Economics 180.367: Investments and Portfolio Management

Solutions to Problem Set I

1. Using simple compounding, a risk-free asset will produce a net return of 2 percent this year, and a net return of 2.5 percent per year over the following two years. Based on this information, if I invested $ 100 in the risk-free asset on January 1, 2008, then how much money will it have grown to after a period of 3 years.

Solution:
At the end of the first year, I will receive $100*(1+0.02)
At the end of the second year, I will receive $100*(1+0.02)*(1+0.025)
At the end of the third year, I will receive $100*(1+0.02)*(1+0.025)*(1+0.025)
So, at the end of three years, I will receive $100*(1 + 0.02)*(1 + 0.025)*(1+0.025) = $107.16

2. Provide an answer to question 1, if continuous compounding is used instead of simple compounding. Show your work.
Solution: (use the formula for continuous compounding, given in class)

At the end of the first year, I will receive $100e^{0.02}$
At the end of the second year, I will receive $100e^{0.02}e^{0.025}$
At the end of the third year, I will receive $100e^{0.02}e^{0.025}e^{0.025}$
So, at the end of three years, I will receive a total of $\$100e^{0.02}e^{0.025}e^{0.025}$ = $107.25

3. The bill yield was 2.825 percent. The yield and price are related via the formula

$$\frac{10000-P}{10000}*\frac{360}{72}=y$$

With the yield of 0.02825, the solution for the price is $9,943.50.

4. XYZ corporation has a current stock price of $ 50. Over the next year its price will rise 10% with probability 0.6, and it will fall 5% with probability 0.4. In addition it will pay a dividend of $ 1 at the end of the next year.

(a) Compute the mean and variance of the net holding period return on XYZ over the next year.

Solution: We have two states here; State 1 in which the price of stock will rise 10%, and State 2 in which the price of stock will fall 5%.

Since the price of stock is $50, then if State 1 occurs a 10% increase of the current price of stock ($P0$) yields a new price ($P1$) of $50(1+0.10) = $55

And if state 2 occurs, then a 5% decrease of the current price of stock ($P0$) yields a new price ($P2$) of $50(1-0.05) = $47.50

Let D=the dividend per share paid out by the company. Then, if State 1 occurs, the gross holding period return is:

$$R_1 = \frac{P_1 + D}{P_0} = \frac{55+1}{50} = 1.12$$

So, the net holding period return is $r_1 = R_1 - 1 = 1.12 - 1 = 0.12$

If State 2 occurs, the gross holding period return is:

$$R_2 = \frac{P_2 + D}{P_0} = \frac{47.5+1}{50} = 0.97$$

So, the net holding period return is $r_2 = R_2 - 1 = 0.97 - 1 = -0.03$

The mean of the net holding period return is therefore:

$$\mu_r = E(r) = \Sigma_{s=1}^{2} \pi(s) r(s) = (0.6)(0.12) + (0.4)(-0.03) = 0.06$$

The variance of the net holding period return is:

$$\sigma_r^2 = \Sigma_{s=1}^{2} \pi(s)[r(s) - \mu_r]^2 = (0.6)(0.12 - 0.06)^2 + (0.4)(-0.03 - 0.06)^2 = 0.0054$$

(b) If Bob Smith invests his wealth $ 10,000 in the stock of XYZ, compute Bob's expected wealth at the end of one year and the standard deviation of his wealth.

Solution: Let W_0 be his initial wealth and let W_1 be his wealth after one year. Let r be the return.

Bob's expected wealth = $E(W_1) = \$W_0(1 + \mu_r) = \$10{,}000(1 + 0.06) = \$10{,}600$

The variance of Bob's wealth is:

$$Var(W_0(1+r)) = Var(W_0 + W_0 r) = Var(W_0 r) = W_0^2 Var(r) = 10{,}000^2 * 0.0054 = 540{,}000$$

So, the standard deviation of his wealth = $(540{,}000)^{1/2}$=734.847.

(c) Suppose Bob has utility function:

$$U(W) = W^{1/3}$$

Compute Bob's expected utility if he pursues the above investment strategy, and the certainty equivalent of the wealth that he receives by following the above investment strategy.

Solution:
If State 1 occurs, Bob will receive a total of $\$10{,}000(1 + 0.12) = \$11{,}200$

If State 2 occurs, Bob will receive a total of \$10,000(1 − 0.03) = \$9,700
So, Bob's expected utility = $EU(W_1) = (0.6*11{,}200^{1/3})+(0.4*9{,}700^{1/3}) = 21.955$

If $U(W_{CE}) = 21.955$, then $W_{CE} = \$21.955^3 = \$10{,}582.79$

So, Bob should receive (approximately) \$10,582.79 to obtain the same utility that he would get from investing in the risky asset.

(d) Suppose the risk-free rate is 2% with simple compounding. If Bob invests all of his wealth in the risk-free asset for 1 year, will he achieve a higher expected utility than if he invested all of his wealth in XYZ for 1 year. Show your work.

Solution:
With 2% simple compounding, at the end of one year, Bob's total wealth = \$10,000*(1 + 0.02) = \$10,200
So, the expected utility from investing in the risk-free asset, $EU(W) = 10{,}200^{1/3}=21.687$

Clearly, this is less than the amount 21.955, the expected utility of the risky asset.
So, Bob will achieve a lower expected utility with the risk-free asset.

5. Idaho foods is a potato producer, and Excela is a producer of luxury cars. The returns of these companies over the next year depend on whether economic growth is strong, moderate, or weak. The returns over the next year, in the various states, are given in the table below.

