

Final Exam

Fall 2019

Econ 180-367

Closed Book.

Formula Sheet Provided. Calculators OK.

Time Allowed: 3 hours

1. (10 points) Suppose that there are three assets: stock A, stock B and the risk-free asset with the following characteristics.

Asset	Expected Return	Variance
Stock A	0.10	0.16
Stock B	0.06	0.04
Risk-free Asset	0.02	

The correlation between the returns on stocks A and B is 0.5. Investors are allowed to borrow and lend at the riskfree rate as much as they would like. They can also go long or short in any stock as much as they would like.

Mary has \$300 to invest and has a utility function $E(r) - \frac{A}{2}\sigma^2$ where $E(r)$ is her expected return, σ^2 is the variance of returns and her risk aversion is $A=2$. How will she split her wealth among these three assets?

2. (10 points). You enter into a five year swap contract where you pay fixed and receive one-year LIBOR with payments occurring once a year. The fixed rate you pay is 3 percent. The notional underlying is \$10 million. After 3 years, and just after the third payment, you ask your dealer to terminate the swap. Here is a table of the swap rates at the time you ask the dealer to terminate the swap.

1 year	0.5 percent
2 year	1 percent
3 year	1.25 percent
4 year	1.5 percent
5 year	2 percent

The dealer is willing to terminate the swap for fair value as discussed in class. What termination fee must you pay the dealer?

3. (10 points). You run an insurance company with a liability of \$1,000,000 due in ten years. All interest rates are 2 percent per annum, quoted with semiannual compounding. You want to immunize this liability with a portfolio of thirty-year STRIPS and a one-year bill. How much do you invest in the thirty-year STRIPS and the one-year bill? For full credit, you must state the dollar amounts invested today, not just the proportions.

4. (10 pts) The price of XYZ corp is currently \$100. There are four possible paths for its price over the next two years:

- (i) It could go up to \$110 in one year and up again to \$121 in two years.
- (ii) It could go up to \$110 in one year and back down to \$100 in two years.
- (iii) It could go down to \$90 in one year and back up to \$100 in two years.
- (iv) It could go down to \$90 in one year and down again to \$81 in two years.

The riskfree rate is 5 percent per year. What is the price of a European call option on the stock of XYZ corp at a strike price of \$100 maturing in two years' time?

5. (5 points) Consider each of the following possible scenarios for the prices of options. Say if it is possible or impossible (no explanation needed). In all cases assume that riskfree rates are zero, the options are European, and the underlying assets pay no dividends.

(a) A call option with a strike of \$20 costs \$8. The underlying asset is worth \$40.

(b) A call option with a strike of \$20 costs \$30. The underlying asset is worth \$40.

(c) A call option with a strike of \$20 costs \$45. The underlying asset is worth \$40.

(d) A call option with a strike of \$20 costs \$12. A call option with a strike of \$25 with the same underlying asset and same maturity costs \$14.

(e) A call option with a strike of \$10 costs \$2. A put option with a strike of \$10 with the same underlying asset and the same maturity costs \$4. The underlying asset is worth \$8.

6. (5 points). Suppose that a stock trades for \$22. At-the-money European call and put options expiring in one year both trade for \$5. The riskfree rate is 10 percent per annum (with annualized compounding). Is there an arbitrage opportunity? If so, what is it exactly? Be sure to be specific as to what asset you go long and short and in what amount.

7. (10 points). A 4% coupon bond has three years left to maturity. It's yield to maturity is 2%. The bond pays coupons twice a year, and yields and the coupon rate are annual rates with semiannual compounding. The face value of the bond is \$100.

(a) What is the price of the bond today?

(b) What is it's duration?

8. (5 points) In March 2000, 3Com announced that at the end of the year it would give their shareholders 1.5 shares in Palm for each 3Com share they owned. After this announcement, the price of Palm shares was \$95.06, and the price of 3Com shares was \$81.11. Assume that you can go long or short either or both shares in any amount, and that you want to exploit any arbitrage.

(a) Would you go long Palm or short Palm?

(b) How many shares?

(c) Would you go long 3Com or short 3Com?

(d) How many shares?

(e) What profit would you expect from your strategy?

9. (5 points) A stock trades for \$90. The stock pays no dividends. The volatility of the stock is 50 percent per annum. The risk-free interest rate is 5 percent. According to the Black-Scholes formula, what is the value of a European call option with a strike price of \$90 maturing in one year?

10. (5 points) Your portfolio has two stocks: Citigroup and Pfizer, with equal weight on both stocks. Citigroup stock has a standard deviation of 20 percent per annum. Pfizer stock has a standard deviation of 30 percent per annum. The two stocks have a correlation of 0.25. What is the standard deviation of the return on your portfolio?

11. (5 points). Consider an investor with a utility function $u(W) = -\frac{1}{W}$ where W is her wealth. What is this investor's coefficient of relative risk aversion?

12. (20 points) Multiple choice questions. Only one option is correct. Please indicate which one it is.

