

## Final Exam

Fall 2021

Econ 180-367

Closed Book.

Formula Sheet Provided. Calculators OK.

Time Allowed: 3 hours

1. (5 points) A stock that pays no dividends trades at \$100. A European at-the-money put option on this stock costs \$10. The risk-free interest rate is zero. What is the price of an at-the-money European call option on that stock with the same maturity date?

2. (10 points) Suppose that a stock is trading for \$30. Its volatility is 0.4. The stock pays no dividends and the risk-free interest rate is zero. You buy an at-the-money European call option on this security with an expiration of 1 year hence.

(a) According to the Black-Scholes formula, what should this call option be worth?

(b) What is the delta of this option?

3. (10 points) The price of a stock today is \$100. There are four possible paths for its price over the next two years:

(i) It could go up to \$120 in one year and up again to \$140 in two years.

(ii) It could go up to \$120 in one year and back down to \$100 in two years.

(iii) It could go down to \$80 in one year and back up to \$100 in two years.

(iv) It could go down to \$80 in one year and down again to \$60 in two years.

The riskfree rate is 5 percent per year. Find the price of a European call option with a strike price of \$100 maturing in two years' time.

4. (5 points) Sue's utility function is  $w^{1/2}$  where  $w$  is wealth in dollars. Sue is offered an asset which has a 30% chance of being worth \$100 and a 70% chance of being worthless. What is the certainty equivalent of this gamble for Sue?

5. (5 points) In February 2020, you entered a short position on a September 2021 Eurodollar futures at a price of \$98.50. At the settlement date in September 2021, the following interest rates were observed (all in percentage points):

Federal Funds Rate	0.10
3 month LIBOR	0.30
1 year LIBOR	0.50
3 month Treasury bill rate	0.20

Do you make or receive a payment? How much?

6. (5 points) Suppose that the price of oil in one year will be an integer. Suppose that there are European options on oil maturing in one year at strikes of \$79, \$80 and \$81, with prices of \$6.60, \$6.30 and \$6.10, respectively. Options traders are risk neutral. What is the probability of the oil price being exactly equal to \$80 in one year?

7. (5 points). Please rank the following four bonds in order of **descending** duration:

Bond	Coupon (%)	Years to Maturity	Yield (%)
A	15	20	10
B	0	25	8
C	0	20	10
D	8	20	10

8. (5 points). The spot exchange rate between the US \$ and the South African Rand is \$1=16 Rand. One-year interest rates are 0.4 percentage points in the US and 4 percentage points in South Africa. According to covered interest parity, what should the one-year forward US\$-South African Rand exchange rate be? For full credit, you must express the answer as Rand per US dollar, as is the market convention for this currency pair.

9. (10 points). Suppose that there are three possible outcomes for the returns on two stocks, Cheese Online and Remsen Tech:

Scenario	Cheese Online	Remsen Tech
1	10%	20%
2	10%	5%
3	-8%	5%

The three scenarios are equally likely.

(a) What is the **variance** of the returns of Cheese Online and Remsen Tech?

(b) What is the **correlation** between the returns of Cheese Online and Remsen Tech?

10. (5 points). Consider a one-factor economy. Portfolio A is well diversified with an expected return of 8% and a beta of 1. Portfolio B is well diversified with an expected return of 7% and a beta of 0.5. The riskfree rate is 2%. Find an arbitrage opportunity. Please be very specific about what assets you go long/short and in what amounts. No credit for vague or ambiguous answers.

11. (5 points). A bond has a 10% annual coupon rate, paying coupons twice a year. The bond matures in two years. The yield to maturity is 1% (annual rate with semiannual compounding). The price is \$117.78. What is the duration of this bond (in years)?

12. Multiple choice questions (2 points each)

(i) What is the name given to a strategy of buying one call option at strike price  $X_1$  and selling a call option a different strike price  $X_2$  with the same underlying and the same maturity date?

- A. Straddle.
- B. Risk Reversal.
- C. Horizontal Spread.
- D. Vertical Spread.
- E. Butterfly

(ii) Which of the following best describes the carry trade?

- A. Borrowing in low interest currencies and investing in high interest currencies.
- B. Borrowing in high interest currencies and investing in low interest currencies.
- C. Buying a currency in the spot market and selling it in the forward market.
- D. Selling a currency in the spot market and buying it in the forward market.
- E. Investing in cargo shipping firms.

(iii) By market convention, which of the following currencies is quoted with the dollar as the base currency?

- A. Swiss Franc.
- B. Euro.
- C. British Pound.
- D. Australian Dollar.
- E. New Zealand Dollar.

(iv) Suppose that the spot oil price is \$70 and the futures price for delivery in December 2022 is \$80. Which of the following terms best describes this situation?

