

Final Exam

Fall 2018

Econ 180-367

Closed Book.

Formula Sheet Provided. Calculators OK.

Time Allowed: 3 hours

- (10 points) A stock costs \$10 today and in one year it will be worth \$7, \$13 or \$16, each with equal probability. The stock pays no dividends. What is the variance of the holding period return?
- (10 points). The spot dollar-yen exchange rate is \$1=110 yen. The one-year interest rate in the US is 2.7 percent, but in Japan it is zero.
 - According to **covered** interest parity, what should the one-year forward dollar-yen exchange rate be?
 - According to **uncovered** interest parity, what is the **expected** spot dollar-year exchange rate one year from now?
- (10 points). You run an insurance company with a liability of \$10,000,000 due in five years. All interest rates are 4 percent per annum, quoted with semiannual compounding. You want to immunize this liability with a portfolio of ten-year STRIPS and a one-year bill. How much do you invest in the ten-year STRIPS and the one-year bill?
- (5 points) Suppose that the economy is described by the APT model with a single factor. The risk-free rate is 2 percent. X is a well-diversified portfolio with a beta of 1 and an 8 percent expected return. Y is also a well-diversified portfolio, but with a beta of 0.5. What is its expected return?

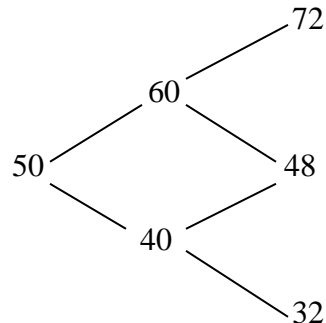
5. (5 points) Suppose that there are two risky assets: A and B, with the following properties.

	Expected Return	Standard Deviation
Asset A	0.08	0.2
Asset B	0.13	0.4

The risk-free rate is 5 percent, and assets A and B have correlation of 0.625. What is the portfolio of A and B that maximizes the Sharpe ratio (please give the weight on each asset)?

- (5 points). Suppose that a stock trades for \$22. At-the-money European call and put options expiring in one year both trade for \$5. The riskfree rate is 10 percent per annum (with annualized compounding). Is there an arbitrage opportunity? If so, what is it exactly? Be sure to be specific of what asset you go long and short and in what amount.
- (5 points). Suppose that a firm has an unlevered beta of 0.3. It has \$400 million in debt and \$200 million in equity. What is the equity beta of the firm?
- (5 points) WARP industries pays a dividend of \$1 per share **this** year. Its current stock price is \$50, and its dividends are expected to grow at a rate of 3% per year forever, so the dividend will be \$1.03 **next** year. Based on the above information, what is the required rate of return on WARP industries' stock?
- (10 points) A stock trades for \$90. The stock pays no dividends. The volatility of the stock is 50 percent per annum. The risk-free interest rate is 5 percent.
 - According to the Black-Scholes formula, what is the value of a European call option with a strike price of \$99 maturing in one year?
 - If the call option is trading at \$12, what arbitrage strategy would you undertake?

10. (10 points). Suppose that a stock trades for \$50 at time zero. In one year, it will either rise to \$60 or fall in price to \$40. If it rises to \$60, in the following year it can either rise to \$72 or fall to \$48. If it falls to \$40, then in the following year it can either rise to \$48 or fall to \$32. The outcomes can be represented by the following binomial tree:



The risk free rate is 5 percent per year. Find the price of an American put option with a strike of \$52 as of time zero.

11 (5 points). For each of the following, say which is **bigger**:

- (i) A: The average return on high book-to-market ratio stocks, *or*
B: The average return on low book-to-market ratio stocks.
- (ii) A: The duration of a 30-year zero coupon bond, *or*
B: The duration of a 30-year 4% coupon bond.
- (iii) A: The price of a call option on a stock with a strike of \$55, *or*
B: The price of a call option on that stock that is identical in all respects, except that the strike is \$50.
- (iv) A: The three-month OIS rate, *or*
B: The three-month LIBOR interest rate.
- (v) A: The three-month repo interest rate, *or*
B: The three-month LIBOR interest rate.