(i) Which of the following currencies are normally quoted with the US\$ as the base (as defined in class)?

- A. Australian dollar.
- B. Hong Kong Dollar.
- C. Euro.
- D. New Zealand Dollar.
- E. British pound.

(ii) An option whose value is calculated by comparing the strike price with the average spot price over the period of the option is called:

- A. A compound option.
- B. A Bermudan option.
- C. A European option.
- D. An American option.
- E. An Asian option.

(iii) Which of the following best describes a warrant?

- A. It gives the owner the right to buy stocks directly from the firm at a fixed price.
- B. It gives the owner the right to buy bonds directly from the firm at a fixed price.
- C. It gives the owner the obligation to sell stocks directly to the firm at a fixed price.
- D. It gives the owner the obligation to buy stocks directly from the firm at a fixed price.
- E. It gives a minimum price for the stock below which the firm promises to convert stock into debt.

(iv) Which of the following best describes average returns on portfolios of stocks

- A. Stocks with large market cap have **higher** average returns than stocks with low market cap, and stocks with high book-to-market ratios have **higher** average returns than stocks with low book-to-market ratios.
- B. Stocks with large market cap have **higher** average returns than stocks with low market cap, and stocks with high book-to-market ratios have **lower** average returns than stocks with low book-to-market ratios.
- C. Stocks with large market cap have **lower** average returns than stocks with low market cap, and stocks with high book-to-market ratios have **higher** average returns than stocks with low book-to-market ratios.
- D. Stocks with large market cap have **lower** average returns than stocks with low market cap, and stocks with high book-to-market ratios have **lower** average returns than stocks with low book-to-market ratios.
- E. Market cap and book-to-market ratios are irrelevant for the average return on stocks.

(v) Suppose that you buy a call option at a strike of \$60, you buy another call option at a strike of \$58 and you sell two call options at a strike of \$59. What is the name for this strategy?

- A. A straddle.
- B. A butterfly
- C. A strangle.
- D. A risk reversal.
- E. A double spread.

(vi) Suppose that Mark enters into a standard interest rate swap contract where the fixed rate is 2 percent. Mark has agreed to pay fixed. The floating rate is 3 month LIBOR and the notional underlying is \$1 million. It is a payment date and the 3 month LIBOR floating rate is 1.5 percent. Which of the following is true on this payment date?

- A. Mark pays \$20,000.
- B. Mark pays \$5,000.
- C. Mark pays \$1,250.
- D. Mark receives \$5,000.
- E. Mark receives \$15,000.

(vii) Which of the following was the most widely used funding currency for the carry trade from 1990 to 2008?

- A. The US dollar.
- B. The British pound.
- C. The Japanese yen.
- D. The Australian dollar.
- E. The Brazilian real.

(viii) You go short a Eurodollar futures contract at a strike of 98.5. At settlement of the contract, the federal funds rate is 1.8 percent and three-month LIBOR is 1 percent. Which of the following is true?

- A. You receive \$1,250.
- B. You owe \$1,250.
- C. You receive \$750.
- D. You owe \$750.
- E. You owe \$2,412.50.

(ix) Which of the following is the best definition of contango?

- A. A stock that trades on the Buenos Aires stock exchange.
- B. A spot price being above the futures price.
- C. A spot price being below the futures price.
- D. A call option price being above the put option price.
- E. A call option price being below the put option price.

(x) Which of the following is the Black-Scholes delta?

- A. The sensitivity of the call option price to the interest rate.
- B. The sensitivity of the call option price to the time to maturity.
- C. The sensitivity of the put option price to the time to maturity.
- D. The sensitivity of the call option price to the stock volatility.
- E. The sensitivity of the call option price to the stock price.

Solutions and Grading Rubric

1. Between stocks A and stock B, the share that she will allocate to A is

$$\frac{0.08 * 0.04 - 0.04 * 0.04}{0.08 * 0.04 + 0.04 * 0.16 - 0.12 * 0.04} = \frac{1}{3}$$

Thus the risky portfolio has a mean return of 0.0733 and a variance of 0.0533. The fraction allocated to the risky portfolio should be

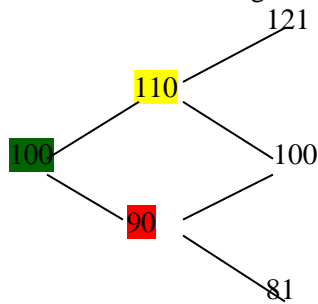
$$\frac{0.0733 - 0.02}{2 * 0.0533} = \frac{1}{2}$$

So, of the \$300, \$150 goes to the risk-free asset and \$50 goes to stock A and \$100 goes to stock B. A purely algebraic error costs 1 point. A misapplication of any formula costs 3 points.

2. You are effectively buying a two year off market swap where you receive fixed of 3 percent and pay floating. This is available but will cost $\frac{200000}{1.01} + \frac{200000}{1.01^2} = \394079 .

