Final Exam

Fall 2009 Econ 180-367

Closed Book.
Formula Sheet Provided. Non-Programmable Calculators OK.
Time Allowed: 3 Hours.
Answer all Ten Questions: All Questions Carry Equal Marks
(Parts *within* each question do not all have equal marks)

1. The price of a one-year zero-coupon bond is $952.38. The price of a two-year zero coupon bond is $890. The price of a three-year zero-coupon bond is $816.30. All of these bonds have $1,000 face value.
(a) What is the price of a three-year bond paying 5 percent coupons with a $1,000 face value?
(b) What is the one-year forward rate from two years to three years?

2. Suppose that the price of a stock is $100. A European at-the-money call option with a maturity of one year hence costs $10. The risk-free rate is 10 percent. There are no dividends.
(a) Find the price of a European at-the-money put option with a maturity of one year.
(b) Will the price of an American at-the-money put option be lower, higher or the same?
(c) Suppose that I take a straddle position, buying both call and put options (European and at-the-money). For what range of values of the stock price in on year's time will I make a profit (net of the cost of buying the options)?

3. (a) What is the delta of an option?
(b) Suppose that the risk-free interest rate is 5 percent and that General Electric share price is $40 today. The volatility (standard deviation) of General Electric is 0.5. Find the delta of a European call option to buy a share in General Electric in one year at a strike price of $45.

4. Johannes Financial Services (JFS) sells a perpetuity paying $5 per year in coupons for ever. The current yield on a perpetuity is 10 percent. There is no default risk.
(a) Find the price of the perpetuity sold by JFS.
(b) If JFS wishes to immunize the interest rate risk on the bond that it has sold by buying Treasury STRIPS, what maturity of STRIPS will JFS buy?

5. The exchange rate between the U.S. dollar and South African rand is 1 Rand =$0.10. The two-year interest rate is 1 percent in the U.S. and 10 percent in South Africa.
(a) Using covered interest parity, find the forward exchange rate for delivery of South African Rand in two years.
(b) If the forward exchange rate for delivery of Rand in two years were instead 1 Rand =$0.10, what arbitrage strategy could you execute?

6. Explain precisely what the LIBOR-OIS spread is and the significance of its widening during the recent financial crisis.

7. The current price of XYZ is 100. Over the next year, it will rise to a price of 120 with probability 3/4, or will fall to a price of 80 with probability 1/4. Based on this information, solve for the expected return on the stock, the variance of the return, and the standard deviation of the return. Show your work.

8. Cheese-on-line, or COL, is an internet start-up company. COL's dividends are now 1 dollar per share. The dividends are forecast to grow at a 3 percent annual rate forever. Suppose that the current market price of COL is 20.6 dollars.

(a) What is the required rate of return on COL's stock?

(b) If the required return of COL satisfies the CAPM, the risk-free rate is 2 percent, and the expected excess return on the market portfolio is 3 percent, what is the beta of COL's stock?

9. Assuming CAPM holds, indicate which of the following situations are possible or impossible. For those that are impossible, explain why.

(a)

Portfolio	Expected Return	Standard Deviation
Market	10%	20%
A	15%	45%
B	15%	55%
C	15%	25%
Risk-Free	5%	0%

(b)

Portfolio	Expected Return	Standard Deviation
Risk-free	4%	0%
Market	10%	20%
A	18%	45%

(c)

Portfolio	Beta	Standard Deviation
A	1.4	10%
B	0.5	55%
Market	1	15%

10. The price of Home Depot shares is now \$35. Next year, the price could increase to \$60 or decrease to \$25. The stock pays no dividend. The borrowing rate is 8%.

Find the price of a European call option with a strike price of \$45 and a maturity of one year hence.

Solutions

1. (a) $P = 50*0.95238 + 50*0.890 + 1050*0.8163 = \$949.23$

(b) The one-, two- and three-year yields are 5, 6 and 7 percent respectively. The required forward rate is $\frac{1.07^3}{1.06^2} - 1 = 9.03\%$

2. (a) From put-call parity, $10 + \frac{100}{1.1} = 100 + P$. Hence, the price of the call option is $0.91.

