

Formula Sheet for Econ 367 Final

- $APR = n * \{ [1 + EAR]^{1/n} - 1 \}$
- Compounding. $V(0)$ today is worth $V(n) = V(0) * \exp(rn)$ in n periods. Hence $r = \frac{\ln(V(n)) - \ln(V(0))}{n}$
- Present Value: $P = \frac{C(1)}{1+r} + \frac{C(2)}{(1+r)^2} \dots + \frac{C(T)}{(1+r)^T}$
- Suppose that X_1, X_2, \dots, X_n are random variables and k_1, k_2, \dots, k_n are constants. Then $E(\sum k_j X_j) = \sum k_j E(X_j)$ and $Var(\sum k_j X_j) = \sum_{i=1}^n \sum_{j=1}^n k_i k_j Cov(X_i, X_j)$
- Minimum Variance Portfolio weights (2 assets): $w_D = \frac{\sigma_E^2 - Cov(R_D, R_E)}{\sigma_E^2 + \sigma_D^2 - 2Cov(R_D, R_E)}$ and $w_E = 1 - w_D$
- Maximum Sharpe Ratio Portfolio weights (2 assets): $w_D = \frac{E(R_D)\sigma_E^2 - E(R_E)Cov(R_D, R_E)}{E(R_D)\sigma_E^2 + E(R_E)\sigma_D^2 - [E(R_D) + E(R_E)]Cov(R_D, R_E)}$
and $w_E = 1 - w_D$ where R_D and R_E are excess returns over the riskfree rate
- Optimal complete portfolio. If an investor has a utility function $E(r) - \frac{A}{2}\sigma^2$ then the weight in the tangent portfolio will be $\frac{E(r_p) - r_f}{A\sigma_p^2}$ where r_p is the return on the tangent portfolio and σ_p^2 is its variance.
- In matrix notation, if w is the vector of weights, μ is the vector of expected excess returns, Σ is the variance-covariance matrix, and i is a vector of ones then

$$\text{Minimum Variance: } w = \frac{\Sigma^{-1}i}{i'\Sigma^{-1}i}$$

$$\text{Maximum Sharpe Ratio: } w = \frac{\Sigma^{-1}\mu}{i'\Sigma^{-1}\mu}$$

- Factor Model

$$r_i = E(r_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \dots + \beta_{ik}F_k + e_i$$

$$E(r_i) = r_f + \lambda_1\beta_{i1} + \lambda_2\beta_{i2} \dots + \lambda_k\beta_{ik}$$

- Convexity

$$\text{Definition of Convexity: } Convexity = \frac{1}{P * (1+y)^2} \sum_{t=1}^n \left[\frac{CF_t}{(1+y)^t} (t^2 + t) \right]$$

$$\text{Correction for Convexity: } \frac{\Delta P}{P} = -D_{MOD} * \Delta y + \frac{1}{2} [Convexity * (\Delta y)^2]$$

- Black Scholes

Formula for call and put options without dividends

$$C_0 = S_0 N(d_1) - X e^{-rT} N(d_2)$$

$$P_0 = X e^{-rT} [1 - N(d_2)] - S_0 [1 - N(d_1)]$$

$$d_1 = [\ln(S_0 / X) + (r + \sigma^2 / 2)T] / \sigma T^{1/2}$$

$$d_2 = d_1 - (\sigma T^{1/2})$$

Formula for call and put options with dividend yield of d

$$C_0 = S_0 e^{-dT} N(d_1) - X e^{-rT} N(d_2)$$

$$P_0 = X e^{-rT} [1 - N(d_2)] - S_0 e^{-dT} [1 - N(d_1)]$$

$$d_1 = [\ln(S_0 / X) + (r - d + \sigma^2 / 2)T] / \sigma T^{1/2}$$

$$d_2 = d_1 - (\sigma T^{1/2})$$

Tables of the Standard Normal Distribution



**Probability Content
from $-\infty$ to Z**

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990