Final Exam

Fall 2017          Econ 180-367

Closed Book.
Formula Sheet Provided. Calculators OK.
Time Allowed: 3 hours

1. (10 points) The price of a stock today is $50. The risk-free rate is zero. One of the following four
scenarios will occur:

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<th>Price in 6 months</th>
<th>Price in 1 year</th>
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<tbody>
<tr>
<td>A</td>
<td>$52</td>
<td>$55</td>
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<td>B</td>
<td>$52</td>
<td>$50</td>
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<tr>
<td>C</td>
<td>$48</td>
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<tr>
<td>D</td>
<td>$48</td>
<td>$46</td>
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Using a binomial tree, price a European call option with an expiration of one year hence and a strike price
of $50.

2. (5 points) The spot Euro-Dollar exchange rate is €1=$1.18. One-year interest rates are 1.25 percent in
the US and 0 in Europe. According to covered interest parity, what should the one-year forward Euro-
Dollar exchange rate be?

3. (5 points) Interest rates are 2 percent per annum. An insurance company has to pay $1,040,040 in two
years. The present value of this liability is $\frac{1,040,040}{1.02^2} = $1,000,000. It wants to immunize this liability
by buying a one-year bill (with a duration of 1 year) and a five-year zero-coupon bond. How much does
the insurance company invest in the five-year zero coupon bond?

4. (10 points) Suppose that there are two stocks. The first stock has a standard deviation of 40%. The second
stock has a standard deviation of 50%. The correlation between the stocks is 0.5. I form a portfolio with a
weight of ¾ in the first stock and ¼ in the second. What is the standard deviation of the portfolio return?

5. (10 points) Suppose that there are two possible assets: debt and equity. Debt has an expected return of
7% and a standard deviation of 10%. Equity has an expected return of 11% and a standard deviation of
20%. Debt and equity are mutually uncorrelated. The risk-free rate is 3%. If Mark is selecting a portfolio
of debt and equity to maximize the Sharpe ratio, what share will he invest in debt?

6. (5 points) Consider a stock that pays no dividends. It trades for $30. A European call option with a
strike price of $24 and an expiration of one year hence trades for exactly $5. The risk-free interest rate is
zero. Assume that there are no transactions costs or borrowing or shorting constraints. Is there an arbitrage
opportunity? If so, say exactly what it is. No credit for vague or ambiguous answers.

7. (5 points) Suppose that I buy at-the-money call and put options on a barrel of oil, when the spot price of
oil is $40 a barrel. The call option costs me $10 and the put option costs me $12. Risk-free interest rates
are zero. For what range of oil prices at the expiration of the contract do I make money on this straddle
position (after accounting for the cost of the options)?

8. (10 points) Suppose that a stock is trading for $30. Its volatility is 0.4. The stock pays no dividends and
the risk-free interest rate is zero. You buy an at-the-money European call option on this security with an
expiration of 1 year hence. According to the Black-Scholes formula, what should this call option be worth?
9. (5 points) Today, the one-year Treasury yield is 0.4 percent, the two-year Treasury yield is 0.6 percent, and the three-year yield is 1 percent. All of these numbers are annual interest rates with annual compounding. If the Expectations Hypothesis is correct, what is the expected one-year Treasury yield two years from today?

10. (10 points) Consider a one-factor economy with a risk-free interest rate of 2 percent. Portfolio X is well diversified with a beta of 1 and an expected return of 5 percent. Portfolio Y is also well diversified with a beta of 2.
(a) According to APT, what is the expected return on portfolio Y?
(b) Suppose that the expected return on portfolio Y were actually 6 percent. Show that there would be an arbitrage opportunity. Be very specific about what you would go long and short and in what amounts. No credit for vague of ambiguous answers.

11. (5 points) A share in Wyman Enterprises is going to pay dividends of $10 next year. Thereafter, dividends will grow by 2 percent per annum. The required return on this stock is 7%. According to the dividend discount model, what is the price of a share in Wyman Enterprises?

12. (20 points) Multiple choice questions: in each case, only one answer is correct.
(i) If a Treasury Bill maturing 90 days after settlement has a face value of $10,000 and a quoted yield of 1 percent per annum, how much will you pay for it?
A. $10,000.
B. $9,975.
C. $9,970.
D. $9,960.
E. $10,100.