12. (20 points) Multiple choice questions. Only one option is correct. Please indicate which one it is.

- (i) Which of the following currencies are normally quoted with the foreign currency as the base (as defined in class)?
A. Australian Dollar.
B. Canadian Dollar.
C. Japanese Yen.
D. Swiss Franc.
E. Hong Kong Dollar.
- (ii) What is the name given to an option position that consists of going long an out-of-the-money call and short and out-of-the-money put on the same underlying security and at the same maturity?
A. A straddle.
B. A risk reversal.
C. A butterfly.
D. A delta hedge.
E. A horizontal spread.

(iii) You go long a Eurodollar futures contract at a strike of 98.5. At settlement of the contract, the federal funds rate is 1.8 percent and three-month LIBOR is 2 percent. Which of the following is true?

- A. You receive \$1,250.
- B. You owe \$1,250.
- C. You receive \$1,750.
- D. You owe \$1,750.
- E. You owe \$2,412.50.

(iv) Which of the following statements is correct about the Black-Scholes implied prices of European options?

- A. The price of a call option is increasing in the volatility of the underlying asset, while the price of a put option is increasing in the volatility of the underlying asset.
- B. The price of a call option is increasing in the volatility of the underlying asset, while the price of a put option is decreasing in the volatility of the underlying asset.
- C. The price of a call option is decreasing in the volatility of the underlying asset, while the price of a put option is increasing in the volatility of the underlying asset.
- D. The price of a call option is decreasing in the volatility of the underlying asset, while the price of a put option is decreasing in the volatility of the underlying asset.
- E. Neither the price of the call option nor the price of the put option depends on the volatility of the underlying asset.

(v) Which of the following is a difference between a futures contract and a forward contract?

- A. A forward contract is traded on an exchange whereas a futures contract is not.
- B. A futures contract is guaranteed by a clearinghouse whereas a forward contract is not.
- C. A forward contract is marked to market whereas a futures contract is not.
- D. Futures contracts are always cash settled whereas forward contracts are physically settled.
- E. Margin has to be posted for a forward contract, but not for a futures contract.

(vi) Empirically, which of the following is currently true of the relationship between the Black-Scholes implied volatility on the S&P 500 and the exercise price?

- A. Implied volatility does not depend on the exercise price.
- B. Implied volatility is an increasing function of the exercise price.
- C. Implied volatility is a decreasing function of the exercise price.
- D. Implied volatility is a U-shaped function of the exercise price.
- E. Implied volatility is an inverse-U-shaped function of the exercise price.

(vii) Which of the following is the name given to an option that can be exercised on multiple fixed dates, but only on those dates?

- A. A Bermudan option.
- B. A European option.
- C. A Japanese option.
- D. A Quanto option.
- E. An American option.

(viii) Which of the following is true of the Treasury yield curve?

- A. It always slopes down.
- B. It always slopes up.
- C. It usually slopes down except that it has sloped up just after economic expansions begin.
- D. It usually slopes up except that it has sloped down just before recessions begin.
- E. It usually slopes down except that it has sloped up just before recessions begin.

(ix) Suppose that you buy a one-year CDS on Wyman enterprises. If they default, the bonds of Wyman enterprises will have a recovery rate of 60 percent. The CDS premium is 1 percent per annum. If investors are risk neutral, which of the following is the probability of Wyman enterprises defaulting in the next year?

- A. 0.4 percent.
- B. 0.6 percent.
- C. 1 percent.
- D. 1.67 percent.
- E. 2.5 percent.

(x) Suppose that gold is costless to store, the spot price is \$1,200 an ounce and the risk-free interest rate is 3 percent. According to spot-futures parity what should the two-year ahead futures price be?