Probability Distribution of Gross Returns

State	Probability	Idaho Foods	Excela
Weak Growth	0.4	1.1	0.9
Moderate Growth	0.4	1.05	0.95
Strong Expansion	0.2	0.98	1.2

(a) Based on the table, compute the mean, variance, covariance, and correlation of the returns on Idaho foods and Excela. Show your work.

Solution:
We have three States here; State 1 in which there is weak growth, State 2 in which there is moderate growth, and State 3 in which there is strong growth.

First consider Idaho Foods. The mean of the gross returns on Idaho Foods is:

$$\mu_I = 0.4*1.10 + 0.4*1.05 + 0.2*0.98 = 1.056$$

The variance of the gross returns on Idaho Foods is:

$$0.4*(1.10-1.056)^2 + 0.4*(1.05-1.056)^2 + 0.2*(0.98-1.056)^2 = 0.001944$$

{note: if you did these as percentage returns, the answers would be 5.6% and 19.44%).

Now consider Excela Foods. The mean of the gross returns on Excela Foods is:

$$0.4*0.9+0.4*0.95+0.2*1.2=0.98$$

The variance of the gross returns on Excela Foods is:

$$0.4*(0.9-0.98)^2+0.4*(0.95-0.98)^2+0.2*(1.2-0.98)^2=0.0126$$

The covariance of the returns on Idaho Foods and Excela Foods is:

$$0.4*(1.10-1.056)(0.9-0.98)+0.4*(1.05-1.056)*(0.95-0.98)+0.2*(0.98-1.056)*(1.2-0.98)$$
$$=-0.00468$$

The correlation of the returns on Idaho Foods and Excela Foods is:

$$\frac{-0.00468}{\sqrt{0.001944*0.0126}}=-0.9456$$

(b) Based on your answers to 4a, suppose that Cassandra Jenrette invests 30 percent of her wealth in Idaho foods, and she invests 70 percent of her wealth in Excela. Based on this information, compute the mean return on her portfolio, and the standard deviation of the return on her portfolio. Use simple algebra to compute your answers. Show the formulas that you use.

Solution:
The mean gross return on her portfolio = $\mu_C = 0.3\mu_I + 0.7\mu_E = 0.3(1.056) + 0.7(0.98) = 1.0028$
The variance of the gross return on her portfolio is
$\sigma_C^2=0.3^2*0.001944+0.7^2*0.0126+2*0.3*0.7*(-0.00468)=0.00438$
So, the standard deviation is $(0.00438)^{1/2} = 0.0662$

(c) Provide the same answers to the previous question, but this time, express your answers using matrix algebra. In your answer, show the vector of portfolio weights, the vector of mean returns, and the variance covariance matrix of the returns on the two assets. Finally, show the formulas that you would use to compute the answer. Note: you don't have to compute the answers again.

Solution:
The vector of portfolio weights is:

$$\begin{pmatrix} 0.3 \\ 0.7 \end{pmatrix}$$

The vector of mean returns is:

$$\begin{pmatrix}\mu_I\\ \mu_E\end{pmatrix}=\begin{pmatrix}1.056\\ 0.98\end{pmatrix}$$

To compute the mean return on Cassandra's portfolio, we multiply the transpose of the portofolio weight vector and the mean returns vector as follows:

$$\begin{pmatrix}0.3 & 0.7\end{pmatrix}\begin{pmatrix}1.056\\ 0.98\end{pmatrix}=1.0028$$

The variance-covariance matrix of returns is

$$\begin{pmatrix}0.001944 & -0.00468\\ -0.00468 & 0.0126\end{pmatrix}$$

To compute the variance of the return on her portfolio, we use the following formula:

$$\begin{pmatrix}0.3 & 0.7\end{pmatrix}\begin{pmatrix}0.001944 & -0.00468\\ -0.00468 & 0.0126\end{pmatrix}\begin{pmatrix}0.3\\ 0.7\end{pmatrix}=0.00438$$

(d) Using matrix algebra compute the covariance between the return on Cassandra's portfolio and a portfolio consisting only of Idaho foods. (Hint: the second portfolio has a weight of 1 on Idaho foods, and 0 on Excela). Show your work.

Solution:

$$\begin{pmatrix}0.3 & 0.7\end{pmatrix}\begin{pmatrix}0.001944 & -0.00468\\ -0.00468 & 0.0126\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix}=-0.00269$$

(e) (6 pts) Bob Jenrette is more risk averse than Cassandra. Assume the net riskfree rate of return with simple compounding is 1 percent per year. Suppose Bob invests 60 percent of his wealth in Cassandra's portfolio and 40 percent in the risk-free asset. Then compute the mean and standard deviation of the return of Bob's portfolio. Also, compute the proportion of Bob's wealth that he invests in Idaho foods, and Excela.

Solution:
The mean of the gross return of Bob's portfolio = μ_B = 0.6(μ_C)+0.4(1+0.01) = 0.6(1.0028) + 0.4(1.01) = 1.00568

Note that the risk-free asset has zero variance. Hence, the variance on its return is zero. So, the variance of the gross return on Bob's portfolio =

$$\sigma_B^2 = 0.6^2 * 0.00438 + 0.4^2 * 0 = 0.001578$$

The standard deviation is = $(0.001578)^{1/2}$ = 0.0397

The proportion of Bob's wealth invested in Idaho foods = 0.6*0.3 = 0.18 or 18%

The proportion of Bob's wealth invested in Excela foods = 0.6*0.7 = 0.42 or 42%