(ii) Which of the following statements concerning options prices is correct.
A. The price of an American call option on any stock must always be greater than the price of a European call option at that same strike and maturity on that same stock.
B. The price of an American call option on any stock must always be the same as the price of a European call option at that same strike and maturity on that same stock.
C. The price of an American call option on any stock must be the same as the price of a European call option at that same strike and maturity on that same stock, as long as the stock pays dividends.
D. The price of an American call option on any stock must be the same as the price of a European call option at that same strike and maturity on that same stock, as long as the stock does not pay dividends.
E. The price of an American call option on any stock must always be less than the price of a European call option at that same strike and maturity on that same stock.

(iii) What name is given to the situation in which a futures price for delivery in one year is higher than the current spot price:
A. Backwardation.
B. Upwardation.
C. Risk reversal.
D. Investment basis.
E. Contango.
(iv) Suppose that in March, I take the long side in a June Eurodollar futures contract for $99.45. Suppose that in June, the effective federal rate is 38 basis points, the one-month LIBOR rate is 55 basis points and the three-month LIBOR rate is 65 basis points. Which of the following describes the payment that I will make or receive:
A. I pay $550.
B. I receive $550.
C. I pay $55.
D. I pay $250.
E. I receive $250.

(v) The five-year CDS spread on Argentinian government bonds is 20 basis points per annum. In the event of default, the recovery rate on Argentine government bonds is 80 percent. Under risk neutrality, which of the following is the approximate probability of Argentina defaulting at some point in the next five years?
A. 0.2 percent.
B. 0.25 percent.
C. 1 percent.
D. 1.25 percent.
E. 5 percent.

(vi) Jane has the utility function \( U = E(r) - \frac{1}{2} A \sigma^2 \) where \( E(r) \) is expected return, \( \sigma^2 \) is the variance of returns and \( A \) is her coefficient of risk aversion. If Jane is splitting money between a risk-free asset, with a return of 7 percent, and a risky asset with an expected return of 15 percent and a standard deviation of 22 percent so as to maximize her utility function, and if her coefficient of risk aversion is 4, what fraction of her wealth will she invest in the risky asset.
A. 0.09.
B. 0.25.
C. 0.41.
D. 0.59.
E. 0.77.

(vii) Suppose that there are many stocks, each with an expected return of 6 percent. Returns on stocks are uncorrelated with each other. Each stock has a standard deviation of 10 percent. The risk-free rate is 4 percent. What is the Sharpe ratio of a portfolio of 25 stocks?
A. 0.2.
B. 0.6.
C. 1.
D. 3.
E. 5.

(viii) Which of the following the price of a 30-year zero coupon bond with a face value of $1,000 with a yield of 2% per annum with semiannual compounding?
A. $304.78.
B. $504.98.
C. $550.45.
D. $552.07.
E. $741.93.
(ix) Which of the following helped to end the 1907 financial crisis?
A. Loans to banks from the Federal Reserve system.
B. The introduction of deposit insurance.
C. The abolition of the gold standard.
D. The election of Theodore Roosevelt.
E. Loans to banks from the New York Clearinghouse.

(x) Which of the following is known as the “carry trade”?
A. Borrowing in high interest currencies, investing in low interest currencies and hedging the currency risk with forward contracts.
B. Borrowing in low interest currencies, investing in high interest currencies and hedging the currency risk with forward contracts.
C. Borrowing in high interest currencies, investing in low interest currencies and NOT hedging the currency risk with forward contracts.
D. Borrowing in low interest currencies, investing in high interest currencies and NOT hedging the currency risk with forward contracts.
E. Borrowing in the currencies of countries with trade surpluses and investing in the currencies of countries with trade deficits.
Solutions and Grading Rubric

1. If in 6 months the price is $48, the option will be worthless. If in 6 months, the price is $52, then the payoff from the option is either $5 or $0. My replication strategy is to buy 1 share and borrow $50. The cost is $2. In state A, this gives me a payoff of $5. In state B, it gives me a payoff of 0. This is exactly equivalent to the call option. So the price of the call option is $2 if the price rises to $52.

Now start at today. The call option will be worth either $2 or $0. The replicating strategy is to buy the stock and borrow $48. This costs $2. It’s payoff is $4 or $0.

So the price of the call option today is $1.

5 points for correctly pricing the option at the $52 node and doing nothing else. 2 points off for simple algebra mistake.