- A. \$1,131.
- B. \$1,165.
- C. \$1,200.
- D. \$1,236.
- E. \$1,273.

Solutions and Grading Rubric

1. The holding period returns are -0.3, 0.3 and 0.6 in the three scenarios. The expected holding period

return is $-\frac{0.3}{3} + \frac{0.3}{3} + \frac{0.6}{3} = 0.2$. The variance is

$$\frac{(-0.3-0.2)^2}{3} + \frac{(0.3-0.2)^2}{3} + \frac{(0.6-0.2)^2}{3} = 0.14$$

So the variance of holding period returns is 14%.

5 points for a mistake in defining variance, such as failing to multiply by the probabilities. 8 points if the variance formula was written out correctly, but there was a purely algebraic error.

2. (a) The formula uses the exchange rate with the foreign currency base. So

$$\frac{1}{110} * (1 + 0.027) = F * 1$$

where F denotes the forward rate. This means that the forward rate is 0.009336. This is an acceptable answer, but it is conventional to quote dollar-yen with the dollar base, which gives an exchange rate of $1/0.009336$, or 107.11.

(b) By the same algebra, under uncovered interest parity, the expected future exchange rate is $\$1 = \text{JPY}107.11$.

5 points for each part. No credit if the wrong formula was used. In particular, no credit if the exchange rate was entered into the formula with dollar-base, because that gives everything backwards. I meant people to write the answer as yen-per-dollar, but accepted the answer of dollar-per-yen. 1 point off on either part for a purely algebraic error.

3. The present value of the liability is $\frac{10,000,000}{1.02^{10}} = 8,203,483$. Of this a fraction ω should be invested in the STRIPS. This solves the equation:

$$10\omega + 1(1 - \omega) = 5$$

which has the solution $\omega = 0.4444$. So invest \$3.65 million in STRIPS and \$4.56 million in one-year bills.

Many people interpreted the question as just asking what fraction to invest in STRIPS and bills, or what fraction of the face value to invest in STRIPS and bills and since I wasn't completely explicit that I wanted the dollar amounts invested today, I accepted these answers for full credit.

4. The expected return on X is $2 + \lambda$ and so $\lambda = 6$. Hence the expected return on Y is $2 + 0.5\lambda$ which is 5 percent.

5. The excess returns on assets A and B are 0.03 and 0.08, respectively. The covariance between assets A and B is $0.625 * 0.4 * 0.2 = 0.05$. So the weight on asset A is:

$$\frac{0.03 * 0.4^2 - 0.08 * 0.05}{0.03 * 0.4^2 + 0.08 * 0.2^2 - [0.03 + 0.08] * 0.05} = 0.32.$$

The weight on asset B is 0.68.

3 points if you used the return instead of the excess return, or the correlation instead of the covariance, or had the wrong formula for the covariance. No points if there were two or more substantive mistakes. 4 points for a purely algebraic error.

6. The put-call parity equation is:

$$C + \frac{X}{1+r_f} = S + P$$

In this case, the left-hand side is cheaper, so my strategy is:

- Go long 1 call option. Cost \$5.
- Invest \$22/1.1=\$20 at the risk-free rate. Cost \$20.
- Short 1 stock. Receive \$22.
- Short 1 put option. Receive \$5.

On net, I receive \$2.

At expiration, suppose that the stock price is $S < 22$. The call option is worthless and my position is worth $22 - S - (22 - S) = 0$.

Now instead suppose that at expiration the stock price is $S > 22$. The put option is worthless and my position is worth $S - 22 + 22 - S = 0$.

And if the stock is worth exactly \$22 at expiration, both options are worthless and my position is worth $22 - S = 0$.

So in all cases I receive \$2 and have a position that is worth \$0 in the future, which is an arbitrage.

