Final Exam

Fall 2010 Econ 180-367

Closed Book.
Formula Sheet Provided. Calculators OK.
Time Allowed: 2 hours
Please write your answers on the page below each question

1. (20 points) Suppose that I buy both put and call European at-the-money options on a stock, which is trading at $100. The put option costs $10. The stock pays no dividends and the risk-free interest rate is zero.
(a) What is the price of the call option (given put-call parity)?
(b) For what range of stock prices at the expiration of the options contracts will I make money (after taking account of the cost of the options)?

2. (25 points) Suppose that a stock is trading for $30. Its volatility is 0.4. The stock pays no dividends and the risk-free interest rate is zero. You buy an at-the-money European call option on this security with an expiration of 1 year hence.
(a) According to the Black-Scholes formula, what should this call option be worth?
(b) What is the delta of this option (i.e. how many stocks would you need to buy to hedge it)?

3. (10 points) Explain precisely what the LIBOR-OIS spread is.

4. (15 points) Explain the arbitrage strategy that the hedge fund Magnetar used to exploit perceived mispricing of Collateralized Mortgage Obligations, as we discussed in class.

5. (20 points) The price of a stock today is $100. Assume that in one year's time, it will either go up to $110 or down to $90. The risk-free interest rate is 5 percent.
(a) Find the price of a put option with a strike price of $100.
(b) Does the answer in (a) depend on whether the investor is risk-neutral, risk-averse, or risk-seeking? Why or why not?

6. (15 points) Normally ten-year Treasury yields are *below* comparable maturity swap rates. However, in the fall of 2008, ten-year Treasury yields were *above* swap rates. Why did this happen?

7. (20 points) Sue's utility function is $w^{1/2}$ where w is wealth in dollars. Sue is offered an asset which has a 30% chance of being worth $100 and a 70% chance of being worthless.
(a) What is the certainty equivalent of this gamble for Sue?
(b) If Sue's initial wealth is $1, what is her coefficient of risk aversion?

8. (10 points) A perpetuity with no default risk pays $5 a year in coupons for ever. Its current yield (annual compounding) is 10 percent. What is the price of this perpetuity?

9. (15 points) (a) Give two examples of a fixed income security with negative convexity.
(b) Two counterparties enter into a standard interest rate swap contract. A pays B a fixed annualized rate of 4 percent on a notional underlying principal of $1 million. B pays A the floating rate every 6 months. Today the 6 month interest rate is 3 percent. What money changes hands between A and B? In 6 months' time, the 6 month interest rate has risen to 5 percent. Now what money changes hands between A and B?

10. (20 points) Suppose that there are three possible values for the price of Google stock next year: $70, $80 and $90. Google pays no dividends. The price of an option to buy at $80 is $4. The price of an option to buy at $70 is $12.
(a) Find the risk-neutral probability density of Google's share price.
(b) How much would a risk-neutral investor pay for an option to buy a share in Google at $75?

11. (10 points) A ten-year zero coupon bond with a maturity value of $100 trades has an annual yield with annualized compounding of 7 percent. After one year, the yield falls to 6 percent (and the bond is then a nine-year bond). Calculate the holding period return on this bond.

12. (20 points)
(a) Suppose that the CAPM holds. Stock XYZ has a beta with respect to the market of 2. The expected market return is 7 percent, and the riskfree rate is 2 percent. What is the expected return on XYZ stock?
(b) To test the CAPM, researchers have run cross-sectional regressions of the return for stock i (r_i) onto the book-to-market ratio of that stock (BM_i), the size of that stock (S_i) and the beta of that stock (β_i):

$$r_i = \gamma_0 + \gamma_1 BM_i + \gamma_2 S_i + \gamma_3 \beta_i + \varepsilon_i$$

According to the CAPM, what should the coefficients γ_0, γ_1, γ_2 and γ_3 be?
(c) When the regression in (b) was run by Fama and French and other researchers, what results did they find?

Solutions

1. (a) \$10
(b) Below \$80 or above \$120.

2. (a) \$4.76
(b) N(0.2)=0.579

3. The OIS rate is the swap rate for a fixed rate of a few months versus the overnight federal funds rate. The LIBOR rate is the rate for unsecured borrowing between banks. Because it is a swap, there is no principal at risk in the OIS contract. The LIBOR-OIS spread measures the difference between these two, and reflects credit and liquidity.

4. Buying the junior tranche and shorting the senior tranche. The coupons offset each other so the strategy was fully self-financing. The strategy made money as long as the default rate was very high or very low. As it turned out, the default rate was high, and the strategy was profitable.

5. (a) Let's first think of the price of a call option with a strike of \$100. My replicating portfolio will buy one stock and borrow \$90/1.05. The cost of this portfolio is \$14.285. If the stock price falls, the replicating portfolio will be worthless. If it rises, it will be worth \$20. Meanwhile, the call option will be worthless if the stock price falls, and worth \$10 otherwise. So the value of the call option is \$7.14.

By put-call parity, the price of the put option satisfies:

$$7.14+\frac{100}{1.05}=100+P$$

and so $P=2.38$.

There's another more direct way of doing it. I construct a portfolio to replicate the put option directly. My replicating portfolio will short one stock and invest \$110/1.05 at the risk-free rate. The cost of this portfolio is \$4.76. If the stock price rises, the replicating portfolio will be worthless. If it falls, it will be worth \$20. Meanwhile the put option will be worthless if the stock price rises and worth \$10 otherwise. So the value of the put option is \$2.38.

(b) No it doesn't matter, because under the given assumptions, the option is priced by constructing a replicating portfolio. The replicating portfolio and option give two ways of getting exactly equivalent payoffs, and so must be priced the same.

6. Lehman had entered into swap contracts where Lehman received fixed and payed floating. The counterparties payed fixed. Once these contracts were effectively voided by the collapse of Lehman, there was demand for swap contracts that payed fixed. To clear the market, the fixed interest rate had to drop.

7. (a) $x^{1/2}=0.3*(100^{1/2})=3$, so the certainty equivalent is just \$9.

(b) Risk-aversion is $-\frac{u''(w)}{u'(w)}=-\frac{-0.25w^{-3/2}}{0.5w^{-1/2}}=\frac{1}{2w}$. So risk aversion is 0.5.

8. \$50.

9. (a) A callable bond, or a mortgage-backed security.
(b) Today A pays B 0.5*$1million$0.01=$5,000.
In 6 months' time, B pays A 0.5*$1million$0.01=$5,000.

10. (a) The probabilities of $70, $80 and $90 are 0.2, 0.4 and 0.4, respectively.
(b) The option at a strike of $75 has a payoff of $0, $5 and $15 if the stock is worth $70, $80 or $90 respectively. So its risk-neutral value is (0.4*5)+(0.4*15)=$8.

11. The price today is $50.83. The price in one year is $59.19. The holding period return is 16.4 percent.

12. (a) 12 percent.
(b) γ_0 should be the risk-free rate, γ_1 and γ_2 should be zero, and γ_3 should be the expected market excess return.
(c) γ_1 is positive, γ_2 is negative, and γ_3 is smaller than the average market excess return.