3. The present value of the liability is $\frac{1,000,000}{1.01^{20}} = 819,544$. The duration of the liability is 10 years. If the portfolio puts a weight of ω on the STRIP then $30\omega + (1 - \omega) = 10$ which means that $\omega = 9/29$. So you should invest \$254,341 in the STRIP and \$565,203 in the bill. 2 points off if you calculate the present value incorrectly by dividing by 1.02^{20} . 5 points off if you give the weights without computing the investment amounts. 1 point off for a purely algebraic mistake.

4. Let's draw the diagram with the nodes



At the node color-coded in red, the call option is worth nothing.

At the node color-coded in yellow, the call option will have a payoff of \$0 or \$21. Buying one share and borrowing $\$ \frac{100}{1.05}$ will cost \$14.76 and also have a payoff of \$0 or \$21. So the European call option is worth \$14.76.

At the node color-coded in green, the option will pay off \$14.76 or 0. If I buy 1 share and borrow $\$ \frac{90}{1.05}$, this costs me \$14.29 and will have a payoff of \$20 or \$0. So the European call option is worth \$10.55.

2 points off for each mistake like getting the payoffs of a replicating portfolio wrong at a particular node, as long as you are on the right tracks.

5. (a) Impossible. Violates $C \geq S_0 - \frac{X}{(1+r_f)^T}$.

(b) Possible.

(c) Impossible. Violates $C \leq S_0$.

(d) Impossible. Call prices cannot be an increasing function of the strike price.

(e) Possible.

1 point for each part. No partial credit.

6. $C + \frac{X}{1+r_f} = 5 + \frac{22}{1.1} = 25$ and $S + P = 22 + 5 = 27$. So the arbitrage is

- Buy 1 call option (cost: \$5)
- Invest \$20 at the riskfree rate (cost: \$20)
- Short 1 stock (get \$22)
- Short 1 put option (get \$5)

That's all that is required. This strategy gives you \$2 up front. If at maturity, the stock price is worth $S < 22$, then the call option is worthless, I get \$22, I buy back the stock for S and my put option is worth $22 - S$ which all adds up to zero. If at maturity, the stock price is worth $S > 22$, then the call option is worth $S - 22$, I get \$22 from the riskfree asset, I buy back the stock for S and the put option is worthless. This all adds up to zero. So it is an arbitrage. 1 point for saying that there is an arbitrage, 1 point for saying that you buy 1 call option, 1 point for saying that you invest \$20 at the riskfree rate, 1 point for saying that you short 1 stock and 1 point for saying that you short 1 put option.

7. (a) The price of the bond is $\frac{2}{1.01} + \frac{2}{1.01^2} + \frac{2}{1.01^3} + \frac{2}{1.01^4} + \frac{2}{1.01^5} + \frac{102}{1.01^6} = 105.80$.

(b) The duration of the bond is $\frac{1}{105.80} \left(\frac{0.5 * 2}{1.01} + \frac{1 * 2}{1.01^2} + \frac{1.5 * 2}{1.01^3} + \frac{2 * 2}{1.01^4} + \frac{2.5 * 2}{1.01^5} + \frac{3 * 102}{1.01^6} \right) = 2.86$ years.

2 points off for replacing 1.01 in the denominators by 1.02 and/ or replacing 2 in the numerators by 4. 5 points for part (a) and 5 points for part (b). 1 point off for a purely algebraic error. However, no credit at all on part (b) if you give an answer bigger than 3 even if this was a purely algebraic error. The reason is that this shows you don't understand what duration is.

8. (a) Short.

(b) 1.5 shares.

(c) Long.

(d) 1 share.

(e) $1.5 * 95.06 - 81.11 = 61.48$.

1 point per part. Of course it is fine to double all the amounts.

9. $d_1 = 0.35$ and $d_2 = -0.15$. So the call option price is \$19.61. 1 point off for a purely algebraic error. No credit for any misapplication of the formula like mixing up variance and standard deviation or replacing the division in the formula for d_1 by a multiplication. The reason is that you are given the formula and so it should be simple to use it correctly.

10. The portfolio variance is $\frac{1}{4} * 0.2 * 0.2 + \frac{1}{4} * 0.3 * 0.3 + 2 \frac{1}{4} 0.25 * 0.2 * 0.3 = 0.04$. So the portfolio standard deviation is 0.2, or 20 percent. 2 points if you make one mistake like not squaring the weights. No credit for 2 or more mistakes. 4 points for a purely algebraic error.

11. The coefficient of relative risk aversion is $-\frac{WU''(W)}{U'(W)}$. Now $U'(W) = \frac{1}{W^2}$ and $U''(W) = -\frac{2}{W^3}$.

So the coefficient of relative risk aversion is $\frac{W(2/W^3)}{1/W^2} = 2$. 1 point off for saying that the answer is -2.

- 12. (i) B.
- (ii) E.
- (iii) A.
- (iv) C.
- (v) B.
- (vi) C.
- (vii) C.
- (viii) B.
- (ix) C.
- (x) E.