7. The formula is $\text{Unlevered Beta} = \frac{E}{E+D} * \text{Equity Beta}$. So $\text{Equity Beta} = \frac{D+E}{E} * \text{Unlevered Beta}$. Thus the equity beta is $\frac{400+200}{200} * 0.3 = 0.9$.

No credit for using the wrong formula. In particular, no credit for getting the formula backwards and giving an answer of 0.1.

8. The required rate of return, r , satisfies the equation $50 = \frac{1 * 1.03}{r - 0.03}$ and so $r = 0.0506$.

9. (a) In the formula, we set $S=90$, $X=90$, $\sigma = 0.5$, $T=0.5$ and $r=0.05$. So

$$d_1 = \left[\ln\left(\frac{90}{99}\right) + \left(0.05 + \frac{0.25}{2}\right) \right] / 0.5 = 0.16$$

$$d_2 = d_1 - 0.5 = -0.34$$

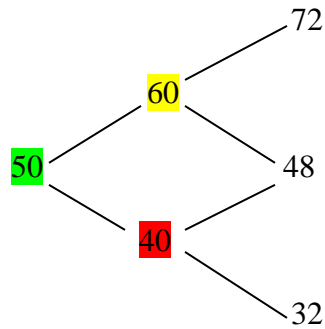
From the normal tables, $N(d_1) = 0.5636$ and $N(d_2) = 0.3669$. So the price of the call option is:

$$C = (90 * 0.5636) - (99 * e^{-0.05} * 0.3669) = 16.17$$

(b) The option is cheap. So I buy the option and short $N(d_1)$ of the underlying as the delta hedge. I buy one option and short 0.5636 shares. This was a delta-hedging question.

6 points for part (a); 4 points for part (b). On part (a), 1 point off for a purely algebraic mistake and 2 points off for a mis-application of the formula (such as $\ln(99/90)$ instead of $\ln(90/99)$). On part (b), 1 point for saying buy the option and 1 point for saying short the underlying (even if it is part of an imaginary put-call parity) without getting the amounts right.

10. Here is the Binomial tree:



- Consider first the node colored in yellow. From this point, a call option with a strike of 52 has payoff of 20 on the upper branch and 0 on the lower branch. If I buy one share and borrow $\frac{48}{1.05}$ this costs me $60 - \frac{48}{1.05} = 14.29$. The payoff is 24 on the upper branch and 0 on the lower branch. If I buy $\frac{24}{20}$ call options this gives the same payoff. So $\frac{24}{20}C = 14.29$ and the call option price is \$11.91. By put-call parity, the put option price is $11.91 + \frac{52}{1.05} - 60 = 1.43$. I would not exercise the option early at the yellow node.
- Next consider the node colored in red. From this point, a call option with a strike of 52 is worthless. So by put-call parity, the put option price is $0 + \frac{52}{1.05} - 40 = 9.52$. However, immediate exercise of the option gives me \$12. So because it is an American option, it's value is \$12.
- And now going back to the green node, the option pays of \$1.43 on the upper branch and \$12 on the lower branch. I replicate this by a strategy of shorting S shares and investing $\frac{R}{1.05}$. I want my portfolio to have the correct value on each branch so I specify that:

$$R - 60S = 1.43$$

$$R - 40S = 12$$

Solving these gives $S=0.5285$ and $R=33.14$. The cost of this replicating portfolio is $\frac{33.14}{1.05} - (0.5285 * 50) = 5.14$. And so the value of the American put option is \$5.14.

6 points for correctly deducing the value of the option treating it as a call instead of an American put. 8 points for correctly deducing the value of a European put option. Some people derived the value of a call and then converted it to a put via put-call-parity which gives the value of a European put.

11. 1 point per correct answer.

- (i) A.
- (ii) A.
- (iii) B.
- (iv) B.
- (v) B.

12. 2 points per correct answer.

(i) A.

(ii) B.

(iii) B.

(iv) A.

(v) B.

(vi) C.

(vii) A.

(viii) D.

(ix) E.

(x) E.