- A. Backwardation.
- B. Forwardation.
- C. Contango.
- D. Novation.
- E. Isomerization.

(v) A preferred stock will pay a dividend of \$2.75 in the upcoming year and every year thereafter; i.e., dividends are not expected to grow. You require a return of 10% on this stock. According to the constant growth dividend discount model, the value of this preferred stock is:

- A. \$0.275
- B. \$27.50
- C. \$31.82
- D. \$56.25
- E. \$275.00

(vi) Which of the following statements are true about the difference between futures and forward contracts?

- A. Futures contracts are traded over the counter, forward contracts are traded on exchanges.
- B. Forward contracts are marked to market daily, whereas futures contracts are not.
- C. Forward contracts require the price for the goods to be paid when the contract is entered into, whereas futures contracts require the price for the goods to be paid at the time of delivery.
- D. Futures contracts require the price for the goods to be paid when the contract is entered into, whereas forward contracts require the price for the goods to be paid at the time of delivery.
- E. Futures contracts are guaranteed by a clearinghouse, whereas forward contracts are not.

(vii) Hopkins hedge fund returns have an expected value of 6 percent per annum with a standard deviation of 8 percent. The market return has an expected value of 8 percent per annum with a standard deviation of 16 percent. The riskfree rate is 1 percent. What is the percent  $M^2$  of the Hopkins hedge fund?

- A. -4
- B. -3
- C. -2
- D. 3
- E. 4.

(viii) A Treasury bill with 180 days to maturity trades for \$9,900. It's face value is \$10,000. What is the quoted interest rate?

- A. 0.5 percent.
- B. 1 percent.
- C. 1.01 percent.
- D. 2 percent.
- E. 2.02 percent.

(ix) The nine-year zero coupon yield is 2 percent and the corresponding ten-year yield is 2.2 percent. What is the nine-to-ten year forward rate? Note: All rates are quoted with annual compounding.

- A. 1 percent.
- B. 2.2 percent.
- C. 3.02 percent.
- D. 4.02 percent.
- E. 5.02 percent.

(x) Which of the following is true of TIPS securities?

- A. The coupons are indexed to inflation, but the principal is not.
- B. The principal is indexed to inflation, but the coupon is not.
- C. The coupons and principal are both indexed to inflation.
- D. TIPS pay no coupons and the principal is indexed to inflation.
- E. TIPS are perpetual securities where the principal is never paid back.

(xi) Researchers have found that in the weeks after a firm has announced stronger-than-expected earnings, the stock outperforms other stocks. Which of the following statements is most accurate?

- A. This is evidence against the weak form of the efficient markets hypothesis.
- B. This is evidence against the semi-strong form of the efficient markets hypothesis.
- C. This is evidence against all forms of the efficient markets hypothesis.
- D. This is evidence for the strong form of the efficient markets hypothesis.
- E. This is evidence for all forms of the efficient markets hypothesis.

(xii) What is meant by the implied volatility of the S&P 500 index?

- A. The actual volatility of the S&P500, computed from historical prices.
- B. The volatility that would set actual options prices equal to their Black-Scholes implied values.
- C. The volatility of the S&P500 implied by the volatilities of indices in other countries.
- D. The volatility of the S&P 500 that is implied by the yield on Treasury securities.
- E. The volatility of the S&P 500 that is implied by stock index futures prices.

(xiii) A two-year CDS contract requires the buyer of credit protection on Risky Bank to pay 1 percent of the notional underlying every year. The recovery rate in default is 50 percent. If the investor is risk neutral what is the approximate probability of a default of Risky Bank at some point in the next two years?

- A. 0.5 percent.
- B. 1 percent.
- C. 2 percent.
- D. 3 percent.
- E. 4 percent.

(xiv) The gold spot price is \$1,800 per ounce and the one-year risk free rate is 1 percent. Assuming that gold is costless to store, what is the one year futures price of an ounce of gold?

- A. \$1,764
- B. \$1,782
- C. \$1,800
- D. \$1,818
- E. \$1,836

(xv) A firm's profits go down by \$200,000 for every 5 cent rise in the pound/dollar exchange rate. Each pound futures contract calls for the delivery of 62,500 pounds. If the firm wishes to use the futures market to exactly hedge the exchange rate exposure, what should it do?

- A. Go short 64 futures contracts.
- B. Go short 32 futures contracts.
- C. Go short 1 futures contract.
- D. Go long 32 futures contracts.
- E. Go long 64 futures contracts.

### Solutions and Grading Rubric

1. From put call parity it is \$10. 3 points if set up correctly but algebra mistake. No credit if you leave the answer as an equation to be solved.