2. From covered interest parity, $1.18 \times 1.0125 = F \times 1$, so the forward rate is €1=$1.19475. 2 points off for algebra error. No credit for having the forward rate lower than the spot rate, even if an algebra error. No credit for using the wrong interest rate.

3. The duration of the liability is 2 years. If the weight on the bond is $ω$, the duration of the asset is $1 - ω + 5ω$. Setting these equal gives $1 - ω + 5ω = 2$ and so $ω = 0.25$. So the firm invests $250,000 in the bonds and $750,000 in the bills. It was enough to say $\frac{1}{4}$ in the bonds. 2 points off for algebra error. 2 points off getting the bonds and bills backwards.

4. The portfolio variance is $\frac{9}{16} \times 0.4^2 + \frac{1}{16} \times 0.5^2 + 2 \times \frac{3}{4} \times 0.4 \times 0.5 \times 0.5 = 0.143125$

The portfolio standard deviation is 0.378 or 37.8%. There were several common mistakes, like not multiplying the covariance by 2, using standard deviation not variance, not squaring the weights, and making algebra mistakes. 3 points off per type of mistake.

5. Expected excess returns are 4% (debt) and 8% (equity). Variances are 0.01 (debt) and 0.04 (equity). Hence, the share in debt is

$$\frac{0.04 \times 0.04}{0.04 \times 0.04 + 0.08 \times 0.01} = \frac{2}{3}$$

6 points for doing everything correctly but with returns not excess returns. 6 points for mixing up the two variances.

6. This violates the arbitrage condition $C \geq S - \frac{X}{(1 + r_f)^T}$. Here is the arbitrage:

- Go long the call option (cost: $5)
- Go short the stock (receive: $30)
- Invest $24 (cost: $24).

On net, I receive $1. At the expiration of the option, if the stock is worth $S_T$, and if $S_T < 24$, my portfolio is worth $24 - S_T > 0$. If $S_T \geq 24$ my portfolio is worth $S_T - 24 - S_T + 24 = 0$. So it is an arbitrage.

Anyone who said go long the option and short the stock gets full credit. Saying that you buy the option and exercise it immediately gets no credit because it is a European option. Getting the arbitrage backwards gets no credit.

7. Suppose that $S_T$ is the value of a barrel of oil at expiration. If $S_T < 40$, I get a payoff only on the put option. My payoff is $40 - S_T$. If $S_T > 40$, I get a payoff only on the put option. My payoff is $S_T - 40$
The straddle position cost me $22, so I only make money if I get a payoff of $22 on either the put or the call. So I make money if \( S_f < 18 \) or \( S_f > 62 \). 2 points off for algebra mistake.

8. The components of the Black-Scholes formula are as follows:

\[
d_1 = \left[ \ln\left( \frac{30}{30} + \frac{0.4^2}{2} \right) \right] / 0.4 = 0.2
\]

\[
d_2 = 0.2 - 0.4 = -0.2
\]

\[
N(d_1) = 0.5793
\]

\[
N(d_2) = 1 - 0.5793 = 0.4207
\]

Hence the price of the call option is:

\[
30 \cdot 0.5793 - 30 \cdot 0.4207 = $4.758
\]

3 points off for algebra mistake.

9. The expected rate is \( \frac{1.01^3}{1.006^2} - 1 = 0.018 \). Or you could simply say that the expected rate is \( 3 \cdot 1 - (2 \cdot 0.6) = 1.8\% \). No partial credit.

10. (a) The expected return on portfolio X is \( 0.02 + \lambda = 0.05 \), so \( \lambda = 0.03 \). So the expected return on portfolio Y is \( 0.02 + 2\lambda = 0.02 + 0.06 = 0.08 \) or 8\%. 5 points. 2 points off for algebra mistake.

(b) Invest $100 in portfolio X. This becomes worth \( 100 \cdot (1.05 + F) \)

Short $50 in portfolio Y. This becomes worth \( -50 \cdot (1.06 + 2F) \)

Borrow $50 at the riskfree rate. This becomes worth \( -51 \).

Adding everything up, I have no cost up front. But my portfolio will be worth $1. 5 points.

11. \( P = \frac{10}{0.07 - 0.02} = \frac{10}{0.05} = 200 \). No partial credit. Anyone who put 10*(1.02) in the numerator gets full credit.

12. (i) B
(ii) D.
(iii) E
(iv) D.
(v) E.
(vi) C.
(vii) C.
(viii) C.
(ix) E.
(x) D.
(2 points per correct answer).