(b) The payoff from this position is $|S - 100|$ where S is the terminal value of the stock. The cost of the two options together is $10.91. So I make an overall profit if $S < 89.09$ or $S > 110.91$

3. (a) The delta of an option measures its sensitivity to the underlying stock price (how much the option increases in value when the stock price rises by $1).

(b) $d_1 = \{\ln(S/X) + (r + \frac{\sigma^2}{2})T\}/(\sigma T^{1/2}) = 0.11$. $N(d_1) = 0.544$ and so this is the delta of the option.

4. (a) $50.

(b) The duration of the perpetuity is $\frac{1+y}{y} = \frac{1.1}{0.1} = 11$. So JFS should by 11 year STRIPS.

5. (a) $0.1*1.01^2 = F*1.1^2$ and so the forward rate is $1=R0.084 [South Africa has the higher interest rate and so the forward value of the rand should be lower than its spot value].

(b) In this case, I would borrow $1, convert it into rand at the exchange rate of $1=R0.10 and sell a forward rand contract for two years hence. I would invest the money (R10) for two years at the South African riskfree rate and receive R$10*1.1^2 = 12.1$. Converting this back to dollars at the forward exchange rate would give me $1.21. I then repay the loan ($1.02, including interest) and receive a profit of $0.19 without risk. [No such arbitrage opportunity is actually observable in reality, because investors would exploit it, driving the forward and spot rates into alignment.]

6. The LIBOR rate is the interbank interest rate for a term of (say) three months. It is the rate banks pay to borrow from each other for a fixed (short) term. The OIS rate is a bet on the level of the overnight interest rate for the same term. No principal changes hands in an OIS contract. The LIBOR-OIS spread therefore measures the extra interest that a bank must pay to secure funding for this period, over and above the expected overnight short-term interest rate as measured by OIS. It represents both the premium for liquidity and compensation for the risk of default. LIBOR ties up capital on the balance sheet and exposes the lender to default risk; OIS does not do either.

Prior to the recent financial crisis, the LIBOR-OIS spread was low and stable, but rose to high levels in the fall of 2007, and to extreme levels in the fall of 2008. It represents some combination of banks desire for the most liquid assets and their fear of default from counterparties.

7. The return is 0.2 with probability ¾ and -0.2 with probability ¼. So the expected return is 0.1.
The variance of returns is $[0.75*(0.2-0.1)^2] + [0.25*(-0.2-0.1)^2] = 0.03$
The standard deviation of return is $\sqrt{0.03} = 0.173$

[This was the net return. Full credit for anyone who used the gross return. The answers would be the same, except that the expected return is 1.1].

8. (a) The required rate of return solves the equation $P = \frac{D_1}{k-g}$, i.e. $20.6 = \frac{1.03}{k-0.03}$ and so k=0.08.

(b) If the required return is equal to the risk-free rate, then beta must be 2.

9. Question 9 was a little unclear as to whether the portfolios could have idiosyncratic risk or not. I marked either interpretation correct (it makes a difference for (b)).
(a) This is impossible. Portfolio C must have a beta of 2 as it has an excess return of twice the market excess return. The standard deviation of the market return is 0.2. So the standard deviation of the return on portfolio C is $0.15^2 = 4*0.2^2 + \sigma_I^2$ where σ_I^2 is the idiosyncratic variance. Solving this gives $\sigma_I^2 = -0.0975$, which is impossible.
(b) This is possible if portfolios can have idiosyncratic risk, but not otherwise.
(c) Impossible. The idiosyncratic variance for portfolio A would have to be -0.034.

10. Suppose I borrow $\frac{\$25}{1.08} = \$23.15$ and buy one share. The net cost is \$35-\$23.15=\$11.85. One year later, I owe \$25 and have a share. If the share price goes up, this portfolio is worth \$35. If the share price goes down, it is worth nothing.

The call option is worth nothing if the share price goes down and worth \$15 if it goes up.

Hence the call option must cost $\frac{15}{35}*11.85 = 5.08$