2. (a) From the Black Scholes formula

$$d_1 = \frac{0.4^2}{2} / 0.4 = 0.2$$

$$d_2 = -0.2$$

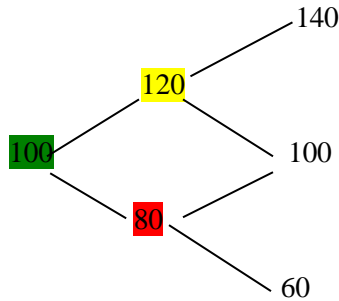
From the normal tables,  $N(d_1) = 0.5793$  and  $N(d_2) = 0.4207$

Hence the call options price is  $C_0 = 30 * 0.5793 - 30 * 0.4207$  which is \$4.76.

6 points for part (a). 2 points off for an algebra mistake. 3 points off for failing to look up normal tables correctly or mixing up standard deviation and variance or otherwise entering incorrect numbers into the formula.

(b)  $N(d_1) = 0.5793$ . 4 points. Full credit on this part if you use the  $N(d_1)$  that you got from (a) even if that was not itself correct. 2 points for approximating the delta by computing the price of the option for two different values of the underlying asset (this is an unnecessarily inaccurate way of doing it).

3. Let's draw the diagram with the nodes



At the node color-coded in red, the call option is worth nothing. There is no partial credit just for noting this correctly.

At the node color-coded in yellow, the call option will have a payoff of \$0 or \$40. Buying one share and borrowing  $\$ \frac{100}{1.05}$  will cost \$24.76 and also have a payoff of \$0 or \$40. So the European call option is worth \$24.76.

At the node color-coded in green, the option will pay off \$24.76 or 0. If I buy 1 share and borrow  $\$ \frac{80}{1.05}$ , this costs me \$23.81 and will have a payoff of \$40 or \$0. So the European call option is worth \$14.74. 5 points off for each conceptual mistake (this includes treating the option as though the strike was \$120). No credit for two or more conceptual mistakes. 3 points off for a pure algebra error.

4.  $x^{1/2} = 0.3 * (100^{1/2}) = 3$ , so the certainty equivalent is just \$9.

5. The settlement price is \$99.70 so the short is on the losing side of the bet and must pay  $\$2,500 \times (99.85 - 99.70) = \$3,000$ . No credit for writing it backwards, or having the wrong settlement price. 2 points for incorrectly multiplying by \$250,000 instead of \$2,500.

6. 10 percent. 7 points for getting it right apart from a pure algebra mistake. Otherwise no partial credit.

7. In order from longest to shortest: B, C, D, A. No partial credit.

8.  $S = 1/16$ , and then  $1.004S = 1.04F$ . Solving this gives  $F = 0.060337$  or 16.57 rand per dollar. 2 points off for expressing as dollars per rand. 2 points off for an algebra mistake. No credit for doing it backwards (using the formula with  $S = 16$ ).

9. For Cheese Online, the expectation is 4% and so the variance is  $\frac{1}{3}(0.06^2 + 0.06^2 + 0.12^2) = 0.0072$ .

For Remsen Tech, the expectation is 10% and so the variance is  $\frac{1}{3}(0.1^2 + 0.05^2 + 0.05^2) = 0.005$ .

The covariance is  $\frac{1}{3}(0.06 \times 0.1 - 0.06 \times 0.05 + 0.12 \times 0.05) = 0.003$ .

The correlation is  $\frac{0.03}{\sqrt{0.005 \times 0.0072}} = \frac{1}{2}$

5 points for the variances; 5 points for the correlation. On the second part, people who got the covariance right but messed up the correlation got 1 point. 2 points off for each pure algebra mistake. No credit on the correlation part if you wrote down that the correlation was greater than +1 or smaller than -1 unless you noted that this must be a mistake.

10. Go long \$100 in portfolio B.

Go short \$50 in portfolio A.

Borrow \$50 at riskfree rate.

This will cost nothing on net and give a sure return of \$2.

No partial credit for getting it backwards or having the wrong mix of portfolios A and B.

11.  $\frac{1}{117.78} \frac{5}{1.005} \times 0.5 + \frac{1}{117.78} \frac{5}{1.005^2} \times 1 + \frac{1}{117.78} \frac{5}{1.005^3} \times 1.5 + \frac{1}{117.78} \frac{105}{1.005^4} \times 2 = 1.87$  years

2 points for setting it up correctly but with the wrong discount rate.

3 points for a pure algebra error.

Deeper mistakes in the formula: no credit.

In all cases, no credit for writing down a duration greater than 2 years unless you noted that this must be a mistake.

12. 2 points per part. No partial credit.

(i). D.

(ii) A.

(iii) A.

(iv) C.

(v) B.

(vi) E.

(vii) D.

- (viii) D.
- (ix) D.
- (x) C.
- (xi) B.
- (xii) B.
- (xiii) E.
- (xiv) D.
- (xv) E.