sue's portfolio
risk-free asset
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
Standard Deviation
Optimal CAL
Sue's CAL